Point-by-Point Response to Referee One

Thank you very much for your careful reading our paper as well as your invaluable comments. We list our response to your comments as follows:
major comments

"*This paper is on manifold learning. It is very confusing to read.*"
Answer:
To the best of our knowledge, manifold learning refers to an approach to non-linear dimensionality reduction, which is a method trying to find the new representation in a lower-dimensional space by approximately keeping the geometry properties of the input which resides in a high-dimensional ambient space. In this paper, we learn the original manifold in the ambient space. Therefore, our work is not on manifold learning.
"*For example, realistically the data generated by the manifold model are generated with noise, and the distribution and properties of the noise in conjunction with the reach of the manifold are critical for the recovery. However, these critical issues are bypassed by making strong assumptions whereby only the local behavior of ridges matters.*"
Answer:
We agree with the referee that the recovery is highly related with the noise and reach of the manifold. In our work, our model in Equation (1) on page assumes the data are generated as

$$x_i = \tilde{x}_i + \epsilon_i, \tag{1}$$

where $\tilde{x}_i$ is the noiseless signal which belong to some manifold, and ϵ_i is the noise. We use the observations x_i to recover the manifold. Therefore, we did not bypass consideration of the noise in our model.
The reach is a global concept which describes the degree of curvature for the manifold. A larger "*reach*" indicates the samples resides further from the manifold can also has a unique projection onto $\mathcal{M}$. Therefore, the manifold is more flat globally.
" *This approach may be interesting in its own rights, but the impact in the bigger picture is rather limited due to the limited perspective and the strong assumptions that are needed. The best part are the computational perspectives provided in the paper, while the theoretical developments are also not very interesting.*"
Answer:
In fact, Assumption 2.2 is more likely to be thought as a "Fact" rather than an Assumption. The main intention of Assumption 2.2 is to say that any low-dimensional manifold can be thought as the ridge of some unknown density function. In this new version of the manuscript, we modify the presentation of Assumption 2.2 to avoid the confusion.
"*A key problem is that the errors in the sampling of the data generated by the manifold are not modeled and the noise-contaminated data are not studied. The paper is not well written, the presentation is substandard and is unfocused.*"
Answer:
As in Equation (1), we model the data as the noiseless $\tilde{x}_i \in \mathcal{M}$ plus the noise ϵ_i. We

consider the noise in the theories and all the cases of the numerical experiments.

"*Problematic assumptions include Assumption 2.2, where parts 1 and 3 are unclear and ambiguous and Assumption 3.4 which is equally unmotivated.*"
Answer:
1. Assumption 2.2 only wants to show that any noiseless manifold can be thought as the ridge of some background density function $p(x)$. And the method to construct $p(x)$ is not complicated.
2. Because of $\mu_s(x, r, h) \in (0, 1), \mu_{s,t}(x, r, h) \in (0, 1)$, we know

$$\mu(r, h) = \sup_x \max\{\max_s \mu_s(x, r, h), \max_{s,t} \mu_{s,t}(x, r, h)\} \in [0, 1].$$

Assumption 3.4 is a bit stronger by requiring $\mu(r, h)$ to be upper-bounded by a number ℓ which is less than 1 for all h.
"*The simulations are too limited as there are many other competing methods to recover the manifolds that are considered, and these manifolds are too simple to be of much interest.*"
Answer:
We compare four manifold fitting algorithms on the one-dimensional ring manifold to show the good recovery property of ℓ-SCRE under the criteria of average margin and Hausdorff distance. The manifold fitting result does not rely on the dataset too much. The comparison results on the other dataset are similar. To justify, we add another numerical result corresponding to a 2-dimensional sphere in the 3D ambient space.