A Development in the Dynamical Approach to General Relativity and Emergent Spacetime Physics: Background-Free Quantum Field Theory

Zachary Hall

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"In my opinion, it would be unsatisfactory if there existed a world without matter. Rather, the $g_{\mu\nu}$ field should *be fully determined by matter and not able to exist without the matter*."

Albert Einstein,
letter to de Sitter, 24 March 1917 [a]

[a] Einstein (1998, 309)

"There is no such thing as an empty space, i.e. a space without field. Space-time does not claim existence on its own, but only as a structural quality of the field."

Albert Einstein,
Relativity and the Problem of Space[a]

[a] Einstein (1992, 155)

"The right side [of $G_{\mu\nu} = \kappa T_{\mu\nu}$] is a formal condensation of all things whose comprehension in the sense of a field theory is still problematic. Not for a moment, of course, did I doubt that this formulation was merely a makeshift in order to give the general principle of relativity a preliminary closed form expression. For it was essentially *no more* than a theory of the gravitational field, which was isolated somewhat artificially from a total field of as yet unkown structure."

Albert Einstein,
Autobriographical Notes[a]

[a] Einstein (1979, 71)

Introduction

In this paper, I offer a development friendly to Harvey Brown's "dynamical approach" to understanding General Relativity (GR) and its background independence by attention to the two-point correlator of quantum field theory (QFT). In doing so, I will offer the beginnings of a *background-free* version of QFT that also proves supportive of the "emergent spacetime" project of Cao, Carroll, and Michalakis (Cao et al., 2017), (Carroll, 2016). In particular, I show that if we identify the metric and affine structure of spacetime with the singular structure of the two-point correlator and do not fix that structure *ab initio*, that structure is, in the case of a free Klein-Gordon field, automatically that of the EFE sourced by inhomogenieties in the smooth contributions to the two-point correlator at coincidence. This result is thus also a background-free recasting of the free Klein-Gordon field of QFT.

In Section 1, I exposit GR's background independence, the dynamical approach to understanding it before registering limitations of some existing resources for developing the dynamical approach (semiclassical gravity and the spin-2 approach). In Section 3, I motivate the two-point correlator as an object of study to improve on these limitations. I then offer the result in Section 4. To conclude, I indicate directions for developing the result further and highlight its significance for the emergentist project.

1 GR's Background Independence

1. GR, through the Einstein Field Equations (EFE)

$$G_{\alpha\beta} + \Lambda g_{\alpha\beta} = \kappa T_{\alpha\beta} \tag{1}$$

with $G_{\alpha\beta} := R_{\alpha\beta} - \frac{1}{2}Rg_{\mu\nu}$ the Einstein tensor, is background independent: some of the structure of spacetime is itself a *solution* to certain field equations, evolving together with the matter within spacetime.

> "instead of thinking of space and time as a stage on which the drama of matter unfolds, we have to imagine some ultramodern theater in which the stage itself becomes one of the actors, changing as the drama unfolds" (Stachel, 1979, 32).

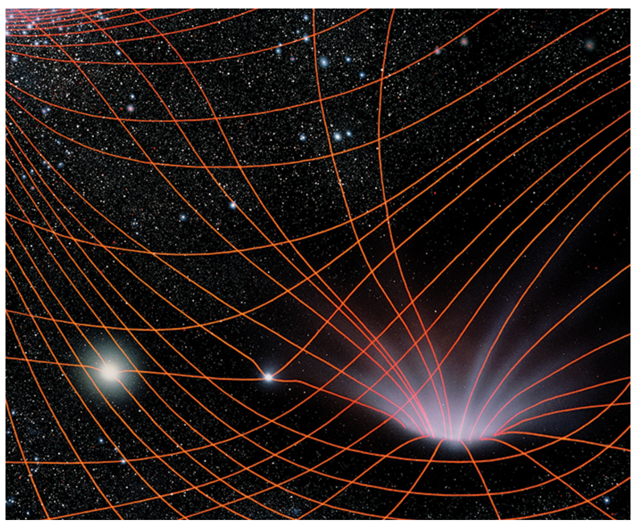

Figure 1: The spacetime structure of GR is dynamically determined by the matter-energy content of the spacetime. Image source: Gribben (2021)

2. Specifically, the *metric* g ("$g_{\mu\nu}$", measuring lengths of curve segments and angles between vectors) and the (curvature of the) *connection* ∇ (encoding "sameness of direction" along curves)—the tensor $R^{\nabla}_{\text{Riemann}}$—are so determined (Misner et al., 1973, 244-277, 404-408).

   - In GR, the the connection is required to be "compatible with" the metric: $\nabla_{\mathbf{u}} g = 0 \ \ \forall \mathbf{u}$, so that lengths and angles are uniformly defined along any curve.

2 The Dynamical Explanation of GR's Background Independence

1. One way of explaining the dynamical depence of the connection and metric on matter is the constructive or dynamical approach championed by Harvey Brown and Oliver Pooley:[1] gravity is a universal attractive force, thus the natural [read: "default"] ways things move can be represented by a connection with curvature; and the behaviors of real rods and clocks (temporal and spatial measures represented by the metric) is systematically deformed because they themselves are gravitionally influenced.

2. Einstein helps drive the point about the metric home:

   > ...strictly speaking, measuring rods and clocks should be represented as solutions of the fundamental equations (objects consisting of moving atomic configurations), not, as it were, as theoretically self-sufficient entities (Einstein, 1979, 54-7).[2]

   - Real temporal and spatial measures are read off from real physical systems.
   - Thus successful cases of chronogeometric representation in GR must be explained by the behaviors of those systems.
   - Thus, fundamentally, the metric's dependence on matter and energy must be explained by the dependence of how real rods and clocks operate on their relations to matter and energy.

3. This explanation is *programmatic* and only completed insofar as one can manifestly recover the empirical successes of GR through theory that interfaces (i) quantum-theoretic description of real systems used as rods and clocks with (ii) gravity as a universally attractive force.

4. Efforts in this direction need not involve "solving quantum gravity."

[1] Brown (2005, Introduction, Ch. 3, Ch. 8-9), Brown and Pooley (2001), Brown and Pooley (2006).

[2] Masstäbe und Uhren müssten eigentlich als Lösungen der Grundgleichungen (Gegenstände aus bewegten atomistischen Gebilden) dargestellt werden, nicht als gewissermassen theoretisch selbständige Wesen.

5. Existing such efforts have defects by the lights of Brown's explanatory program.

   (a) Semiclassical gravity feeds the expectation value of the stress energy tensor $< T_{\mu\nu} >$ into Einstein's Field Equations (EFE) to determine a metric and connection that is then used as a curved background for the equations of standard QFT generalized to curved spaces, which then determines $< T_{\mu\nu} >$ more precisely in that region, and so on—it is a mixed, iterative application of GR and QFT (Birrell and Davies, 1982), (Wald, 1994), (Kuo and Ford, 1993).

      - This *works* well in making sense of systems where both quantum theoretic effects and gravitational effects are relevant. But it is not an *explanation* from gravity as a universal attractive force at the quantum level to the empirical successes of GR. It takes the EFE as a *given*, then finds some mathematically tractable way of iteratively relating the structures of QFT and the EFE, rather than recovering the EFE as an effective description of gravitation-involving quantum-theoretic systems.

(b) The spin-2 approach theorizes a universal attractive coupling in the framework of Quantum Field Theory (QFT) via a [self-interacting] "massless spin-2 field" h mediating the gravitational force by coupling to the stress energy tensor T, and gives the EFE under certain natural assumptions. Schematically, one starts with a flat-space QFT (metric = η) with Lagrangian $\mathcal{L}_{\text{total}}$, then adds a term $\mathcal{L}_{\text{total}} \to \mathcal{L}_{\text{total}} + h_{\mu\nu}T^{\mu\nu}$, and consistency arguments show h must self-interact so that $S_{\text{total}} \to S_{\text{total}} + S_{\text{EH}}$, from which the EFE follow by the variational principle, $G_{\mu\nu} = \kappa T^{\mu\nu}$ with $G_{\mu\nu}$ computed from g in the standard way (the connection is the Levi-Civita connection) (Gupta, 1954), (Deser, 1970).

- This plays well with QFT formally, but crucially *does not show that the the microphysical dynamics of quantum fields reflects the curved metric given by h.* More specifically, none of the dynamics for the matter fields ϕ_i are required by the self-consistency argument to use a connection compatible with the variable metric, $\nabla^{R(g)}$.[3] The dynamical approach requires that the microphysical dynamics of rods and clocks *give rise to* the metric g of the EFE, but that those microphysical dynamics even track g at all can only be stipulated by fiat after spin-2 delivers its result, thus the spin-2 approach does not deliver for the dynamical explanatory strategy, *pace* Salimkhani (2020).

6. Thus, neither semiclassical gravity nor spin-2 has the resources to execute the dynamical explanatory strategy. My aim is to see if we can get some traction here by attention to some formally geometric structure in the microphysical details of QFT (the two-point correlator and its singular structure).

[3] The matter Lagrangian $\mathcal{L}$ begins formulated with the flat connection ∇^η, and the self-consistency argument simply gives a non-matter Lagrangian from which $G_{\mu\nu} = \kappa T_{\mu\nu}$ is derived—the consistency condition is that the total stress energy $T^{\text{total}}_{\mu\nu} = T^{\text{matter}}_{\mu\nu} + T^{\text{gravitational}}_{\mu\nu}(h)$ is conserved *with respect to the flat connection*, $\nabla^\eta_\mu T^{\mu\nu}_{\text{total}} = 0$. This leaves the matter Lagrangian completely untouched, and thus all microphysical quantum dynamics untouched. One could motivate changing the connection in $\mathcal{L}_{\text{matter}}$ to $\nabla^{R(g)}$ because otherwise we do not have $\nabla^{R(g)}_\mu T^{\mu\nu}_{\text{matter}} = 0$, which is implied by the EFE which the consistency argument has forced. However, $\nabla^{R(g)}_\mu T^{\mu\nu} = 0$ is forced by the EFE *only by utilizing a geometric identity involving $G_{\mu\nu}$*, namely $\nabla^{R(g)}_\mu T^{\mu\nu} = 0$, but the whole point of the spin-2 approach is not to treat h/g as a metric in the first place, but to show that matter only "sees" an effective metric g from the universal dynamics of the self-interacting h. Nor does $\nabla^\eta_\mu(T^{\text{matter}}_{\mu\nu} + T^{\text{gravitational}}_{\mu\nu}(h)) = 0$ analytically imply $\nabla^{R(g)}_\mu T^{\mu\nu}_{\text{matter}} = 0$ on its own, as the only way to get an expression for $T^{\text{gravitational}}_{\mu\nu}(h)$ in the first place is, apparently, by *assuming* $\nabla^{R(g)}_\mu T^{\mu\nu}_{\text{matter}} = 0$ (Padmanabhan, 2008), (Linnemann et al., 2026) (forthcoming). Therefore, the only way to motivate the transition from $\nabla^\eta_\mu T^{\mu\nu}_{\text{total}} = 0$, which the self-consistency argument implies, to $\nabla^{R(g)}_\mu T^{\mu\nu}_{\text{matter}} = 0$, which it does not, is to illicitly treat g as the geometry of the spacetime substantively, such that fluxes of stress-energy are divergence-free with respect to its geometry; but this is precisely not to give a microphysical, quantum field theoretic explanation of the dynamics of GR, but to postulate that those dynamics are specified by the geometry of GR.

3 The Two-Point Correlator, Hadamard States

1. A natural object of study for universal, geometrically representable microphysical dynamics is the (time-symmetric) two-point correlator $W(x, y) = \langle S| \phi(x)\phi(y) |S\rangle$, where $\phi(x)$ is a quantum field of certain kind and S is either the vacuum ($S = 0$) or an excitation thereof for that field-kind. $W(x, x')$ represents how correlated a field is with itself between two points x and x' in the vacuum state or in an excited state (Wald, 1994, 43-45, 74-75).

2. One motivation is that it is the simplest object that represents how quantum fields exist over arbitrarily small spacetime regions, as they are ill-defined pointwise (Wald, 1994, 35, 45), (Haag, 1996, 45). (They are operator-valued distributions rather than classical fields.)

3. Another is that the mathematical structure of $W(x, y)$ for all physically admissible states of all fields (i) is universal and (ii) reflects metric and causal relations (Wald, 1977, 14-16),(Fulling et al., 1981, 243-244), (Birrell and Davies, 1982, 23), (Wald, 1994, 91-93); (Wald, 1994, 91), (Fulling, 1989, 176).

4. Roughly, $W(x, y)$ is the following form:

$$W(x, y) \;\simeq\; \frac{U(x, y)}{\sigma(x, y)} \;+\; S(x, y)\,,$$

where $\frac{U(x,y)}{\sigma(x,y)}$ is the universal, geometric-dependent part, with Synge's world function $\sigma(x, y)$ equal to half the geodesic distance squared, and $S(x, y)$ is the state-dependent part that encodes difference from the geometric baseline structure. $\frac{U(x,y)}{\sigma(x,y)}$ is *singular*—it diverges as $y \to x$. $S(x, y)$ is *smooth*: it never diverges, and is distributionally differentiable at all orders in both arguments, $S(x, y) \in C^\infty(M \times M)$ (Friedlander and Joshi, 1998, Ch. 1-2). The universal part is the generalization of the singular structure of the quantum vacuum of flat-space QFT to QFT in curved spacetime (QTFCS) (Wald, 1994, 94). Thus:

$$W(x, y) \;\simeq\; \overbrace{\frac{U(x, y)}{\sigma(x, y)}}^{\text{singular, reflects short distance and causal structure (universal)}} \;+\; \overbrace{S(x, y)}^{\text{smooth (state dependent)}}$$

- The singular part reflects short-distance structure in that as geodesic distance on timelike or spacelike curves approaches 0, the field becomes infinitely self-correlated. Intuitively: this reflects infinitely energetic vacuum fluctuations at short distance (Wald, 1994, 94). Mathematically, this reflects the distributional character of quantum fields (Wald, 1994, 35, 45), (Haag, 1996, 45).

- The singular part reflects causal structure because along null geodesics (along which causal signals propagate), $\sigma(x,y) = 0$. This reflects that the singularity of $W(x,y) = \langle S| \phi(x)\phi(y) |S\rangle$ at coincidence ($x = y$) must propagate along the bicharacteristic flow of the kinetic differential operator $\mathcal{P}$ when that field propagates in accordance with $\mathcal{P}\phi = 0$, i.e., precisely along null cones. This was proved by Duistermaat and Hörmander (1972, 194-197) as the "propagation of singularities theorem." The key conceptual upshot for our purposes: this result allows both the time-like and spacelike distance structure and the causal-kinetic structure of fields to be represented in the universal part of the two-point function $W(x,y)$.
- In summary: that $W(x,y)$ of any physically well-behaved quantum field ϕ is singular at coincidence ($x = y$) mathematically encodes the short-distance structure of such fields; that it is singular away from coincidence ($x \neq y$) precisely along null cones (geodesic distance equals 0) reflects those fields' being internally causally connected by null cones.

5. More precisely, for all physically admissible states,

$$W(x,y) \;=\; \frac{U(x,y)}{\sigma(x,y)} \;+\; V(x,y)\ln\sigma \;+\; S(x,y)\,,$$

where the $V(x,y)\ln\sigma$ term is included to cancel unwanted terms resulting from applying the kinetic differential operator $\mathcal{P}$ to $\frac{U(x,y)}{\sigma(x,y)}$ when applying it to $W(x,y)$ (Zelditch, 2017, 125)—this ensures that $\mathcal{P}_x W(x,y) = \mathcal{P}_y W(x,y) = 0$ as it must when $\mathcal{P}\phi = 0$.[4] $U(x,y)$ and $V(x,y)$ are coefficients that obey transport equations such that those unwanted terms cancel at arbitrary $x \neq y$ (Poisson et al., 2011, 71).

6. It is standard to collect the U and V terms into one geometry-representing, singular bidistribution, $H(x,y) := \frac{U(x,y)}{\sigma(x,y)} \; + \; + V(x,y)\ln\sigma$ (Wald, 1994). Thus we have, for all physically admissible states,

$$W(x,y) \;=\; \overbrace{H(x,y)}^{\text{singular, reflects short distance and causal structure (universal)}} \;+\; \overbrace{S(x,y)}^{\text{smooth (state dependent)}}\,.$$

Such a state is called a *Hadamard state*.

7. Only Hadamard states are thought to be physically admissible because only they allow identification of finite, local, state-dependent quantities (Wald, 1977), (Kay et al., 1997), (Fewster and Verch, 2002), (Fewster and Verch, 2013).

[4] If $\mathcal{P}\phi = 0$, then $\mathcal{P}_x W(x,y) = \langle S| P_x\phi(x)\phi(y) |S\rangle = \langle S| 0\phi(y) |S\rangle = 0$ and $\mathcal{P}_y W(x,y) = \langle S| \phi(x)P_y\phi(y) |S\rangle = \langle S| \phi(x)0 |S\rangle = 0$.

4 Non-Interacting Background-Free QFT

1. In QFTCS, the metric g is taken as given, and then properties of e.g. the two-point correlator $W(x,y)$ is studied, with $\sigma(x,y)$ computed with g. See e.g. Wald (1977).

2. This allows representation of the microphysical dynamics of quantum fields as obeying the geometry specified by the EFE, but leaving it as it stands undermines the dynamical explanatory strategy: there is no explanation of why microphysical dynamics give rise to the macroscopic effective spacetime geometry successfully represented by the EFE, but a stipulation that they do.

3. Is there a way to affect this strategy by attention to the geometric structure of the two-point correlator $W(x,y)$ *itself*, without supplying the metric from elsewhere?

4. Consider, the universal, singular part of $W(x,y)$, i.e. $H(x,y)$, already encodes geometric information and, by the propagation of singularties theorem, causal information. For this reason, given that such an $H(x,y)$ exists with a suitable real function $\sigma(x,y)$, the metric can be defined from $\sigma(x,y)$ itself: $g_{\alpha\beta} = \lim_{y\to x} \nabla_\alpha \nabla_\beta \sigma(x,y)$ with ∇ acting on the x-slot (Poisson et al., 2011, 37). Perhaps we can think of it *as* the spacetime geometry, rather than merely reflecting it. This suggestion is consonant with Carroll et. al.'s program that seeks to define spacetime structure from entanglement entropy (Cao et al., 2017): perhaps, just as Carroll (2016) suggests, we can think of *spacetime geometry as in the quantum state*—specifically in how it is self-correlated—rather than the quantum state as in spacetime. Carroll's result showing EFE-like dependence on the curvature defined from the entanglement entropy under perturbations of the system further encourages looking in this direction, given the close connection of the two-point correlator to entanglement entropy (Lin et al., 2024).

5. Now we must ask: are there any principles holding of $W(x,y)$ itself that put constraints on the geometry encoded by $\sigma(x,y)$? If not, it is difficult to see how the dynamical explanatory strategy can succeed.

6. Let us first consider the structure of the two-point correlator for a free field, i.e. $\mathcal{P}\phi = 0$. In this case, beyond the Hadmard-form of $W(x,y)$, we have that $W(x,y)$ is a bisolution of $\mathcal{P}$: $\mathcal{P}_x W(x,y) = \mathcal{P}_y W(x,y) = 0$.[5] I will henceforth abbreviate both conditions by $\mathcal{P}_g W(x,y) = 0$.

    - The subscript g marks that the kinetic operator $\mathcal{P}_g$ is defined with the metric g: it always includes a (Levi-Cevita) connection ∇.
    - Bär and Ginoux (2012, Ch. 1.5) gives the general form of all such operators. In the simplest case—-the massless scalar field—$\mathcal{P}_g = \Box_g = g^{\alpha\beta}\nabla_\alpha\nabla_\beta$ (Wald, 1994, 54). This is a hyperbolic operator with wavelike solutions to $\mathcal{P}\phi = 0$ travelling at the speed of light c—in flat space, $\mathcal{P} = -c^2\partial_t^2 + \sum_i \partial_i^2$.

[5] See (Birrell and Davies, 1982, 10, 20-21), (Wald, 1994, 91), and foonote 4.

7. Bär et al. (2007, 34-36) prove that the Levi-Civita connection—the unique torsion-free metric compatibe connection—is the unique connection for all normally hyperbolic operators; together with Duistermaat and Hörmander (1972, 194-197)'s Propagation of Singularities theorem, this implies that the Levi-Civita connection is required for the definition of Hadamard states—otherwise, the singularities do not propagate along the null cones, and $H(x, y)$ singular away from coincidence ($x \neq y$) cannot encode the causal structure of the field ϕ. I will thus henceforth require the connection ∇ to be Levi-Civita.

8. When $\mathcal{P}_g\phi = 0$,
$$\mathcal{P}_g W(x,y) \;=\; \mathcal{P}_g\big(H(x,y) \;+\; S(x,y)\big) = 0\,.$$
$H(x, y)$ encodes the spacetime geometry/metric (both distance structure and causal relatedness), and $S(x, y)$ encodes the excitation content of the quantum field. Thus we have
$$\mathcal{P}_g \overbrace{H(x,y)}^{\text{geometry}} + \mathcal{P}_g \overbrace{S(x,y)}^{\text{matter}} = 0$$
when $\mathcal{P}_g\phi = 0$.

9. The equation
$$\mathcal{P}_g \overbrace{H(x,y)}^{\text{geometry}} + \mathcal{P}_g \overbrace{S(x,y)}^{\text{matter}} = 0 \tag{2}$$
is valid in QFTCS. Again, QFTCS places no *a priori* restrictions on g—σ—whatsoever; the geometry is given from without. Thus QFTCS treats g as input, and Eq. (2) as a linear differential equation governing the propagation of the smooth part S given that geometry. As far as QFTCS is concerned, g is specified freely, and Eq. (2) specifies the admissible S given g.

10. One might think the free field law $\mathcal{P}_g\phi = 0$ is thus insufficient for the dynamical explanatory strategy: we will have to include interactions ($\mathcal{P}_g\phi \neq 0$), ultimately gravitational interactions, to constrain the universal geometry encoded by σ. But consider: if we treat g not as externally specifiable, but as a *variable to be determined* by Eq. (2)—signify g with this standing by $g[W]$—then Eq. (2) takes on a different mathematical character: it is a non-linear differential equation of second order in g—the propagation operator in general (for arbitrary fields) contains $\nabla_\alpha\nabla_\beta$ to leading order—and ∇ and g are nonlinearly codetermining), restricting the admissible *pairs* (W, g).

11. Thus the equation
$$\mathcal{P}_{g[W]} W = 0 \tag{3}$$
is equivalent to
$$\mathcal{P}_{g[W]} \overbrace{H(x,y)}^{\text{geometry}} + \mathcal{P}_{g[W]} \overbrace{S(x,y)}^{\text{matter}} = 0$$
and is therefore a nonlinear, second-order coupling of g to the excitation content S that is just as background-free as GR. It is a *fixed point law*: its solutions, thus all

possible ways excitation data and geometric data can interrelate, are consistent pairs $(W, g[W])$ where $\mathcal{P}_{g[W]}$ propagates the excitation data of W consistently with its total structure and W Hadamard form.

4.1 Local Geometric Consequences of $\mathcal{P}_{g[W]}W = 0$

III.2.1.i - The Balance Law

1. For simplicity in the following, I will restrict my attention to Klein-Gordon fields, where $\mathcal{P} = g^{\alpha\beta}\nabla_{\alpha\beta} - m^2$.

2. Some local geometric consequences of the coupling $\mathcal{P}_{g[W]}W = 0$ can be unpacked by defining a divergence-free correlator current from W. But

$$J^\alpha_W(x,y) := i\big(W^*(x,y)\nabla^\alpha_x W(x,y) - W(x,y)\nabla^\alpha_x W^*(x,y)\big)\,.$$

is divergence-free *only away from coincidence* because of the singularity in $W(x,x)$. Because we want local geometric consequences, we need a current that is divergence-free and pointwise defined. Now, mathematically, for a given g, there are an infinite number of Hadamard bisolutions of $\mathcal{P}_g$, $\{B_c\}_g$, each differing from any other by a smooth bisolution (Lewandowski, 2022, §1.B). As a renormalization trick, we can use some reference $W_0 \neq W$, $W_0 \in \{B_c\}_{g[W]}$ to subtract from W to obtain a smooth bisolution on the geometry $g[W]$: $S_B(x,y) := W(x,y) - W_0(x,y)$. Now we have $\mathcal{P}S_B(x,y) = 0$, and we can define a correlation current

$$J^\alpha_{S_B}(x,y) := i\big(S^*_B(x,y)\nabla^\alpha_x S_B(x,y) - S_B(x,y)\nabla^\alpha_x S^*_B(x,y)\big)\,.$$

3. $J^\alpha_{S_B}(x,y)$ is smooth everywhere, including at coincidence, and divergence free everywhere: it is mathematically the same as the divergence-free Klein-Gordon current with an extra y index floating around, $S(x,y)$ is a bisolution of the same $\mathcal{P}$ that ϕ solves, and $\mathcal{P}\phi$ is the reason the Klein-Gordon current is divergence free.[6]

4. Therefore,
$$\nabla_\alpha J^\alpha_S(x) = 0 \tag{4}$$
with $J^\alpha_{S_B}(x) := \lim_{x\to y} J^\alpha_{S_B}(x,y)$. (As per Bär and Ginoux (2012, Ch. 1), when a bidistribution $A(x,y)$ is smooth, covariant differentiation and the limit-taking $x \to y$ commute—the only thing that would spoil this is singular behavior at the diagonal.)

5. Thus, by Gauss' law,
$$\int_V \nabla_\alpha J^\alpha_{S_B} = \oint_{\partial V} J^\alpha_{S_B} n_\alpha d\Sigma = 0\,.$$

6. Now for point p, pick a null direction k^α (let k^α be dimensionless) and arbitrarly small space-like 2-surface S_0 transverse to k^α, and consider the family of null geodesics emanating from S_0. We may choose S_0 to be a small disc of radius ϵ. This defines a small "null pencil" with each null geodesic parametrizable by some λ, generating a small cross-sectional area $A_\epsilon(\lambda)$ defined by the metric g. Write $J^\alpha_{S_B} = \rho V^\alpha$ with V^α unit magnitude and dimensionless, and define $\rho_{||} := J^\alpha_{S_B} k_\alpha$ with k_α null and dimensionless. Thus ρ is the magnitude of $J^\alpha_{S_B}$ and $\rho_{||}$ is its magnitude projected along the null vector chosen.

[6] See the proof by Gingrich (2004) and simply substitute $\partial_\mu \to \nabla_\mu$.

7. Now consider the pencil segment generated by λ ranging from λ_1 to λ_2. We may make a closed four-volume by moving off the 3-volume by small magnitude Δz. By Gauss' law, the surface integral over the surface enclosing this 4-volume vanishes. This integral has four parts: the two caps of the pencil—call these pieces $-\Theta(\lambda_1)$ and $\Theta(\lambda_2)$—, the null-oriented 2-surface boundary of the pencil segment, and the additional boundary generated by Δz. Thus we have

$$\oint_{\partial V} J^{\alpha}_{S_B} n_\alpha d\Sigma = \Theta(\lambda_2) - \Theta(\lambda_1) + \Theta_{\text{null boundary}} + \Theta_{\Delta z} = 0\,.$$

In the $\Delta z \to 0$ limit,

$$\lim_{\Delta z \to 0} \oint_{\partial V} J^{\alpha}_{S_B} n_\alpha d\Sigma = \Theta(\lambda_2) - \Theta(\lambda_1) + \Theta_{\text{null boundary}} = 0\,,$$

as the surface contributed by Δz becomes vanishingly small. But the null-oriented boundary has zero surface area, because each of its infinitesimal areas is spanned by a null vector k^α which has zero length according to the metric, $g_{\alpha\beta}k^\alpha k^\beta = 0$, and therefore there is zero flux of $J^{\alpha}_{S_B}$ through it. Therefore

$$\lim_{\Delta z \to 0} \oint_{\text{pencil segment}} J^{\alpha}_{S_B} n_\alpha d\Sigma = \Theta(\lambda_2) - \Theta(\lambda_1) = 0\,.$$

8. Now we may write $\Theta_\lambda = \bar{\rho}_{||}(\lambda)A_\epsilon(\lambda)$, and the above shows us that $\bar{\rho}_{||}A_\epsilon =$ const. along any null pencil. Therefore

$$\frac{1}{\bar{\rho}_{||}A_\epsilon}\frac{d}{d\lambda}(\bar{\rho}_{||}A_\epsilon) = \frac{1}{A_\epsilon}\frac{dA_\epsilon}{d\lambda} + \frac{1}{\bar{\rho}_{||}}\frac{d\bar{\rho}_{||}}{d\lambda} = 0\,.$$

9. $\frac{1}{A_\epsilon}\frac{dA_\epsilon}{d\lambda}$ has a standard name—the *expansion scalar* θ (more precisely θ_ϵ), so called because it encodes how cross-sectional areas of geodesic congruences expand (or contract) given the curvature of g (Wald, 1984, 216-222), (Poisson, 2004, 34). Thus we have

$$\theta_\epsilon + \frac{1}{\bar{\rho}_{||}}\frac{d\bar{\rho}_{||}}{d\lambda} = 0\,,$$

therefore

$$\frac{d\theta_\epsilon}{d\lambda} + \frac{d}{d\lambda}\left(\frac{1}{\bar{\rho}_{||}}\frac{d\bar{\rho}_{||}}{d\lambda}\right) = 0\,. \tag{5}$$

10. Eq. (5) is a balance law: it says that changes in how the cross-sectional area of a null congruence change along the parameter must be balanced by changes in normalized changes of the correlation density $\bar{\rho}_{||}$ along that parameter.

11. It is well known that, for a null pencil chosen with zero initial expansion or shear at point p—such a pencil always exists (Jacobson, 1995, 1261)—, in the limit $\epsilon \to 0$, $\frac{d\theta_\epsilon}{d\lambda} = -R_{\alpha\beta}k^\alpha k^\beta$, where $R_{\alpha\beta}$ is the Ricci curvature tensor for metric g (Wald, 1984, 216-222), (Poisson, 2004, 34, 40-51)—intuitively, $R_{\alpha\beta}$ controls focusing or defocusing

of geodesics, so it controls $\frac{d\theta_\epsilon}{d\lambda}$. In the $\epsilon \to 0$ limit, $\bar{\rho}_{||} \to \rho_{||}$. Thus the balance law (5) becomes

$$-R_{\alpha\beta}k^\alpha k^\beta + \frac{d}{d\lambda}\Big(\frac{1}{\rho_{||}}\frac{d\rho_{||}}{d\lambda}\Big) = 0\,. \tag{6}$$

12. Now, $\frac{d\rho_{||}}{d\lambda}$ is the same as the covariant derivative of ρ in direction k^α, as $k^\alpha = \frac{dx^\alpha}{d\lambda}$ for coordinates $x^\alpha(\lambda)$ of any null geodesic in the congruence. (Note that there is uniquely one such $k^\alpha(\lambda)$ in the $\epsilon \to 0$ limit.) Thus

$$-R_{\alpha\beta}k^\alpha k^\beta + \nabla_k\Big(\frac{\nabla_k\rho}{\rho}\Big) = 0\,.$$

Some algebra—noting that $k^\alpha\nabla_\alpha k^\beta$ vanishes because k^α picks out a null geodesic congruence—leads to

$$-R_{\alpha\beta}k^\alpha k^\beta + Q_{\alpha\beta}k^\alpha k^\beta = 0$$

where

$$Q_{\alpha\beta} := \frac{1}{\rho}\nabla_\alpha\nabla_\beta\rho - \frac{1}{\rho^2}\nabla_\alpha\rho\nabla_\beta\rho = 0\,. \tag{7}$$

13. Therefore the balance law Eq. (6) is

$$R_{\alpha\beta}k^\alpha k^\beta = Q_{\alpha\beta}k^\alpha k^\beta\,. \tag{8}$$

14. Eq. (8) expresses a local, null-contracted tensorial relation between geometric data of W and some Q_{ab} constructed from $W(x,y)$ at coincidence.

4.1.1 Comment on the Balance Law

1. $Q_{\alpha\beta}k^\alpha k^\beta > 0$ just in case $\nabla_k(\frac{\nabla_k\rho}{\rho}) > 0$. I.e., $Q_{ab}k^\alpha k^\beta$ is a measure of the change in the normalized change in correlational density ρ along null direction k^α. When it is positive, this normalized second derivative is positive. Thus $Q_{ab}k^\alpha k^\beta > 0$ means: $\nabla_k\rho$ is increasing in the direction k^α relative to what it already was, i.e. a concavity in $\ln\rho$ evaluated in null direction k.

4.1.2 Uncontracting the Balance Law - The EFE as the Fixed Point Law for the Free KG Field

1. We may re-write Eq. (8) as

$$(R_{\alpha\beta} - Q_{\alpha\beta})k^\alpha k^\beta = 0\,.$$

Now, eq. (8) is valid at *all* points in the manifold for *all* vectors k^α that are null with respect to g—its derivation was with respect to any g-null k^α WLOG. Thus we have

$$(R_{\alpha\beta} - Q_{\alpha\beta})k^\alpha k^\beta = 0 \;\; \forall k^\alpha \text{ null w.r.t. } g\,.$$

Thus $R_{\alpha\beta} - Q_{\alpha\beta}$ and $g_{\alpha\beta}$ have precisely the same null cones: $(R-Q)(k,k) = 0$ just in case $g(k,k) = 0$. Beem et al. (1996, §2.3) prove that in such a case, $(R-Q)$ must be conformally related to g, i.e.

$$R_{\alpha\beta} - Q_{\alpha\beta} = \Phi_n(x) g_{ab}$$

or

$$R_{\alpha\beta} = Q_{\alpha\beta} + \Phi_n(x) g_{ab} \,. \tag{9}$$

2. Now consider the Einstein tensor $G_{\alpha\beta} := R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta}$. Substituting Eq. (9), it quickly follows that

$$G_{\alpha\beta} + \Phi(x) g_{\alpha\beta} = Q_{\alpha\beta} \tag{10}$$

with $\Phi(x) := \frac{1}{2}R - \Phi_n(x)$.

3. Now, Eq. (10) is an analytic consequence of $P_{g[W]}W = 0$, which is a non-linear coupling of the geometric data to the excitation data. Therefore, any geometric term that appears in any of its local-geometric analytic consequences must be locally constructable from the metric and all of its derivatives. But Hollands and Wald (2001, §5) show that the only such term including just the metric is a global constant times the metric. Therefore, it must be that $\Phi(x) = \Lambda =$ const. and, as a consequence $\nabla^\alpha Q_{\alpha\beta} = 0$. That is,

$$\boxed{G_{\alpha\beta} + \Lambda g_{\alpha\beta} = Q_{\alpha\beta}} \tag{11}$$

is an analytic consequence of $P_{g[W]}W = 0$ for the Klein-Gordon field. This is just the EFE with the matter source in units of curvature—we could define $T_{\alpha\beta} := \frac{Q_{\alpha\beta}}{\kappa}$ to bring Eq. (11) into identical form as Eq. (1).

   - Note that this implies that the $W_0 \neq W$ that is a bisolution on $g[W]$ used in our renormalization trick is in fact not a solution of the fixed-point law $P_{g[W]}W = 0$. But this is benign, it was just used as an auxilliary mathematical object to unpack the local geometric consequences of that law.
   - Changing the choice of W_0 to another $W_0' \neq W$ that is a bisolution of $\mathcal{P}_{g[W]}$ changes Q in a way that can be absorbed into the cosmological constant Λ: if $W_0 \to W_0'$, $J_{S_B}(x) \to J_{S_B'}(x)$, thus $\rho \to \rho'$, *therefore* Eq. (8) changes to $R_{\alpha\beta}k^\alpha k^\beta = Q'_{\alpha\beta}k^\alpha k^\beta$, so we have $(Q_{\alpha\beta} - Q'_{\alpha\beta})k^\alpha k^\beta = 0$; combined with $\nabla^\alpha Q_{\alpha\beta} = \nabla^\alpha Q'_{\alpha\beta} = 0$, this shows that $Q_{\alpha\beta} - Q'_{\alpha\beta} = c_1\Lambda$. Thus $W_0 \to W_0'$ corresponds to $\Lambda \to (1-c_1)\Lambda$ for some c_1.
   - The various $W_0 \neq W$, $W_0 \in \{B_c\}_{g[W]}$ therefore belong together with corresponding Λ to a gauge orbit $\{(W_\Lambda, \Lambda)\} | W_\Lambda \neq W, W_\Lambda \in \{B_c\}_{g[W]}$; it is plausible that each member of such an orbit represents the same solutions of Eq. (11).

4. The EFE are therefore the geometry of the two-point correlator of the free Klein-Gordon field when that geometry is defined internally to the field.

Conclusion

Imagining for a moment that the world were only populated by free Klein-Gordon fields, the above result tells us the following: the spacetime internal to those fields—that of its two-point correlator—is automatically governed by Einstein's Field Equations sourced by local inhomogenities in the smooth part of that two-point correlator. This result thus strongly supports Carroll's speculative suggestion—independently supported by his co-authored results in Cao et al. (2017)—that "gravity isn't hard to obtain" in quantum theory, but rather "automatic" (Carroll, 2016). This result is a step towards finding gravity *in* the quantum world (namely in the $W(x,y)$ defined on $M \times M$), rather than quantizing gravity.

Of course, the world is not only populated by free Klein-Gordon fields: it features a zoo of interacting fields at least as diverse as those of the Standard Model. This limitation notwithstanding, the result is technically significant for the emergentist project: it finds the full EFE automatically—including for null and time-like directions—from a microlocal consistency condition, thus opening up an inroad for the emergentist story to interface with the geometric foundations of QFT. Incidentally, if gravity is to be found within the structure of quantum fields generally along these lines, the cosmological constant problem dissolves, or is rather re-framed—on this view, "vacuum energy" is a category mistake: the divergence of the two-point correlator at coincidence itself represents the geometry internal to quantum fields, and the only possible content that manifests as gravitational are the inhomogeneties in the state-dependent smooth part of the correlator $Q_{\alpha\beta}$. Thus the 120-of-orders-of-magnitude-too-big estimate of vacuum energy from QFT cannot possibly be the origin of the cosmological constant, whose observed value would have to be explained some other way.

There are a few different ways one could try to generalize the result to other fields and to the interacting case. The most straightforward parallel derivation would be to generalize the current $J_{S_B}(x,y)$ suitably for other fields such that each such current for each such field is divergence-free. Another would be to give a structural derivation of the EFE in the form of Eq. (11) directly from $P_{g[W]}W = 0$ without need of defining a divergence-free current, drawing on the results e.g. of Hollands and Wald (2001) classifying objects locally constructible from the germ of the metric. This is already desirable, as the local geometric consequences of $P_{g[W]}W = 0$ should be analyzable without relying on auxiliary bisolutions of the operator $\mathcal{P}_{g[W]}$. One could then investigate first, whether such a structural derivation works for the total two-point correlator for arbitrary kinds of quantum fields, and second, whether it works when there are interactions modifying the smooth part of that total correlator. This latter investigation would certainly benefit from engagement with the results of Cao et al. (2017) showing the dependence of spatial curvature on the entanglement entropy of real excitations, given the close relation of entanglement entropy and the two-point correlator (Lin et al., 2024).

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