Adversarial Estimation of Network Formation Models

Consider a nutshell network formation model, the adjacency matrix $\boldsymbol{A}$ is generated as

$$a_{ij} = \mathbb{1}\left\{\theta_{0,i} + \theta_{0,j} > u_{ij}\right\}, \, i < j,$$

where θ_i is the fixed effect for each $i = 1, 2, \cdots, n$, and u_{ij} follows i.i.d. standard logistic distribution. Let $a_{ij} = a_{ji}$ for $i < j$ and $a_{ii} = 0$.

Denote $\boldsymbol{\theta_0} = (\theta_{0,1}, \theta_{0,2}, \cdots, \theta_{0,n})$. For given $\boldsymbol{\theta}$, we can generate

$$\widetilde{u}_{ij,k} \sim \text{i.i.d.}\Lambda\left(\cdot\right), \, i < j, \, k = 1, 2, \cdots, m,$$

where $\Lambda\left(u\right) = \left(1 + e^{-u}\right)^{-1}$ is the CDF of the standard logistic distribution. Transform $\widetilde{u}_{ij,k}$ to $\widetilde{d}_{ij,k}$ by

$$a_{ij,k,\boldsymbol{\theta}} = \mathbb{1}\left\{\theta_i + \theta_j > \widetilde{u}_{ij,k}\right\}.$$

When $m = 1$, a single network is generated based on parameter $\boldsymbol{\theta}$.

The oracle discriminator,

$$D_{\boldsymbol{\theta}}\left(a_{ij,k}\right) = \frac{p_0\left(a_{ij,k}\right)}{p_0\left(a_{ij,k}\right) + p_{\boldsymbol{\theta}}\left(a_{ij,k}\right)},$$

where

$$p_0\left(a_{ij,\cdot}\right) = \begin{cases} \Lambda\left(\theta_{0,i} + \theta_{0,j}\right) & \text{if } a_{ij,\cdot} = 1, \\ 1 - \Lambda\left(\theta_{0,i} + \theta_{0,j}\right) & \text{if } a_{ij,\cdot} = 0, \end{cases}$$

and

$$p_{\boldsymbol{\theta}}\left(a_{ij,\cdot}\right) = \begin{cases} \Lambda\left(\theta_i + \theta_j\right) & \text{if } a_{ij,\cdot} = 1, \\ 1 - \Lambda\left(\theta_i + \theta_j\right) & \text{if } a_{ij,\cdot} = 0. \end{cases}$$

The estimation follows Kaji et al. (2022, KMP) with the oracle discriminator,

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \frac{1}{n\left(n-1\right)} \sum_{i<j} \log D_{\boldsymbol{\theta}}\left(a_{ij}\right) + \frac{1}{mn\left(n-1\right)} \sum_{k=1}^{m} \sum_{i<j} \log\left(1 - D_{\boldsymbol{\theta}}\left(a_{ij,k,\boldsymbol{\theta}}\right)\right).$$

0.1 Identification

Chatterjee et al. (2011), Gao (2020).

References

Chatterjee, S., P. Diaconis, and A. Sly (2011). Random graphs with a given degree sequence. *The Annals of Applied Probability 21*(4), 1400 – 1435.

Gao, W. Y. (2020). Nonparametric identification in index models of link formation. *Journal of Econometrics 215*(2), 399–413.

Kaji, T., E. Manresa, and G. Pouliot (2022). An adversarial approach to structural estimation. *arXiv preprint arXiv:2007.06169*.