Identification and Estimation of Categorical Random Coefficient Models

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RANDOM COEFFICIENT MODELS

$$y_i = a + \beta_i x_i + u_i$$

- Nonparametric identification and estimation of the dist. of β_i and u_i w/ i.i.d. β_i and u_i (Beran and Hall, 1992)
- Beran (1993); Beran and Millar (1994); Beran, Feuerverger, and Hall (1996); Hoderlein, Klemelä, and Mammen (2010); Hoderlein, Holzmann, and Meister (2017); Breunig and Hoderlein (2018)

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- Scenario 1: estimate the consumer surplus based on a linear demand system (Hausman and Newey, 1995; Foster and Hahn, 2000)

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$\uparrow$

parametric assumptions on β_i

CATEGORICAL RANDOM COEFFICIENT MODEL

- Suppose the single cross-section observations $\{y_i, x_i, \boldsymbol{z}_i\}_{i=1}^{n}$ follow the linear model

$$y_i = x_i \beta_i + {\boldsymbol{z}_i}' \boldsymbol{\gamma} + u_i$$

CATEGORICAL RANDOM COEFFICIENT MODEL

$$y_i = x_i \beta_i + \boldsymbol{z}_i{}' \boldsymbol{\gamma} + u_i$$

- $\beta_i \in \{b_1, b_2, \cdots, b_K\} \sim_{\text{i.i.d.}}$ a categorical distribution,

$$\beta_i = b_k, \ \text{w.p. } \pi_k,$$

with $\pi_k \in (0,1)$, $\sum_{k=1}^{K} \pi_k = 1$, $b_1 < b_2 < \cdots < b_K$

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    - K is assumed to be known.

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- $\boldsymbol{\gamma} \in \mathbb{R}^p$ is homogeneous, which may include the intercept.

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- The idiosyncratic shock $u_i \sim \left(0, \sigma_i^2\right) \perp \boldsymbol{w}_i = (x_i, \boldsymbol{z}_i')'$.

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- $\boldsymbol{\gamma} \in \mathbb{R}^p$ is homogeneous, which may include the intercept.
- The idiosyncratic shock $u_i \sim \left(0, \sigma_i^2\right) \perp \boldsymbol{w}_i = (x_i, \boldsymbol{z}_i')'$.
- Allow for distributions of $\boldsymbol{w}_i$ and u_i are not identical across i while independence across i is maintained. Details

CATEGORICAL RANDOM COEFFICIENT MODEL

Goal: Identify and estimate $\boldsymbol{\gamma}$ and the distribution of β_i,

$$\boldsymbol{\theta} = \left(\boldsymbol{\pi}' = (\pi_1, \pi_2, ..., \pi_K), \boldsymbol{b}' = (b_1, b_2, ..., b_K)\right).$$

IDENTIFITION OF THE MOMENTS OF β_i

- Let $\tilde{y}_i = y_i - \mathbf{z}_i'\boldsymbol{\gamma}$, consider

$$\tilde{y}_i^r = (x_i\beta_i + u_i)^r \,,$$
$$\tilde{y}_i^r x_i^r = (x_i\beta_i + u_i.)^r \, x_i^r$$

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- Take expectations, sum over i and take limits,

$$\rho_{0,r}\mathrm{E}\left(\beta_i^r\right) + \sigma_r = \rho_{r,0} - \sum_{q=2}^{r-1}\binom{r}{q}\rho_{0,r-q}\sigma_q\mathrm{E}\left(\beta_i^{r-q}\right),$$
$$\rho_{0,2r}\mathrm{E}\left(\beta_i^r\right) + \rho_{0,r}\sigma_r = \rho_{r,r} - \sum_{q=2}^{r-1}\binom{r}{q}\rho_{0,2r-q}\sigma_q\mathrm{E}\left(\beta_i^{r-q}\right).$$

Def. of $\rho_{r,s}$ and σ_r

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▸ Def. of $\rho_{r,s}$ and σ_r

- Recursively solve the linear systems for $\mathrm{E}\left(\beta_i^r\right)$ and σ_r, $r = 2, 3, \cdots, 2K-1$,

IDENTIFICATION OF THE DISTRIBUTION OF β_i

- Show the system

$$\mathrm{E}\left(\beta_i^r\right) = \sum_{k=1}^{K} \pi_k b_k^r,\ r = 0, 1, 2, \cdots, 2K-1,$$

has a unique solution $\left(\boldsymbol{\pi}', \boldsymbol{b}'\right)$ based on a linear recurrence structure, under the conditions

$$b_1 < b_2 < \cdots < b_K,\ \text{and}\ \pi_k \in (0, 1)\,.$$

$\sqrt{n}$-CONSISTENT ESTIMATOR FOR γ

- Let $\boldsymbol{w}_i = (x_i, \boldsymbol{z}_i')'$, $\boldsymbol{\phi} = (\mathrm{E}(\beta_i), \boldsymbol{\gamma}')'$, the OLS estimator

$$\hat{\boldsymbol{\phi}} = \left(\frac{1}{n}\sum_{i=1}^{n} \boldsymbol{w}_i \boldsymbol{w}_i'\right)^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} \boldsymbol{w}_i y_i\right)$$

- Under regularity conditions,
  - [Consistency] $\hat{\boldsymbol{\phi}}$ is a consistent estimator for $\boldsymbol{\phi}$
  - [Asymptotic normality]

$$\sqrt{n}\left(\hat{\boldsymbol{\phi}} - \boldsymbol{\phi}\right) \to_d N\left(\mathbf{0}, \boldsymbol{Q}_{ww}^{-1} \boldsymbol{V}_{w\xi} \boldsymbol{Q}_{ww}^{-1}\right)$$

GMM ESTIMATOR FOR $\boldsymbol{\theta} = \left(\boldsymbol{\pi}', \boldsymbol{b}'\right)'$

- The moment conditions,

$$\mathrm{E}\left(\tilde{y}_i^r x_i^{s_r}\right) - \sum_{q=0}^{r} \binom{r}{q} \mathrm{E}\left(x_i^{r-q+s_r}\right) \mathrm{E}\left(u_i^q\right) \underbrace{\mathrm{E}\left(\beta_i^{r-q}\right)}_{=\sum_{k=1}^{K} \pi_k b_k^{r-q}} = 0$$

  $s_r = 0, 1, \cdots, S - r$, where $S > 2K - 1$.

- Plug in $\hat{\boldsymbol{\gamma}}$ for $\boldsymbol{\gamma}$ in the sample analogue, $\hat{\boldsymbol{g}}_n\left(\boldsymbol{\theta}, \boldsymbol{\sigma}, \hat{\gamma}\right)$.
- GMM estimator

$$\left(\hat{\boldsymbol{\theta}}', \hat{\boldsymbol{\sigma}}'\right)' = \arg \min_{\boldsymbol{\theta} \in \Theta, \boldsymbol{\sigma} \in \mathcal{S}} \hat{\boldsymbol{g}}_n\left(\boldsymbol{\theta}, \boldsymbol{\sigma}, \hat{\gamma}\right)' \boldsymbol{A}_n \hat{\boldsymbol{g}}_n\left(\boldsymbol{\theta}, \boldsymbol{\sigma}, \hat{\gamma}\right)$$

GMM ESTIMATOR FOR $\boldsymbol{\theta} = \left(\boldsymbol{\pi}', \boldsymbol{b}'\right)'$

Under regularity conditions,

- [Consistency] Let $\boldsymbol{\eta} = \left(\boldsymbol{\theta}', \boldsymbol{\sigma}'\right)'$ and $\boldsymbol{\eta}_0 = \left(\boldsymbol{\theta}_0', \boldsymbol{\sigma}_0'\right)'$,

$$\hat{\boldsymbol{\eta}} \to_p \boldsymbol{\eta}_0,$$

  as $n \to \infty$.

- [Asymptotic normality] Let $\sqrt{n}\,(\hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma}) \to_d \zeta_\gamma \sim N(0, V_\gamma)$,

$$\sqrt{n}\,(\hat{\boldsymbol{\eta}} - \boldsymbol{\eta}_0) \to_d \left(\boldsymbol{G}_0' \boldsymbol{A} \boldsymbol{G}_0\right)^{-1} \boldsymbol{G}_0' \boldsymbol{A} \left(\boldsymbol{\zeta} + \boldsymbol{G}_{0,\gamma} \boldsymbol{\zeta}_\gamma\right),$$

  as $n \to \infty$.

MULTIPLE REGRESSORS WITH RANDOM COEFFICIENTS

- The model

$$y_i = \boldsymbol{x}_i' \boldsymbol{\beta}_i + \boldsymbol{z}_i' \boldsymbol{\gamma} + u_i,$$

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$$y_i = \boldsymbol{x}_i' \boldsymbol{\beta}_i + \boldsymbol{z}_i' \boldsymbol{\gamma} + u_i,$$

- $\boldsymbol{\beta}_i \in \mathbb{R}^p$ follows the multivariate distribution

$$\Pr\left(\beta_{i1} = b_{1k_1}, \beta_{i2} = b_{2k_2}, \cdots, \beta_{ip} = b_{pk_p}\right) = \pi_{k_1,k_2,\cdots,k_p},$$

with $k_j \in \{1, 2, \cdots, K\}$, $b_{j1} < b_{j2} < \cdots < b_{jK}$, and $\sum_{k_1,k_2,\cdots,k_p \in \{1,2,\cdots,K\}} \pi_{k_1,k_2,\cdots,k_p} = 1$.

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- Identification and estimation of the marginal distributions of $\boldsymbol{\beta}_i$ follow as corollaries.
- The joint distribution can be identified in special cases, $p = 2$ and $K = 2$ for example.

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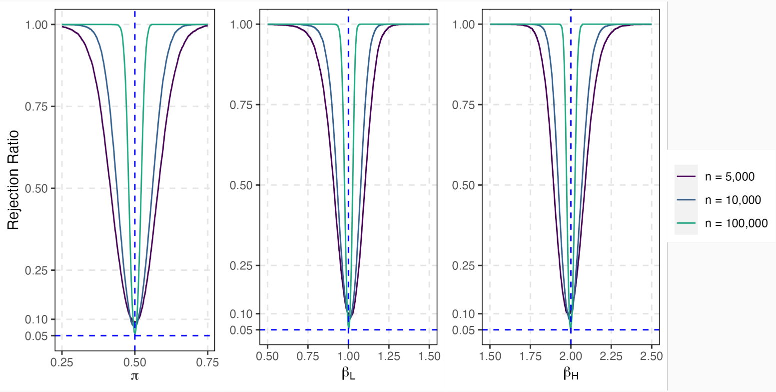
	$\pi = 0.5$			$\beta_L = 1$			$\beta_H = 2$		
n	Bias	RMSE	Size	Bias	RMSE	Size	Bias	RMSE	Size
500	-0.0234	0.2384	0.3678	-0.0882	0.6297	0.3217	-0.0216	0.5816	0.2186
1K	-0.0185	0.1769	0.2981	-0.0362	0.4285	0.2767	-0.0198	0.3216	0.2083
2K	-0.0069	0.1233	0.2376	-0.0151	0.2274	0.2370	-0.0123	0.1574	0.1828
5K	-0.0029	0.0677	0.1586	-0.0020	0.0988	0.1504	-0.0060	0.0735	0.1434
10K	-0.0010	0.0414	0.1112	0.0008	0.0535	0.1060	-0.0032	0.0463	0.1050
100K	0.0001	0.0114	0.0610	0.0006	0.0135	0.0666	-0.0003	0.0135	0.0620

Data generating process: $y_i = \alpha + x_i\beta_i + z_{i1}\gamma_1 + z_{i2}\gamma_2 + u_i$,for $i = 1, 2, ..., n$. generate $x_i = (\tilde{x}_{1i} - 2)/2$ where $\tilde{x}_{1i} \sim \text{IID}\chi^2(2)$ for $i = 1, 2, \cdots, \lfloor n/2 \rfloor$, and $x_i = (\tilde{x}_{2i} - 2)/4$ where $\tilde{x}_{2i} \sim \text{IID}\chi^2(4)$, for $i = \lfloor n/2 \rfloor + 1, \cdots, n$. The additional regressors, z_{ij}, for $j = 1, 2$ with homogeneous slopes are generated as $z_{i1} = x_i + v_{i1}$ and $z_{i2} = z_{i1} + v_{i2}$, with $v_{ij} \sim \text{IID } N(0, 1)$, for $j = 1, 2$. The error term u_i is generated as $u_i = \sigma_i\varepsilon_i$, where σ_i^2 are generated as $0.5(1 + \text{IID}\chi^2(1))$, and $\varepsilon_i \sim \text{IID}N(0, 1)$.

MONTE CARLO



Empirical Power Functions

HETEROGENEOUS RETURN TO EDUCATION

- Estimate the Mincerian equation with repeated cross-sectional data by education groups,

$$\log \text{wage}_{it} = \alpha_t + \beta_{it}\text{edu}_{it} + \rho_{1t}\text{exper}_{it} + \rho_{2t}\text{exper}_{it}^2 + \tilde{\mathbf{z}}_{it}'\tilde{\boldsymbol{\gamma}}_t + u_{it},$$

where $\beta_{it} = \begin{cases} b_{tL} & \text{w.p. } \pi_t, \\ b_{tH} & \text{w.p. } 1-\pi_t. \end{cases}$

HETEROGENEOUS RETURN TO EDUCATION

- Estimate the Mincerian equation with repeated cross-sectional data by education groups,

$$\log \text{wage}_{it} = \alpha_t + \beta_{it}\text{edu}_{it} + \rho_{1t}\text{exper}_{it} + \rho_{2t}\text{exper}_{it}^2 + \tilde{\mathbf{z}}_{it}'\tilde{\boldsymbol{\gamma}}_t + u_{it},$$

$$\text{where } \beta_{it} = \begin{cases} b_{tL} & \text{w.p. } \pi_t, \\ b_{tH} & \text{w.p. } 1 - \pi_t. \end{cases}$$

- Data: The May and Outgoing Rotation Group (ORG) supplements of the Current Population Survey (CPS) data, 1973 - 2003.

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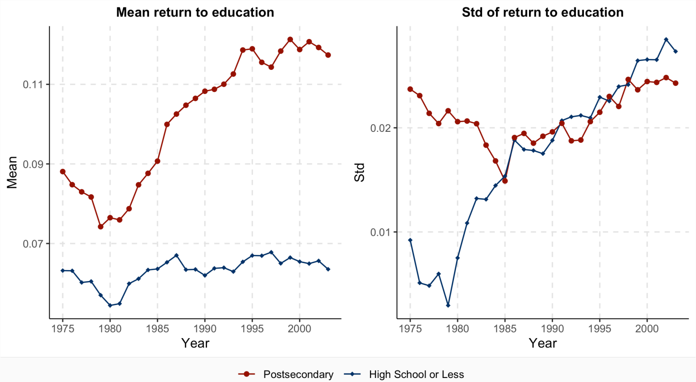
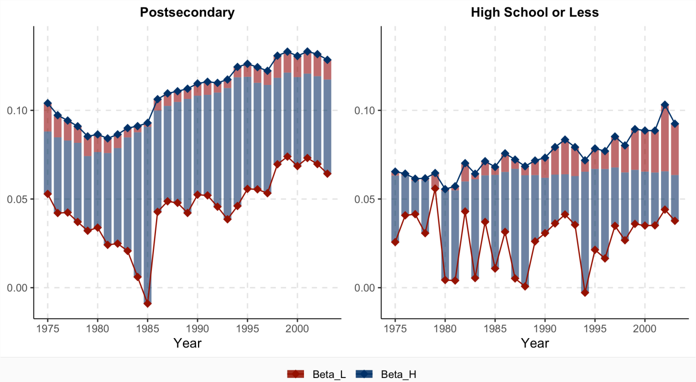
- Data: The May and Outgoing Rotation Group (ORG) supplements of the Current Population Survey (CPS) data, 1973 - 2003.
- Between group heterogeneity $\uparrow$ due to the postsecondary education group; Within group heterogeneity $\uparrow$ in high school or less education group.

HETEROGENEOUS RETURN TO EDUCATION



- Red line: the proportion of low return group.
  Blue line: the proportion of high return group.
- Left panel: the mean return to education
  Right panel: the std of the return to education.

HETEROGENEOUS RETURN TO EDUCATION



- Red bar represents the proportion of low return group. Blue bar represents the proportion of high return group.
- ◇ / ◇ represent the estimated high / low return to education.
- Left panel: Postsecondary / Right panel: High school or less group.

Thank you!

Questions and comments are welcomed

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(LIMITED) DEGREE OF HETEROGENEITY IN MOMENTS

- Let $\boldsymbol{\phi}_i = (\beta_i, \boldsymbol{\gamma}')'$, $\boldsymbol{\phi} = E(\boldsymbol{\phi}_i) = (E(\beta_i), \boldsymbol{\gamma}')'$, then

$$\mathrm{E}(\boldsymbol{w}_i y_i) = \mathrm{E}\left(\boldsymbol{w}_i \boldsymbol{w}_i'\right) \phi.$$

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$$\mathrm{E}(\mathbf{w}_i y_i) = \mathrm{E}\left(\mathbf{w}_i \mathbf{w}_i'\right) \phi.$$

- Average over i,

$$\frac{1}{n}\sum_{i=1}^{n} E(\mathbf{w}_i y_i) = \left[\frac{1}{n}\sum_{i=1}^{n} E\left(\mathbf{w}_i \mathbf{w}_i'\right)\right] \phi$$

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- Let $n \to \infty$, $\boldsymbol{\phi}$ is identified as

$$\boldsymbol{\phi} = \boldsymbol{Q}_{ww}^{-1} \boldsymbol{q}_{wy}$$

Back

(LIMITED) DEGREE OF HETEROGENEITY IN MOMENTS

- Let $\tilde{y}_i = y_i - \mathbf{z}_i'\boldsymbol{\gamma}$. Assumptions in use:

$$\left| \frac{1}{n} \sum_{i=1}^{n} \mathrm{E}\left(\tilde{y}_i^r x_i^s\right) - \rho_{r,s} \right| = O\left(n^{-1/2}\right),$$

$$\left| \frac{1}{n} \sum_{i=1}^{n} \mathrm{E}\left(u_i^r\right) - \sigma_r \right| = O\left(n^{-1/2}\right).$$

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$$\left| \frac{1}{n} \sum_{i=1}^{n} \mathrm{E}\left(u_i^r\right) - \sigma_r \right| = O\left(n^{-1/2}\right).$$

- Example. Let $e_{ir} = \mathrm{E}\left(u_i^r\right) - \sigma_r$,

$$\left| n^{-1} \sum_{i=1}^{n} \mathrm{E}\left(u_i^r\right) - \sigma_r \right| \leq n^{-1} \sum_{i=1}^{n} |e_{ir}| \leq O\left(n^{-1/2}\right)$$

if $\sum_{i=1}^{n} |e_{ir}| = O\left(n^{\alpha_r}\right)$ with $\alpha_r < \frac{1}{2}$, where α_r measures the degree of heterogeneity. Back