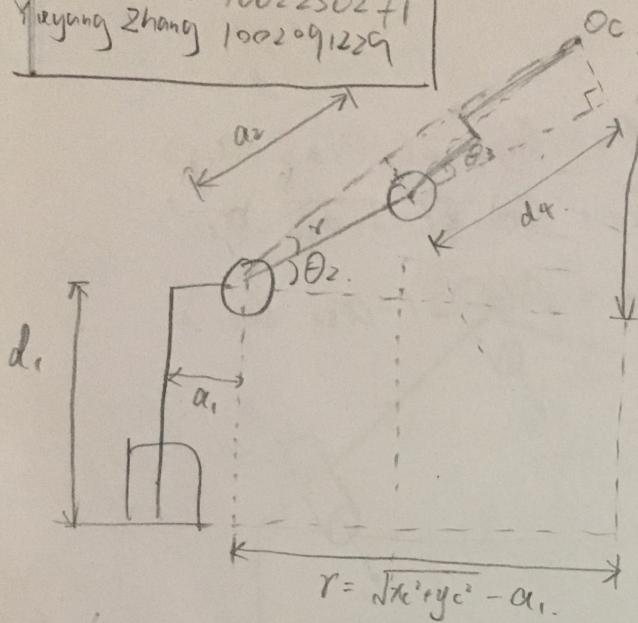


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$$\theta_3: r^2 + s^2 = \alpha_2^2 + d_4^2 + \alpha_3^2 - 2\alpha_2 d_4 \sqrt{\alpha_4^2 + \alpha_3^2}$$

$$\cos(\pi - \theta_3 + \frac{\pi}{2} - \text{atan2}(\alpha_3, d_4))$$

$$r^2 + s^2 = \alpha_2^2 + d_4^2 + \alpha_3^2 + 2\alpha_2 d_4 \sqrt{\alpha_4^2 + \alpha_3^2}$$

$$\sin(\theta_3 + \text{atan2}(\alpha_3, d_4))$$

$$\sin(\theta_3 + \text{atan2}(\alpha_3, d_4)) = \frac{r^2 + s^2 - \alpha_2^2 - d_4^2 - \alpha_3^2}{2\alpha_2 d_4 \sqrt{\alpha_4^2 + \alpha_3^2}} = D$$

$$\boxed{\theta_3 = \text{atan2}(D, \sqrt{1-D^2}) - \text{atan2}(\alpha_3, d_4)}$$

$$\theta_2: \theta_2 + \gamma = \text{atan2}(s, r)$$

$$\gamma = \text{atan2}(\sqrt{\alpha_3^2 + d_4^2} \sin(\theta_3 - \frac{\pi}{2} + \text{atan2}(\alpha_3, d_4)), \alpha_2 + \sqrt{\alpha_3^2 + d_4^2} \cos(\theta_3 - \frac{\pi}{2} + \text{atan2}(\alpha_3, d_4)))$$

$$\boxed{\theta_2 = \text{atan2}(s, r) - \gamma}$$

Continue in the other side.

$$V_r = \sin(\theta_3) \cdot s$$

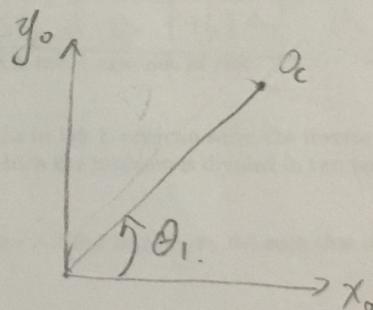
$$= \sin(\theta_3) \cdot (d_4 \sin(\theta_2) + d_2 \sin(\theta_2 - \alpha_2 - \alpha_3 + \theta_3))$$

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = O_d^\circ - R_d \begin{bmatrix} -d_6 \\ 0 \\ d_6 \end{bmatrix}$$

$$= O_d^\circ - R_d \begin{bmatrix} -296.23 \\ 0 \\ 161.44 \end{bmatrix}$$

$$\boxed{\theta_1 = \text{atan2}(y_c, x_c)}$$

Top View



$$R_6^3(\theta_4, \theta_5, \theta_6) = (R_3^0)^T R_d.$$

$$\left\{ \begin{array}{l} \theta_4 = \text{Atan} 2(-\alpha_{23}, \alpha_{13}) \\ \theta_5 = \text{Atan} 2(\sqrt{1-\alpha_{33}^2}, \alpha_{33}) \\ \theta_6 = \text{Atan} 2(\alpha_{32}, -\alpha_{31}) \end{array} \right.$$

or

$$\left\{ \begin{array}{l} \theta_4 = \text{Atan} 2(-\alpha_{23}, -\alpha_{13}) \\ \theta_5 = \text{Atan} 2(-\sqrt{1-\alpha_{33}^2}, \alpha_{33}) \\ \theta_6 = \text{Atan} 2(-\alpha_{32}, \alpha_{31}) \end{array} \right.$$

*For 32
Alpha*

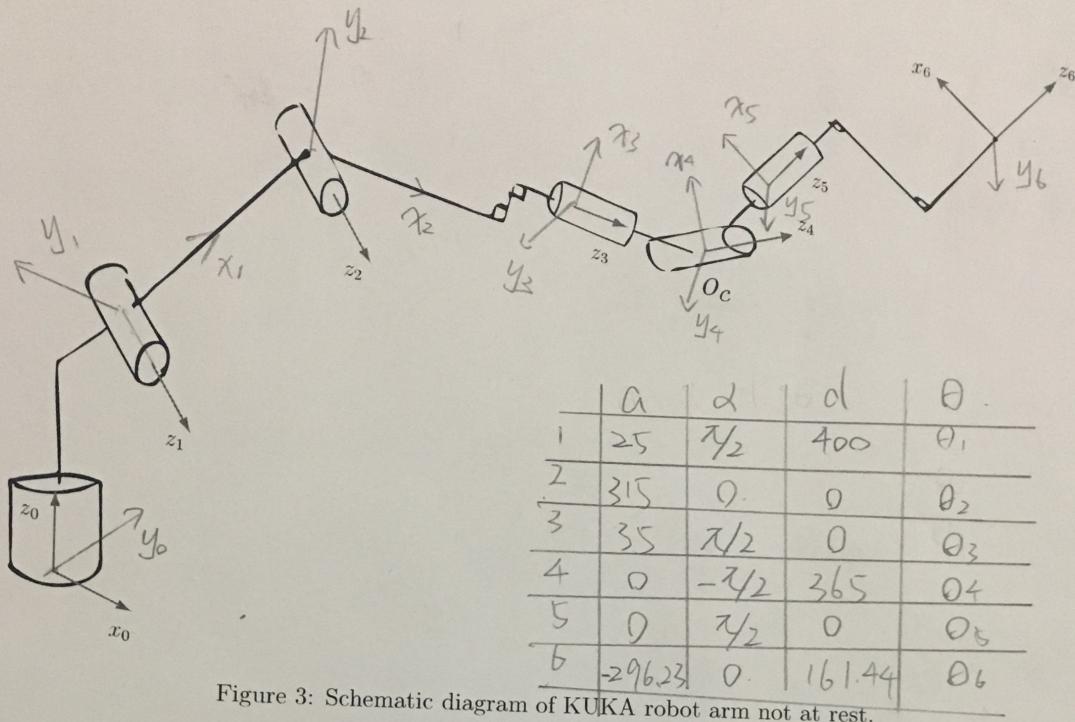


Figure 3: Schematic diagram of KUKA robot arm not at rest.

up solution. Write your derivations neatly on paper. As in lab 1, one can solve the inverse kinematics problem by the technique of kinematic decoupling in which the problem is divided in two parts: inverse position and inverse orientation.

- The position of the wrist centre o_c is shown in Figure 2. First find $(\theta_1, \theta_2, \theta_3)$ such that $o_c^0(\theta_1, \theta_2, \theta_3) =$

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = o_d^0 - R_d \begin{bmatrix} -a_6 \\ 0 \\ d_6 \end{bmatrix}.$$

Then solve the equation

$$R_6^3(\theta_4, \theta_5, \theta_6) = (R_3^0)^\top R_d.$$

for $(\theta_4, \theta_5, \theta_6)$.

- Modify your inverse kinematics function from lab 1 to incorporate the changes of this setup. Specifically, write Matlab functions `mykuka.m`, `forward_kuka.m`, and `inverse_kuka.m` as follows.

`myrobot = mykuka(DH)` defines the robot structure of the KUKA robot with the 6×4 DH table you found earlier.

`H = forward_kuka(q,myrobot)` returns the homogeneous transformation matrix `H` of the end effector, where `q` is the 6×1 vector of joint angles, and `myrobot` is the robot structure defined above.

`q = inverse_kuka(H,myrobot)` returns the 6×1 vector of joint angles `q = ($\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$)`, where `H` is the 4×4 homogeneous transformation matrix $H = \begin{bmatrix} R_d & o_d^0 \\ 0 & 1 \end{bmatrix}$.

- Test your software: you should get

```
>> kuka=mykuka(DH);
>> forward_kuka([pi/5 pi/3 -pi/4 pi/4 pi/3 pi/4]',kuka)
```

`ans =`