① $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$$L(p) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n} \binom{n}{x_i} p^{x_i} (1-p)^{1-x_i}$$

$$\ln(L(p)) = \ln\left(\prod_{i=1}^{n} \binom{n}{x_i} p^{x_i} (1-p)^{1-x_i} \right)$$

$$= \sum_{i=1}^{n} \left(\ln\binom{n}{x_i} + x_i \ln p + (1-x_i)\ln(1-p) \right)$$

$$= \sum_{i=1}^{n} \left(\ln\binom{n}{x_i} \right) + \left(\sum_{i=1}^{n} x_i \right) \ln p + \left(n - \sum_{i=1}^{n} x_i \right) \ln(1-p)$$

$$\frac{\partial \ln(L(p))}{\partial p} = 0 + \frac{1}{p} \sum_{i=1}^{n} x_i + - \frac{1}{1-p} \left(n - \sum_{i=1}^{n} x_i \right) = 0$$

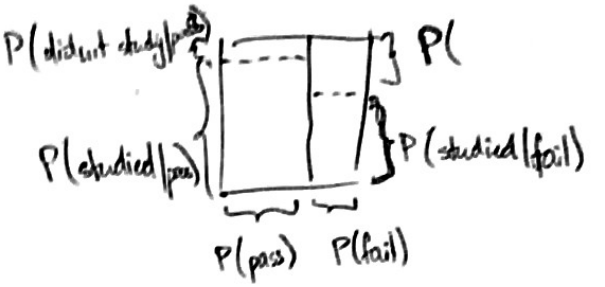
$$\frac{(1-p) \sum_{i=1}^{n} x_i - p\left(n - \sum_{i=1}^{n} x_i \right)}{p(1-p)} = 0$$

$$\sum_{i=1}^{n} x_i - p \sum_{i=1}^{n} x_i - pn + p \sum_{i=1}^{n} x_i = 0$$

$$pn = \sum_{i=1}^{n} x_i \quad \Rightarrow \quad p = \frac{\sum_{i=1}^{n} x_i}{n}$$

② $P(pass) = 0,6$

$P(fail) = 0,4$



$P(\text{studied} | \text{pass}) = 95\% = 0,95 \Rightarrow P(\text{didn't study} | \text{pass}) = 0,05$

$P(\text{studied} | \text{fail}) = 60\% = 0,6 \Rightarrow P(\text{didn't study} | \text{fail}) = 0,4$

(a) $P(\text{pass} | \text{studied}) - ?$

$$P(\text{pass} | \text{studied}) = \frac{P(\text{pass}) \times P(\text{studied} | \text{pass})}{P(\text{studied})} = \frac{0,6 \times 0,95}{P(\text{studied})} =$$

$$P(\text{fail}) = \frac{0,6 \times 0,95}{P(\text{pass}) \times P(\text{studied} | \text{pass}) + P(\text{fail}) \times P(\text{studied} | \text{fail})} =$$

$$= \frac{0,6 \times 0,95}{0,6 \times 0,95 + 0,4 \times 0,6} = \frac{0,57}{0,81} \approx \boxed{0,7037}$$

(b) $P(\text{pass} | \text{didn't study}) = \frac{P(\text{pass}) \times P(\text{didn't study} | \text{pass})}{P(\text{pass}) \times P(\text{didn't study} | \text{pass}) + P(\text{fail}) \times P(\text{didn't study} | \text{fail})} =$

$$= \frac{0,6 \times 0,05}{0,6 \times 0,05 + 0,4 \times 0,4} = \frac{0,03}{0,19} \approx \boxed{0,1579}$$

$Y=y$ \ $X=x$	$P(3\le X\le 5)$	$P(5\le X\le 9)$	$P(Y=y)$
$P(1\le Y\le 5)$	0,2	0,2	0,4
$P(5\le Y\le 9)$	0,2	0,4	0,6
$P(X=x)$	0,4	0,6	1

$C=c$ \ $X=x$	$3\le X\le 5$	$5\le x\le 9$	$P(C=c)$
$C=0$	0,12	0,18	0,3
$C=1$	0,28	0,42	0,7
$P(X=k)$	0,4	0,6	1

$C=c$ \ $Y=y$	$1\le Y\le 5$	$5\le Y\le 9$	$P(C=c)$
$C=0$	0,06	0,24	0,3
$C=1$	0,34	0,36	0,7
$P(Y=y)$	0,4	0,6	1

b)

$Y=y$ \ $X=x$	$(3\le X\le 5 \mid C=0)$	$(5\le X\le 9 \mid C=0)$	$P(Y=y \mid C=0)$
$(1\le Y\le 5 \mid C=0)$	0,08	0,12	0,2
$(5\le Y\le 9 \mid C=0)$	0,32	0,48	0,8
$P(X=x \mid C=0)$	0,4	0,6	1

$Y=y$ \ $X=x$	$(3\le X\le 5 \mid C=1)$	$(5\le X\le 9 \mid C=1)$	$P(Y=y \mid C=1)$
$(1\le Y\le 5 \mid C=1)$	0,132	0,228	0,48
$(5\le Y\le 9 \mid C=1)$	0,208	0,312	0,52
$P(X=x \mid C=1)$	0,4	0,6	1

$\forall x,y,c \quad P(X=x, Y=y \mid C=c) = P(X=x \mid C=c) \cdot P(Y=y \mid C=c)$

$\forall x,y \quad P(X=x, Y=y) \neq P(X=x) \cdot P(Y=y)$

(4)

(a) $P(X=2) = \binom{7}{2} \cdot 0{,}75^2 \cdot 0{,}25^5 = \boxed{0{,}0115}$

(b) $P(X \geq 4) = \sum_{i=4}^{7} \binom{7}{i} \cdot 0{,}75^i \cdot 0{,}25^{7-i} = 0{,}173 + \ldots = \boxed{0{,}9234}$

(c) $E[X] = 7 \cdot 0{,}75 = \boxed{5{,}25}$