7. Introduction to Quantum Mechanics

Grade 12 Physics

Olympiads School

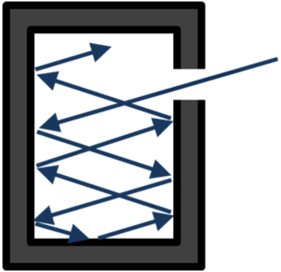
Summer 2018

Light is a Wave

- In Unit 5, we firmly established that light is a wave. After all, light has all the properties of waves:
  - Diffraction
  - Refraction
  - Constructive and destructive interference
- We even know what *kind* of a wave it is
  - Electromagnetic (“EM”) wave
  - Same as: radio waves, microwave, infrared, ultraviolet, x-ray…
- Travels in vacuum with a speed of 2.998×10^8 m/s, independent of the velocity of the object emitting the light

Now we’re going to find out that things aren’t as simple as it seems.

Black-body Radiation



- An idealized physical object that absorbs all incident electromagnetic radiation, regardless of frequency or angle of incidence
- A black body is in thermal equilibrium, i.e. all absorbed radiation (energy) is immediately radiated back
- A black body at room temperature appears black, as most of the energy it radiates is infrared and cannot be perceived by the human eye
- Thermal radiation spontaneously emitted by many ordinary objects can be approximated as black-body radiation
- The concept was coined by Gustav Kirchkoff in 1860

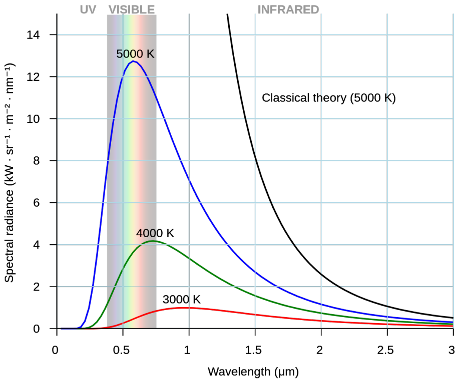
Raleigh-Jeans Law and the Ultraviolet Catastrophe

- Based on classical thermodynamics

$$P(\lambda, T) = 8\pi kT\lambda^{-4}$$

T=temperature, λ=wavelength, k=Boltzmann's constant

- Agrees with experimental results for long wavelengths, but the equation predicts that short wavelengths (e.g. ultraviolet radiation, x-ray) will have infinite intensity
- Known as "**ultraviolet catastrophe**"



According to classical thermodynamics, we should all be dead. But yet we are not. Why? How?

“Quantization” of Energy

Max Planck

- Made a strange modification in the classical calculations
- Derived a function of $P(\lambda, T)$ that agreed with experimental data for all wavelengths
- First found an empirical function to fit the data
- Then searched for a way to modify the usual calculations
- Energy emitted by black body not continuous but discrete
- When energy is emitted from the harmonic oscillator, it drops to the next lower energy level

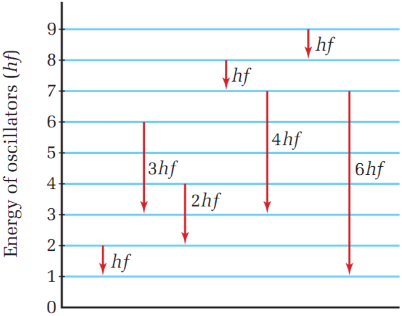
"Quantization" of Energy

When a black-body emits radiation, it must drop down one or more energy levels and emit a unit of energy equal to the difference between the allowed energy levels of the oscillator.

$$\boxed{E = hf}$$

Variable	Symbol	SI Unit
Energy	E	J (joules)
Planck's constant	h	J s (joule seconds)
Frequency	f	Hz (hertz)

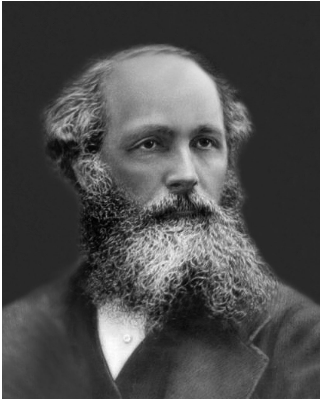
where $h = 6.626 \times 10^{-34}$ J s.



As for his formula, it's called Planck's Law:

$$P(\lambda, T) = \frac{2hc^2}{\lambda^5}\left[1/\left(e^{\frac{hc}{\lambda k_B T}} - 1\right)\right]$$

Maxwell's Equations



James Clerk Maxwell

- Classical laws of electrodynamics
- When you have an alternating current, or a oscillating charge, it generates a fluctuating electric field and magnetic field.
- The disturbance travels through space in an "electromagnetic" wave with speed:

$$\boxed{c = \frac{1}{\sqrt{\varepsilon_o \mu_o}} = 2.998 \times 10^8 \text{ m/s}}$$

- Physicists knew of the speed of light already, so is light an electromagnetic wave then?

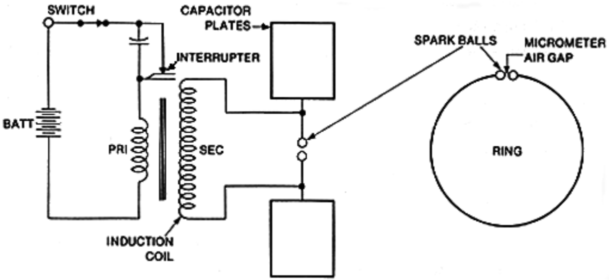
How do you prove it?

- In order to prove that light is an electromagnetic wave, we must generate an alternating current with a frequency of 10^{14} oscillations per second.
- Technology of that time can only generate frequencies around 10^8 /s (much higher than the 60 /s that our electrical outlet uses, but still 10^6 times too low)

Heinrich Hertz

- German physicist (1857-1894)
- Devised the “spark gap experiment” to generate high frequencies
- The unit for frequency is named after him in his honour

The Spark Gap Experiment



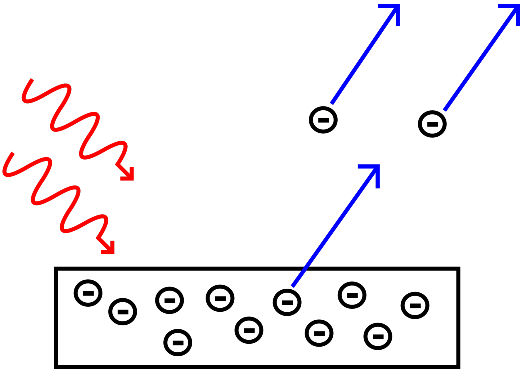
- Produced EM waves with frequency 10^{14} oscillations per second
- Also showed that light waves have the same wavelengths as predicted by Maxwell's equations
- Finally, proof that light is an EM wave!

Discovery of the Photoelectric Effect

- Terse remark in Hertz's results:
  It is essential that the pole surfaces of the spark gap should be frequently repolished to ensure reliable operation of the spark.
- Caused by the ultraviolet radiation
- This is now known as the **photoelectric effect**
- Hertz and other physicists who repeated his experiments did not have a good explanation. . .
- **In fact, this is the first evidence that light isn't a wave after all.**

Photoelectric Effect

When electromagnetic waves (e.g. light) hits certain metals, electrons are knocked off the surface



Photoelectric Effect

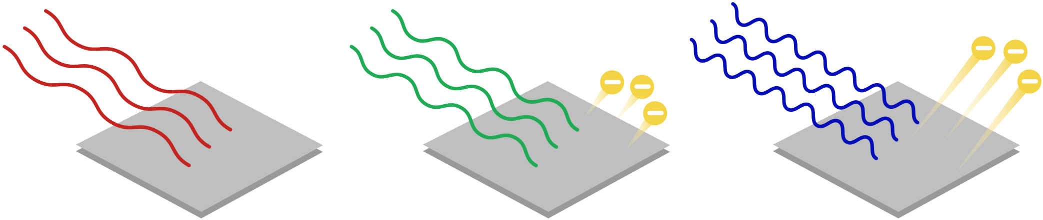
The Classical View

- Classical electrodynamics: energy is transferred from light wave to the electrons
- The kinetic energy of the emitted electrons should be proportional to **intensity**
- Increasing intensity:
  - Increases the number of electron emitted
  - Increases the electrons' kinetic energy
- Even light with low intensity (a dim light) should eventually transfer enough energy to an electron be emitted

But this isn't what is happening!

Photoelectric Effect

What actually happened



- Increasing intensity of light knocked off more electrons, but doesn't change their kinetic energy, but
- Changing the frequency of the light did change K though, although
- Below a certain frequency, *no* electrons were emitted

1905: *Annus Mirabilis* (The Miraculous Year)

The Year That Einstein Became Very Famous

- **Photoelectric effect:** “On a Heuristic Viewpoint Concerning the Production and Transformation of Light”
- **Brownian motion:** “On the Motion of Small Particles Suspended in a Stationary Liquid, as Required by the Molecular Kinetic Theory of Heat”
- **Special relativity:** “On the Electrodynamics of Moving Bodies”
- **Mass-energy equivalence:** “Does the inertia of a body depend upon its energy content?”

The Photon: Packets of Energy

- Light is not a continuous wave, but instead it is **a collection of discrete energy packets** called “photons”, each has energy $E = hf$.
- A simple relationship between kinetic energy of electrons and the photons:

$$\boxed{K = \begin{cases} hf - \varphi & \text{if } hf > \varphi \\ 0 & \text{otherwise} \end{cases}}$$

Quantity	**Symbol**	**SI Unit**
Kinetic energy of “photo-electrons”	K	J (joules)
Planck’s constant	h	J s (joule seconds)
Frequency of the EM wave	f	Hz (hertz)
Work function of the metal	φ	J (joules)

The Photon: Packets of Energy

The energy of the photon is determined by its *frequency*, in agreement with Planck's equation

- Therefore the higher the frequency of the light, the higher the kinetic energy K the photo-electrons

The *number* of photons is the intensity (brightness) of light

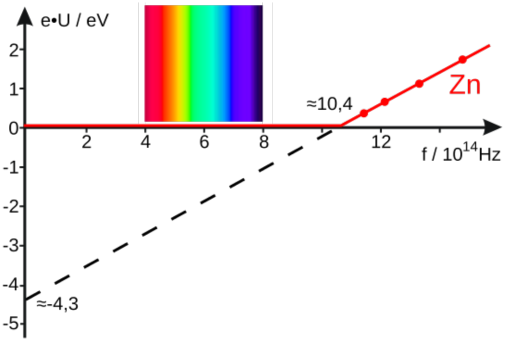
- Therefore the brighter the light, the higher *number* of electrons are knocked off the metal surface

The "work function" that determines how much energy is absorbed until an electron is knocked off

Bottom line: Classical concept of light does not work

Work Function φ

Work function is the minimum energy required to remove an electron from the metal to a point immediately outside the metal surface



Metal	Work function (eV)
aluminum	4.28
calcium	2.87
cesium	2.14
copper	4.65
iron	4.50
lead	4.25
lithium	2.90
nickel	5.15
platinum	5.65
potassium	2.30
tin	4.42
tungsten	4.55
zinc	4.33

Slope is h no matter what metal it is.

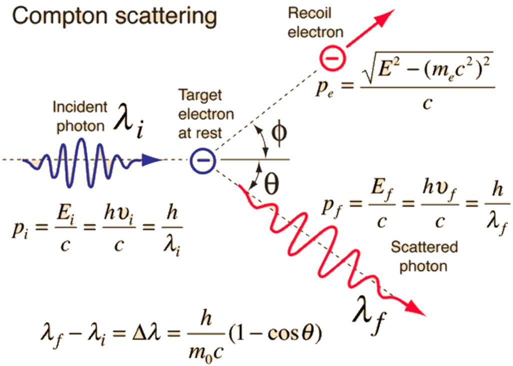
Compton Scattering

Arthur Compton

- American physicist Arthur Holly Compton, studying x-ray scattering by free electrons
- Classical theory cannot account for the scattering behaviour
- Frequency shift only depends on scattering angle
- Prediction possible if treating the x-ray as photons with momentum–just like a particle

$$\boxed{p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}}$$

Compton Scattering



If we treat the x-ray as a photon with momentum $p = h/\lambda$ then we can use Newton's laws of motion to predict both the recoil electron and scattered x-ray!

Momentum of a Photon

The momentum of a photon is proportional to Planck's constant and inversely proportional to its wavelength.

$$\boxed{p = \frac{h}{\lambda}}$$

Quantity	Symbol	SI Unit
Momentum	p	kg m/s (kilogram metres per second)
Planck's constant	h	J s
Wavelength	λ	m (metres)

This is an odd expression, which treats photon both as a particle (with momentum) and a wave (with a wavelength λ).

Example Problem

Example 1: Calculate the momentum of a photon of light that has frequency of 5.09×10^{14} Hz.

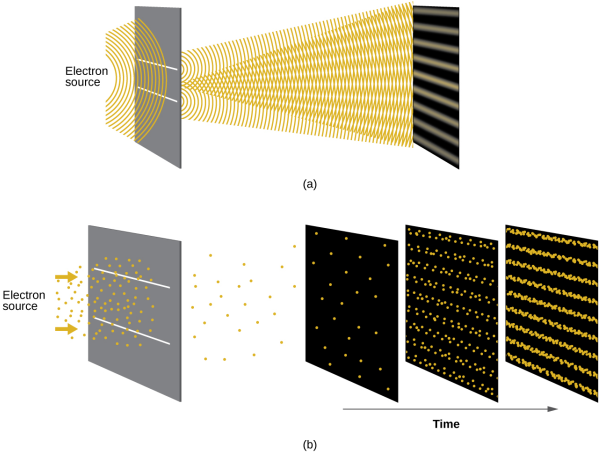
Matter Waves

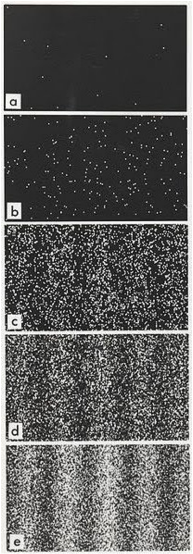
Louis De Broglie

- **If electromagnetic waves are really particles of energy, then are particles (e.g. electrons) a wave of some sort?**
- The De Broglie hypothesis in 1924: a particle can also have a wavelength
- Confirmed by the Davisson-Germer Experiment in 1927 (beam of electron scattering on nickel crystal surface)

Electron Interference

If I perform double-slit experiment with a beam of electrons, will I get an interference pattern?





De Broglie Wavelength

If matter is also a wave, then what would be its wavelength?

- Solve momentum equation for λ:

$$p = \frac{h}{\lambda} \quad \rightarrow \quad \lambda = \frac{h}{p} \quad \rightarrow \quad \boxed{\lambda = \frac{h}{mv}}$$

Quantity	**Symbol**	**SI Unit**
Wavelength of a particle	λ	m (metres)
Planck's constant	h	J s (joule seconds)
Mass	m	kg (kilograms)
Velocity	v	m / s (metres per second)

Example Problem

Example 2: Calculate the wavelength of an electron moving with a velocity of 6.39×10^6 m/s.

Particle-Wave Duality

The Copenhagen interpretation:

- Accepted view: wave-particle duality
- An experiment can either show:
  - The wave nature: diffraction, refraction (e.g. light)
  - The particle nature: scattering (Compton effect), photoelectric effect
  - But not both.

Heisenberg Uncertainty Principle

Because of the wave properties of particles, you can never be completely certain of the relationship between an object's momentum p and position x:

$$\boxed{\sigma_p \sigma_x \geq \frac{1}{2}\hbar} \quad \text{where} \quad \hbar = \frac{h}{2\pi} = 1.054 \times 10^{-34}\,\mathrm{J\,s}$$

Quantity	Symbol	SI Unit
Uncertainty in momentum	σ_p	kg m/s (kilogram metres per second)
Uncertainty in position	σ_x	m (metres)
Reduced Planck's constant	$\hbar$	J s (joule seconds)

The more you know about an object's position, the less you know about its momentum, and vice versa.

Atomic Model

J. J. Thomson: plum-pudding model (1897)

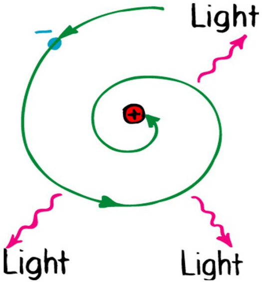
- Developed along with William Crookes
- Negatively-charged electrons are like raisins in a positively-charged "dough"

Ernest Rutherford: planetary model (1911)

- The atom is mostly empty space
- Negatively-charged electrons orbiting a fixed, positively-charged nucleus in set, predictable paths (orbits)

The Rutherford model explains a lot more than the Thomson model, but misses out on a very important feature of an accelerating electron: **it radiates energy as electromagnetic waves!**

Why the Planetary Model Doesn't Work



- An accelerating electron has an oscillating electric field and an oscillating magnetic field
- Therefore it emits electromagnetic radiation
- The electron will lose energy and the orbit decays
- Eventually it'll collapse into the nucleus

Bohr Atomic Model

Bohr postulated that electron can move in certain “non-radiating” orbits, corresponding to energy levels:

$$\boxed{E_n = -\frac{k^2 e^4 m}{2\hbar^2}\frac{Z^2}{n^2}}$$

Quantity	Symbol	SI Unit
Energy at level n	E_n	J (joules)
Coulomb's constant	k	N m^2 / C^2
Elementary charge	e	C (coulombs)
Atomic mass	m	kg (kilograms)
Reduced Planck's constant	$\hbar$	J s (joule seconds)
Atomic number	Z	integer; no units
Energy level	n	integer; no units (joule seconds)

Bohr Atomic Model

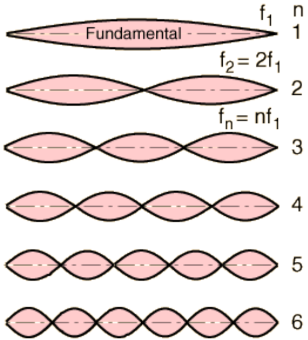
Successful in describing the behaviour of hydrogen atoms (but fails for heavier atoms), although it still relies on

- Coulomb forces between electrons and protons (classical)
- Centripetal forces (classical)
- Quantization of energy (new physics!)

De Broglie’s hypothesis gives us a glimpse of what Bohr is missing

- The “orbits” correspond to a standing wave around the nucleus
- A standing wave does not lose energy

Standing Wave on a String

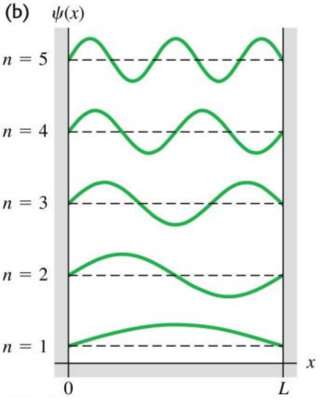


- We have studied standing waves in Grade 11
- If electron is to be in a “stable orbit” around a nucleus, it has to be in a standing wave pattern
- Otherwise, it will interfere with itself

Circular Standing Wave

Electron resonance states $n = 3, 4, 5, 6$

Particle in a Box



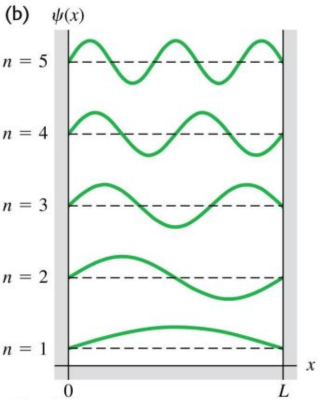
A particle in a 1D box has to behave like a standing wave. The resonance modes (frequencies where a stable standing wave exists) are the same as what we studied in Physics 11:

$$\lambda = \frac{2L}{n} \quad \text{where} \quad n = 1, 2, 3, 4$$

and the momentum of the particle is:

$$p = \frac{h}{\lambda} = \frac{nh}{2L}$$

Particle in a Box



Express kinetic energy of the particle in terms of momentum:

$$E_n = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{n^2h^2}{8mL^2}$$

In 3D where the box has 3 dimensions, the energy is

$$E = E_x + E_y + E_z = \frac{(n_x^2 + n_y^2 + n_z^2)h^2}{8mL^2}$$

Bottom line: The energy of the particle can never be zero, as long as it is confined inside the box. Therefore it cannot have zero velocity!

Example

Example 3: A $0.150\,\mathrm{kg}$ billiard ball is confined to the pool table $1.42\,\mathrm{m}$ wide. How long (in seconds) will it take to travel from one side of the table to the other? (Use fundamental mode.)

What Else Can You Learn from Quantum Mechanics

- Schrödinger’s Equation:
  - The differential equations that governs the quantum state of a quantum system changes in time
  - It’s like what Newton’s second law of motion and conservation of energy to classical mechanics
  - Gives full details of the behaviour of electrons in atoms
  - “Schrödinger’s Cat” thought experiment