2. Momentum, Impulse and Energy

Grade 12 Physics

Olympiads School

Summer 2018

Files for You to Download

Please download from the school website if you have not done so:

- `2-momentumImpulse.pdf`—This presentation (for screen display)
- `2-momentumImpulse_print.pdf`—The print version of the handout for this unit. I recommend printing 4 slides per page
- `2-Homework.pdf`—The homework assignment for this unit

Reminder: Always download/print the PDF file for the unit before each class. There is no point copying notes that are already printed out for you. Instead, take notes on things I say that aren't necessarily on the slides.

Where Are We In the Course

1. Fundamentals of Dynamics
2. Momentum, Impulse and Energy
3. Motion in Space
4. Gravitational, Electric and Magnetic Fields
5. Wave Nature of Light
6. Theory of Special Relativity
7. Introduction to Quantum Mechanics

U Have Some Potential There

To Avoid Some Confusion

- For the rest of the course, we will use the symbol "U" for "potential energy" and "K" for kinetic energy
- A common notation in university-level physics textbooks and technical papers

Gravitational Potential Energy:	Electric Potential Energy:	Elastic Potential Energy:
U_g	U_q	U_e

- We need to use "E" for something else later
- This is better anyway…

A New Concept: Momentum

You can't stop a train by yourself. Why not?

- A train is *fast*
- A train is *massive*

A New Concept: Momentum

You can't stop a bullet either (unless you wear a bullet-proof vest). Why not?

- A bullet isn't massive
- A bullet sure is *fast*!

A New Concept: Momentum

What makes it so difficult to stop a train, or a speeding bullet, or a car?

- Both the train, the speeding bullet and the car have a lot of *momentum*
- Momentum is related to both the *mass* and *velocity* of an object
- “Mass in motion”: the tendency for the object to remain in the same state of motion
- Newton referred to momentum as the “quantity of motion”

Then What is Momentum?

Momentum is proportional to both the object's **mass** and its **velocity**.

$$\boxed{\mathbf{p} = m\mathbf{v}}$$

Quantity	Symbol	SI Unit
Momentum	$\mathbf{p}$	kg m/s (kilogram metres per second)
Mass	m	kg (kilograms)
Velocity	$\mathbf{v}$	m/s (metres per second)

- Momentum $\mathbf{p}$ is a vector
- Does not have it's own unit (unlike force or energy)

A Very Simple Example: Hockey Puck

Example 1: Determine the momentum of a $0.300\,\mathrm{kg}$ hockey puck travelling across the ice at a velocity of $5.55\,\mathrm{m/s}$ [N].

Impulse $\mathbf{J}$

Impulse is the change in momentum:

$$\boxed{\mathbf{J} = \Delta\mathbf{p} = \mathbf{p}_2 - \mathbf{p}_1}$$

Assuming m is constant, we can find out how $\Delta\mathbf{p}$ relates to $\mathbf{F}$:

$$\sum \mathbf{F} = m\mathbf{a} = m\frac{\Delta\mathbf{v}}{\Delta t} = \frac{\Delta(m\mathbf{v})}{\Delta t} = \frac{\Delta\mathbf{p}}{\Delta t}$$

In fact, this is actually how Newton's second law is defined: the net force is the *rate of change of momentum*. That means that:

$$\boxed{\Delta\mathbf{p} = \sum \mathbf{F}\Delta t}$$

Impulse

$$\boxed{\mathbf{J} = \Delta\mathbf{p} = \sum\mathbf{F}\Delta t}$$

Quantity	Symbol	SI Unit
Impulse	$\mathbf{J}$	N s (newton seconds)
Average net force	$\mathbf{F}$	N (newtons)
Time interval	Δt	s (seconds)

- Impulse is also a vector...
- The sum of the net force $\sum\mathbf{F}$ is averaged over the entire interval Δt

Pro tip: The symbol $\mathbf{I}$ is sometimes used for impulse in some textbooks and by engineers.

For the over-achievers amongst you. . .

NOTE: You don't have to know this for this class. If you know calculus, or if you're studying it right now, you can get a more precise definition of impulse:

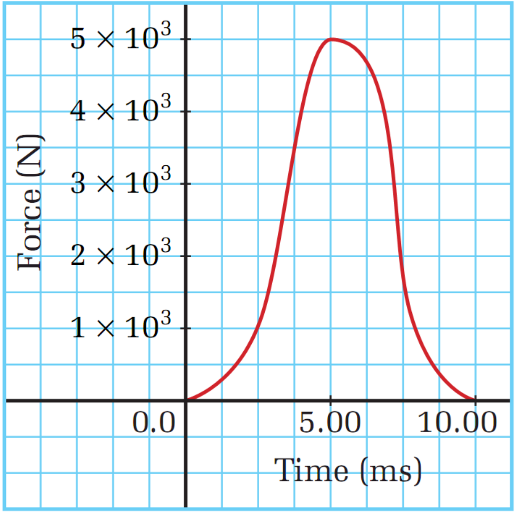
$$\mathbf{J} = \int_{t_1}^{t_2} \mathbf{F}_{\text{net}}(t)\,dt$$

This can get very complicated if $\mathbf{F}_{\text{net}}$ is changing in both magnitude and direction with time

Example Problem: Golf Club

Example 2: If a golf club exerts an average force of 5.25×10^3 N $[\mathrm{W}]$ on a golf ball over a time interval of 5.45×10^{-4} s, what is the impulse of the interaction?

Why Average Force?



Why did the previous example use **average force**?

A Slightly Longer Example

Example 3: A student practices her tennis volleys by hitting a tennis ball against a wall.

- If the $0.060\,\mathrm{kg}$ ball travels $48\,\mathrm{m/s}$ before hitting the wall and then bounces directly backward at $35\,\mathrm{m/s}$, what is the impulse of the interaction?
- If the duration of the interaction if $25\,\mathrm{ms}$, what is the average force exerted on the ball by the wall?

Conservation of Momentum

- Like **mass**, and **energy**, momentum can be conserved as well
- Conservation of momentum is derived through Newton's third law
- When objects interact (forces are created), the total momentum *before* the interaction is the same as *after* the interaction:

$$\sum_i \mathbf{p}_i = \sum_i \mathbf{p}'_i$$

- Examples:
  - Collision of 2 or more objects
  - A rocket expelling gas from the engine nozzle
  - Skaters pushing on each other on a friction-less ice surface
  - An exploding bomb

Conservation of Momentum

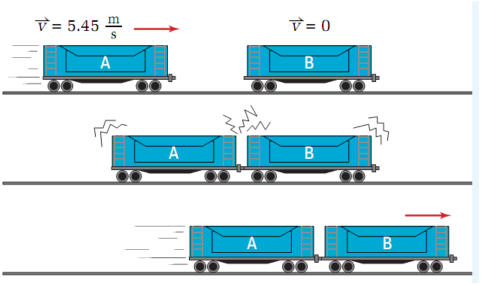
For a collision between two objects A and B:

$$\boxed{m_A \mathbf{v}_A + m_B \mathbf{v}_B = m_A \mathbf{v}'_A + m_B \mathbf{v}'_B}$$

Quantity	Symbol	SI Unit
Mass of objects A and B	m_A, m_B	kg (kilograms)
Velocities of A and B before	$\mathbf{v}_A$, $\mathbf{v}_B$	m/s (metres per second)
Velocities of A and B after	$\mathbf{v}'_A$, $\mathbf{v}'_B$	m/s (metres per second)

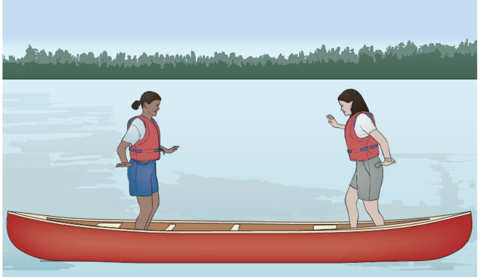
Example Problem: Boxcars

Example 4: A 1.75×10^4 kg boxcar is rolling down a track towards a stationary boxcar that has a mass of 2.00×10^4 kg. Just before the collision, the first boxcar is moving east at $5.45\,\mathrm{m/s}$. when the boxcars collide, they lock together and continue to down the track. What is the velocity of the two boxcars immediately after the collision?



Example Problem: Canoe

Example 5: Two people A and B stand in a canoe on top of the water. Find the velocity of the canoe and person B at the instant that person A start to take a step, if her velocity is $0.75\,\mathrm{m/s}$ [forward]. Assume person A has a mass of $65\,\mathrm{kg}$ and the combined mass of the canoe, A and B, is $115\,\mathrm{kg}$.



Conservation of Momentum

What happens if I let go of this balloon?

We will talk about this more in Unit 3 when we talk about propulsion in space

A More Difficult Example: Glancing Collision

A glancing collision involves motion in 2D. Since momentum is a vector, the calculation will involve some vector arithmetic.

Example 6: A billiard ball of mass $0.155\,\mathrm{kg}$ ("cue ball") moves with a velocity of $12.5\,\mathrm{m/s}$ towards a stationary billiard ball ("eight ball") of identical mass and strikes it with a glancing blow. The cue ball moves off at an angle of 29.7° clockwise from its original direction, with a speed of $9.56\,\mathrm{m/s}$.

(a) What is the final velocity of the eight ball?

(b) Is the collision elastic?

We can't answer the second question yet (be patient, we'll come back to this example later), but we can definitely solve the first part.

Conservation of Momentum

- We can only solve *some* collision problems using conservation of momentum alone
- For complicated problems, we may have to also think about the conservation of energy

Go Back to Work!

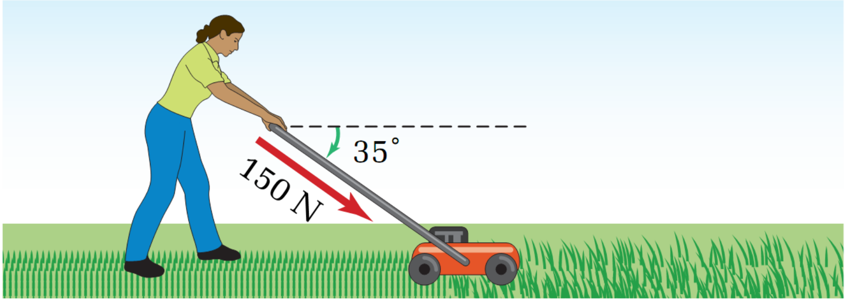
- Work is the mechanism in which:
  - Energy is transferred from one place to another
  - Energy is transformed from one form to another
- Work itself is not energy

$$\boxed{W = F\Delta d \cos\theta}$$

Quantity	**Symbol**	**Unit**
Work	W	J (joules)
Magnitude of force	F	N (newtons)
Magnitude of displacement	Δd	m (metres)

Example Problem

Example 7: A woman pushes a lawnmower with a force of $150\,\mathrm{N}$ at an angle of 35° down from the horizontal. The lawn is $10.0\,\mathrm{m}$ wide and required 15 complete trips across the back. How much work does she do?



Example Problem

Example 8: You drive a nail horizontally into a wall, using a $0.448\,\mathrm{kg}$ hammerhead. If the hammerhead is moving horizontally at $5.5\,\mathrm{m/s}$ and in one blow drives the nail into the wall a distance of $3.4\,\mathrm{cm}$, determine the average force acting on

- The hammerhead
- The nail

Work-Kinetic Energy Theorem

In the work-kinetic energy theorem, Work is required to change kinetic energy:

$$\boxed{W = \Delta K = K_2 - K_1}$$

where K is defined as:

$$\boxed{K = \frac{1}{2}mv^2}$$

(The derivation for K is done through integrating the equations for work and kinematic equations. If you know calculus, it's a straightforward calculation. For Physics 11 and 12, we will just accept that this expression is correct.)

The Work-Potential Energy Theorem

Work equals to change in potential energy:

$$\boxed{W = \Delta U = U_2 - U_1}$$

Potential energies U can be in different forms:

- Gravitational U_g
- Elastic (spring) U_e
- Electrical U_q
- Other forms of potential energy

Gravitational Potential Energy

Near Earth's surface, gravitational potential energy is given by

$$\boxed{U_g = mgh}$$

where $g = 9.81\,\mathrm{m/s^2}$ is the acceleration due to gravity, and h is the height. At a "reference level" where $h = 0$, $U_g = 0$.

Example 9: A gas-powered winch on a rescue helicopter does 4.20×10^3 J of work while lifting a $50.0\,\mathrm{kg}$ swimmer at a constant speed up from the ocean. Through what height was the swimmer lifted?

Elastic Potential Energy

Elastic potential energy is stored while the material *deforms*. It is transformed back into kinetic energy while the material returns to its original shape (state). Examples of how elastic potential is stored:

- Diving board
- Rubber band
- Exercise ball
- Spring

A deformation that cannot be restored is called a "plastic deformation". Every material will deform elastically, then plastically, then fracture, but the mechanics of *how* this happens is complex. In Physics 12, we will only look at elastic deformation of springs.

Hooke's Law

In an **ideal spring**, applied force equals to the product of the spring constant and the amount of extension or compression of the spring.

$$\boxed{\mathbf{F}_a = k\mathbf{x}}$$

Quantity	Symbol	SI Unit
Applied force	$\mathbf{F}_a$	N (newtons)
Spring constant	k	N/m (newtons per metre)
Amount of extension/compression	$\mathbf{x}$	m (metres)

- Spring constant k (also known as "force constant") is the stiffness of the spring
- To *stretch* the spring by x requires applied force of $F_a = kx$
- To *compress* the spring by $-x$ requires a force of $F_a = -kx$

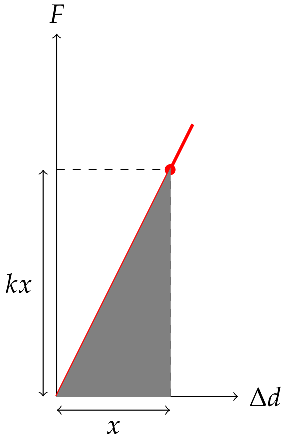
Mass-Spring Simulation

Click for external link: Hooke's Law

Example Problem

Example 10: A typical compound archery bow requires a force of $133\,\mathrm{N}$ to hold an arrow at "full draw" (pulled back $71\,\mathrm{cm}$). Assuming that the bow obeys Hooke's law, what is its spring constant?

Elastic Potential Energy



- Work done to extend/compress a spring is the area under the force-displacement graph
- If we apply the work-potential energy theorem, we can see that work done is the potential stored in the spring

$$\boxed{W = U_e = \frac{1}{2}kx^2}$$

- This equation looks very similar to the equation for kinetic energy; make sure you don't mistaken one for the other.

Example Problem

Example 11: A spring with spring constant of $75\,\mathrm{N/m}$ is resting on a table.

- If the spring is compressed by $28\,\mathrm{cm}$, what is the increase in its potential energy?
- What force must be applied to hold the spring in this position?

The Work-Energy Theorem and Conservation of Energy

Work is equal to the sum of the changing potential and kinetic energy. K is the kinetic energy of objects, and U are all the potential energies.

$$\boxed{W = \Delta U + \Delta K}$$

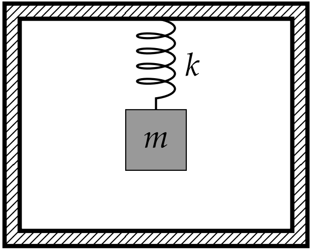
For an *isolated system*, the net amount of work done to the system by its surrounding must be zero, i.e. $W = 0$, therefore energy in the system must be conserved

$$\boxed{U + K = U' + K'}$$

Let's take a closer look at isolated systems

Isolated Systems and the Conservation of Energy

- An isolated system is a system of objects that does not interact with its surroundings
- "Interaction" can be in the form of
  - Friction
  - Exchange of heat
  - Sound emission
- Think of an isolated system as a bunch of objects inside an insulated box

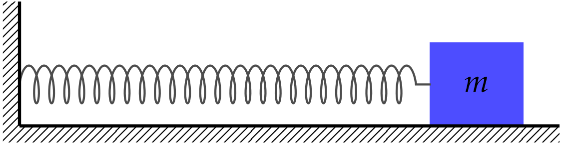


Isolated Systems and Conservation of Energy

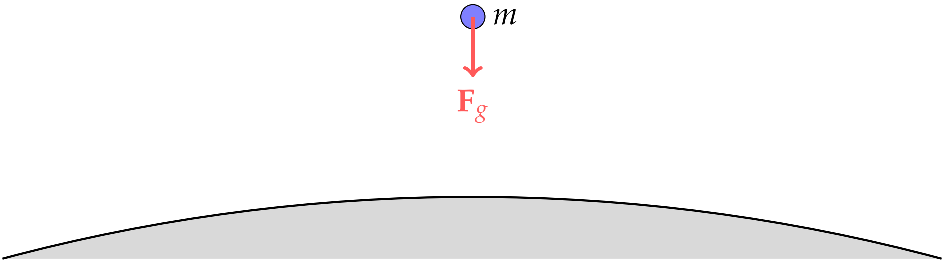
- Since the system is isolated from the surrounding environment, the environment can't do any work on it, by definition!
- Likewise, the energy inside the system cannot escape either
- Therefore energy is conserved
- There are *internal* force inside the system that is doing work, but the work only converts kinetic energy into potential energies, and vice versa.

Example: Mass sliding on a spring

- Assuming that there is no friction in any part of the system
- The isolated system consists of the mass and the spring
- Energies:
    - Kinetic energy of the mass
    - Elastic potential energy stored in the spring

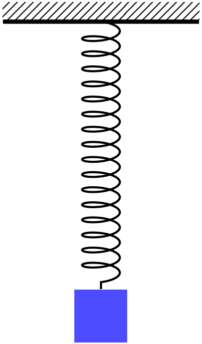


Example: Gravity



- The isolated system consists only of the mass and Earth.
- Assuming no friction
- Energies:
  - Kinetic energy of the mass
  - Gravitational potential energy of the mass

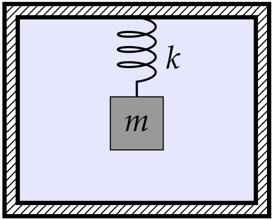
Example: A vertical spring-mass system



- The system consists of a mass, a spring and Earth
- Energies:
  - Kinetic energy of the mass
  - Gravitational potential energy of the mass
  - Elastic potential energy stored in the spring
- The total energy of the system is conserved if there is no friction

What if there is friction?

Energy is always conserved as long as your system is defined properly



- The system consists of a mass, a spring, Earth and all the air particles inside the box
- As the mass vibrates, friction with air slows it down
- While the mass loses energy, the temperature of the air rises due to friction
- Energies:
  - Kinetic and gravitational potential energies of the mass
  - Elastic potential energy stored in the spring
  - Kinetic energy of the vibration of the air molecules
- Total energy is conserved even as the mass stops moving

Example Problem

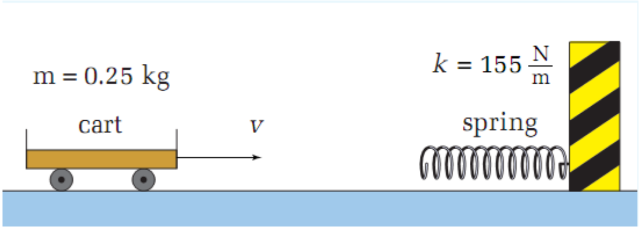
Example 12: A skier is gliding along with a speed of $2.00\,\mathrm{m/s}$ at the top of a ski hill, $40.0\,\mathrm{m}$ high. The skier then begins to slide down the icy (friction-less) hill.

(a) What will be the skier's speed at a height of $25.0\,\mathrm{m}$?

(b) At what height will the skier have a speed of $10.0\,\mathrm{m/s}$?

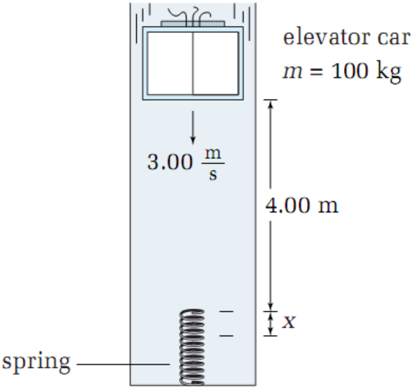
Example Problem

Example 13: A cart with a mass of $0.25\,\mathrm{kg}$ travels along a frictionless horizontal track and collides head on with a spring that has a spring constant of $155\,\mathrm{N/m}$. If the spring was compressed by $6.0\,\mathrm{cm}$, how fast was the cart initially travelling?



Example Problem

Example 14: A freight elevator car with a total mass of $100.0\,\mathrm{kg}$ is moving downward at $3.00\,\mathrm{m/s}$, when the cable snaps. The car falls $4.00\,\mathrm{m}$ onto a huge spring with a spring constant of $8.00 \times 10^3\,\mathrm{N/m}$. By how much will the spring be compressed when the elevator car reaches zero velocity?



Back to Collision

- There are two types of collisions: **elastic** and **inelastic**
- Elastic collisions
  - Both momentum and kinetic energy are conserved
  - Usually in these collisions the two objects don't actually make contact with each other
  - **You must NOT assume that a collision is elastic unless you are told!**
- Inelastic collisions (the majority of all collisions)
  - Kinetic energy is lost due to heat, friction or sound.
  - Only momentum is conserved
  - **Special case:** In a *completely inelastic* collision, the objects stick together
  - Energy is conserved to motion just before and/or just after the collision (depends on the situation)

Example Problem

Example 15: A $0.0520\,\mathrm{kg}$ golf ball is moving velocity of $2.10\,\mathrm{m/s}$ when it collides, head on, with a stationary $0.155\,\mathrm{kg}$ billiard ball. If the golf ball rolls directly backwards with a velocity of $-1.04\,\mathrm{m/s}$, is the collision elastic?

To solve this type of problem, using the conservation of momentum to find the velocities of the golf ball and the billiard ball. Then, sum the total kinetic energies of both balls, and compare that to the total kinetic energy of the balls before the collision.

Example Problem

Example 16: A car $(1000\,\mathrm{kg})$ travels at a speed of $20\,\mathrm{m/s}$ towards a truck $(3000\,\mathrm{kg})$. The car rear ends the truck elastically What is the velocity of the truck after the collision?

For this example we are not concerned with the implausibility of such a collision. We just want to figure out what happens *if* it actually happens.

Solving the Example Problem

In order to solve this question we need *both* the conservation of kinetic energy and the conservation of momentum. Here, A is the car, B is the truck.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$

$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2$$

Solving the Example Problem

Momentum

$$m_A v_A + \cancel{m_B v_B} = m_A v'_A + m_B v'_B$$

We can eliminate v_B, since the truck wasn't moving:

$$m_A v_A = m_A v'_A + m_B v'_B$$

Move terms with m_A to the left:

$$\boxed{m_A(v_A - v'_A) = m_B v'_B} \quad (1)$$

Kinetic Energy

$$\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}m_A v_A'^2 + \frac{1}{2}m_B v_B'^2$$

We can multiply every term by 2, then remove the v_B term, since it's zero:

$$m_A v_A^2 = m_A v_A'^2 + m_B v_B'^2$$

Again, moving m_A terms to the left:

$$\boxed{m_A(v_A^2 - v_A'^2) = m_B v_B'^2} \quad (2)$$

Solving the Example Problem

Dividing (2) by (1), we get:

$$\frac{(2)}{(1)} = \frac{m_A(v_A^2 - v_A'^2)}{m_A(v_A - v_A')} = \frac{m_B v_B'^2}{m_B v_B'} = v_B'$$

We can cancel out the m_A terms on the left, then expand on the squared terms on top:

$$\frac{(v_A + v_A')(v_A - v_A')}{(v_A - v_A')} = v_B'$$

Now we get:

$$v_B' = v_A + v_A'$$
$$v_A' = v_A - v_B'$$

We substitute back to the momentum equation (1)

Solving the Example Problem

$$\boxed{v'_A = \frac{m_A - m_B}{m_A + m_B} v_A} \qquad \boxed{v'_B = \frac{2m_A}{m_A + m_B} v_A}$$

These equations work for *all* elastic impact where object B (in this example, the truck) is stationary when impact occurs. Substituting values for m_A, m_B and v_A, we get:

$$v'_A = \frac{m_A - m_B}{m_A + m_B} v_A = \frac{(1000 - 3000)}{(1000 + 3000)} \times 20 = \boxed{-10m/s}$$

$$v'_B = \frac{2m_A}{m_A + m_B} v_A = \frac{(2 \times 1000)}{(1000 + 3000)} \times 20 = \boxed{10m/s}$$

This Example tells us much more!

$$\boxed{v'_A = \frac{m_A - m_B}{m_A + m_B} v_A}$$

$$\boxed{v'_B = \frac{2m_A}{m_A + m_B} v_A}$$

Let's say $m_A = m_B = m$, then:

$$v'_A = \frac{m_A - m_B}{m_A + m_B} v_A$$
$$v'_A = \frac{m - m}{m + m} v_A$$
$$v'_A = 0$$

$$v'_B = \frac{2m_A}{m_A + m_B} v_A$$
$$v'_B = \frac{2m}{m + m} v_A$$
$$v'_B = v_A$$

If both masses are the same, then *all* of the momentum and energy are transferred from A to B!

This Example tells us much more!

$$\boxed{v'_A = \frac{m_A - m_B}{m_A + m_B} v_A} \qquad \boxed{v'_B = \frac{2m_A}{m_A + m_B} v_A}$$

Let's say $m_A \gg m_B$, then we can "ignore" m_B:

$$v'_A = \frac{m_A - m_B}{m_A + m_B} v_A \qquad v'_B = \frac{2m_A}{m_A + m_B} v_A$$

$$v'_A \approx \frac{m_A}{m_A} v_A \qquad v'_B \approx \frac{2m_A}{m_A} v_A$$

$$v'_A \approx v_A \qquad v'_B \approx 2v_A$$

Object A continues to move like nothing happened, but object B is pushed to move at an even higher speed.

This Example tells us much more!

$$\boxed{v_A' = \frac{m_A - m_B}{m_A + m_B} v_A} \qquad \boxed{v_B' = \frac{2m_A}{m_A + m_B} v_A}$$

Similarly, if $m_A \ll m_B$ then we can “ignore” m_A:

$$v_A' = \frac{m_A - m_B}{m_A + m_B} v_A \qquad v_B' = \frac{2m_A}{m_A + m_B} v_A$$

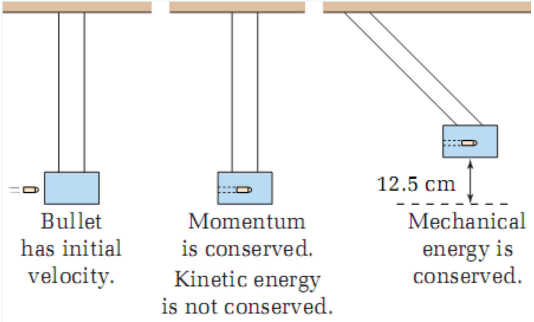
$$v_A' \approx \frac{-m_B}{m_B} v_A \qquad v_B' \approx \frac{0}{m_B} v_A$$

$$v_A' \approx -v_A \qquad v_B' \approx 0$$

Object A bounces off B, and travels in the opposite direction with the same velocity magnitude.

Example Problem

Example 17: A forensic expert needed to find the velocity of a bullet fired from a gun in order to predict the trajectory of a bullet. He fired a $5.50\,\mathrm{g}$ bullet into a ballistic pendulum with a bob that had a mass of $1.75\,\mathrm{kg}$. The pendulum swung to a height of $12.5\,\mathrm{cm}$ above its rest position before dropping back down. What was the velocity of the bullet just before it hit and became embedded in the pendulum bob?



Example Problem

Example 18: A block of wood with a mass of $0.500\,\mathrm{kg}$ slides across the floor towards a $3.50\,\mathrm{kg}$ block of wood. Just before the collision, the small block is travelling at $3.15\,\mathrm{m/s}$. Because some nails are sticking out of the blocks, the blocks stick together when they collide. Scratch marks on the floor show that they slid $2.63\,\mathrm{cm}$ before coming to a stop. What is the coefficient of friction between the wooden blocks and the floor?

We don't have to use any kinematic or dynamic equations to solve this problem. We only need the conservation momentum equation, and the definition of kinetic energy.