3. Motion in Space

Grade 12 Physics

Olympiads School

Summer 2018

Files for You to Download

Please download from the school website if you have not done so:

- `3-planetaryMotion.pdf`—This presentation (for both screen display and printing on paper)
- `3-Homework.pdf`—This unit's homework assignment

Reminder: Always download/print the PDF file for the unit before each class. There is no point copying notes that are already printed out for you. Instead, take notes on things I say that aren't necessarily on the slides.

Before Newton, there was Kepler

Johannes Kepler

- German mathematician, astronomer & astrologer
- Published his first 2 laws in 1609 (79 years before Newton published his work)
- Published third law in 1619
- His work was controversial, but by 1670, most have accepted his findings

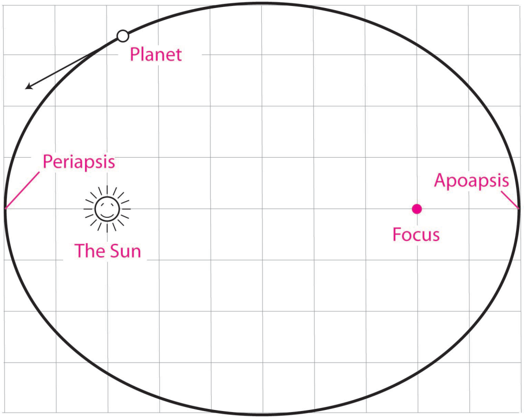
Kepler's Laws of Planetary Motion

Kepler's Laws

- "Empirical": based purely on observed data
- No physics theory behind the equation
- "Fitting the curve" to the data
- Often provide insight to scientists to come up with new theory to be tested further.

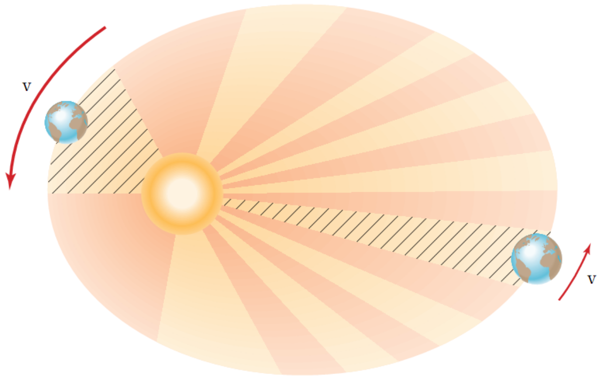
Kepler's Laws of Planetary Motion

1. The orbit of a planet is an ellipse with the Sun at one of the two foci.



Kepler's Laws of Planetary Motion

2. A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.



Kepler's Laws of Planetary Motion

3. The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.

It means this:

$$\boxed{\frac{T^2}{r^3} = \text{constant}} \quad \text{or} \quad \boxed{\frac{T_A^2}{r_A^3} = \frac{T_B^2}{r_B^3}}$$

where A and B are two different planets circulating the same sun

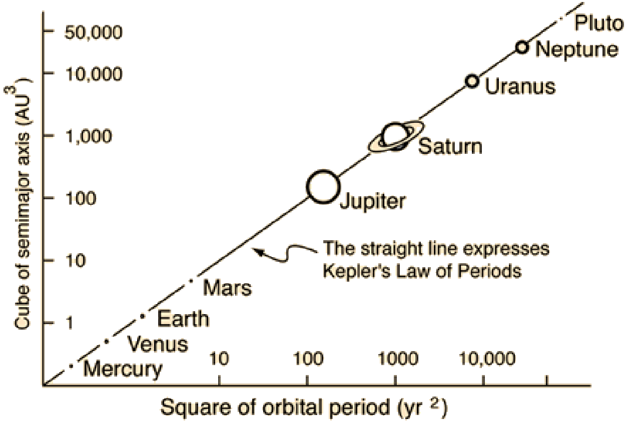
Kepler's Laws of Planetary Motion

- Kepler had *no* idea why planets move like that
- Also, the ellipses aren't perfect; Kepler knew hat something is missing
- But it'll all be explained later by someone else . . .

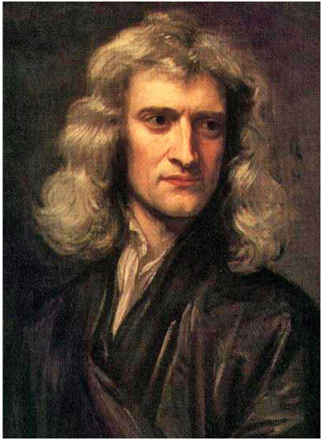
Kepler's Laws of Planetary Motion

Orbital Radii and Periods of Different Planets

Planet	R (AU)	T (days)
Mercury	0.389	87.77
Venus	0.742	224.70
Earth	1.000	365.25
Mars	1.524	686.98
Jupiter	5.200	4332.62
Saturn	9.150	10,579.20



Then Came Newton. . .



Sir Issac Newton
in 1689

- 1687: *Philosophiæ Naturalis Principia Mathematica* (“Mathematical Principles of Natural Philosophy”)
  - One of the most important publications in science
  - Laid out the foundation of what we now call “classical” (“Newtonian”) mechanics and calculus
- Based on works by Galileo and Kepler

Newton's Law of Universal Gravitation

$$\boxed{F_g = G\frac{m_1 m_2}{r^2}}$$

- Newton theorized that gravity must exist between all mass
- He also figured out that gravity must be inversely proportional to the squared of the distance r between the masses
- But he didn't know that G should be
- That came later, by Henry Cavendish, 70 years after Newton died

Newton's Law of Universal Gravitation

What we know now

$$\boxed{F_g = G\frac{m_1 m_2}{r^2}}$$ where $G = 6.674 \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$

Quantity	**Symbol**	**SI Unit**
Force of gravity	F_g	N (newtons)
First and second mass	m_1, m_2	kg (kilograms)
Distance between the centres of the masses	r	m (metres)
Universal gravitational constant	G	$\mathrm{N\,m^2/kg^2}$

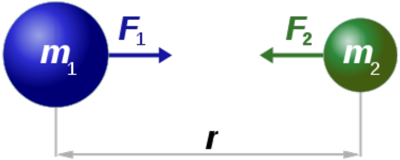
A Simple Example Problem

Example 1: A $65.0\,\mathrm{kg}$ astronaut is walking on the surface of the moon, which has a mean radius of $1.74 \times 10^3\,\mathrm{km}$ and a mass of $7.35 \times 10^{22}\,\mathrm{kg}$. What is the weight of the astronaut?

Newton's Law of Universal Gravitation

$$\boxed{F_g = G\frac{m_1 m_2}{r^2}}$$

If m_1 exerts a force F on m_2, then m_2 also exerts the same force on m_1:



$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

Newton's Universal Gravitation

By knowing the mass of Earth is 5.97×10^{24} kg and the radius of Earth is 6.37×10^{6} m, we can show that acceleration due to gravity is $g = 9.81\,\mathrm{m/s^2}$:

$$F_g = \underbrace{\left[G \frac{m_{\text{Earth}}}{r_{\text{Earth}}^2} \right]}_{=g} m = mg$$

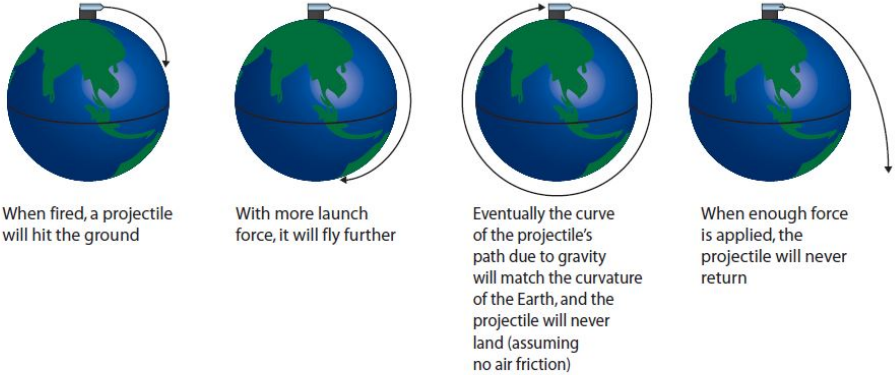
In the next unit we will look at g in depth.

Example Problem

Example 2: How far apart would you have to place two $7.0\,\mathrm{kg}$ bowling balls so that the force of gravity between them would be $1.25 \times 10^{-8}\,\mathrm{N}$?

Orbital Velocity

Newton's Thought Experiment



Orbital Velocity

Application of Gravitational and Centripetal Force

- The velocity required for an object to reach an orbital path without falling back onto the surface. For example:
  - A spy satellite orbiting around Earth
  - The moon orbiting around Earth
  - The Earth orbiting around the Sun
- Assume an approximately circular orbit
- The centripetal force is equal to the gravitational force:

$$\underbrace{\frac{GMm}{r^2}}_{\mathrm{F_g}} = \underbrace{\frac{mv^2}{r}}_{\mathrm{F_c}}$$

r is the distance between the centres of the two objects

Orbital Velocity

Cancelling m, and r in both sides of the equation, and solving for v, we get:

$$\boxed{v_{\text{orbit}} = \sqrt{\frac{GM}{r}}}$$

- Orbital velocity does not depend on the mass of the object in orbit
- A 1.5×10^{-13} kg speck of cosmic dust and the $419\,600$ kg International Space Station both have the same v_{orbit} around Earth if they are at the same altitude.

Example Problem

Example 3: At what velocity and altitude must a satellite orbit in order to be geostationary?

(Explanation: “Geostationary” means that the position of the satellite is over the same position on earth all the time.)

Example Problem

Example 4: What is the orbital velocity of a satellite at a height of $300\,\mathrm{km}$ above the surface of Earth? (mass of Earth is $5.97 \times 10^{24}\,\mathrm{kg}$ and the radius of Earth is $6370\,\mathrm{km}$)

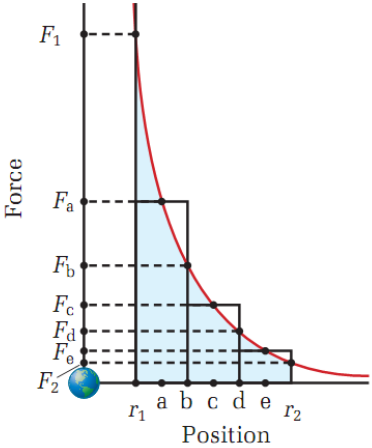
1. $5.42 \times 10^{1}\,\mathrm{m/s}$
2. $1.15 \times 10^{6}\,\mathrm{m/s}$
3. $7.7 \times 10^{3}\,\mathrm{m/s}$
4. $6.0 \times 10^{6}\,\mathrm{m/s}$
5. $3.0 \times 10^{8}\,\mathrm{m/s}$

Make sure you use the right r!

Work for Lift-Off

The work required to move an object with mass m from r_1 to r_2 is given by:

$$\boxed{W = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$



M = mass of the planet
m = mass of object
r_1 = initial position relative to planet's centre
r_2 = final position relative to planet's centre
If r_2 is sufficiently large, then W the binding energy/escape energy

Escape Velocity

An object can leave the surface of Earth at any velocity. But when all the kinetic energy of that object is converted to gravitational potential, it'll return back to the surface of the earth. There is, however, a *minimum* velocity at which the object would not fall back to Earth because of gravity.

Let's work this out.

Escape Velocity

Here's the equation for the work required to bring an object from r_1 to r_2:

$$\boxed{W = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

Starting on the surface of the planet, $r_1 = r_p$, then the most amount of work that you need to do is when $r_2 = \infty$, then

$$\frac{1}{r_2} = \frac{1}{\infty} = 0$$

That means that the most amount of work that can be done is:

$$W_{\mathrm{max}} = \frac{GMm}{r_p}$$

Escape Velocity

If you start with *more* kinetic energy than is required to do all the work, after the all the work is done, the object will still have kinetic energy left! Let's set K to equal to W:

$$K = \frac{1}{2}mv^2 = \frac{GMm}{r} = W$$

The mass of the object (m) on both sides of the equation cancel, and we can now solve for escape velocity v:

$$\boxed{v_{\mathrm{esc}} = \sqrt{\frac{2GM}{r}}}$$

For any $v > v_{\mathrm{esc}}$ the object can break free of the planet's gravitational pull.

Comparing Orbital Velocity to Escape Velocity

Orbital and escape velocities differ by a factor of $\sqrt{2}$:

$$v_{\text{orbit}} = \sqrt{\frac{GM}{r}} \qquad v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

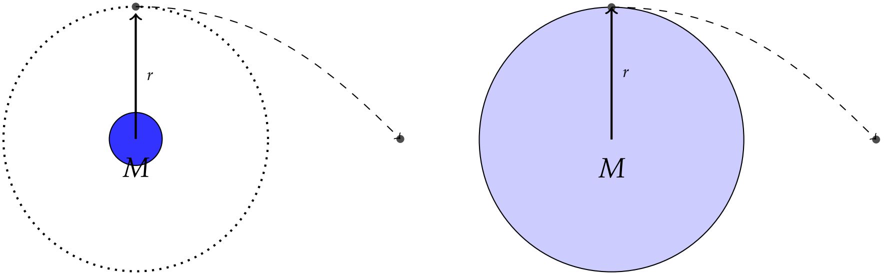
- What happens if $v_{\text{orbit}} < v < v_{\text{esc}}$?
- What happens if $v < v_{\text{orbit}}$?

Example Problem

Example 5: Determine the escape energy and escape velocity for a 1.60×10^4 kg rocket leaving the surface of Earth.

What if I'm not escaping from the surface?

These two objects have the same escape velocity:



In both cases, the object is escapeing from distance r from the centre of the planet. The difference is that an object in orbit (left) already has orbital speed v_{orbit}, so escaping from that orbit requires an additional speed of

$$\Delta v = v_{\text{esc}} - v_{\text{orbit}} = (\sqrt{2} - 1)v_{\text{orbit}}$$

Orbital Kinetic Energy

We can obtain the **orbital kinetic energy** by applying the orbital speed in our expression of kinetic energy:

$$K_{\text{orbit}} = \frac{1}{2} m v_{\text{orbit}}^2 = \frac{1}{2} m \left(\sqrt{\frac{GM}{r}} \right)^2 = \boxed{\frac{GMm}{2r}}$$

Gravitational Potential Energy

Let's look at gravitational potential again. Up until now, we've been using

$$U_g = mgh$$

where m is the mass of the object, g is acceleration due to gravity, and h is the height above a reference level. But now that we know a bit more about gravity, let's consider the equation:

$$\boxed{U_g = -\frac{GMm}{r}}$$

Gravitational Potential Energy

$$\boxed{U_g = -\frac{GMm}{r}}$$

- Proportional to the universal gravitational constant (G), the mass of the planet or celestial body (M), and the mass of the object (m)
- Inversely proportional to the distance from the centre of the planet (r)
- The negative sign is chosen with respect to infinity (i.e. $U_g = 0$ at $r = \infty$)

Total Orbital Energy

Total energy is the sum of kinetic and gravitational potential energies:

$$E_{\text{tot}} = K + U_g$$

Substituting the the K and U_g terms that we derived earlier:

$$E_{\text{tot}} = \frac{GMm}{2r} + \left(-\frac{GMm}{r}\right)$$

We now have:

$$\boxed{E_{\text{tot}} = -\frac{GMm}{2r}}$$

Orbital Energies

Simple relationships between K, U_g and E_{tot}

Orbital kinetic energy:

$$K_{\text{orbit}} = \frac{GMm}{2r}$$

gravitational potential energy:

$$U_g = -\frac{GMm}{r} = -2K_{\text{orbit}}$$

Total orbital energy:

$$E_T = K + U_g = -\frac{GMm}{2r} = -K_{\text{orbit}}$$

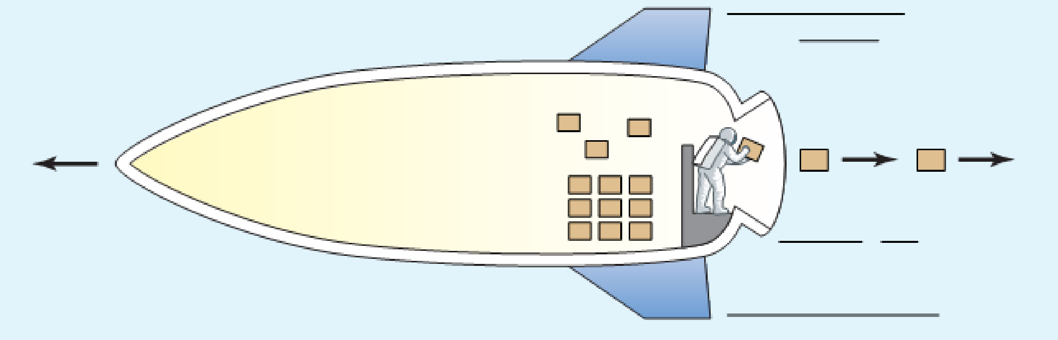
Example Problem

Example 6: On March 6, 2001, the Mir space station was deliberately crashed into Earth. At the time, its mass was 1.39×10^3 kg and its altitude was $220\,$km.

- Prior to the crash, what was its binding energy to Earth?
- How much energy was released in the crash? Assume that its orbit was circular

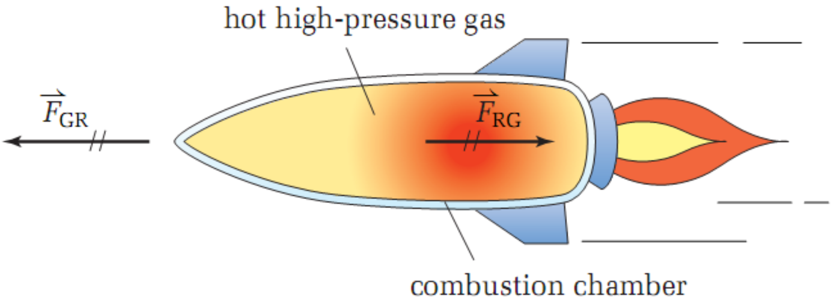
Energy and Momentum in Space

How does a spacecraft move in space anyway? Well, like this:



Energy and Momentum in Space

Okay, we're not throwing out blocks of cargo out the back of the spacecraft, but we *do* throw out burnt fuel from the rockets. . .



$\vec{F}_{GR}$ = force exerted by the gas on the rocket (thrust) [forward]

$\vec{F}_{RG}$ = force exerted by the rocket on the gas [backward]

$$\vec{F}_{GR} = -\vec{F}_{RG}$$

Energy and Momentum in Space

Let's go back to the impulse-momentum theorem from Unit 2:

$$\mathbf{F}\Delta t = m\Delta\mathbf{v}$$

Therefore

$$\mathbf{F}_{(\text{on gas})}\Delta t = m\Delta\mathbf{v}_{\text{gas}}$$

Or

$$\boxed{\mathbf{F}_{(\text{on gas})} = \left(\frac{m}{\Delta t}\right)\Delta\mathbf{v}_{\text{gas}}}$$

The Space Shuttle

- Two solid-fuel booster rockets during take-off
  - Once turned on, they can't be turned off
  - Retrived from the ocean to be re-used again
- One single-use external liquid-fuel tank ("ET") to supply additional fuel to the shuttle's main rocket engines ("SSME")

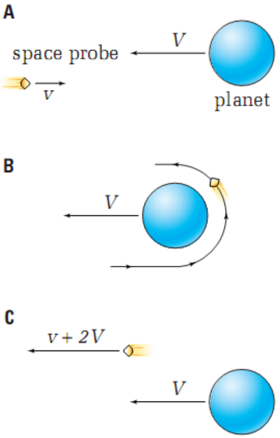
Example Problem

Example 7: A rocket engine consumes $50.0\,\mathrm{kg}$ of hydrogen and $400.0\,\mathrm{kg}$ of oxygen during a $5.00\,\mathrm{s}$ burn.

- If the exhaust speed of the gas is $3.54\,\mathrm{km/s}$, determine the thrust of the engine
- If the rocket has a mass of $1.5 \times 10^4\,\mathrm{kg}$, calculate the acceleration of the rocket

Gravitational Assist or Slingshot

The interaction between the objects is a a very very very VERY elastic “collision”



Gravitational Assist or Slingshot

The interaction between the objects is very very very VERY elastic