First Name:_____ Last Name: _____ Student ID: _____

Derivatives (2)

1. Differentiate:

a. f(x) = $\frac{3x+1}{x-1}$	b. f(x) = $\frac{x^2-1}{x^2+1}$
c. f(x) = $\sqrt{3x^2-1}$	d. f(x) = $\sqrt{\frac{x}{x+2}}$
e. f(x) = $(5x-1)^3(x^2+1)^5$	f. f(x) = $(\frac{x-1}{x+1})^4$

2.

a. Suppose that $h(x)$ is an odd, differentiable function. Show that $h'(x)$ is an even function.
b. Suppose that $g(x)$ is an even, differentiable function. Show that $g'(x)$ is an odd function.

3. *Cotangent Curves*

Two curves which both pass through the point *P* are said to be cotangent at *P* if they have the same tangent line at *P*. For what values of the constants *a* and *b* are the curves C_1: $y = \frac{8}{x} - 2$, and C_2: $y = x^2 + ax + b$ cotangent at the point *P* at which $x = 2$?

4. Suppose that *f* is a function satisfying the equation $f(x+y)= f(x)+ f(y)$ for all real *x* and *y*, and that $f'(0)=c$, a constant.

i. Show that $f(0)=0$, and hence that $f'(0) = \lim_{h\to 0} \frac{f(h)}{h}$

ii. Use the definition of the derivative, $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, and the given equation for $f(x+y)$, to show that $f'(x)=c$ for all real *x*, i.e., that the graph of $y=f(x)$ has slope *c* for all real *x*.

5. Find the equation of the normal line to the graph of the given function at the given point:

f(x) = −2x + $2x^2$; P(1, 0)

6. Find $f^{(n)}$(x) for the following functions

a. f(x) = x^3 + x^2 + x + 1

b. f(x) = $\frac{1}{x-a}$

7.

a. Let $f(x) = x|x|$. Find $f'(0)$, or show that it does not exist.
b. Let $f(x)=|x|$, and show that $f'(x) = \frac{x}{|x|}$.
c. Consider the function $f(x) = ||x|-1|$.
   i. Sketch the graph of $y = |x|-1$.
   ii. Use your graph in part i) to sketch the graph of $y = f(x)$.
   iii. Find an explicit piecewise definition of $f(x)$.
   iv. Indicate any points where $f'(x)$ does not exist, giving reasons. Find an explicit expression for $f'(x)$ wherever it does exist. (It will be a piecewise-defined derivative function.)

8. Find the points on the curve y = x^3 - x^2 – x + 1 where the tangent lines are horizontal.

9. Given y = x^3 – 8x + 7 where x = f(t). If f(0) = 3 and f'(0) = 2, what is the value of $\frac{dy}{dt}$ when t = 0?

10. Determine the slope of the tangent line to the curve x^2 + 2xy - y^3 = 0 at (1, -1).