First Name:_____ Last Name: _____ Student ID: _____

Applications of vectors

1. Calculate the dot product, $\vec{u} \cdot \vec{v}$, to one decimal place accuracy, given that

a. $|\vec{u}|=10, |\vec{v}|=2$, and the angle between $\vec{u}$ and $\vec{v}$ is 40∘
b. $\vec{u} = 3\hat{\imath} - \hat{\jmath} + 4\hat{k}$ and $\vec{v} = -\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$

2. Use the dot product to prove the relation:

$(\vec{a} + \vec{b})(\vec{a} - \vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$

3. If the vectors $\overrightarrow{2a} + \vec{b}$ and $\frac{1}{2}\vec{a} - \vec{b}$ are perpendicular to each other and $2\ |\vec{b}| = 3\ |\vec{a}|$ find the angle

$\theta = \angle(\vec{a}, \vec{b})$.

4. Find the angle between each pair of vectors:

a. $\vec{u}=3\hat{\imath}-\hat{\jmath}$ and $\vec{v}=-\hat{\imath}+2\hat{\jmath}$
b. (2,1,−3) and (1,0,4)

5. For each of the following pairs of vectors, find the value of a which makes $\vec{u}$ orthogonal to $\vec{v}$:

a. $\vec{u}=(3,-4)$ and $\vec{v}=(a,6)$
b. $\vec{u}=2\hat{\imath}+\hat{\jmath}+3\hat{k}$ and $\vec{v}=a\hat{\imath}+2\hat{\jmath}-\hat{k}$
c. $\vec{u}=(3,a,-2)$ and $\vec{v}=(1-a,-3,4)$

6. Use the dot product to determine if $\triangle ABC$ is right-angled, given the coordinates of its vertices. If it is, state which angle measured 90∘.

a. A (3,−1), B (0,−2), C (2,0)
b. A (1,−1,4), B (−2,5,3), C (3,0,4)

7. The parallelogram *PQRS* has vertices *P* (7,12),*R* (20,5), and *S* (4,3).

a. Find the coordinates of *Q*.

b. Find the measure of $\angle PSR$

c. Calculate the area of the parallelogram.

8. If $\vec{u}$ has magnitude 11, $\vec{v}$ has magnitude 5, and the angle between $\vec{u}$ and $\vec{v}$ is 140∘, what is the magnitude of $\vec{u} \times \vec{w}$ to one decimal place accuracy?

9. Find the cross product $\vec{u} \times \vec{v}$ given that

a. $\vec{u} = 3\hat{\imath} - \hat{\jmath} + 4\hat{k}$ and $\vec{v} = -\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$
b. $\vec{u} = (1,2,3)$ and $\vec{v} = (4,-1,5)$
c. $\vec{u} = (-2,1,3)$ and $\vec{v} = (4,-2,-6)$

10. Given the vectors $\vec{u}=(-2,1,-1)$ and $\vec{v}=(-1,2,-1)$

a. Find a unit vector perpendicular to both $\vec{u}$ and $\vec{v}$.
b. Find two vectors of magnitude 11 which are perpendicular to both $\vec{u}$ and $\vec{v}$.

11. For each pair of vectors $\vec{u}$ and $\vec{v}$, find the vector projection of $\vec{u}$ on $\vec{v}$.

a. $\vec{u}=(-2,1)$ and $\vec{v}=(3,4)$
b. $\vec{u}=(1,5)$ and $\vec{v}=(-5,1)$
c. $\vec{u}=(-2,1,-1)$ and $\vec{v}=(2,1,3)$
d. $\vec{u}=(-2,1,-1)$ and $\vec{v}=(4,-2,2)$

12. For each pair of vectors $\vec{u}$ and $\vec{v}$ in Question 11, find the scalar projection of $\vec{u}$ on $\vec{v}$.

13. Determine if the vectors (1, 3, 2), (5,0,−1), and (−4,3,3) are coplanar.

14. Find the volume of the parallelepiped defined by the vectors $\vec{a}$= (0, 1, −3), $\vec{b}$= (1, 2, 3) and $\vec{c}$ = (−1,0,1).

15. Vectors $u^{\rightarrow}$, $v^{\rightarrow}$, $w^{\rightarrow}$ are perpendicular to each other and $|u^{\rightarrow}|$=1, $|v^{\rightarrow}|$=3, $|w^{\rightarrow}|$=4. Find the magnitude of the vector $(u^{\rightarrow} \times v^{\rightarrow}) + (v^{\rightarrow} \times w^{\rightarrow}) + (w^{\rightarrow} \times u^{\rightarrow})$.

16. Consider the vectors $a^{\rightarrow}$, $b^{\rightarrow}$, and $c^{\rightarrow}$. If $u^{\rightarrow} = (a^{\rightarrow} \cdot b^{\rightarrow})c^{\rightarrow} - (a^{\rightarrow} \cdot c^{\rightarrow})b^{\rightarrow}$, prove that $a^{\rightarrow}$ is perpendicular to $u^{\rightarrow}$.

17. Find all unit vectors perpendicular to (1, 2, 3) that make equal angles with the unit vectors $\hat{\imath}$ and $\hat{\jmath}$.