5. The Wave Nature of Light

Grade 12 Physics

Olympiads School

Summer 2018

So What is Light Anyway?

$$\boxed{\textbf{Is light a wave or a particle?}}$$

- If it's a particle, it should behave like particles (e.g. billiard balls) and we can apply our equation of motion to describe its behaviour
- If it's a wave it should behave like all other waves (e.g. ocean waves) and we can apply our wave equations to describe its behaviour
- **In this unit, I'll argue that light is a *wave*, and I'll show you how to work with light by treating it as a wave.**

In This Unit

In this unit, we will be discussing some important properties of light:

- Light waves passing through a medium
  - Reflection
  - Refraction
  - Dispersion
- Light waves passing through an opening
  - Diffraction
  - Interference
  - Optical resolution
- The nature of light? (What kind of wave is light?)
  - Maxwell's equations
  - Electromagnetic waves
  - Polarization of light
  - Speed of light

Some of the topics that we are discussing are reviews. . . but with new insights and more information.

Huygens' Principle

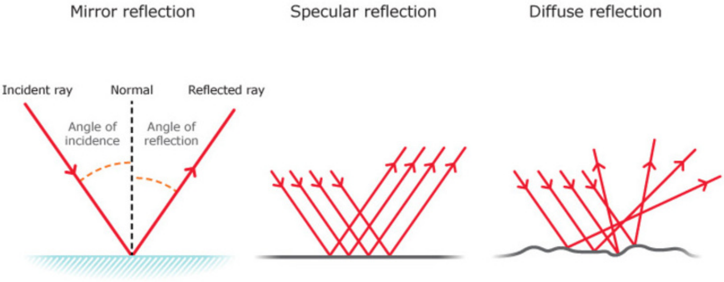
In the 1600's there were two competing theories of light...

- Some, including Issac Newton, believed that light is a particle
- Others, including Christiaan Huygen (Dutch) and Augustin-Jean Fresnel (French), believed that light is a wave

Huygen's Princple: all waves are in fact an infinite series of circular wavelets

Reflection of Light

Law of Reflection
The incident ray, the reflected ray, and the normal to the surface of the mirror all lie in the same plane, and the angle of reflection is equal to the angle of incidence.



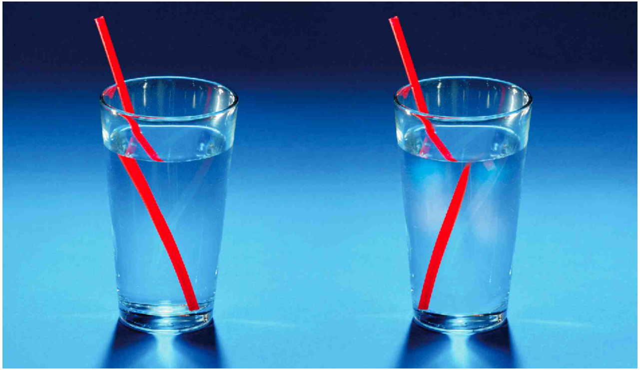
Specular Reflection

Example: Lake Reflection

This photo of Lake Matheson shows specular reflection in the water of the lake with reflected images of Aoraki/Mt Cook (left) and Mt Tasman (right). The very still lake water provides a perfectly smooth surface for this to occur.

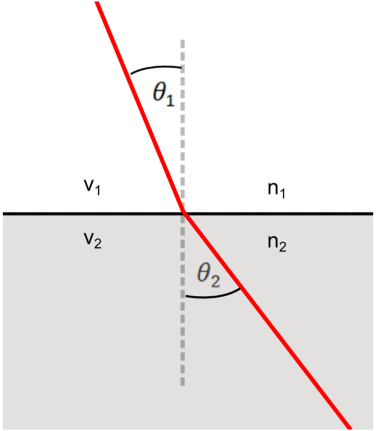
Refraction of Light Through a Medium

- When a wave enters another medium, the wave speed changes
- When entering at an angle, the change of speed causes the wave to change direction (e.g. from air to water, air to glass, glass to air etc)
- The amount of bending depends on the **indices of refraction of the two media**
- Responsible for **image formation** by lenses and the eye



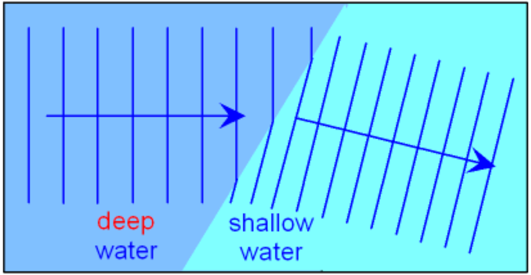
Refraction of Light Through a Medium

You have probably all seen this diagram of light entering from one medium to another.



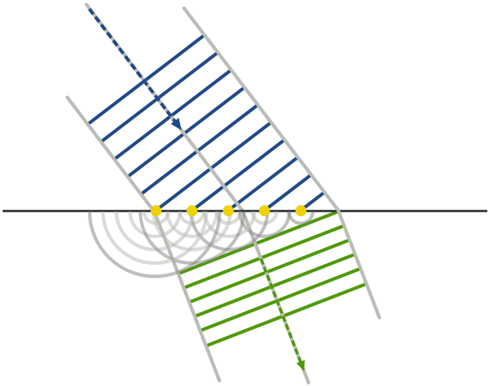
Light could be going in either direction, from top to bottom (n_1 to n_2) or from bottom to top (n_2 to n_1)

Refraction Happens in Ocean Waves Too!



Refraction happens not only with light, we see the same behaviour in ocean waves, when the wave travel from deeper water (faster waves) to shallow depths.

Refraction and Huygens Principle



We can explain the refraction phenomenon using Huygens' Principle

Snell's Law

Snell's Law relates the indices of refraction n of the two media to the directions of propagation in terms of the angles to the normal.

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

Variable	Symbol	SI Unit
Indices of refraction of the media	n_1, n_2	(no units)
Incident angle of light	θ_1	(no units)
Refraction angle of light	θ_2	(no units)

Index of Refraction

- Index of refraction (n) is defined as the speed of light in vacuum (c) divided by the speed of light in the medium (v).

$$\boxed{n = \frac{c}{v} = \frac{\lambda_{\text{vacuum}}}{\lambda}}$$

- When light enters a second medium, the *frequency* remains unchanged (i.e. the colour doesn't change!) but since the speed changes, the *wavelength* also changes:

$$\boxed{\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1}}$$

(Work this out using the equation: $v = f\lambda$)

Index of Refraction of Common Materials

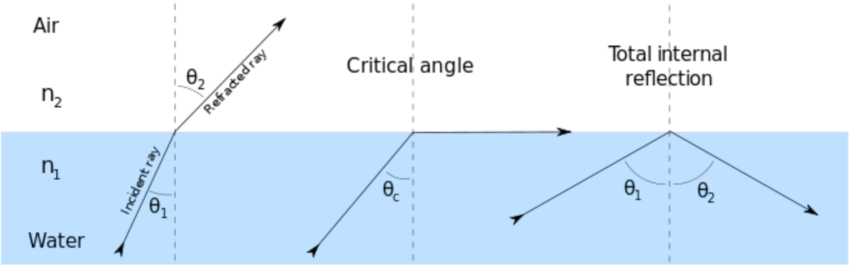
Material	n	Material	n
Vacuum	1	Ethanol	1.362
Air	1.000277	Glycerine	1.473
Water at $20\,^{\circ}\mathrm{C}$	1.33	Ice	1.31
Carbon disulfide	1.63	Polystyrene	1.59
Methylene iodide	1.74	Crown glass	1.50-1.62
Diamond	2.417	Flint glass	1.57-1.75

The values given are *approximate* and do not account for the small variation of index with light wavelength which is called **dispersion**. We'll get to that later!

Total Internal Reflection

From High Index to Low Index

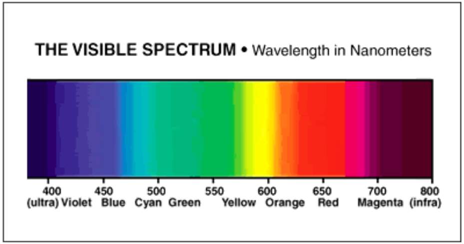
Snell's Law still holds, but something weird can happen:



Critical angle θ_c for water-air interface is 48.6°. If incident angle is greater $\theta_1 > \theta_c$, we have **total internal reflection**. TIR can only happen going from a higher index to a lower index, $n_1 > n_2$.

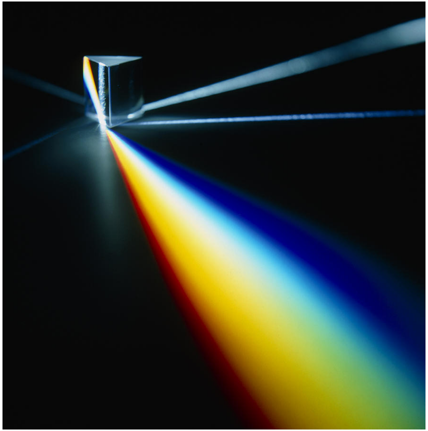
Colour of Light and Wavelength

Human eyes perceive different frequencies of light as different colours. The visible spectrum of light:



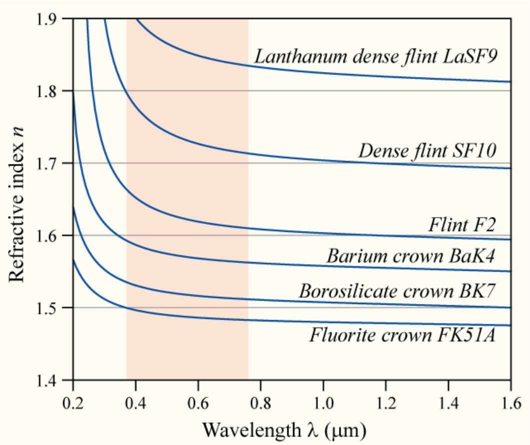
- The “colour” of the light depends on its frequency (& wavelength when it’s in a vacuum)
- *White light* is light that contains waves in all frequencies.

Dispersion of Light Through Refraction



- When white light passes through a prism it is separated into different colours (spectrum) through refraction.
- This is because the index of refraction n is slightly different for different wavelengths
- Otherwise, we will never see a rainbow

Wavelength Dependency of Index of Refraction



Chromatic Aberration

This is What Disperson Can Do!

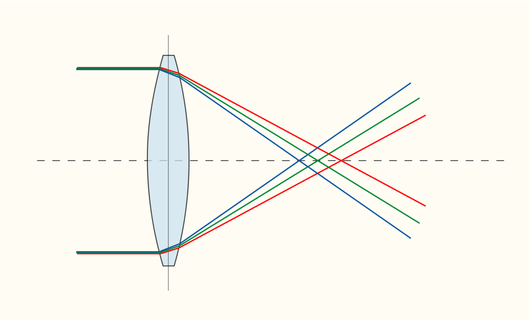
When looking at an image through a low-quality binocular, magnifying glass, or telescope, we often see the edges of images blurred a bit. Sometimes we see a rainbow-coloured edge:

Chromatic Aberration can occur even with high-quality camera lenses, particularly with wide-angle lenses where light from high angles have to bend towards the camera sensor/film, requiring high lens curvature.

Chromatic Aberration

This is What Disperson Can Do!

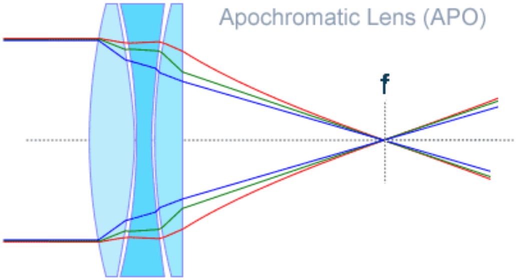
The reason that chromatic aberration happens is the same reason that prisms work: **dispersion of light**



The focal lengths for different frequencies (colour) of light are different, thus blurring the image. So how do we fix it?

Chromatic Aberration: Camera Lens Design

By lining different lenses of different materials and geometries, we can correct for the chromatic aberration.



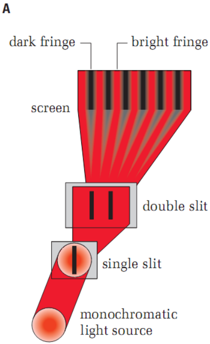
- Lens design is a closely guarded secret by camera companies
- Shape of the lens, material and coating are all factors
- A “lens” on a DSLR camera can have up to 30 lens “elements”

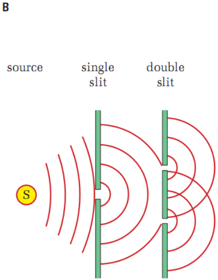
Dispersion

Although dispersion through refraction is the most commonly discussed in physics, it is not the only way that light can split into its spectrum.

Thomas Young's Double-Slit Experiment

First definitive evidence that light is a wave

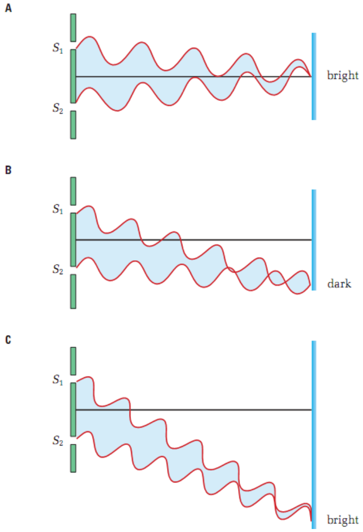




- **Monochromatic light** light with a single colour (frequency); the light source can be a laser, LED , or gas lamp (most likely what Young used)
- **Slit:** an opening; also called an **aperture**
- The **screen** far away from the slits is also called the **projection**

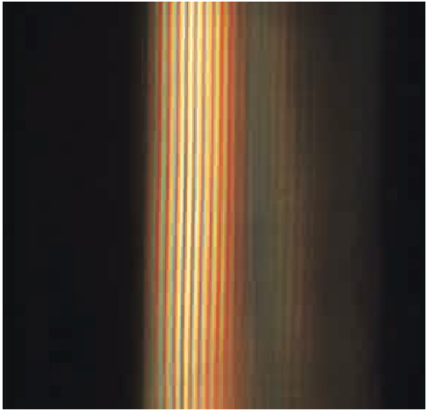
Double-slit experiment showed that light causes interference, just like any other wave

Thomas Young's Double-Slit Experiment



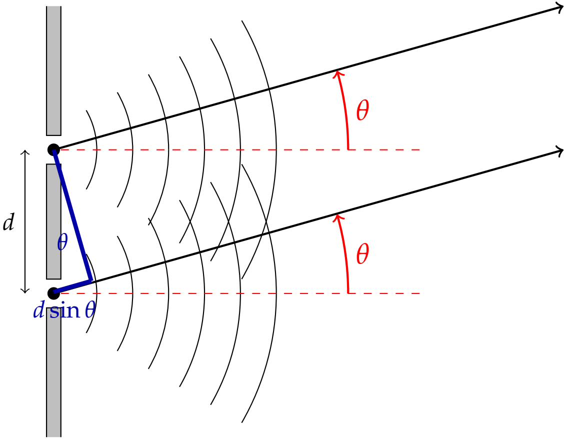
- At **A**, the path from slits S_1 and S_2 are the same, therefore we have **constructive interference** and the projection is bright
- At **B**, the path from S_1 and S_2 are diffed by half a wavelength, and therefore there is **destructive interference** and the projection is dark
- At **C**, the path from S_1 and S_2 are diffed by one wavelength, and therefore there is **constructive interference** again, and again, the projection is bright

Interference Pattern: Bright and Dark Fringes



The “bright fringes” are from constructive interference; the “dark fringes” are from destructive interference.

Let's Work This Out!



- We have two slits at distance d apart, emitting *coherent* light
- Huygens' Principle: light passing through the slits become point sources
- Assume that the projection (screen) is far enough from the slits that we can treat the two beams of light from the slits as being parallel
- Using basic geometry, we can see that the path difference from the the two slit to the projection is $d \sin \theta$

Double-Slit Interference

Constructive Interference

A bright fringe (constructive interference) will happen if the path length difference ($d \sin \theta$) is an integer (n) multiple of wavelength (λ), i.e.

$$\boxed{n\lambda = d \sin \theta_n}$$

where $n = 0, \pm 1, \pm 2, \pm 3 \ldots$

Quantity	Symbol	SI Unit
Integer number of full wavelengths	n	(none)
Wavelength of light	λ	m (metres)
Distance between slits	d	m (metres)
Angle between slit separation and line perpendicular to light rays	θ	(unit less)

Double-Slit Interference

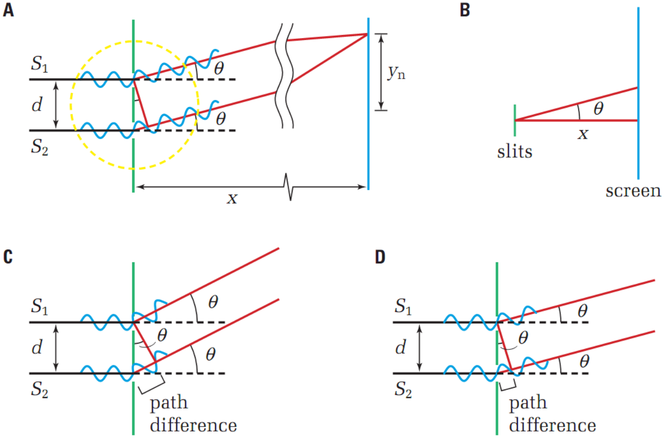
Destructive Interference

Conversely, a dark fringe (destructive interference) will happen if the path length difference ($d\sin\theta$) is an half-number ($n+\frac{1}{2}$) multiple of wavelength (λ), i.e.

$$\boxed{\left(n+\frac{1}{2}\right)\lambda = d\sin\theta_n}$$

where $n = 0, \pm 1, \pm 2, \pm 3 \ldots$

Double-Slit Interference



Approximation of The Wavelength of Light

We can actually estimate the wavelength of light based on the distances between bright fringes, by applying the **small-angle approximation**:

$$\theta \approx \tan\theta \approx \sin\theta$$

The angle θ must be measured in **radians**, not degrees. (As an exercise, compute $\tan\theta$ and $\sin\theta$ for $\theta = 0.001$)

We can already relate the distance from slits to the screen (x), and the distance of the n-th bright fringe from the centre (y_n) to θ_n. Applying the approximation, we have:

$$\tan\theta_n = \frac{y_n}{x} \approx \sin\theta_n$$

Approximation of The Wavelength of Light

Now the Fun Begins

We can substitute our approximation into the constructive interference equation:

$$n\lambda \approx \frac{y_n d}{x}$$

For $n = 0$, $y_0 = 0$ as well, therefore for $n = 1$, $y_1 = \Delta y$, the distance between bright fringes. The wavelength can be approximated:

$$\boxed{\lambda \approx \frac{\Delta y d}{x}}$$

Approximation of The Wavelength of Light

This equation applies equally to dark fringes (nodal lines) as well as bright fringes.

$$\boxed{\lambda \approx \frac{\Delta y d}{x}}$$

Quantity	Symbol	SI Unit
Wavelength	λ	m (metre)
Distance between fringes	Δy	m (metre)
Distance between slits	d	m (metre)
Distance from source to screen	x	m (metre)

Since the approximation is based on small angles, we generally apply this to Δy close to the centre, where light from both slits are deflected by a small angle.

Important Notes

- We have applied the double-slit problem specifically to light, but it can be applied to any wave (e.g. ocean waves) as well
- The “slits” don’t actually need to be slits; any point source will do
- The projection/screen doesn’t need to be a real screen either; it just has to be a line where wave intensity can be measured

Important Question

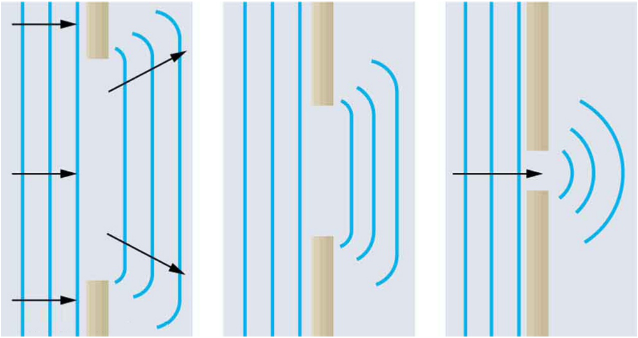
We know that when light passes through the double slit, it spreads out. The single slit also does the same thing, but **why**?

Diffraction of Waves

When a wave goes through an small opening, it **diffracts**. This happens with sound waves, ocean waves. . . and light.

(The photo is from the Port of Alexandria in Egypt. The shape of the entire harbour is created because of diffraction of ocean wave.)

Diffraction of Waves



The smaller the opening (compared to the wavelength of the incoming wave) the greater the diffraction effects.

There are Two Types of Diffraction

- **Fresnel diffraction**
  - “Near-field” diffraction
  - The distance between aperture and the projection is small
  - The short distance to the projection causes the diffraction pattern observed to differ in size and shape
- **Fraunhofer diffraction**
  - “Far-field diffraction”
  - The distance between the aperture and the projection is large
  - Will only focus on this form of diffraction in Physics 12 because the pattern is easier to understand

Fresnel Number

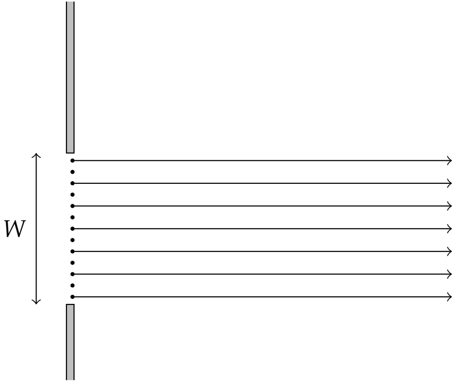
The Fresnel Number tell us when to use Fresnel diffraction (difficult!) when to use Fraunhofer (easier):

$$F = \frac{W^2}{\lambda L}$$

Quantity	Symbol	SI Unit
Fresnel Number	F	(no units)
Characteristic length of the aperture	W	m (metres)
Wavelength of light	λ	m (metres)
Distance from aperture to projection	L	m (metres)

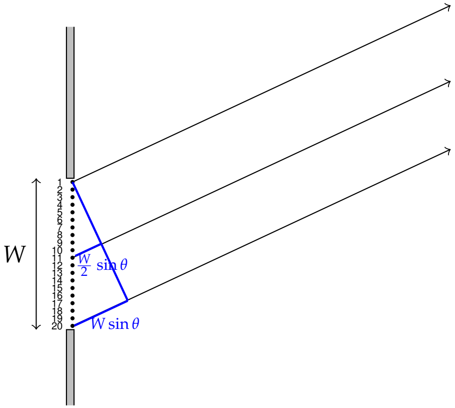
- Fresnel diffraction if $F \gg 1$; Fraunhofer diffraction if $F \ll 1$
- In Physics 12, we will only deal with Fraunhofer diffraction

Let's Work This Out Again!



- Similar to the double-slit problem, we apply Huygens' Principle again
- This time we treat the slit as wide enough that there is a series (an infinite series, actually) of point waves at the slit
- We can easily see that the light from the wavelet that travel perpendicular to the slit (aperture) will not interfere with one another
- i.e. a bright fringe at the middle. **This is called the central maximum.**

At Some Other Angle θ



- Like what we did with double-slit, we can find the path difference between the wavelet on the top (1) and bottom (20): $W \sin\theta$
- At some θ, the path difference between 1 and 20 will be an integer multiple of the wavelength ($m\lambda$)
- In this case, the path difference between 1 and 11 is a half-number multiple of the wavelength (i.e. destructive interference) and they cancel each other
- Similarly, 2 cancels 12, 3 cancels 13...

RESULT: COMPLETE DESTRUCTIVE INTERFERENCE

Dark Fringes: Destructive Interference

Dark fringes exists on the screen at regular, whole-numbered intervals ($m = 1, 2, 3 \ldots$):

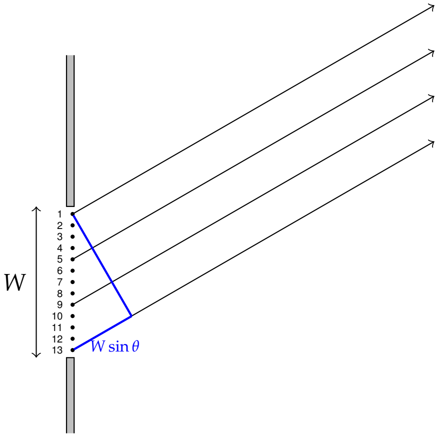
$$\boxed{\pm m\lambda = W \sin\theta_m}$$

Again, we can apply our small-angle approximation equation, and end up with:

$$\boxed{y_m = \frac{m\lambda L}{W}}$$

Pro-tip: This equation looks very similar to the double-slit equation for *bright* fringes, so be *very* careful when you use them!

At Some Other Angle θ



- Again, we follow what we did with the the previous case, and we find that at some angle θ, the path difference between the top and bottom is $W \sin\theta = \frac{3}{2}\lambda$
- Beam from (1) and (5) differ by $\frac{\lambda}{2}$, so they have destructive interference; similarly 2 and 6, 3 and 7, 4 and 8, 9 and 13 will all interfere destructively
- But some of the beams will not, so we have a bright fringe at the projection
- This bright fringe is not as bright as the central one because of the destructive interference

Bright Fringes: Constructive Interference

Bright fringes exist on the screen at regular, half-numbered intervals ($m = 1, 2, 3 \ldots$):

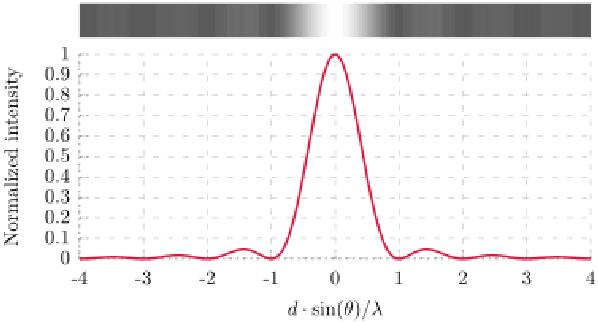
$$\boxed{\pm\left(m + \frac{1}{2}\right)\lambda = W \sin\theta_m}$$

Again, similar to the dark fringes, we apply our small-angle approximation equation:

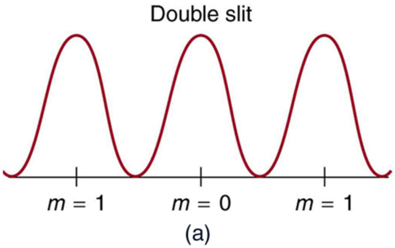
$$\boxed{y_m = \pm\left(m + \frac{1}{2}\right)\frac{\lambda L}{W}}$$

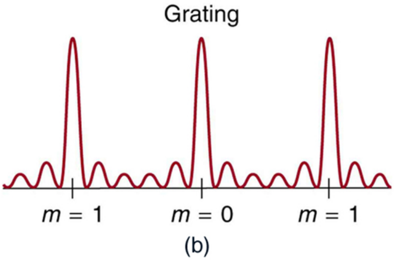
Single-Slit Diffraction, A Summary

- Similar to the double-slit interference, single-slit diffraction projects a series of alternating bright fringes (“maxima”) and dark fringes (“minima”) in the far field
- The bright fringe in the middle (“central maximum”) is twice as wide and very bright
- Subsequent bright fringes on either side (“higher-order maxima”) are much dimmer because of the partial destructive interference



Diffraction Grating: What if there are more than 2 slits?





- We can apply the same analysis from double-slit to a diffraction grating
- Use equation for double-slit interference to locate bright fringes

$$n\lambda = d \sin \theta_n$$

- Interference pattern is sharper
- Bright fringes are narrower

Example Problem

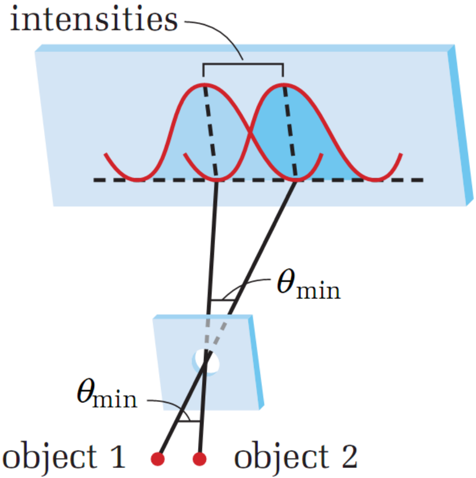
Example 1: Viewing a $645\,\mathrm{nm}$ red light through a narrow slit cut into a piece of paper yields a series of bright and dark fringes. You estimate that five dark fringes appear in a space of $1.0\,\mathrm{mm}$. If the paper is $32\,\mathrm{cm}$ from your eye, calculate the width of the slit.

Resolving Power

The ability of an optical instrument (e.g. the human eye, microscope, camera) to distinguish two distinct objects.

WHY? When light from any object passes through an “optical instrument”, it **diffracts**, therefore “blurring” the object.

Resolving Power



Rayleigh limit: Two objects are resolved if the angle $\theta > \theta_{\min}$, where $\theta_{\min}$ is when the first minimum (dark fringe) from object 1 overlaps with the central maximum (bright fringe in the middle) from object 2.

Resolving Power

In order to resolve two objects, the minimum angle between rays from the two objects passing through a rectangular aperture is the quotient of the wavelength and the width W of the aperture. For a circular aperture, the minimum angle is the quotient of 1.22 times the wavelength and the diameter D of the aperture.

Rectangular aperture:

$$\boxed{\theta_{\text{min}} = \frac{\lambda}{W}}$$

Circular aperture:

$$\boxed{\theta_{\text{min}} = \frac{1.22\lambda}{D}}$$

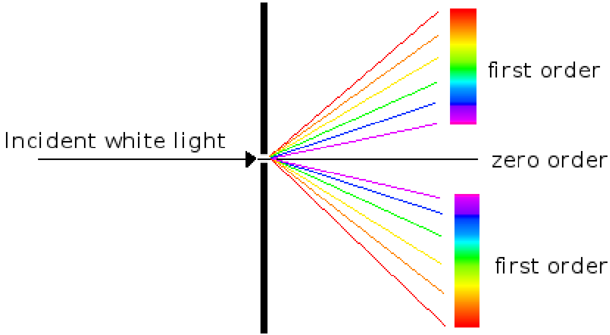
Note: The angle θ_{min} is measured in **radians** not degree.

Example Problem

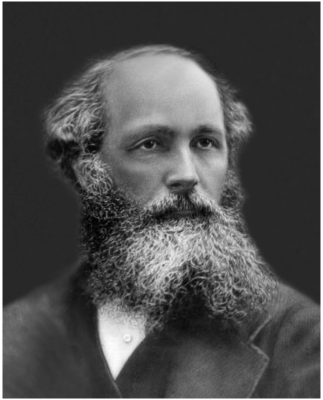
Example 2: A skydiver is falling toward the ground. How close to the ground will she have to be before she is able to distinguish two yellow baseballs lying $25.0\,\mathrm{cm}$ apart, reflecting $625\,\mathrm{nm}$ light in air? Her pupil diameter is $3.35\,\mathrm{mm}$. Assume that the speed of light inside the human eye is $2.21 \times 10^{8}\,\mathrm{m/s}$.

Dispersion of Light Through Diffraction

The examples for single- and double-slit patterns that have all been based on a single wavelength of light, but we know that the equations depends on wavelength. So what happens to our diffraction pattern when the light source is a white light?



New Physics: Maxwell's Equations



James Clerk Maxwell

- Classical laws of electrodynamics
- Published in 1861 and 1862
- Explains the relationship between
    - Electricity
    - Electric Circuits
    - Magnetism
    - Optics
- Previously these disciplines are thought to be separate and not related

Maxwell's Equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_o}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = -\mu_o \mathbf{J} + \mu_o \varepsilon_o \frac{\partial \mathbf{E}}{\partial t}$$

That's a lot of symbols that you won't recognize. Solving them require *a lot* of difficult calculus that even most science students in university don't need to learn. (**i.e. you don't need to learn this**)

Maxwell's Equations

Major Findings

- Electric fields starts/ends at a charge
- Magnetic fields runs in a loop, and has no beginning or ends
- A changing electric field creates a magnetic field
- A changing magnetic field creates an electric field
- Disturbances in the electric and magnetic fields propagate as a wave with speed

$$\boxed{c = \frac{1}{\sqrt{\varepsilon_o \mu_o}} = 2.998 \times 10^8 \,\mathrm{m/s}}$$

. . . the speed of light!

Speed of Electromagnetic Radiation

Electric Permittivity ε_0

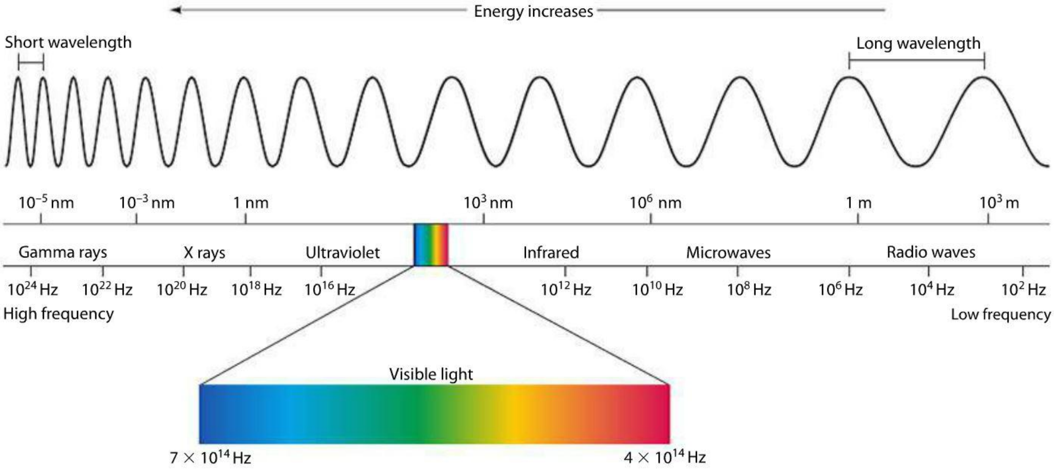
The ability of a medium to resist the formation of an electric field within it. The constant is directly related to the Coulomb constant in Coulomb's law.

Magnetic Permeability μ_o

A measure of the ability of the medium to become magnetized.

- Scientist have previously measured the speed of light to good accuracy
- Maxwell's Wquations show that light is (probably) an electromagnetic ("EM") wave
- *Proofing* that though, brought up a lot of new insights into physics

The Electromagnetic Spectrum



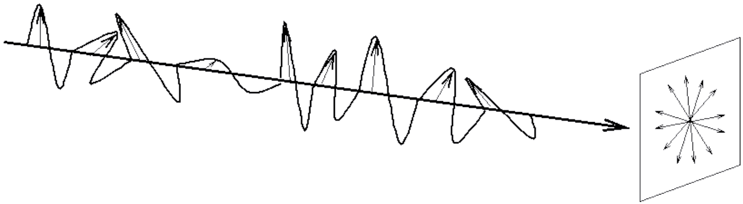
On Polarization of Light

Let's Combine Everything We Know

- Light is an electromagnetic wave, generated by
  - An oscillating charged particle (e.g. shaking an electron violently)
  - An alternating ("A/C") current (i.e. lots of oscillating charged particles)
  - Through black-body radiation
- The EM wave has both an oscillating electric field ($\mathbf{E}$) and magnetic field ($\mathbf{B}$), because
  - A charged particle creates an electric field, and
  - A moving charged particle creates a magnetic field
- $\mathbf{E}$ and $\mathbf{B}$ are always perpendicular to one another, according to Maxwell's Equations

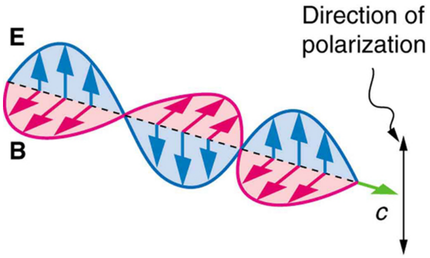
On Polarizaion of Light

- A charged particle can vibrate in any direction, so the oscillating $\mathbf{E}$ and $\mathbf{B}$ can look quite chaotic. We can only guarantee that no matter what happen, $\mathbf{E}$ and $\mathbf{B}$ are:
    - Always perpendicular to each other
    - Always perpendicular to the direction of wave travel
    - This kind of light (or general EM wave) is “unpolarized”
    - Most EM waves you experience in life are this kind:



On Polarization of Light

But if we can confine $\mathbf{E}$ and $\mathbf{B}$ to one plane, then we have a “polarized” light:

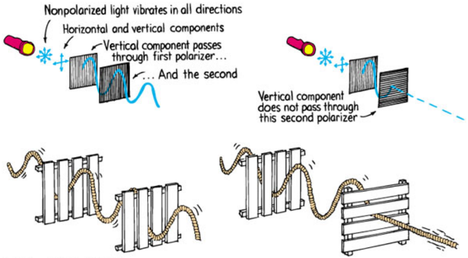


There are a few ways to do this. . .

On Polarization of Light

Using Polarizer

- A polarizer is really just a grill that only lets in vibration in one direction through:

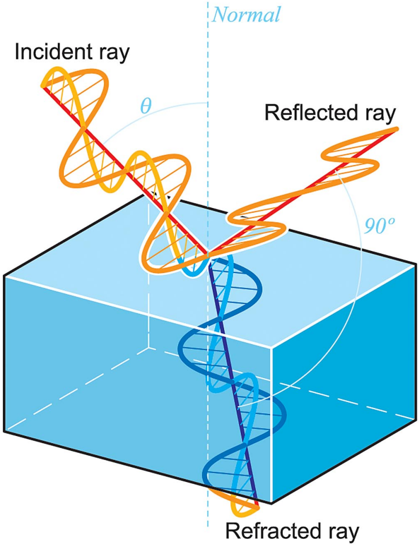


Hewitt, *Conceptual Physics*, Ninth Edition.

- The incoming wave can be vibrating in any direction, but outgoing wave only vibrates in one direction.
- Sunglasses with polarizing lens
- Polarizer filters on cameras

On Polarization of Light

Polarization by Reflection



At **Brewster's angle**, the light reflected off a medium (e.g. glass, water) is also polarized

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

- Incident light is non-polarized
- Reflected light is polarized
- Refracted light is partially polarized
- For water ($n = 1.33$), $\theta_B = 53^\circ$
- For glass ($n = 1.5$), $\theta_B = 56^\circ$