

# 1. Fundamental of Dynamics

Grade 12 Physics

Olympiads School

Summer 2018

# Where Are We In the Course

1. Fundamentals of Dynamics
2. Momentum, Impulse and Energy
3. Motion in Space
4. Gravitational, Electric and Magnetic Fields
5. Wave Nature of Light
6. Theory of Special Relativity
7. Introduction to Quantum Mechanics

# Files for You to Download

Download from the school website:

- 0-courseOutline.pdf—The course outline
- equations.pdf—An equations sheet for this course
- 1-funDynamics\_print.pdf—The print version of my presentation slides for this unit. I recommend printing 4 slides per page.
- 1-Homework.pdf—This homework assignment for this unit

Please download/print the PDF file before each class. There is no point copying notes that are already printed out for you. Instead, take notes on things I say that aren't necessarily on the slides.

## Notes on Writing Vectors

Most university textbooks *print* vectors using a bold face font, but we still use the “arrow on top” format when *writing*.

**In print (books, journal papers)**

**$\mathbf{v}$     $\mathbf{F}_g$     $\mathbf{p}$     $\mathbf{I}$**

**Handwritten (used by some books)**

$\vec{v}$     $\vec{F}_g$     $\vec{p}$     $\vec{I}$

When we write the magnitude of these vectors, we have two options:

**With absolute-value sign**

$|\mathbf{v}|$     $|\mathbf{F}_g|$     $|\mathbf{p}|$     $|\mathbf{I}|$

**Or as a scalar value**

$v$     $F_g$     $p$     $I$

This actually makes sense because the magnitude of a vector is a scalar

# It's All Greek To Me

## Use of Greek letters in Physics

Angles are labelled with:

$\theta$     $\phi$     $\alpha$     $\beta$

Capital letter *delta* ( $\Delta$ ) means “the change in”, e.g.: “change in velocity” is

$\Delta \mathbf{v}$

Capital letter *sigma* ( $\Sigma$ ) means “the sum of”. e.g. “sum of all forces” is

$\Sigma \mathbf{F}$

# Frame of Reference

A **frame of reference** is a hypothetical mobile “laboratory” an observer uses to make measurements (e.g. mass, lengths, time). At a minimum, it must include:

- A ruler to measure lengths
- A clock to measure the passage of time
- A scale to compare forces
- A balance to measure masses

High-school textbooks often refer to the frame of reference as a “coordinate system”. While it certainly includes that, it’s much more, as we will see in the unit on relativity.

# Frame of Reference

- We assume that the hypothetical laboratory is *perfect*
- What instruments are used are unimportant
- What matters is the *motion* (at rest, uniform motion, acceleration etc) of your laboratory, and how it affects the measurement that you make
- “From the point of view of. . .”

## Frame of Reference

An **inertial** frame of reference moving in uniform motion (constant velocity, without acceleration)



### The Principle of Relativity:

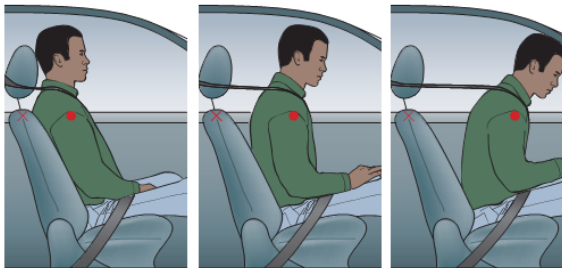
The laws of motion must be obeyed in all inertial frames of reference



## Frame of Reference

**Non-inertial frame of reference:** accelerating (non-constant velocity)

- Newton's 1st and 2nd laws are not valid
- Require “fictitious force” to account for the observations
  - Hypothetical force
  - Does not exist in inertial frame of reference



## Example: Reference Frame



**Example 1:** Passengers in a high-speed elevator feel as though they are being pressed heavily against the floor when the elevator starts moving up. After the elevator reaches its maximum speed, the feeling disappears.

## Example: Reference Frame

- When do the elevator and passengers form an
  - inertial frame of reference?
  - A non-inertial frame of reference?
- Before the elevator starts moving,
  - what forces are acting on the passengers?
  - How large is the external (unbalanced) force?
- Is a person standing outside the elevator in an inertial or non-inertial frame of reference?

# Frame of Reference

**Example 1a:** Is Earth an inertial frame of reference?

# Kinematics

- A branch of mechanics that describes the motion of points, bodies (objects), and systems of bodies (groups of objects)
- Relationship between
  - Position & displacement
  - Velocity
  - Acceleration
- Kinematics does not deal with what causes the motion

## Relating Motion

**Position** is where you are relative a “reference point”, which is the origin of your coordinate system:

**d**

**Displacement** is the change in position from 1 to 2 (make sure you use the same coordinate system!):

$$\Delta \mathbf{d} = \mathbf{d}_2 - \mathbf{d}_1$$

**Velocity** is how quickly your displacement is changing with time. In math, we call it the “rate of change in displacement”:

$$\mathbf{v} = \frac{\Delta \mathbf{d}}{\Delta t}$$

# Relating Motion Quantities

**Acceleration** is the rate of change in velocity, or how quickly velocity is changing with time:

$$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t}$$

- Remember that  $\mathbf{d}$ ,  $\Delta \mathbf{d}$ ,  $\mathbf{v}$  and  $\mathbf{a}$  are all *vector* quantities
- Time  $t$  and time interval  $\Delta t$  are scalar quantities

# Working with Vectors

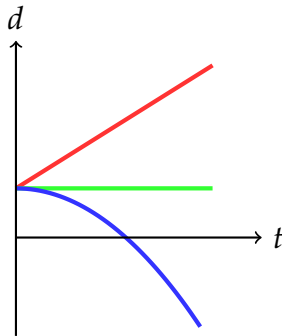
- A change in a vector can mean a change magnitude and/or direction
  - Think about what happens if a car is turning at constant speed
- Vectors obey the *principle of superposition*, which is a fancy way of saying that they add together. Methods for adding vectors include:
  - Using **Pythagorean theorem** (for vectors at right angles of each other)
  - Using **cosine and sine laws**
  - Decomposing vectors in to **components**, then reassemble them using Pythagorean theorem
- For 1D problems, (+) and (-) signs are sufficient to indicate direction
  - Remember to indicate which way is positive though!



# Motion Graph: Position–Time Graph

For 1D motion, we can plot the position of any object against time in a **position–time** (“ $d-t$ ”) graph

- Velocity is the slope
- If the slope is **positive**, the object is moving in the positive direction, and the velocity is positive
- If the slope is **negative**, the object is moving in the negative direction, and the velocity is negative
- If the slope is **zero**, the object is not moving



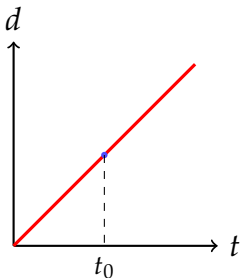
For 2D or 3D motion, there is a motion graph for *each* of the axes

# Motion Graph: Position–Time Graph

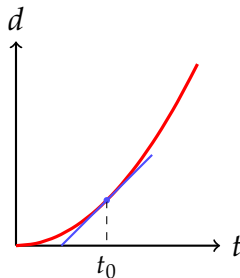
Velocity is the slope of the position–time graph

- Average velocity is the slope of the secant
- Instantaneous velocity is the slope of the tangent

Constant Velocity (straight line)



Constant Acceleration (parabola)

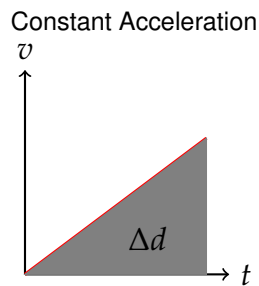
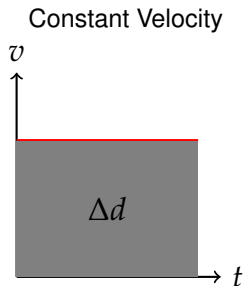


Never assume that just because a graph *looks* like a parabola that it must be the case!

# Motion Graphs: Velocity–Time Graphs

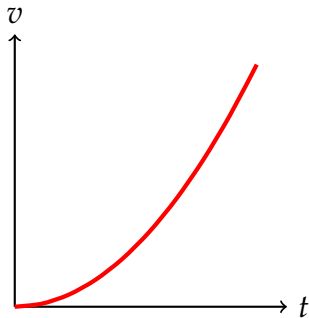
Sometimes it is more advantageous to plot *velocity* against time, because in many cases, it is easier to extract information about the motion

- Area under the curve: displacement
- Slope of the curve: acceleration



## Non-uniform Acceleration

In real life, acceleration does not have to be constant. In this case, acceleration is increasing. (Pay attention to the fact that this is a velocity–time graph.)



## Side Note: Relating Kinematics to Calculus

If you know calculus, or if you are learning calculus right now, you may already know that velocity  $\mathbf{v}$  is the time derivative of position  $\mathbf{d}$ , and acceleration  $\mathbf{a}$  is the derivative of velocity:

$$\mathbf{v}(t) = \frac{d}{dt}\mathbf{d}(t) \quad \mathbf{a}(t) = \frac{d}{dt}\mathbf{v}(t)$$

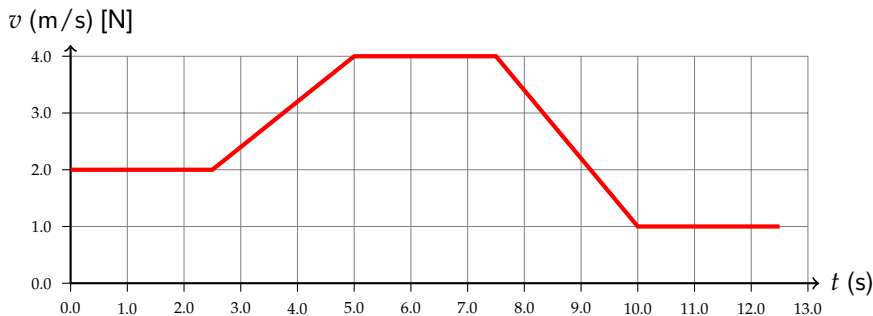
In reverse, integrating acceleration gives you velocity, and integrating velocity gives you displacement:

$$\mathbf{v}(t) = \int \mathbf{a} dt \quad \mathbf{d}(t) = \int \mathbf{v} dt$$

Don't worry about it if you don't understand it right now (we don't need calculus for Physics 12) but this will come in handy in university.

## Example Problem

**Example 2:** Change the following graph to a **position–time** graph. Assume displacement starts at zero.



Please do a sanity check: the object is **NOT** moving east at first!!

# 1D Kinematic Equations

$$\Delta d = v_1 \Delta t + \frac{1}{2} a \Delta t^2$$

$$\Delta d = v_2 \Delta t - \frac{1}{2} a \Delta t^2$$

$$\Delta d = \frac{v_1 + v_2}{2} \Delta t$$

$$v_2 = v_1 + a \Delta t$$

$$v_2^2 = v_1^2 + 2a \Delta d$$

- Only for constant acceleration
- Variables are:

$$\Delta d \quad v_1 \quad v_2 \quad \Delta t \quad a$$

- For 1-object problems, you are usually given 3 of the 5 variables, and you are asked to find a 4th one
- For 2-object problems, the motion of the two objects are connected by time interval  $\Delta t$  and displacement  $\Delta d$

# Projectile Motion: Applying Kinematic Equations in 2D

- For 2D problems, resolve the problem into its horizontal ( $x$ ) and vertical ( $y$ ) directions, and apply these equations independently
- For projectile motion, there is no acceleration in the  $x$  direction, i.e.  $a_x = 0$ , therefore the kinematic equations reduce to just

$$\Delta x = v_x \Delta t$$

- The only acceleration is in the  $y$  direction:

$$a_y = 9.81 \text{ m/s}^2 \text{ [down]}$$

We *usually* define the (+) direction to be [up], so  $g = -9.81 \text{ m/s}^2$ , but it can change depending on the problem

- The variable that connects the two directions is the time interval  $\Delta t$

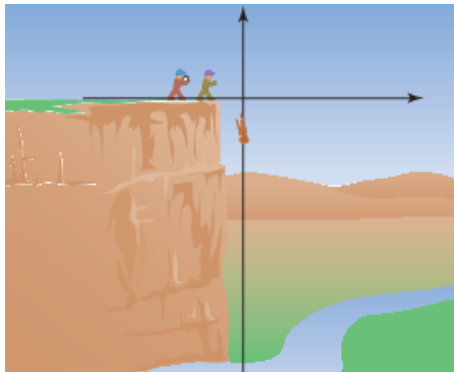


# Projectile Motion

## Let's Start With an Example

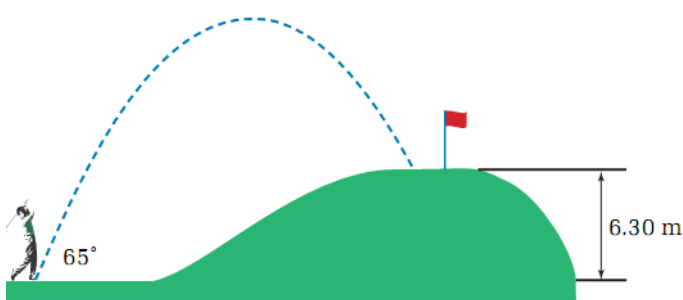
**Example 3:** While hiking in the wilderness, you come to a cliff overlooking a river. A topographical map shows that the cliff is 291 m high and the river is 68.5 m wide at that point. You throw a rock directly forward from the top of the cliff, giving the rock a horizontal velocity of 12.8 m/s.

- (a) Did the rock make it across the river?
- (b) With what velocity did the rock hit the ground or water?



## Projectile Motion Example: Golfer

**Example 4:** A golfer hits the golf ball off the tee, giving it an initial velocity of  $32.6 \text{ m/s}$  at an angle of  $65^\circ$  with the horizontal. The green where the golf ball lands is  $6.30 \text{ m}$  higher than the tee, as shown in the illustration. Find the time interval when the golf ball was in the air, and the distance to the green.



## Projectile Motion Example: Tossing a Ball

**Example 15:** You are playing tennis with a friend on tennis courts that are surrounded by a 4.8 m fence. Your opponent hits the ball over the fence and you offer to retrieve it. You find the ball at a distance of 12.4 m on the other side of the fence. You throw the ball at an angle of  $55.0^\circ$  with the horizontal, giving it an initial velocity of 12.1 m/s. The ball is 1.05 m above the ground when you release it. Did the ball go over the fence, hit the fence, or hit the ground before it reached the fence?

## Symmetrical Trajectory

Trajectory is symmetrical if the object lands at the same height as when it started.

- Time of flight

$$t_{\max} = \frac{2v_i \sin(\theta)}{g}$$

- Range

$$R = \frac{v_i^2 \sin(2\theta)}{g}$$

- Maximum height

$$h_{\max} = \frac{v_i^2 \sin^2(\theta)}{2g}$$

The angle  $\theta$  is the **above the horizontal**

## Example Problem

**Example 6:** A player kicks a football for the opening kickoff. He gives the ball an initial velocity of  $29 \text{ m/s}$  at an angle of  $69^\circ$  with the horizontal. Neglecting friction, determine the ball's maximum height, hang time and range?

# Dynamics

Now that we can mathematically describe the motion of any object, we have to be able describe *what* causes motion.

- Newton's three laws of motion

# Newton's First Law

**An object at rest or in uniform motion will remain at rest or in uniform motion unless acted on by an external force.**

- Uniform motion: constant velocity
- e.g. spacecraft in “deep space”
- e.g. hockey puck sliding on very smooth ice

## Newton's Second Law

**The sum of the forces acting on an object is proportional to its mass and its acceleration.**

$$\mathbf{F}_{\text{net}} = \Sigma \mathbf{F} = m \mathbf{a}$$

Quantity	Symbol	SI Unit
Net force (sum of all forces)	$\mathbf{F}_{\text{net}}$	N
Mass	$m$	kg
Acceleration	$\mathbf{a}$	m/s <sup>2</sup>

(Actually, we will find out later that this isn't exactly what Newton said. But it requires us to learn another concept first, which we will do in the next unit!)



# Newton's Third Law

**For every action there is an equal and opposite reaction.**

For every action force on an object (B) due to another object (A), there is a reaction force which is equal in magnitude but opposite in direction, on object (A), due to object (B):

$$\mathbf{F}_{A \text{ on } B} = -\mathbf{F}_{B \text{ on } A}$$

**The reaction forces act on different objects!**

# Example Problem

## A Blast From the Past

**Example 7:** In old-style television picture tubes and computer monitors (cathode ray tubes), light is produced when fast-moving electrons collide with phosphor molecules on the surface of the screen. The electrons (mass  $m = 9.1 \times 10^{-31}$  kg) are accelerated from rest in the electron “gun” at the back of the vacuum tube. Find the velocity of an electron when it exits the gun after experiencing an electric force of  $5.8 \times 10^{-15}$  N over a distance of 3.5 mm.

# Forces

- **Force** is the interaction between the objects.
  - When there is interaction, then forces are created
  - A “push” or a “pull”
- Newton considered all forces acting at a single point of an object called the centre of gravity (“CG”)
  - The centre of gravity is also called the centre of mass (“CM”)
  - If the density of an object is constant, then the CG is also the geometric centre (centroid) of the object

# Forces

- There are two types of forces:
  - **Contact forces** act between two objects that are in contact with one another
  - **Non-contact forces** act between two objects without them touching each other.  
They are also called “action-at-a-distance” force
- If the net force on an object is zero ( $\Sigma \mathbf{F} = \mathbf{0}$ ) then the object is in a *state of equilibrium*
  - Dynamic equilibrium: the object is moving relative to us
  - Static equilibrium: the object is not moving relative to us

# Common Forces

Common everyday forces that we encounter in Physics 12 include:

- Gravitational force (weight)  $\mathbf{F}_g$
- Normal force  $\mathbf{F}_N$
- Friction (static  $\mathbf{F}_s$  and kinetic  $\mathbf{F}_k$ )
- Tension  $\mathbf{F}_T$
- Applied force  $\mathbf{F}_a$
- Air resistance (drag)  $\mathbf{D}$
- Electrostatic force  $\mathbf{F}_q$  (discussed in Unit 4)
- Magnetic force  $\mathbf{F}_M$  (discussed in Unit 4)

# Gravity

- The force of attraction between all objects with mass
- Near the surface of Earth:

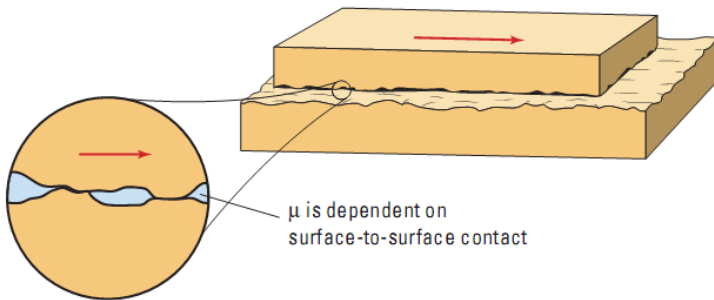
$$\boxed{\mathbf{F}_g = m\mathbf{g}} \quad \text{where} \quad \mathbf{g} = 9.81 \text{ m/s}^2 \text{ [down]}$$

- $\mathbf{F}_g$  always points **down**
- Newton's law of universal gravity:

$$\boxed{F_g = \frac{Gm_1m_2}{r^2}} \quad \text{where} \quad G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2$$

# Friction

- A force that opposes the sliding of two surface across one another
- Always act in a the direction opposite to motion or attempted motion
- Two types: *static friction* and *kinetic friction*



# Static Friction

- The friction between the two surfaces when there is no relative motion between them
- Static friction increases with increasing applied force  $F_a$ . It is at maximum when the object is just about to move.

$$\max F_s = \mu_s F_N$$

- $\mu_s$  is the static friction coefficient (does not have a unit)
- $F_N$  is the normal force
- I didn't use vector notation. This equation deals with the magnitude only

**(Pro tip:** The symbol for the coefficient of friction  $\mu$  is the Greek letter *mu*)



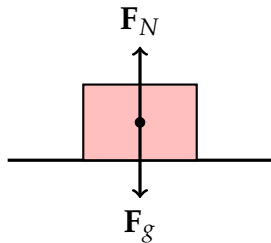
# Kinetic Friction

$$F_k = \mu_k F_N$$

- The friction between two surfaces moving relative to each other
- $F_k$  is constant along the path of movement as long as the normal force stays constant
- $\mu_k$  is the kinetic friction coefficient (does not have a unit). It is always lower than the static coefficient, otherwise nothing will ever move

$$\mu_k \leq \mu_s$$

# Normal Force

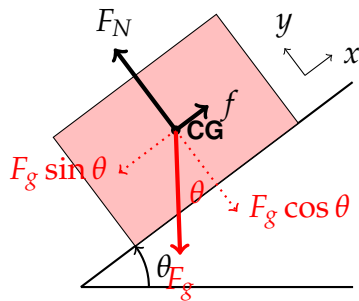


$$\mathbf{F}_g = m\mathbf{g} = -\mathbf{F}_N$$

- A force a surface exerts on another object that it is in contact with
- Always **perpendicular** to the contact surface
- **Special case:** When an object is on a horizontal surface with no additional applied force, the magnitude of the normal force is equal to the magnitude of the weight of the object, i.e.  $F_N = F_g$

## Normal Forces on a Slope

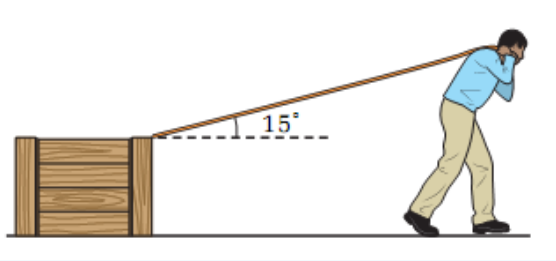
For this case, we label the  $x$ -axis to be along the slope, and  $y$ -axis to be perpendicular to the slope.



- If on a slope:  $F_N = F_g \cos \theta$ 
  - $F_N$  decreases as ramp angle  $\theta$  increases
  - Obviously, at  $\theta = 90^\circ$ ,  $F_N = 0$ !
- $F_g$  has a component along the ramp  $F_g \sin \theta$  that wants to slide the block down.
- Friction force  $f$  opposes the motion
  - Be careful: if the block is moving *up* the ramp with an applied force, then  $f$  will point *down* the ramp

## Example Problem

**Example 8:** To move a 45 kg wooden crate across a wooden floor ( $\mu = 0.20$ ), you tie a rope onto the crate and pull on the rope. While you are pulling the rope with a force of 115 N, it makes an angle of  $15^\circ$  with the horizontal. How much time elapses between the time at which the crate just starts to move and the time at which you are pulling it with a velocity of 1.4 m/s?



## Example Problem

**Example 9:** You are holding an 85 kg trunk at the top of a ramp that slopes from a moving van to the ground, making an angle of  $35^\circ$  with the ground. You lose your grip and the trunk begins to slide.

- If the coefficient of friction between the trunk and the ramp is 0.42, what is the acceleration of the trunk?
- If the trunk slides 1.3 m before reaching the bottom of the ramp, for what time interval did it slide?

## Example: Vertical Motion

**Example 10:** A 55 kg person is standing on a scale in an elevator. If the scale is calibrated in *newtons*, what is the reading on the scale when the elevator is not moving? If the elevator begins to accelerate upward at  $0.75 \text{ m/s}^2$ , what will be the reading on the scale?

# Tension in a Cable

**Tension:** The magnitude of the force exerted on and by a cable, rope, or string. How do engineers determine the amount of tension needed for a specific object (bridges, floors or light fixtures)?

- You can't push on a rope
- Assume the cable/rope/string to be mass less
- Force can change direction when used with pulleys

## Example Problem

**Example 11:** An elevator filled with people has a total mass of 2245 kg. As the elevator begins to rise, the acceleration is  $0.55 \text{ m/s}^2$ . What is the tension in the cable that is lifting the elevator?



# Applying Newton's Third Law on Connected Bodies



- Usually the objects are connected by a cable or a solid linkage with negligible mass
- All object have the same acceleration
- Require multiple free-body diagrams

# Solving Connected-Bodies Problems

To solve a connected-bodies problem, you can follow these procedures:

1. Draw a FBD on each of the objects
2. Sum all the forces on all the objects along the direction of motion
  - Direction of motion are usually very obvious
  - All the tension forces should cancel, because they are “internal” forces and not “external forces”
3. Compute the acceleration of the entire system using Newton's second law
  - Remember that every object has the same acceleration!
4. Go back to the FBD of each of the objects and compute the unknown forces (usually tension)

## Connected Bodies: Example

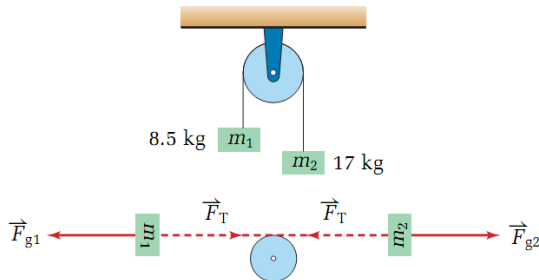
**Example 12:** A tractor-trailer pulling two trailers starts from rest and accelerates to a speed of 16.2 km/h in 15 s on a straight, level section of highway. The mass of the truck itself (T) is 5450 kg, the mass of the first trailer (A) is 31 500 kg, and the mass of the second trailer (B) is 19 600 kg.

- What magnitude of force must the truck generate in order to accelerate the entire vehicle?
- What magnitude of force must each of the trailer hitches withstand while the vehicle is accelerating?

For this problem we will assume that frictional forces are negligible in comparison with the forces needed to accelerate the large masses.

## Example Problem: Atwood Machine

An **Atwood machine** is made of two objects connected by a rope that runs over a pulley. The pulley allows the direction of force and direction of motion to change between two objects.

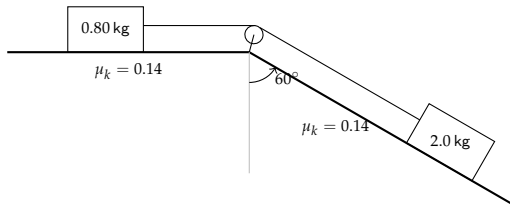


**Example 13:** The object on the left ( $m_1$ ) has a mass of 8.5 kg and the object on the right ( $m_2$ ) has a mass of 17 kg.

- What is the acceleration of the masses?
- What is the tension in the rope?

## A More Typical Problem

**Example 14:** More typically, an Atwood machine problem is one where two objects are sliding on a surface. These surfaces may have (or may not) have friction. In this example, two blocks are connected by a mass-less string over a friction-less pulley as shown in the diagram.



- (a) Determine the acceleration of the blocks.
- (b) Calculate the tension in the string.
- (c) If the string broke, for what minimum value of the coefficient of static friction would the  $2.0 \text{ kg}$  block not begin to slide?

# Uniform Circular Motion

- Constant speed (magnitude of velocity)
- Changing direction (direction of velocity)
- Changing velocity, i.e. always accelerating
- Examples:
  - Roller coaster rides
  - Motors
  - Swinging a key chain

# Centripetal Acceleration

- aka “centre-seeking acceleration”. It’s called that because in Latin, “centripetal” literally means “looking for the centre”
- Direction of the acceleration is always pointing towards the centre

$$a_c = \frac{v^2}{r}$$

Quantity	Symbol	SI Unit
Centripetal acceleration	$a_c$	m/s <sup>2</sup>
Speed (magnitude of velocity)	$v$	m/s
Radius (of the circular path)	$r$	m

Let’s do a “unit analysis” to make sure that we’re correct. . .

# Centripetal Force

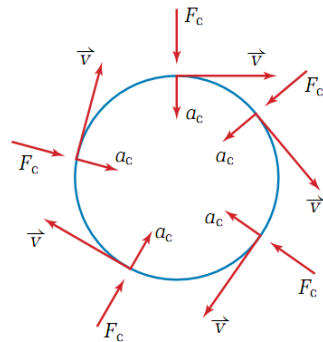
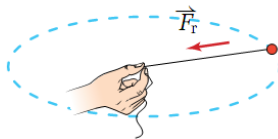
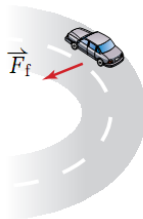
- If there is an acceleration, there must be a force as well!
- With centripetal acceleration, there is a **centripetal force**!

$$F_c = \frac{mv^2}{r}$$

Quantity	Symbol	SI Unit
Centripetal force	$F_c$	N
Mass	$m$	kg
Speed (magnitude of velocity)	$v$	m/s
Radius (of the circular path)	$r$	m



# Centripetal Force

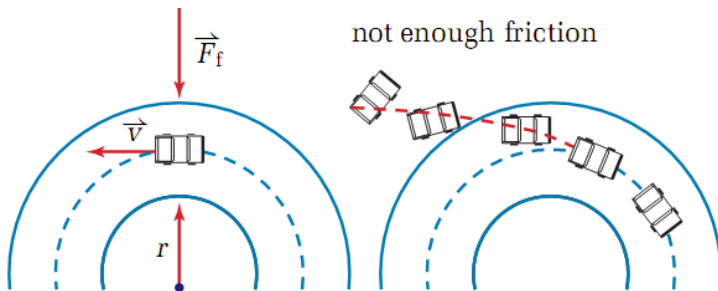


# Centripetal Force

- Supplied by various forces
  - Gravity
  - Friction
  - Tension
  - Etc.
- A bit “different” from previous example with friction or gravity
  - Necessary for an object to move in a circular path.
  - $\mathbf{a}$  is *a*lways acting perpendicular to  $\mathbf{v}$

## Example Problem

**Example 15:** A car with a mass of  $m = 2135$  kg is rounding a curve on a level road. If the radius of curvature of the road is  $r = 52$  m and the coefficient of friction between the tires and the road is  $\mu = 0.70$ , what is the maximum speed at which the car can make the curve without skidding off the road?



## Example Problem

**Example 16:** You are playing with a yo-yo with a mass of 225 g. The full length of the string is 1.2 m. You decide to see how slowly you can swing it in a vertical circle while keeping the string fully extended, even when the yo-yo is at the top of its swing.

- Calculate the minimum speed at which you can swing the yo-yo while keeping it on a circular path.
- At the speed that you determined, find the tension in the string when the yo-yo is at the side and at the bottom of its swing.

## Centrifugal Force??

- Centrifugal force is a **fictitious forces** i.e. NOT REAL!
- Caused by the inertia of the object (e.g. person) and the changing direction of the enclosure (e.g. a car)

**AND WE SHALL NEVER  
TALK ABOUT IT AGAIN**

## Equations to Describe Rotational Motion

**Period**  $T$  is the time it takes to do one complete revolution (distance  $2\pi r$  at speed  $v$ ), and **frequency**  $f$  shows how many revolutions per second. They are reciprocals of one another:

$$T = \frac{2\pi r}{v} = \frac{1}{f} \quad f = \frac{1}{T} = \frac{v}{2\pi r}$$

We can write centripetal acceleration in terms of frequency or period:

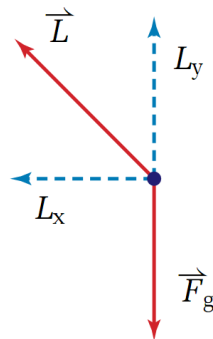
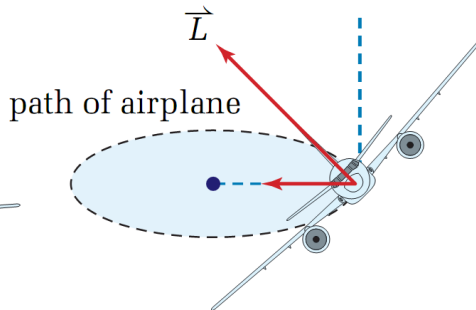
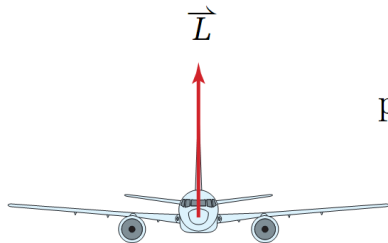
$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 r f^2$$

and centripetal force is just mass times acceleration:

$$F_c = ma_c = 4\pi^2 mr f^2$$

# How does an airplane turn in the sky?

Centripetal forces are required!



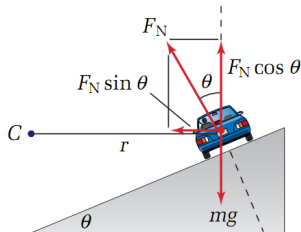
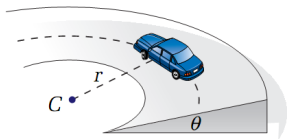
## How does an airplane turn in the sky?

The “aileron” is deflected, so that one wing generates a bit more lift than the other, and this rotates (“rolls”) the aircraft to the left or right.





# Banked Curves on Highways and Racetracks



$y$ -direction (vertical)

$$F_N \cos \theta = F_g = mg$$

$x$ -direction (towards C)

$$F_N \sin \theta = F_c = \frac{mv^2}{r}$$

Combine the two equations together:

$$\frac{F_N \sin \theta}{F_N \cos \theta} = \frac{\frac{mv^2}{r}}{mg}$$

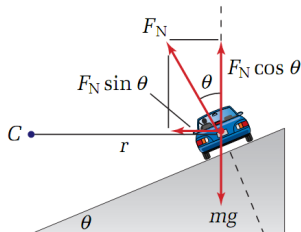
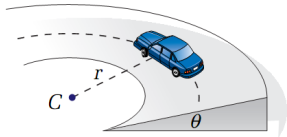
Cancel out  $F_N$  and  $m$  terms:

$$\frac{\sin \theta}{\cos \theta} = \frac{v^2}{rg} \longrightarrow \boxed{\tan \theta = \frac{v^2}{rg}}$$

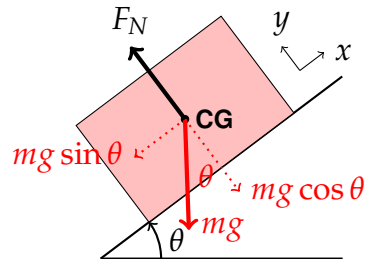
# Banked Curves on Highways and Racetracks

Whoa! Wait a minute! That FBD doesn't look right!

Circular motion:



Mass Sliding on a Slope:



# Banked Curves on Highways and Racetracks

Whoa! Wait a minute! That FBD doesn't look right!

Normal forces are different in both cases because the nature of the motion is different. Block sliding on a ramp is a 2D problem, but the race car around the race track is a pseudo-3D problem. **You'll just have to remember when to use which one.**

## Example Problem

**Example 17:** Canadian Indy racing car driver Paul Tracy set the speed record for time trials at the Michigan International Speedway (MIS) in 2000. Tracy averaged 378.11 km/h in qualifying. The ends of the 3.0 km oval track at MIS are banked at  $18.0^\circ$  and the radius of curvature is 382 m.

- At what speed can the cars round the curves without needing to rely on friction to provide a centripetal force?
- Did Tracy rely on friction for some of his required centripetal force?

# Making Sense of The Last Example Problem

In Real Life:

- $18^\circ$  banked curve is very high!
- Without relying on friction, max speed is only 126 km/h

Normal Highway:

- Banked a few degree max
- Speed Limit on on/off ramps are lower than the posted 100 km/h limit
- Large radius
- Ramp is different
- Not designed to drive as fast as the raceway

## One Last Example

**Example 18:** A car exits a highway on a ramp that is banked at  $15^\circ$  to the horizontal. The exit ramp has a radius of curvature of 65 m. If the conditions are extremely icy and the driver cannot depend on any friction to help make the turn, at what speed should the driver travel so that the car will not skid off the ramp?