

4. Gravitational, Electric and Magnetic Fields

Grade 12 Physics

Olympiads School

Summer 2018

Files for You to Download

Now downloadable from the school website:

- 4-forceFields_print.pdf—The print version of the slides for this unit. I recommend printing 4 slides per page
- 4-Homework.pdf—This unit's homework assignment

Please download/print the PDF file before each class. There is no point copying notes that are already printed out for you. Instead, take notes on things I say that aren't necessarily on the slides.

Where Are We In the Course

1. Fundamentals of Dynamics
2. Momentum, Impulse and Energy
3. Motion in Space
4. Gravitational, Electric and Magnetic Fields
5. Wave Nature of Light
6. Theory of Special Relativity
7. Introduction to Quantum Mechanics

Force Fields

When we think of a “force field”, many of us start thinking about fantastic crazy technologies in science fiction movies



What We Are REALLY Talking About

We're really dealing with is actually:

- Gravitational force (\mathbf{F}_g)
- Electrical force (\mathbf{F}_q)
- Magnetic force (\mathbf{F}_m)

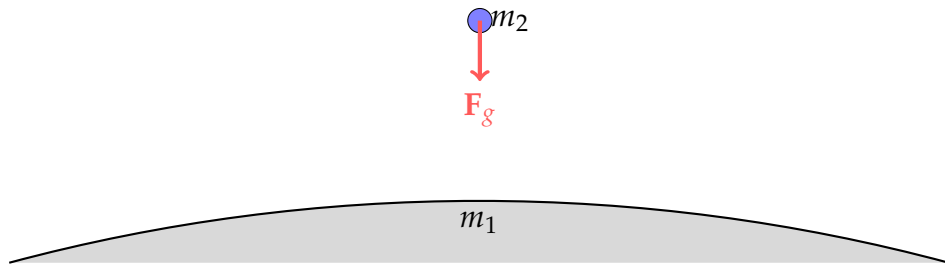
(Sorry if you thought otherwise)

Let's Start with Gravitational Force

We've seen it many times already

- In Units 1 and 3, we studied *how* we calculated the value of g (acceleration due to gravity) on Earth's surface
- In Physics 11, we studied how g changes at various points on/near Earth
- So g isn't really a constant. But if it's not a constant, then what is it?

Newton's Law of Universal Gravitation



$$F_g = \frac{Gm_1m_2}{r^2}$$

where $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$ is the universal gravitation constant

Universal Gravitation

- If m_1 exerts a force \mathbf{F}_g on m_2 , then m_2 also exerts a force $-\mathbf{F}_g$ on m_1 .
 - These two forces are equal in magnitude and opposite in direction (Newton's 3rd law)
- Assumption: m_1 and m_2 are *point masses* that do not occupy any space
- For the universal gravitation equation to work, the two objects haven't collided into one another, i.e.:

$$r > (r_1 + r_2)$$

Think Gravitational Field: What is g ?

We can describe the force of gravity as

$$\mathbf{F}_g = m\mathbf{g}$$

To find g , we group the variables in Newton's universal gravitation equation:

$$F_g = \underbrace{\left[\frac{Gm_1}{r^2} \right]}_{=g} m_2 = m_2 g$$

On/near Earth's surface, we use $m_1 = m_{\text{Earth}}$ and $r = r_{\text{Earth}}$ to compute $g = 9.81 \text{ m/s}^2$, or $g = 9.81 \text{ N/kg}$ (both units are equivalent)

Gravitational Field

A closer look at g shows that it's a function of a source mass m_s and the distance r from it. This is called the **gravitational field**.

$$g(m_s, r) = \frac{Gm_s}{r^2}$$

It's a mapping of how source mass m_s influences the gravitational forces on other masses

Quantity	Symbol	SI Unit
Gravitational field intensity	g	N/kg
Universal gravitational constant	G	$\text{N m}^2/\text{kg}^2$
Mass of source (a point mass)	m_s	kg
Distance from centre of source	r	m

Relating Gravitational Field & Gravitational Force

\mathbf{g} itself doesn't do anything until there is another mass m . At which point, m experiences a gravitational force related to \mathbf{g} by:

$$\mathbf{g} = \frac{\mathbf{F}_g}{m}$$

\mathbf{F}_g and \mathbf{g} are vectors in the same direction: towards the centre of the source mass that created the field, therefore all vector operations apply

Quantity	Symbol	SI Unit
Gravitational field	\mathbf{g}	N/kg
Gravitational force on a mass	\mathbf{F}_g	N
Mass inside the gravitational field	m	kg

Gravitational Potential Energy

In the last unit, we used a more general equation for **gravitational potential energy**:

$$U_g = -\frac{Gm_1m_2}{r}$$

- Obtained by integrating \mathbf{F}_g by a distance r to find the work done
- U_g is the work required to move two objects from r to ∞
- $U_g = 0$ at $r = \infty$ and *decrease* as r decreases

Relating Gravitational Potential Energy to Force

It Helps to Know Calculus

If you know vector calculus, you can easily see that gravitational force (\mathbf{F}_g) gravitational potential energy (U_g) are related by the gradient operator:

$$\mathbf{F}_g = -\nabla U_g = -\frac{dU_g}{dr}\hat{\mathbf{r}}$$

Since none of you know vector calculus, we use a simpler explanation: \mathbf{F}_g **always points from high to low potential**

- A falling object is always decreasing in U_g
- “Steepest descent”: the direction of \mathbf{F} is the shortest path to decrease U_g
- Objects travelling perpendicular to \mathbf{F} has constant U_g

Relating U_g , F_g and g

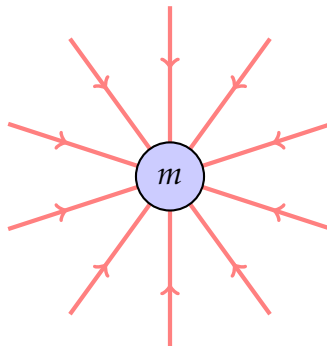
Knowing that F_g and g only differ by a constant, we can also relate gravitational field to U_g by the gradient operator:

$$\mathbf{g} = \frac{\mathbf{F}_g}{m} = -\nabla \left(\frac{U_g}{m} \right) = -\frac{d}{dr} \left(\frac{U_g}{m} \right) \hat{\mathbf{r}}$$

We already know that the direction of g is the same as F_g , i.e.

- The direction of g is the shortest path to decrease U_g
- Objects travelling perpendicular to g has constant U_g

Gravitational Field Lines



- The direction of \mathbf{g} is towards the centre of the object that created it
- Field lines do not tell the intensity (i.e. magnitude) of \mathbf{g} , only the direction

Gravitational field is pretty straightforward.
Actually, the most interesting stuff that we
can do is actually with electric fields.

The Charges Are

Let's Review Some Basics

We already know quite a bit about charge particles:

- A **proton** carries a **positive** charge
- An **electron** carries a **negative** charge
- A *net charge* of an object means an excess of protons or electrons
- From Physics 11: electric force is one of the fundamental forces in the universe

We will start with electrostatics:

- Charges that are not moving relative to one another

Coulomb's Law for Electrostatic Force

The magnitude of the **electrostatic force** between two point charges is given by:

$$F_q = \frac{k |q_1 q_2|}{r^2}$$

Quantity	Symbol	SI Unit
Electrostatic force	F_q	N (newtons)
Coulomb's constant (electrostatic constant)	k	$\text{N m}^2 / \text{C}^2$
Point charges 1 and 2 (occupies no space)	q_1, q_2	C (coulombs)
Distance between point charges	r	m (metres)

- **Coulomb's Constant:** $k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N m}^2 / \text{C}^2$
- The equation has a very similar form compared to gravitational force!

Comparing F_g to F_q

Electric Force (point charge):

$$F_q = \frac{kq_1q_2}{r^2}$$

Gravitational Force (point mass):

$$F_g = \frac{Gm_1m_2}{r^2}$$

- Similarities:
 - Both inversely proportional to r^2
 - Both are scaled by a constant
- Difference:
 - For gravity only positive mass
 - Electric charge can be either positive or negative

Think Electric Field

We can get **electric field** by repeating the same procedure as with gravitational field. Again, let's group the variables in Coulomb's equation:

$$F_q = \underbrace{\left[\frac{kq_1}{r^2} \right]}_E q_2$$

We can say that charge q creates an “electric field” (E) with an intensity

$$E(q, r) = \frac{kq}{r^2}$$

Similar to gravitational field, electric field \mathbf{E} created by q is a function (“vector field”) that shows how it influences other charged particles around it

Electric Field Intensity Near a Point Charge

The electric field intensity from a point charge is proportional to the charge q and inversely proportional for the square of the distance r from the charge. **The direction of the field is radially outward from a positive point charge and radially inward towards a negative charge.**

$$E = \frac{kq_s}{r^2}$$

Quantity	Symbol	SI Unit
Electric field intensity	E	N/C (newtons per coulomb)
Coulomb's constant	k	$\text{N m}^2/\text{C}^2$
Source charge	q_s	C (coulombs)
Distance from source charge	r	m (metres)

Think Electric Field

E doesn't do anything until another charge interacts with it. And when there is a charge q , the electric force \mathbf{F}_q that it experiences in the presence of \mathbf{E} is:

$$\boxed{\mathbf{F}_q = q\mathbf{E}}$$

\mathbf{F}_q and \mathbf{E} are vectors, and following the principle of superposition, i.e.

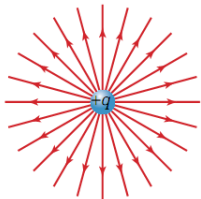
$$\mathbf{F}_q = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 \dots$$

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \mathbf{E}_4 \dots$$

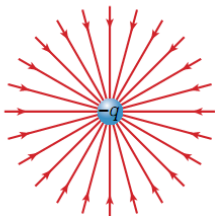
Field Lines

Electric Field lines: if you place a positive charge in an electric field, the force on the charge will be in the direction of the electric field.

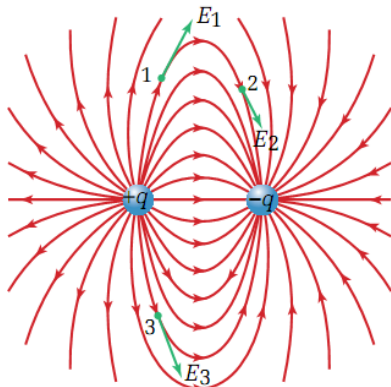
A



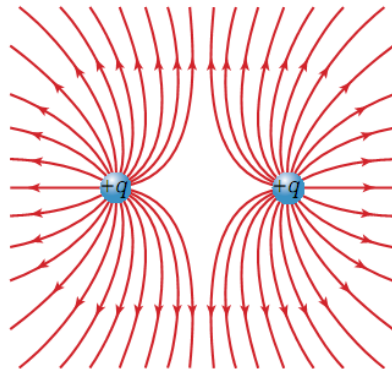
B



C



D

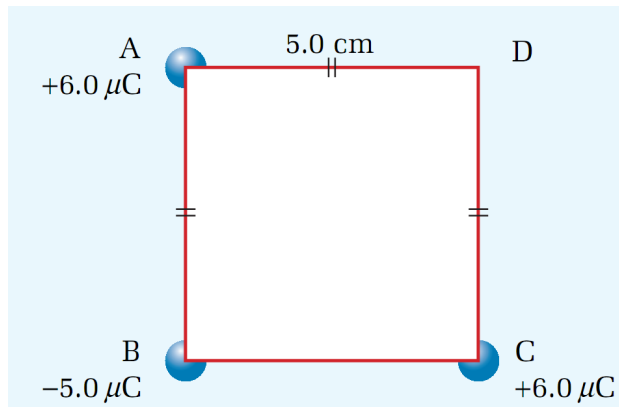


Example Problem

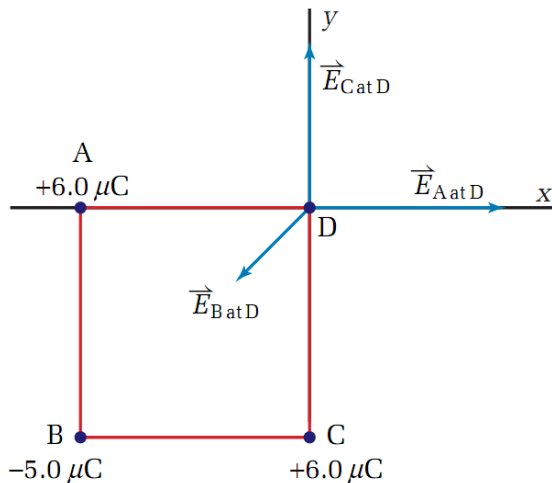
Example 2: What is the electric field intensity at a point 30.0 cm from the centre of a small sphere that has a positive charge of $2.0 \times 10^{-6} \text{ C}$? ($k = 8.99 \times 10^9 \text{ N m}^2 / \text{C}^2$)

Example Problem

Example 3: Three charges, A($6.0\ \mu\text{C}$), B($-5.0\ \mu\text{C}$), and C($6.0\ \mu\text{C}$), are located at the corners of a square with sides that are $5.0\ \text{cm}$ long. What is the electric field intensity at point D?



Example Problem (Cont.)



Electric Potential Energy

When bring two charges together, I have to do work against electrostatic force, therefore, there is a gain/lost in **electric potential energy**

$$U_q = \frac{kq_1q_2}{r}$$

Quantity	Symbol	SI Unit
Electric potential energy	U_q	J (joules)
Coulomb's constant	k	$\text{N m}^2 / \text{C}^2$
Electric charges	q_1, q_2	C (coulombs)
Distance	r	m (metres)

U_q can be positive or negative, because charges can be positive or negative

How it Differs from Gravitational Potential

Two positive charges:

$$U_q > 0$$

Two negative charges:

$$U_q > 0$$

One positive and one negative charge:

$$U_q < 0$$

- $U_q > 0$ means that I have to do work to bring two charges together (true if both charges are positive, or both are negative)
- $U_q < 0$ means that I have to do work to pull two charges apart (true if the charges are opposite)
- For gravitational potential U_g is always < 0

Electric Potential

Start with an Analogy

When I lift an object of mass m from one height to another, the work (change in gravitational potential energy) that I do is proportional to m , i.e. there is a “constant” in that scales with *any* mass, as long as they move between those same two points:

$$W = Km$$

In the simple case (small changes in height, no change in g), this constant is just

$$K = \frac{W}{m} = g\Delta h$$

Electric Potential

This is also true for moving a charged particle against an electric force, and the constant is called the **electric potential**. For a point charge, it is defined as

$$V = \frac{U_q}{q} = \frac{kq}{r}$$

The unit for electric potential is a *volt* which is *one joule per coulomb*:

$$1 \text{ V} = 1 \text{ J/C}$$

Potential Difference (Voltage)

The change in electric potential is called the **potential difference** or **voltage**:

$$\Delta V = \frac{\Delta U_q}{q}$$

Here, we can relate ΔV to an equation that we knew from Physics 11, which related to the energy dissipated in a resistor in a circuit ΔU_q to the voltage drop ΔV :

$$\Delta U = q\Delta V$$

Electric potential difference also has the unit *volts* (V)

Getting Those Names Right

Remember that these three quantities are all scalars, as opposed to electric force \mathbf{F}_q and electric field \mathbf{E} which are vectors

- Electric potential energy:

$$U = \frac{kq_1q_2}{r}$$

- Electric potential:

$$V = \frac{kq}{r}$$

- Electric potential difference (voltage):

$$\Delta V = \frac{\Delta U_q}{q}$$

Relating U_q , F_q and E

Our Integrals In Reverse

Using vector calculus, we can relate electric force (F_q) to electric potential energy (U_q), and electric field (E) to the electric potential (V):

$$\mathbf{F}_q = -\nabla U_q = -\frac{dU_q}{dr}\hat{\mathbf{r}} \quad \mathbf{E} = -\nabla V = -\frac{dV}{dr}\hat{\mathbf{r}}$$

But since you haven't learned vector calculus, the simpler explanation:

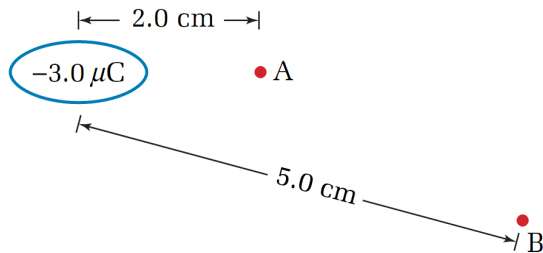
- Electric force F_q always points from high potential to low potential energy
- Electric field can also be expressed as the change of electric potential per unit distance, which has the unit

$$1 \text{ N/C} = 1 \text{ V/m}$$

- Electric field is also called “potential gradient”

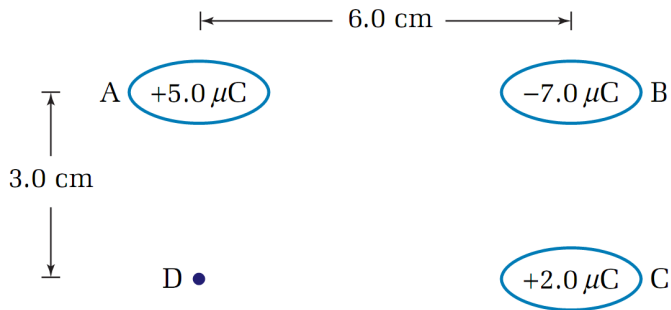
Example Problem

Example 4: A small sphere with a charge of $-3.0\ \mu\text{C}$ creates an electric field. Calculate the electric potential difference at point A, located 2.0 cm from the source charge, and at point B, located 5.0 cm from the same source charge. Which point is at higher potential?



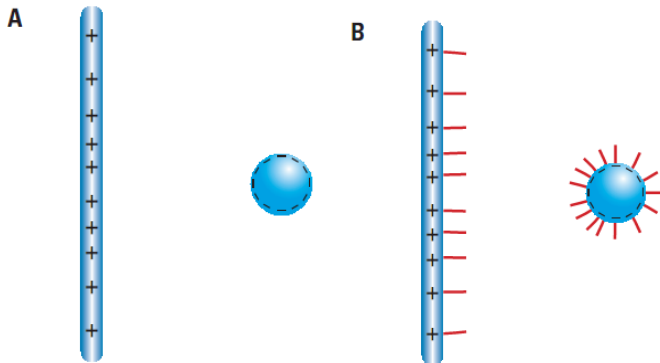
Example Problem

Example 5: The diagram shows three charges, A($5.0\ \mu\text{C}$), B($-7.0\ \mu\text{C}$), and C($2.0\ \mu\text{C}$), placed at three corners of a rectangle. Point D is the fourth corner. What is the electric potential at point D?

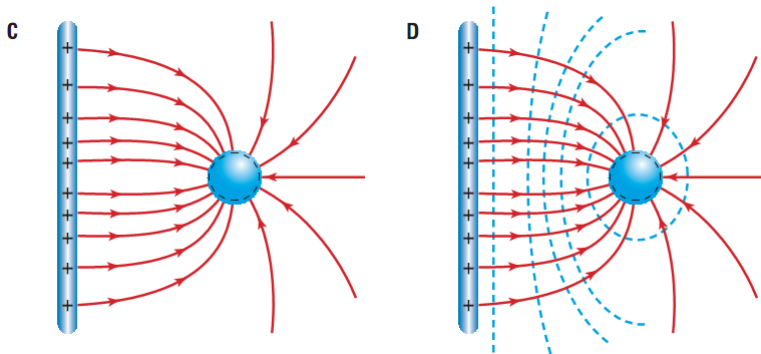


Field Structure

How should we draw the field lines?

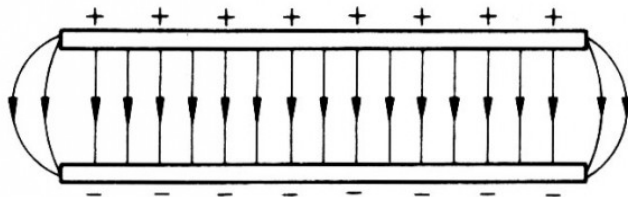


Field Structure



The dotted blue lines are called “equipotential lines”. They’re always *perpendicular* to the electric field lines. Charges moving in the direction of the equipotential lines do not lose potential energy

Electric Field between Two Parallel Plates



- E is uniform at all points between the parallel plates, independent of position
- E is proportional to the charge density (charge per unit area) on the plates:

$$E \propto \sigma \quad \text{where} \quad \sigma = \frac{q}{A}$$

- E outside the plates is very low (close to zero), except for the fringe effects at the edges of the plates.

Example Problem

Example 6: An identical pair of metal plates is mounted parallel on insulating stands 20 cm apart and equal amounts of opposite charges are placed on the plates. The electric field intensity at the midpoint between the plates is 400 N/C.

1. What is the electric field intensity at a point 5.0 cm from the positive plate?
2. If the same amount of charge was placed on plates that have twice the area and are 20 cm apart, what would be the electric field intensity at the point 5.0 cm from the positive plate?
3. What would be the electric field intensity of the original plates if the distance of separation of the plates was doubled?

Electric Field and Potential Difference

- A few slides ago we introduced (with some calculus that you haven't learned) the relationship between electric field (\mathbf{E}) and electric potential difference (V):

$$\mathbf{E} = -\frac{dV}{dr}$$

But calculus is way too complicated for Physics 12!

- Fortunately in a uniform electric field (e.g. parallel plate) it simplifies to a very simple equation:

$$E = \frac{\Delta V}{d}$$

Electric Field and Potential Difference

The intensity of the (uniform) electric field between two parallel plate is directly proportional to the electric potential difference V , and inversely proportional to the distance between the two plates:

$$E = \frac{\Delta V}{d}$$

Quantity	Symbol	SI Unit
Electric field intensity	E	N/C (newtons per coulomb)
Electric potential difference	ΔV	V (volts)
Distance	d	m (metres)

Example Problem

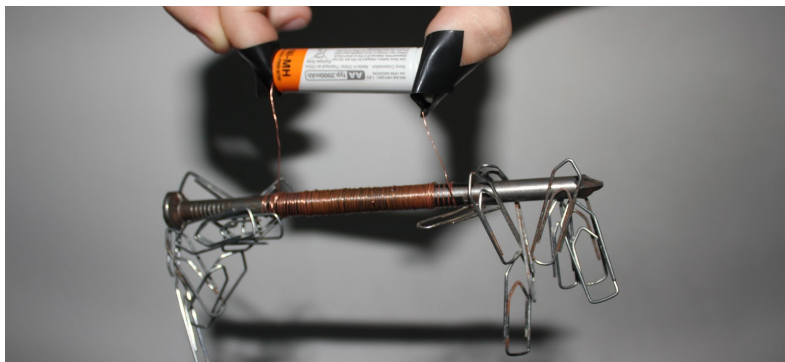
Example 7: Two parallel plates 5.0 cm apart are oppositely charged. The electric potential difference across the plates is 80.0 V.

- What is the electric field intensity between the plates?
- What is the potential difference at point A?
- What is the potential difference at point B?
- What is the potential difference between points A and B?
- What force would be experienced by a small $2.0\ \mu\text{C}$ charge placed at point A?

Magnetic Field

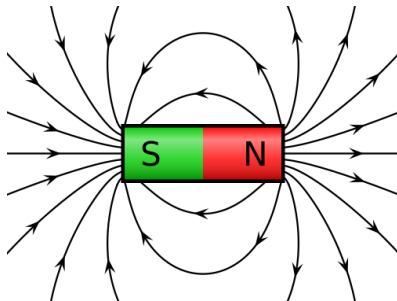
Let's Review Some Basics

- A magnetic field is created by charges that are moving
 - a single charge, or
 - a current along a wire
- Example: building electromagnets using a battery, copper wires and a nail



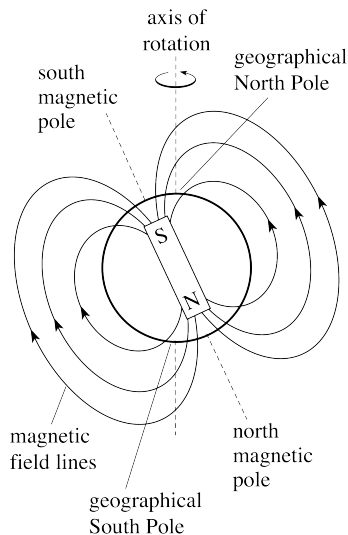
We're Also Familiar with Permanent Magnets

- Generally made of iron, nickel, cobalt, some alloys of rare-earth materials, some minerals (e.g. lodestone)
- Atoms in these materials can be organized such that the electrons are always creating a small current inside
- We're told that the magnetic field runs from “north” to “south” pole, like the diagram shown below. If this is the case, someone lied to you. . .



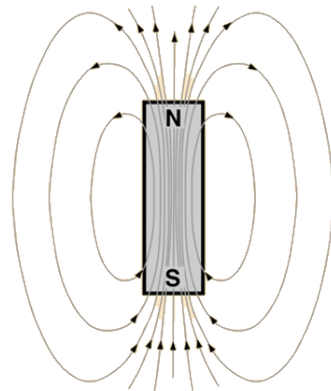
A Gigantic “Permanent” Magnet

- Earth is also a “permanent” magnet
 - Magnetic field generated by electric currents in the conductive material in its core
 - Current created by convection currents due to heat escaping from the core
- Magnetic field lines run from south to north
- By our convention, our South Pole is actually the “magnetic north pole”, and our North Pole is the “magnetic south pole”



What About Permanent Magnet?

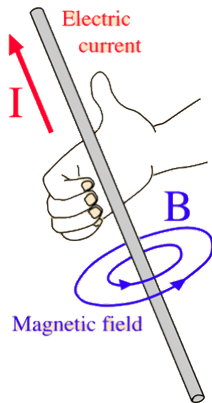
- Magnetic fields don't actually run from a "North" pole to a "South" pole
- In fact, they run in a loop (see right)
- The magnetic field lines continues inside the bar magnet



Magnetic Field Generated By a Wire

In fact, the magnetic field generated by a current also runs in a loop, given by the equation:

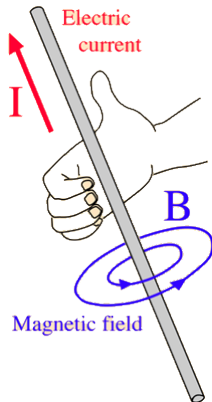
$$B = \frac{\mu_0 I}{2\pi r}$$



Quantity	Symbol	SI Unit
Magnetic field intensity	B	T (teslas)
Current	I	A (amperes)
Radial distance from the wire	r	m (metres)

The constant $\mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$ is the “permeability of free space”

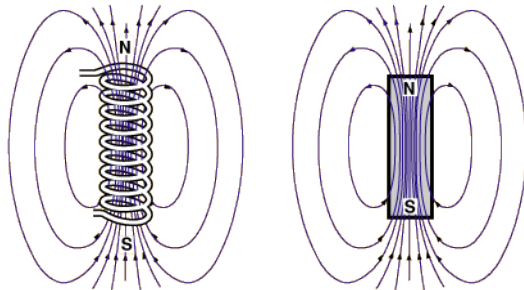
Magnetic Field Generated By a Wire



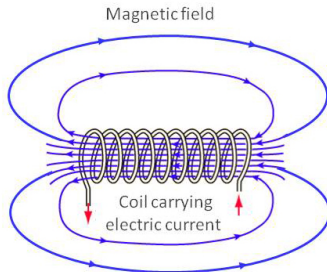
- Both electric current (I) and magnetic field (B) are vectors
- Direction of I based on movement of positive charges
 - In an *actual* wire, negative charges (electrons) are moving in the opposite direction
- Direction of B is determined using **right hand rule**

Wounding Wires Into a Coil

- A **solenoid** is when you wound a wire into a coil
- You create a magnet very similar to a bar magnet, with an effective north pole and a south pole
- Magnetic field inside the solenoid is uniform
- Magnetic field strength can be increased by the addition of an iron core



Magnetic Field Inside a Solenoid



- The magnetic field **inside** the solenoid given by:

$$B = \mu n I$$

- Direction of **B** determined by **right hand rule**

Quantity	Symbol	SI Unit
Magnetic field intensity	B	T (teslas)
Number of coils	n	integer, no units
Current	I	A (amperes)
Effective permeability	μ	T m / A (tesla metres per ampere)

Tesla

The unit of magnetic field, not the car!

- The strength of a magnetic field
- A charge of one coulomb, travelling with a speed of 1 m/s perpendicular to the magnetic field ($\theta = 90^\circ$ and $\sin \theta = 1$) experiences a force of 1 N

$$\text{tesla} = \frac{\text{newton}}{\text{coulomb} \frac{\text{metre}}{\text{second}}}$$

So What Does the Magnetic Field Do?

Gravitational Field g

- Generated by any object that has a mass
- Affects everything that has mass

Electric Field E

- Generated by all charged particles
- Affects all charged particles
- The charged particle can be at rest or moving

Magnetic Field B

- Generated by **electric currents** and **moving charged particles**
- Affects electric currents and moving charged particles

Force on a Moving Charge in a Magnetic Field

When a moving charge (q) enters a magnetic field (\mathbf{B}) with a velocity \mathbf{v} , the magnetic field exerts a force (\mathbf{F}_M) on the charge:

$$F_M = qvB \sin \theta$$

Quantity	Symbol	SI Unit
Magnetic force on the moving charge	F_M	N (newtons)
Electric charge of the particle	q	C (coulombs)
Speed of the charged particle	v	m/s (metres per second)
Magnetic field strength	B	T (teslas)
Angle between particle and magnetic field	θ	

Pro tip: For those who know vectors, the magnetic force on a moving charged particle is:

$$\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$$

Convention for Diagrams



View: X



Field into page



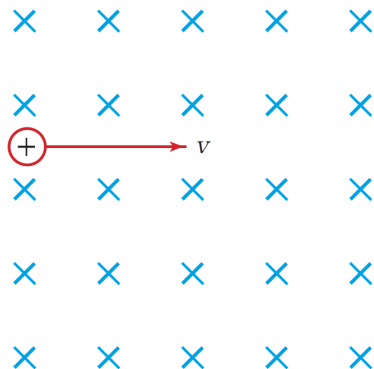
View: •



Field out of page

Example Problem

Example 8: A particle carrying a charge of $2.50\ \mu\text{C}$ enters a magnetic field travelling at $3.40 \times 10^5\ \text{m/s}$ to the right of the page. If a uniform magnetic field is pointing directly into the page and has a strength of $0.500\ \text{T}$, what is the magnitude and direction of the force acting on the charge as it just enters the magnetic field?



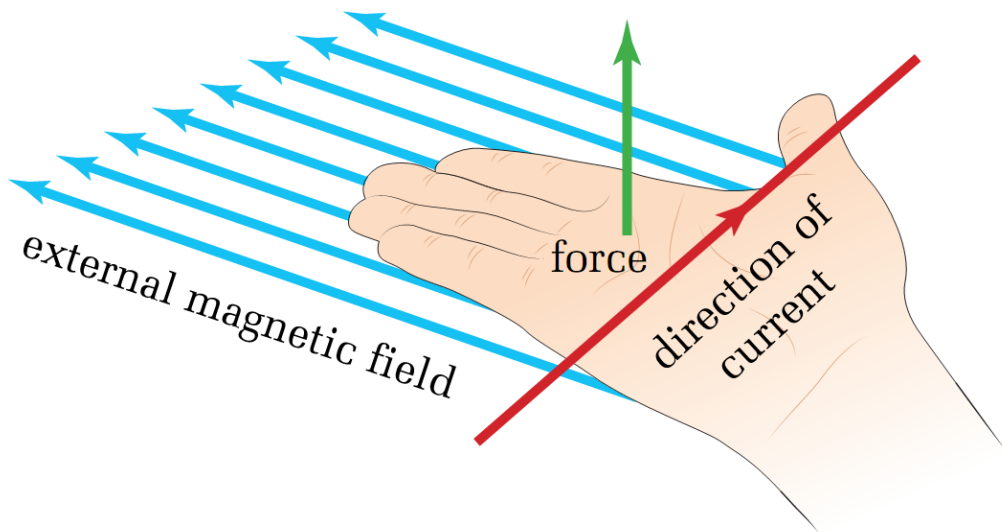
Force on a Current-Carrying Conductor in a Magnetic Field

The magnetic field also exerts a force on a conductor carrying a current.

$$F_M = IlB \sin \theta$$

Quantity	Symbol	SI Unit
Magnetic force on the conductor	F_M	N (newtons)
Electric current in the conductor	I	A (amperes)
Length of the conductor	l	m (metres)
Magnetic field strength	B	T (teslas)
Angle between conductor and magnetic field	θ	

Right Hand Rule for Induced Magnetic Force



Example Problem

Example 9: A wire segment of length 40.0 cm, carrying a current of 12.0 A, crosses a magnetic field of 0.75 T [up] at an angle of [up 40° right]. What magnetic force is exerted on the wire?

Circular Motion Caused by a Magnetic Field

When a charged particle enters a magnetic field at right angle. . .

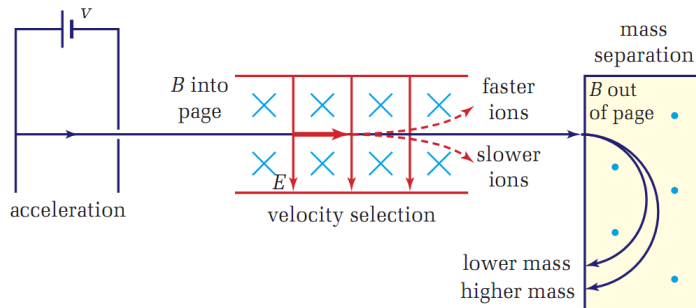
- Magnetic force \mathbf{F}_M perpendicular to both velocity \mathbf{v} and magnetic field \mathbf{B} .
- Results in circular motion

Centripetal force \mathbf{F}_c given by the magnetic force \mathbf{F}_M . We can solve for the radius r of the motion:

$$\frac{mv^2}{r} = qvB$$
$$r = \frac{mv}{qB}$$

Mass Spectrometer

- Separates particles of different mass
- Measures the mass of the particle by measuring displacement
- Three major components:
 - Particle accelerator
 - Velocity selector
 - Mass separator



Simple Particle Accelerator

- Made of:
 - A pair of parallel plates
 - Accelerating potential difference
 - Particle source
- Ionized particle source passes through the plates and get accelerated by the potential difference then get shot out of the other end of the plate.
- e.g.: particle gun or electron gun

$$\frac{1}{2}mv^2 = qV$$

Velocity Selector

- Often associates with the parallel plate particle accelerator.
- Filters the beam of particles to let particles with the same velocity to pass by only.
- Consists of a crossed (perpendicular) electric and magnetic field.
- When electric and magnetic forces are balanced, particle travels straight through.

Velocity Selector

The particle will travel straight through when magnetic force \mathbf{F}_M and electric force \mathbf{F}_q are balanced:

$$F_M = F_q$$

Substitute the expressions for F_M and F_q , then solve for v :

$$qvB = qE$$

$$v = \frac{E}{B}$$

We can adjust which particle velocity can go straight through by adjusting the relative strength of the electric field \mathbf{E} and magnetic field \mathbf{B} .

Mass Separation

Particles of different masses are separated by allowing to go into circular motion inside a magnetic field. Centripetal force F_c given by magnetic force F_M

$$F_c = F_M$$

Substitute the expressions for F_c and F_M , then solve for m :

$$\frac{mv^2}{r} = qvB$$
$$m = \frac{rqB}{v}$$

Can tell the mass of the particle by knowing its charge q , speed v , magnetic field strength B and the radius of the circular motion r .

Example Problem

Example 9: A positive ion, having a charge of 3.20×10^{-19} C, enters at the extreme left of the parallel plate assembly associated with the velocity selector and mass spectrometer shown previously.

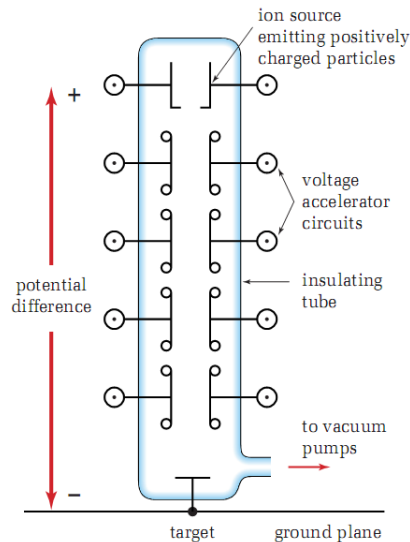
- If the potential difference across the simple accelerator is 1.20×10^3 V, what is the kinetic energy of the particle as it leaves through the hole in the right plate?

Example Problem (cont.)

- The parallel plates of the velocity selector are separated by 12.0 mm and have an electric potential difference across them of 360.0 V. If a magnetic field of strength 0.100 T is applied at right angles to the electric field, what is the speed of the particles that will be selected to pass on the mass spectrometer?
- When these particles then enter the mass spectrometer, which shares a magnetic field with the velocity selector, the radius of the resulting circular path followed by the particles is 6.26 cm. What is the mass of the charged particles?

The Cockcroft-Walton Proton Accelerator

- Capable of 1 MeV (mega electron volt)
- $1 \text{ eV} = 1.6021 \times 10^{-19} \text{ J}$



The Cyclotron

- Large number of small increases in potential.
- 30 MeV is very common

