Lesson 7 – Review units 1, 2, 3, and 4.

1. Consider the following vectors $\vec{a}$ = $2\vec{\imath}$ +$3\vec{\jmath}$ and $\vec{b}$ = $-2\vec{\imath}$ + $3\vec{k}$, and $\vec{c}$ = (-3, 0, 2). Compute the following:

a. $\vec{a} \cdot \vec{b}$ b. $\vec{b} \times \vec{c}$ c. $(\vec{a} \times \vec{b})\cdot \vec{c}$ d. $(\vec{c} \times \vec{b})\times \vec{a}$

2. Given that $|\vec{a}|$ = 10, $|\vec{b}|$ = 14, and $\alpha = \angle(\vec{a}, \vec{b})$ = 150^0 find $\vec{a} \cdot \vec{b}$ and $|\vec{a} \times \vec{b}|$.

3. Consider the parallelogram ABCD where A(0, 1, 2), B(1, -2, 3), and C(2, 1, 0).

Find:

a. angle ∠B

b. area of the triangle ΔABC

c. the angle between the diagonals of parallelogram ABCD

4. Find the algebraic vector $\overrightarrow{AB}$ and a unit vector collinear with $\overrightarrow{AB}$ where A(2, -3, 4) and B(0, -2, 3).

5. Find a unit vector perpendicular on both $\vec{a}$ = (1, 2, -1) and $\vec{b}$ = $2\vec{\imath}$ - $\vec{k}$.

6. Prove the following identity involving vectors.

$$\vec{a} \cdot \vec{b} = \frac{1}{4}\left|\vec{a} + \vec{b}\right|^2 - \frac{1}{4}\left|\vec{a} - \vec{b}\right|^2$$

7. Convert the Cartesian equation of the plane -2x + 3y – 4z + 12 = 0 to:

a. parametric equations b. vector equation

8. Find the Cartesian equation of the plane that passes through the points A(-3, -2, 0), B(1, 3, -1), and C(-1, 2, 4).

9. Find the angle between the intersecting

a. lines $L_1 : \vec{r} = (0, 1, -2) + s(2, -1, 3)$, $s \in R$ and $L_1 : \vec{r} = (2, 0, 1) + t(0, 1, -2)$, $t \in R$

b. planes π_1: 2x – 3y + z – 1 = 0 and π_2: -4x + 6y – 3z = 0

10. Find the distance between

a. the point P(0, -2) and the line L: -2x + 3y – 6 = 0

b, the point P(-1, 3, 0) and the plane π: x – 2y + 3z – 12 = 0

11. Find the point(s) of intersection between

a. the lines $L_1 : \begin{cases} x = 2t \\ y = 2 - 4t \\ z = -1 \end{cases}$ and $L_2 : \frac{x+4}{-2} = \frac{y-10}{1} = \frac{z+1}{3}$

b . the line $L : \frac{x-1}{2} = \frac{y+2}{-1} = z$ and the plane π: x -2y + z – 4 = 0

c. the planes π_1: x – 2y + z + 4 = 0 and π_2: 2x – 3y – z – 6 = 0

12. For the vectors $\vec{a} = (2, -1, -2)$ and $\vec{b} = (3, -4, 12)$ determine the following:

a. the angle between the two vectors

b. the scalar and vector projections of $\vec{a}$ on $\vec{b}$

c. the scalar and vector projections of $\vec{b}$ on $\vec{a}$

13.
a. Determine the line of intersection between π_1: 4x + 2y + 6z – 14 = 0 and π_2: x - y + z – 5 = 0
b. Determine the angle between the two planes.

14. If $\vec{x}$ and $\vec{y}$ are unit vectors, and the angle between them is 60° determine the value of each of the following:
a. $|\vec{x} \cdot \vec{y}|$
b. $|\overrightarrow{2x} \cdot \overrightarrow{3y}|$
c. $|(2\vec{x} - \vec{y}) \cdot (\vec{x} + 3\vec{y})|$
d. $|\vec{x} \times \vec{y}|$

15. Expand and simplify each of the following, where $\vec{\imath}$, $\vec{\jmath}$, and $\vec{k}$ represent the standard basis vectors in R^3
a. $2(\vec{\imath} - 2\vec{\jmath} + 3\vec{k}) - 4(2\vec{\imath} + 4\vec{\jmath} + 5\vec{k}) - (\vec{\imath} - \vec{\jmath})$
b. $-2(3\vec{\imath} - 4\vec{\jmath} - 5\vec{k}) \cdot (2\vec{\imath} + 3\vec{k}) + 2\vec{\imath} \cdot (3\vec{\jmath} - 2\vec{k})$
c. $\vec{\imath} \cdot (\vec{\jmath} \times \vec{k}) - \vec{k} \cdot (\vec{\jmath} \times \vec{k})$

16. Determine the angle that the vector $\vec{a}$ = (4, -2, -3) makes with the positive *x*-axis, *y*-axis, and *z*-axis.

17. If $\vec{a}$ = (1, -2, 3), $\vec{b}$ = (-1, 1, 2), and $\vec{c}$ = (3, -4, -1) determine each of the following:
a. $\vec{a} \times \vec{b}$
b. the area of the parallelogram determined by $\vec{a}$ and $\vec{b}$
c. $2\vec{a} \times 3\vec{b}$
d. $\vec{c} \cdot (\vec{b} \times \vec{a})$

18. Determine the coordinates of the unit vector that is perpendicular to $\vec{a}$ = (1, -1, 1), and $\vec{b}$ = (2, -2, 3).

19.
a. Determine vector and parametric equations for the line that contains A(2, -3, 1) and B(1, 2, 3)
b. Verify that C(4, -13, -3) is on the line that contains *A* and *B*.

20. Show that the lines L_1: $\vec{r}$ = (2, 0, 9) + t(-1, 5, 2) and L_2: $x - 3 = \frac{y+5}{-5} = \frac{z-10}{-2}$ are parallel and distinct.

21. Determine vector and parametric equations for the line that passes through (0, 0, 4) and is parallel to the line with parametric equations x = 1, y = 2 = t, z = -3 + t, t ∈ R.

22. Determine the value of *c* such that the plane with equation 2x + 3y + cz – 8 = 0 is parallel to the line with equation $\frac{x-1}{2} = \frac{y-2}{3}$ = z + 1.

23.

a. Determine the equation of the plane that passes through the points A(1, 2, 3), B(-2, 0, 0), and C(1, 4, 0)

b. Determine the distance from O(0, 0, 0) to this plane.

24. Determine a Cartesian equation for each of the following planes:

a. the plane through the point A(-1, 2, 5) with $\vec{n}$ = (3, -5, 4)

b. the plane through the point K(4, 1, 2) and perpendicular to the line joining the points (2, 1, 8) and (1, 2, -4)

c. the plane through the point (3, -1, 3) and perpendicular to the *z*-axis

d. the plane through the points (3, 1, -2) and (1, 3, -1) and parallel to the y-axis