

First Name: _____ Last Name: _____ Student ID: _____

Relationships Between Points, Lines and Planes (1)

1. Without solving, determine if the given line intersects the given plane at a single point or not:

a. $\pi: 3x - y + 4z - 8 = 0$

$l: (x, y, z) = (3, 0, 5) + t(7, -11, 18)$

b. $\pi: \vec{r} = (0, 0, 1) + t(1, 1, -1) + u(13, 3, 2)$

$l: \vec{r} = (5, -1, 4) + t(1, -2, 3)$

c. $\pi: 6x - 2y + 2z + 6 = 0$

$l: \vec{r} = (4, 12, -19) + t(2, -3, -9)$

2. For each part of Question 1, find the intersection of the line with the plane.

3. Find the intersection of the line and the plane.

a. $\frac{x+4}{3} = z, y = 0$ and $x - 2y - 3z + 4 = 0$

b. $\begin{cases} x = 2t \\ y = 1 - t \\ z = -4 + t \end{cases}$ and $\begin{cases} x = 2 + 2m + 2n \\ y = 3 - m \\ z = -5 + m - n \end{cases}$

4. Which of the following lines lie in the plane $3x - y - 3z - 12 = 0$?

a. $\vec{r} = (2, -9, 1) + t(1, 0, 1)$

b. $\vec{r} = (4, 0, 0) + t(2, 3, -1)$

c. $\vec{r} = (1, 5, -2) + t(3, 2, 1)$

5. Find the point on the line $\vec{r} = (6, 7, -8) + t(-2, 4, 1)$ that is also

a. on the xy plane.

b. on the yz plane.

6. Find the point on the plane $x-2y+z-8=0$ that is closest to $(10,12,4)$ (Hint: it is on a line that passes through the given point, perpendicular to the plane.)

7. Find the value(s) of a and b such that the given plane and line
 $4x-2y-z+12=0$, $\vec{r} = (3,4,a)+t(7,b,6)$

- a. Intersect at every point on the line.
- b. Intersect at a single point.
- c. Do not intersect.

8. For each pair of planes, determine whether they intersect in a line, are coincident, or are parallel and distinct:

- a. $2x-y+3z-8=0$ and $6x-3y+9z+3=0$
- b. $4x+3y+z-8=0$ and $3x-y-2z-2=0$
- c. $12x-6y+15z-9=0$ and $20x-10y+25z-15=0$

9. Find the intersection of the two planes given.

- a. $2x-2y+5z+10=0$ and $2x+y-4z+7=0$
b. $3x-2y+5z+3=0$ and $6x-4y+10z+7=0$

10. Find the Cartesian equation of the plane that is perpendicular to the plane $\vec{r}=(4,-5,2)+s(2,1,3)+t(-1,4,0)$ and intersects it at the line $\vec{r}=(4,-5,2)+t(1,-1,1)$.

11. Given the two planes below, find the value(s) of k that make the two planes intersect in the desired way, if possible. Explain your reasoning.

$$6x-9y+15z=21 \quad \text{and} \quad 10x-15y+kz=35$$

- a. Along a whole plane. b. Along a single line.

c. No intersection. d. At a single point.