Dear [Prof/Dr] [Name of the Editor] (or just Dear Editor),

The authors appreciate reviewers' comments and have worked very hard to address all suggestions and comments.
The revised manuscript was just submitted to the Journal's portal. Also, please find below the response to reviewers' comments, highlighting the main changes in the manuscript.

Best Regards

[Your Name]

Reviewer 1

The paper studies the creation of viscous fingering using the CVFEM with different element pairs and also dynamic mesh optimisation. I think the paper is very interesting by addressing a very complicated effect as viscous fingering with different numerical methods and formulations while also giving a very good background on the work done by others and I will strongly recommend its publication after some important modifications to the paper, as in its current state it looks like a draft. Therefore, my comments are towards improving the quality of the text and presentation more that the content itself, which I consider good enough for publication.

Major comments:

R1.1: In the introduction it first appears what I consider to be one the biggest of the paper. They try to explain too many mathematical concepts but they realise that it would take too much effort and they leave it unfinished. What does it mean, for example, "The dual pressure and velocity fields are represented simultaneously in FE and CV spaces"? More importantly, does it matter to the "Numerical Investigation of Viscous Flow Instabilities in Multiphase Heterogeneous Porous media"??? Isn't it better to send the interested reader that want to know more about the discretisation to the discretisation paper of the method as you already mention in some parts of the paper? I would like to see a paper more focused on what is presented and less diffuse.

We are thankful for the reviewer's comments. We acknowledge that the Introduction is slightly long (in particular the literature review on numerical methods for porous media flows) and should be shortened. Also, **R1.16** highlighted that literature review on viscous fingering was spread over Introduction and Section 3 and they should be merged. Finally, part of the reviewer' comments outline some confusing and unnecessary sentences for a full appreciation of this work, therefore Introduction and Section 3 were partially merged. Thus,

Before:

"1. Introduction

Numerical investigation of multiphase flows in porous media has attracted the attention of the scientific community over the past 40 years. Characterisation of such fluid flows serves as the foundation of reservoir engineering and groundwater studies (White et al., 1981). Underground coal gasification is another important field of interest and, more recently, due to the role of green house gases (GHG) emissions in the global climate change, several research work have focused on CO 2 migration and trapping mechanisms in carbon capture utilisation and storage (CCUS) operations (Spycher et al., 2003; Self et al., 2012; Jiang, 2011).

Description of physics and mechanisms of multiphase porous media flows were reported by Wooding and Morel-Seytoux (1976) with focus on capillary pressure and flow regimes. A comprehensive review of force balances at the interface between immiscible fluids and resulting mechanisms for flow instabilities can be found in Homsy (1987). Flow instabilities can be classified as macroscopic and microscopic, the former is due to imposed boundary conditions, whereas the later is associated to local phenomena at the fluids interface (e.g., Kelvin-Helmholtz and Saffman-Taylor instabilities, Saffman, 1959). This work focuses on flow instabilities in two-phase systems due to viscous and stress forces (often referred as viscous instabilities or viscous fingering, Section 3).

In oil and gas reservoir exploration, viscous and density instabilities are relatively common during water-injection processes. As water and oil interacts, the interface between these fluids moves creating an uneven or fingered flow profile (see Fig. 3). Viscous fingering results in inefficient flow sweeping which can bypass significant quantities of recoverable oil and may lead to early breakthrough of water into

neighbour production wells. Viscous instabilities are mainly controlled by the mobility ratio (MR) between displacing and displaced fluids. Other conditions that may also influence the severity of viscous fingering are: heterogeneity (i.e., wide spatial porosity and/or permeability distribution), gravitational forces, anisotropic dispersion, non-monotonic viscosity profile etc (Budek et al., 2017; Nicolaides et al., 2015).

Multi-fluid flow dynamics in porous media are described by continuity and momentum (extended Darcy's law) conservative equations for each fluid (or phases) with coupling mass terms (i.e., density and saturation) appearing in both sets of equations. Advanced numerical methods are in continuous development to accurately represent force balances in Darcean flows. Finite difference methods (FDM) have been extensively used in most industry-standard reservoir simulators (Aziz and Settari, 1986; Chen et al., 2005; Chang et al.,1990) with relative success. However, they are often limited to relatively simple geometries representing idealised geological formations through structured quadrilateral (2-D) and hexahedral (3-D) grid cells (Mlacnik et al.,2004; King and Mansfield, 1999). Additionally, FDM schemes often result in excessive numerical dispersion when strong heterogeneity (represented by permeability and porosity fields) is present (Chavent and Jaffŕe, 1986).

The geometrical flexibility associated with high-order numerical accuracy of finite element methods (FEM) has proven to be more efficient than FDM to solve fluid flow dynamics in complex geometries. Among FEM-based formulations for porous media, the control volume finite element methods (CVFEM, Fung et al., 1992) has been widely used as it can guarantee local mass conservation and high-order numerical accuracy as well as being able to use tetrahedral geometry-conforming elements. In traditional CVFEM formulations, pressure and velocity are interpolated using piece-wise linear FE basis functions, while material properties and flow conditions (e.g., phase saturation, density, temperature, species concentration etc) are represented with CV basis functions (Voller, 2009). Saturation equations are solved explicitly after solving for the dual pressure-velocity at each non-linear iteration (a detailed description of the implicit pressure explicit saturation, IMPES, formulation can be found in Chen et al., 2006; Lux and Anguy, 2012).

Since geometries are captured by finite elements, constructed control volumes typically extend on each side of the interface which may have different properties. Therefore, some average values of the coupled velocity-pressure 3fields (defined in the FE space but projected onto the CV space) are applied across the CVs at these interfaces. These often lead to excessive numerical dispersion especially in highly heterogeneous media (represented by spatial-dependent permeability and porosity fields). In order to overcome such artificial numerical dispersion, a discontinuous hybrid finite element finite volume method (DFEFVM) formulation was introduced by Nick and Matthai (2011a,b). This novel discretisation scheme was designed to simulate flows through discrete fractured rocks in which CVs are divided along the interfaces of different materials. Cumming et al. (2011) demonstrated that CVFEM discretisation could also be used to solve Richards' equations (coupled mass conservation and Darcy equation) in heterogeneous media with relatively small computational overhead (compared with traditional coupled velocity-pressure based formulations, see also Cumming, 2012). Fluxes over CVs were calculated based upon material properties, whereas the saturation field was volume-averaged at the interface of the materials, enforcing mass balance as described by Kirkland et al. (1992).

In this work, a novel CVFEM formulation, previously introduced by Gomes et al. (2017) (see also Jackson et al., 2015; Salinas et al., 2015), is used to numerically investigate formation and growth of viscous fingers in heterogeneous porous media. The continuity equation is embedded into the pressure equation to enforce mass conservation whilst ensuring that the force balance is preserved. A hybrid family of $P_nDG\text{-}P_m$ triangular and tetrahedral FE pairs is used to discretise velocity and pressure. A sketch of the $P_1DG\text{-}P_2$ FE-pair is shown in Fig. 1, in which velocity is represented by discontinuous and piecewise linear basis functions whereas pressure is interpolated through continuous and piecewise quadratic basis functions. Scalar fields are stored in CV space (Fig. 2). The dual pressure and velocity fields are represented simultaneously (through non-linear projections) in FE and CV spaces.

A brief description of the numerical formulation and viscous fluid instabilities are introduced in Sections 2 and 3, respectively. Model set up and results including initial model-benchmark are presented in Section 4. Impact of mobility ratio on the fingers formation is also included in this section. Finally, concluding remarks are presented in Section 5."

Now:

"1. Introduction

Numerical investigation of multiphase Darcy flows has attracted the attention of the porous media community over the past 40 years. Characterisation and prediction of such flows are the foundation of groundwater and oil & gas reservoirs studies.

Description of physics and mechanisms of multiphase porous media flows was reported by Wooding and Morel-Seytoux (1976) with focus on capillary pressure and flow regimes. A comprehensive review of force balances at the interface between immiscible fluids and resulting mechanisms for flow instabilities can be found in Homsy (1987). This work focused on flow instabilities in two-phase systems due to viscous and stress forces, often referred as viscous instabilities or viscous fingering.

Muskat (1934) used Hele-Shaw cells (i.e., parallel flat plates separated by an infinitesimal gap) to study fluid flow and the impact of the capillary number (N_c) on the flow dynamics. This experimental apparatus enabled instabilities to be qualitatively investigated by simplifying the flow (in both porous and non-porous media) to a 2D problem. McLean and Saffman (1981) developed a semi-analytic solution for flows in Hele-Shaw cells which were later used by Guan and Pitchumani (2003) to investigate fingers' formation, dimensions and branchiness.

More recently, Howison (2000) and Praud and Swinney (2005) provided a comprehensive description of the mathematical formulation of immiscible two-phase flows in Hele-Shaw cells. For a Hele-Shaw cell of a given size, flow development depends only on the capillary number, therefore if N_c is too high, Saffman (1959a) (see also Saffman, 1959b; Homsy, 1987; Tabeling et al., 1987) demonstrated that the flow develops to a single steady-state finger which moves through the cell with constant velocity.

Multi-fluid flow dynamics in porous media are described by continuity and momentum conservative equations for each fluid with coupling terms appearing in both sets of equations. Finite difference methods (FDM) have been extensively used in most industry-standard reservoir simulators to solve this set of equations, however they are often limited to relatively simple geometries and often lead to excessive numerical dispersion when strong heterogeneity is present (Chavent and Jaffre, 1986).

The geometrical flexibility associated with high-order numerical accuracy of finite element methods (FEM) has proven to be more efficient than FDM to solve fluid flow dynamics in complex geometries. Among FEM-based formulations for porous media flows, the control volume finite element method (CVFEM, Fung et al., 1992) was designed to guarantee local mass conservation and high-order numerical accuracy as well as being able to use geometry-conforming elements (e.g., triangles, tetrahedra, pyramids etc). In traditional CVFEM formulations, pressure and velocity are interpolated using piecewise linear FE basis functions, while material properties and flow conditions (e.g., phase saturation, density, temperature, species concentration etc) are interpolated with CV basis functions (Voller, 2009).

Since geometries are captured by FE, constructed CVs typically extend on each side of the interface which may have different properties. Therefore, average values of coupled velocity-pressure variables are applied across the CVs at the interfaces. This often leads to excessive numerical dispersion especially in highly heterogeneous media. In order to reduce artificial numerical diffusion associated to the upwind weighting of the advected quantity in regions of strong heterogeneity, a discontinuous hybrid finite element finite volume method (DFEFVM) formulation was introduced by Nick and Matthai (2011a,b). This discretisation scheme was designed to simulate flows through discrete fractured rocks in which CVs are divided along the interfaces of different materials. Coupled control-volume distributed (CVD) flux-limiting method and high-order convection schemes were introduced by Edward (2006) to reduce anisotropic numerical diffusion in flows through semi-impervious barriers (i.e., flow across regions of large permeability gradient). Solution for such particular numerical diffusion problem was recently revisited by Salinas et al. (2018) using the model formulation outlined in Section 2.

The impact of viscosity ratio and flow rate on fingering development was experimentally investigated (with coreflood and micromodels) studied by Doorwar and Mohanty (2016). Through numerical experiments (based on pore-network model), they demonstrated that fingers' dynamics (i.e., growth, splitting and coalescence) are also strongly dependent on the domain's aspect-ratio and capillary forces (see Doorwar and Mohanty, 2014), and proposed a new instability dimensionless number to assess breakthrough recovery of a number of an extensive set of experimental data.

Therefore, the main aim of this work is to numerically investigate formation and growth of viscous fingers in heterogeneous porous media through a novel computational multi-fluid dynamics model (Gomes et al., 2017). The two main aspects of this work are: (a) qualitative and quantitative validation of the model (Section 4.3) for viscous flow instabilities and, (b) impact of mesh resolution on capturing the onset formation and growth of viscous fingers (Section 4.4).

A brief description of the numerical formulation and viscous flow instabilities are introduced in Sections 2 and 3, respectively. Model set up and results including initial model-benchmark are presented in Section 4. Finally, concluding remarks are presented in Section 5."

R1.2: Equation 1. The permeability K is a tensor as far I understand, and therefore if you want to divide by it, it has to be K^-1. Also in equation 1 you say it is define for t > 0. However, in the current state that equation does not depend on time and therefore that assumption is unnecessary. In the paragraph after equation one, you say S and S_alpha are this and that. However in the equations you have S_u,alpha and S_cty,alpha. This needs to be corrected.

Authors thank the reviewer's comment and all suggestions were implemented, thus,

Before:

"(...) saturation equations,

$$\left(\frac{\mu_\alpha S_\alpha}{\mathbf{K}\mathcal{K}_{r\alpha}}\right)\mathbf{u}_\alpha = \underline{\underline{\sigma}}_\alpha \mathbf{u}_\alpha = -\nabla p + \mathcal{S}_{u,\alpha} \qquad \text{with } x_i \in \Omega,\ t > 0, \qquad (1)$$

$$\phi\frac{\partial S_\alpha}{\partial t} + \nabla\cdot(\mathbf{u}_\alpha S_\alpha) = \mathcal{S}_{cty,\alpha} \qquad \text{with } x_i \in \Omega,\ t > 0, \qquad (2)$$

respectively, where μ, K, p and ϕ are viscosity, absolute permeability, pressure and porosity, respectively. u_α is the saturation-weighted Darcy velocity of the α-phase and $K_{r,\alpha}$ is the relative permeability. $\mathcal{S}$ is the source term associated with the Darcy and continuity equations. S_α represents the saturation of the α-phase with mass conservation constraints of $\sum_{\alpha=1}^{\mathcal{N}_p} S_\alpha = 1$, where $\mathcal{N}_p$ denotes the number of phases. $\underline{\sigma}_\alpha$ is an absorption-like term that represents the implicit linearisation of the viscous frictional forces."

Now:

"(...) saturation equations,

$$\mu_\alpha S_\alpha \left(\mathbf{K}\mathcal{K}_{r\alpha}\right)^{-1}\mathbf{u}_\alpha = \underline{\underline{\sigma}}_\alpha \mathbf{u}_\alpha = -\nabla p + \mathcal{S}_{u,\alpha}, \qquad (1)$$

$$\phi\frac{\partial S_\alpha}{\partial t} + \nabla\cdot(\mathbf{u}_\alpha S_\alpha) = \mathcal{S}_{cty,\alpha}, \qquad (2)$$

with $x_i \in \Omega, t > 0$. μ, K, p and ϕ are viscosity, absolute permeability, pressure and porosity, respectively. u_α is the saturation-weighted Darcy velocity of the α-phase. $\mathcal{S}_{u,\alpha}$ and $\mathcal{S}_{cty,\alpha}$ are source terms of the α-phase related to the momentum (e.g., gravity, capillarity forces etc) and continuity (e.g., mass transfer across fluids/phases, geochemical reactions etc) equations, respectively. Finally, $K_{r,\alpha}$ is the relative permeability, and S_α is the α-phase saturation with mass conservation constraints of of $\sum_{\alpha=1}^{\mathcal{N}_p} S_\alpha = 1$, where $\mathcal{N}_p$ denotes the number of phases. $\underline{\sigma}_\alpha$ is an absorption-like term that represents the implicit linearisation of the viscous frictional forces."

R1.3: In section 2 you mention that high-order fluxes are limited on CV boundaries, which are limited to yield bounded fields. Again you mention something without enough details, you say that the saturation is bounded between 0 and 1. This is clearly not enough to guarantee stability of a high-order flux, as starters because if the irresidual oil and water saturation are non-zero then a saturation of a phase of 1 or 0 would be unphysical and therefore wrong.

The authors acknowledge that sentence lacks the required level of detail on the methods used. This was changed as:

Before:

"(...) Mass balance (continuity) equations are solved in CV space and a Petrov-Galerkin FEM is used to obtain high-order fluxes on CV boundaries, which are limited to yield bounded fields (e.g., positive densities, saturations bounded between 0 and 1, etc)."

Now:

"(...) Mass balance (continuity) equations are solved in CV space and a Petrov-Galerkin FEM is used to obtain high-order fluxes on CV boundaries. Smooth transition between first- and high-order fluxes are enforced through an extrema detection scheme based on normalised variable diagram (NVD, Jasak et al., 1999; Darwish and Moukalled, 2003) applied across faces of neighbouring CVs. Resulting normalised

upwind face values are used to calculate flux-limited solutions (based on total variation diminishing criteria, TVD, Piperno and Depeyre, 1998) to yield bounded fields (e.g., positive densities, saturations bounded between 0 and 1 etc. See Gomes et al., 2012, for more details on the flux-limitin g scheme within the model)."

With respect to the second query on non-physical saturation when irreducible wetting and residual non-wetting phase saturations are non-zero, maybe there is a bit of confusion here. The flux-limiting scheme embedded in the model bounds scalar fields (e.g., solution between 0 and 1 for normalised quantities such as saturation, positive density etc) when the system of equations is solved together. Experimental minimum phase saturations (i.e., irreducible wetting and residual non-wetting phase saturations) implicitly bound the solution through the parametrisation of the relative permeability, embedded into the linearised viscous frictional force, $\underline{\sigma}_{\alpha}$, (Eqns. 1, 8 and 9 in the main text) .

R1.4: You mention in section two that you use P1DGP1 and P1DGP2, however later on you use P1DGP1DG. This leads to two questions: what have you used finally? And second why use an unstable element pair like P1DGP1DG? In Gomes et al 2017 they do not use P1DGP1DG.

P_1DG-P_1 and P_1DG-P_2 were used for 2D cases whereas the P_1DG-P_1DG was used for the 3D case (Section 4.5). The last sentence of the second paragraph of page 5,

Before:

"(...) Simulations performed for this work were conducted using two types of elements: P_1DG-P_1 and P_1DG-P_2."

Now:

"(...) Simulations performed for this work were conducted using three types of elements: P_1DG-P_1, P_1DG-P_2 and P_1DG-P_1DG (Section 4.5)."

Also, P_1DG-P_1DG and P_2DG-P_1DG element-pairs were used in 3D fluvial channel simulations (Section 4.4) in Gomes et al. (2017), and they concluded that solutions obtained from the former have more diffusion than solutions obtained from the latter. In this manuscript we opted to use P_1DG-P_1DG rather than P_1DG-P_2 due to larger computational costs associated to 3D simulations using the later.

R1.5: Velocities within and between elements is not described in Jackson et al 2013, as they only use classical CVFEM in that paper. Moreover, in Gomes et al 2017 they explain that they use two different methods for within elements and between elements. In the same paragraph you talk about sigma instead of sigma with two bars and bold, this needs to be corrected. Again this part is not even necessary in this paper, as it confuses from the main objective of the paper.

The authors acknowledge that the reference was not accurate and this was corrected along with the sigma symbol, thus

Before:

"(...) Velocities across CV interfaces (within and between elements) are calculated through a directional-weighted flux-limited scheme based on upwind value of σ at individual CV as described by Jackson et al. (2013)."

Now:

"(...) Velocities across CV interfaces (within and between elements) are calculated through a directional-weighted flux-limited scheme based on upwind value of $\underline{\sigma}$ at individual CV as described by Gomes et al. (2017)."

R1.6: The mention that the discretised global mass and force balance equations are solved using a multigrid-like approach as described in Pavlidis et al. is not correct as this is not described in that paper. Again this statement feels unnecessary in this paper.

The authors acknowledge that the reference was not accurate and removed it,

Before:

"The discretised global mass and force balance equations are solved using a multigrid-like approach as described by Pavlidis et al. (2016) (see also Salinas et al., 2016). The numerical formulation is fully described by Gomes et al. (2017) (see also Salinas et al., 2015; Adam et al., 2016)."

Now:

"The discretised global mass and force balance equations are solved using a multigrid-like approach. The numerical formulation is fully described by Gomes et al. (2017) (see also Salinas et al., 2015; Adam et al., 2016)."

R1.7: In section 4.4 you present and give a lot of information about dynamic mesh optimisation. I think that all this background information should not go in the results section, but in the introduction or the methods section.

The authors thank the Reviewer for her/his comments and they agree that paragraphs on dynamics mesh optimisation may not be relevant for the AWR's readers. Thus, the Section 4.4,

Before:

"In most numerical simulations involving geo-fluid dynamics, local geometric constraints (e.g., faults, fractures etc) and spatial multi-scale variability of flow properties are often ignored as the underlying computational mesh grid is too coarse to reliably represent any of these features. Whilst structured grids often struggle to conform to complex domain boundaries with consistent mesh connectivity, unstructured mesh techniques often relax cells' neighbourhood relationship constraints by dividing the domain into polytopes in which elements share adjacent faces. This leads to mesh grids that conform to the domain topography and can make the best use of state-of-the-art self-adaptive computational methods.

In Section 4.3, the impact of viscosity ratio on the growth of fingers and the triggering effect of heterogeneity on instability's nucleation were demonstrated in modified simulated Hele-Shaw cells. In order to capture the continuous development of viscous instabilities, mesh grids with sufficient resolution need to be applied over the interface between fluids. Figure 11 shows a numerical simulation performed with the same geometry, boundary and initial conditions as the one shown in Fig. 10, but with noticeable lower resolution (3734 instead of 26313 elements). As it can be seen, the lower the resolution the more difficult is to capture the fingers formations, growth and branching.

Traditional computational geo-fluid dynamics (CGFD) models often rely on fixed mesh with sufficient resolution throughout the domain to capture specific flow dynamics (e.g., fluid instabilities, flow re-circulation, heat and mass transfers etc), however computational overhead may be prohibitive for simulations involving complex geometries and heterogeneous properties. Mesh adaptivity methods have been extensively used by the CGFD community to help capturing detailed flow dynamics, compositional non-equilibrium fluid displacement and solid-fluid interactions (Paluszny et al., 2007; Pietro et al., 2014; Su et al., 2016; Melnikova et al., 2016). In these methods, the mesh grid is continuously modified (i.e., adjustments of the number and distribution of the degrees of freedom to reduce solution error) to focus resolution where is necessary as the simulation evolves in time and space. There are four main families of mesh adaptivity methods: adaptive mesh refinement (AMR), edge and face element manipulations whereas keeping the order of the element basis function fixed (h-adaptivity), mesh deformation (r-adaptivity) and changes of element basis function order (p-adaptivity). Detailed description and analysis of mesh adaptivity methods are beyond the scope of this manuscript but can be readily found in Lo (2015) (see also Plewa et al., 2004; Frey and George, 2008).

The dynamic mesh adaptive algorithm embedded in the Fluidity/IC-FERST model utilises a metric tensor field dependent on solution interpolation error-estimates which locally control the topology of elements in the metric (Pain et al., 2001; Power et al., 2006). Mesh optimisation generates unstructured finer mesh in regions where flow properties change faster, and coarser mesh in regions where properties change more slowly (Piggott et al., 2006). The mesh adapts in three stages: metric formation, mesh optimisation and fields' interpolation from the pre- to post-adapted mesh (see Hiester et al., 2014). In the simulations shown in this work, mesh will adapt as a response to oscillations in the phase saturation field with prescribed interpolation error estimate (Mostaghimi et al., 2016).

Here, two numerical simulations were conducted with fixed and adaptive mesh resolutions to qualitatively assess model capability to capture fingers dynamics. In order to trigger the formation of fingers, regions with sharp permeability gradient were introduced. The computational domain, Fig. 12a, consists of a rectangular geometry fully saturated with a fluid (except by a squared region containing 50% of a second fluid – wetting fluid phase, Fig. 12b) and divided into 5 regions with prescribed permeability distribution. A no-flux boundary condition was imposed across upper and lower borders, whilst pure wetting fluid phase (i.e., $S_1 = 1$) is driven into the domain from the left-hand side face at velocity $u_1 = 1$. Viscosity ratio was set to 10.

Figures 13 and 14 show fluid displacement in simulations performed with fixed and adaptive meshes. Both simulations started with a mesh of 13068 triangular ($P_1DG\text{-}P_2$) elements, however as flow dynamics evolve the number of elements of the simulation conducted with adaptive mesh oscillates from a

minimum of 4400 to a maximum of 16430 (Fig. 15). In both cases, fingers' development (formation, growth and coalescence) and fluid cross flows (through regions of distinct permeabilities at the top of the domain, K = {2, 3}) can be readily noticed. The simulation performed with a fixed and relatively fine mesh was able to capture the progressive development of fingers (as shown in Fig. 16b) prompted by sharp permeability gradient at the interface of regions with K = {1, 5}. In regions with no permeability gradient, formation and development of fingers (Fig. 16c) were not captured by the simulation conducted with fixed mesh. In both cases, dynamic adaptive mesh based on perturbation of phase saturations with imposed interpolation error estimate proved to be able to capture the onset of instabilities and fingers' development with relatively little computational overhead, as indicated in Fig. 15."

Now:

"In most numerical simulations involving geo-fluid dynamics, local geometric constraints (e.g., faults, fractures etc) and spatial multi-scale variability of flow properties are often ignored as the underlying computational mesh grid is too coarse to reliably represent any of these features. Whilst structured grids often struggle to conform to complex domain boundaries with consistent mesh connectivity, unstructured mesh techniques often relax cells' neighbourhood relationship constraints by dividing the domain into polytopes in which elements share adjacent faces. This leads to mesh grids that conform to the domain topography and can make the best use of state-of-the-art self-adaptive computational methods.

In Section 4.3, the impact of viscosity ratio on the growth of fingers and the triggering effect of heterogeneity on instability's nucleation were demonstrated in modified simulated Hele-Shaw cells. In order to capture the continuous development of viscous instabilities, mesh grids with sufficient resolution need to be applied over the interface between fluids. Figure 11 shows a numerical simulation performed with the same geometry, boundary and initial conditions as the one shown in Fig. 10, but with noticeable lower resolution (3734 instead of 26313 elements, i.e., marginally above the solution mesh-independent resolution of 3.5k elements shown in Fig. 4). As it can be seen, the lower the resolution the more difficult is to capture the fingers formations, growth and branching.

The dynamic mesh adaptive algorithm embedded in the Fluidity/IC-FERST model utilises a metric tensor field dependent on solution interpolation error-estimates which locally control the topology of elements in the metric (Pain et al., 2001). Mesh optimisation generates unstructured finer mesh in regions where flow properties change faster and coarser mesh in regions where properties change more slowly (Piggott et al., 2006; Hiester et al., 2014; Mostaghimi et al., 2016). In the simulations shown in this work, mesh grid adapts as a response to oscillations of phase saturations with: (a) minimum value for length of elements of 0.05 unit-length, (b) maximum number of nodes of $5x10^4$, and (c) prescribed interpolation error estimate of 10^{-2}.

Here, two numerical simulations were conducted with fixed and adaptive mesh resolutions to qualitatively assess model capability to capture fingers dynamics. In order to trigger the formation of fingers, regions with sharp permeability gradient were introduced. The computational domain, Fig. 12a, consists of a rectangular geometry fully saturated with a fluid (except by a squared region containing 50% of a second fluid – wetting fluid phase, Fig. 12b) and divided into 5 regions with prescribed synthetic permeability distribution with values between 1 and 5. A no-flux boundary condition was imposed across upper and lower borders, whilst pure wetting fluid phase (i.e., $S_1 = 1$) is driven into the domain from the left-hand side face at velocity $u_1 = 1$. Viscosity ratio was set to 10.

Figures 13 and 14 show fluid displacement in simulations performed with fixed and adaptive meshes. Both simulations started with a mesh of 13068 triangular (P_1DG-P_2) elements, however as flow dynamics evolve the number of elements of the simulation conducted with adaptive mesh oscillates from a minimum of 4400 to a maximum of 16430 (Fig. 15). In both cases, fingers' development (formation, growth and coalescence) and fluid cross flows (through regions of distinct permeabilities at the top of the domain, K = {2, 3}) can be readily noticed. The simulation performed with a fixed and relatively fine mesh was able to capture the progressive development of fingers (as shown in Fig. 16b) prompted by sharp permeability gradient at the interface of regions with K = {1, 5}. In regions with no permeability gradient, formation and development of fingers (Fig. 16c) were not captured by the simulation conducted with fixed mesh. In both cases, dynamic adaptive mesh based on perturbation of phase saturations with imposed interpolation error estimate proved to be able to capture the onset of instabilities and fingers' development with relatively little computational overhead, as indicated in Fig. 15."

R1.8: Section 4.4 there are a couple of references to Figures with question marks.

All references have been corrected.

R1.9: Figure 12, check the scale of the permeability.
Permeability is expressed in cm^2 and the caption for this figure has been corrected.

R1.10: Currently the sizes of many numerical test cases are missing; this should be included to allow for reproducibility.
The authors thanks the Reviewer for outlining the missing dimensionality of computational domains for simulations described in Sections 4.4 and 4.5 (Figures 12-23). For simulations performed for Section 4.4 (Figs. 12-16), we added a sketch of the computational domain in Fig. 12. For Section 4.5, we added the following sentence

Before:

"(...) In this Section, 3D simulations are conducted to investigate preferential flow pathways through semi-pervious and impervious geological layers.

The computational domain with permeability distribution of $10^{-4} \leq \mathbf{K} \leq 10^{5}$ milidarcy (Fig. 17) was designed to naturally capture preferential flow pathways through channels of large absolute permeability (...)"

Now:

"(...) In this Section, 3D simulations are conducted to investigate preferential flow pathways through semi-pervious and impervious geological layers.

Simulations were performed in a hexahedron domain of 4 cm × 2.5 cm × 10 cm, with permeability distribution of $10^{-4} \leq K \leq 10^{5}$ milidarcy (Fig. 17). The computational domain was designed to naturally capture preferential flow pathways through channels of large absolute permeability (...)"

Minor comments:

R1.11: In the abstract a "-" is used but it is not closed with a "-". A comma should be used instead as it is a clarification and the first level for clarification is a comma. If the authors insists to use a "-" then for consistency the sentence should be closed with a "-" as well.
The "-" has been removed and the sentence has been corrected/rephrased to "Due to large viscosity ratios, flow instabilities at fluids' interface may arise leading to the formation of fingers therefore creating an uneven front with elongation at the outside edge of fluids interface with strong impact on the displacement efficiency".

R1.12: It has struck me the fact that despite using dynamic mesh optimisation this does not appear in the title. The authors should consider giving more emphasis to the use of dynamic mesh optimisation for fingering processes.
The authors thank the Reviewer's suggestion to focus the work on mesh adaptivity technology to capture viscous fingering. However, this was never the aim of the authors. Numerical investigation of immiscible viscous fingering through dynamic mesh adaptivity (with the same flow simulation model as used in this work) was the focus of Mostaghimi et al. (2016), referenced in Section 4.4.

R1.13: In the introduction the authors present a problem with classical CVFEM leaking to low permeable regions. Michael Edwards (Edwards, M.: Higher-resolution hyperbolic-coupled-elliptic flux-continuous cvd schemes on structured and unstructured grids in 2-d. (2006)) have done a good job presenting this problem and I believe it would be useful for the interested reader in pointing to this work. What surprised me is that the authors are very humble by not presenting their own solution to this problem. In Gomes et al 2017 I believe that they present their solution as well and I think they should present it as well in this paper.
The authors thank the Reviewer for point this out. Edwards' work –re flux-limiting in CVD schemes was added into the Introduction as outlined in the response **R1.1** above.

R1.14: You seem to imply that the classical CVFEM requires IMPES, after the citation of Voller. This assumption is wrong and needs to be corrected, or better explained.
The authors thank the Reviewer for outlining this misunderstanding, as this was not our intent to imply a requirement of IMPES for any CVFEM formulation. As part of the shortening of the Introduction section, the above sentence has been removed.

R1.15: In the introduction you cite the work of Jackson et al. 2014 and later on of Jackson et al 2013. I have checked and I think that the correct citation is actually Jackson et al. 2015 in all the cases.
The authors acknowledge that the reference was not accurate and this was corrected.

R1.16: Section 3 seems like a proper introduction. As a suggestion the authors may think in merging this section with the introduction to also reduce the size of the paper, which is currently "threatening".

The authors thank the Reviewer's suggestion and they had partially merged the literature review of Section 3 with Introduction (see **R1.1** for the new Section 1). A few sentences were also removed to reduce the size of the manuscript. Thus, for Section 3:

Before:

"**3. Brief Summary of Viscous Instabilities**

The study of viscous flow instabilities (i.e., fingering) is particularly important in oil exploration due to heterogeneities (i.e., natural fractures, permeability and/or porosity characteristics in different zones) of geological formations. A major problem associated with water-flooding processes is the early water-breakthrough caused by high-permeability layers and unfavourable mobility ratios. Water breakthrough and volumetric sweep efficiency (ratio of volumes between the recovered and the injected fluids) are the main determinants of the productive life of a reservoir (Riaz and Tchelepi, 2004; Tavassoli et al., 2015). During immiscible CO_2-flooding (i.e., in CO_2 enhanced oil recovery, CO_2-EOR, processes), viscosity of super-critical CO_2 is lower than crude oil, thus viscous fingering and/or channelling may occur.

The efficiency of fluid displacement depends upon the ratio of viscous and capillary forces (or capillary number, N_c, Eqn. 5). When viscous force of the injected fluid overcomes the capillary force, hydrodynamic instabilities may occur, resulting in the collapse of the interface between fluids and fingers start to form.

Viscous flow instabilities can be found across several disciplines and scales, from chemical separation processes to geological reservoir fluids. Muskat (1934) investigated fluid flow in Hele-Shaw cells, i.e., parallel flat plates separated by an infinitesimal gap, and the impact on the capillary number (N_c),

$$N_c = \frac{\mu U}{\gamma}. \qquad (5)$$

In Eqn. 5, γ is the surface tension and U is the characteristic velocity of the moving interface. This experimental apparatus enables instabilities to be qualitatively investigated by simplifying the flow in both porous and non-porous media into a 2D problem.

More recently, Howison (2000) and Praud and Swinney (2005) provided a comprehensive description of the mathematical formulation of immiscible two-phase flows in Hele-Shaw cells (also know as Saffman & Taylor problem). For a Hele-Shaw cell of a given size, flow development depends only on the capillary number. Thus if N_c is too high, Saffman (1959), Homsy (1987) and Tabeling et al. (1987) determined that the flow develops into a single steady-state finger which moves through the cell with constant velocity U.

Under the assumption that fluids remain immiscible along an interface, surface tension plays an important role in determining the shape and progress of the fingers (Howison, 2000). During the displacement of a fluid by a less viscous one, the expected uniform front (Buckley and Leverett, 1942; Sheldon and Cardwell, 1959) is perturbed leading to an uneven front with elongations at the outside edge of the fluid interface (Fig. 3), often referred as fingers. In homogeneous domains, fingers start to develop when the surface tension acting on the interface between the fluids exert an opposite force towards the change of shape of the interface. The interface becomes unstable and collapses, taking a curved shape (Homsy, 1987; Jackson et al., 2017).

In heterogeneous domains, such instability may be triggered by permeability differences across regions as demonstrated in Section 4.3. The higher the velocity of the low viscosity fluid, the less wide (tip-splitting behaviour) the finger is. Pressure differences acting on the interface produces a net pressure force,

$$\Delta P = -\gamma \nabla . \hat{n}. \qquad (6)$$

This expression is also know as the Young-Laplace equation, a relation describing the capillary pressure across the interface between two fluids, with ΔP denoting the pressure difference and $\hat{n}$ is the unit normal vector out of the surface.

As demonstrated by Habermann (1960) (...)"

Now:

"**3. Brief Summary of Viscous Instabilities**

Viscous flow instabilities are relatively common in waterflooding for heavy oil reservoirs, resulting in inefficient flow sweeping which can bypass significant quantities of recoverable oil. This may also lead to early breakthrough of water in neighbour production wells.

Under the assumption that fluids remain immiscible along the interface, surface tension plays an important role in determining shape and progress of the fingers (Howison, 2000). During the displacement of a fluid by a less viscous one, the expected uniform front (Buckley and Leverett, 1942; Sheldon and

Cardwell, 1959) is perturbed leading to an uneven front with elongations at the outside edge of the fluid interface. In immiscible displacements, viscous fingering occurs when the viscosity ratio is greater than unity. As surface tension becomes weak, the interface is stressed and becomes unstable leading to the formation of fingers. The interface of the main finger collapses and starts splitting into new lobes of fingers. One of these new fingers may eventually outgrow the others and then spreads to occupy an increasingly larger width. In the process, the finger reaches a critical width while the saturation gradient at the front becomes steep as a result of stretching caused by the cross-flow, causing the tip of the finger to become unstable and splitting again, and the pattern repeats.

Therefore, surface tension plays an essential dual role, it must be weak enough for the tip front to be unstable, but it is also the physical force causing the spreading and ensuing repeated branching (Tan and Homsy, 1988; Carvalho et al., 2013). In heterogeneous domains, such instability may be triggered by permeability differences across regions as shown in Section 4.3.

As demonstrated by Habermann (1960) (...)"

R1.17: In section 3 you mention that MR can be reduced to VR. However, it seems that all the studies have been done for MR, why is this reduction done? Maybe this needs to be explained better. Also , after explaining in very detail that MR controls fingering you mention all of a sudden that the Peclet number is also important. I believe that this should be presented earlier. At the end of section 3 you present what is going to be presented in section 4, like it is normally done in the introduction. As I said before section 3 feels like a second introduction.

Mobility ratio (MR, Eqn. 5 in the new version of the Manuscript) is defined as the ratio of mobility ($\mathcal{K}_{r,\alpha}/\mu_\alpha$) of the displacing and displaced fluid. In the simulations performed for this work, phase relative permeability is expressed by the modified Brooks&Corey expression (Eqns 6-7) which is a function of the local (CV-wise, time-dependent) phase saturation. Therefore, fixing a value of MR would not reflect the spatial-/temporal variability of the problem. The authors are aware that the majority of the literature assumes constant MR (though relative permeability varies with phase saturation) for simulations. However, we prefer to reduce this parametrisation to the viscosity ratio (assuming phase viscosities are invariant). Mechanisms of finger growth and splitting (penultimate paragraph of the Section in the original manuscript) was modified and moved to the beginning of the Section (see **R1.15**). The authors decided to add a short summary of Section 4 (numerical results) at the end of Section 3 to improve readability of the manuscript for potential readers that may not be fully familiar to topic. Thus the last paragraphs of Section 3,

Before:

"(...) are both set to 2.

From Eqns. 7-9, it is clear that during fluid displacement, the mobility ratio changes as phase saturation, $S_w(x_i,t)$ and $S_{nw}(x_i,t)$, varies in time and space (Eqn. 2). Therefore, with no lack of generality, the MR can be reduced to the viscosity ratio (VR),

$$VR = \frac{\mu_i}{\mu_j},$$

which will be used in the parametrisation of the numerical simulations conducted in Section 4. Analysis performed in the following sections will make use of this simplified definition as phase saturation (S_α) is a time- and spatial-dependent prognostic field which is calculated along with pressure (p) and velocity (u α) fields.

In immiscible displacements, viscous fingering occurs when the viscosity ratio is greater than unity. As surface tension becomes weak, the interface is stressed and becomes unstable leading to the formation of fingers. At this point it should be mentioned that there are two parameters – Peclet number (Pe) and mobility ratio (MR), that determine the flow stability characteristics. There are always a few dominant fingers that spread and shield the growth of other fingers. The interface of the main finger collapses and starts splitting into new lobes of fingers. One of these new fingers may eventually outgrow the others and then spreads to occupy an increasingly larger width. In the process, the finger reaches a critical width while the saturation of its front becomes steep as a result of stretching caused by the cross-flow, causing the tip of the finger to become unstable and splits again, and the pattern repeats itself. Therefore, surface tension plays an essential dual role, it must be weak enough for the tip front to be unstable, but it is also the physical force causing the spreading and ensuing repeated branching (Tan and Homsy, 1988; Carvalho et al., 2013).

In the next section, the numerical formulation used to simulate multi-fluid flow in porous media is briefly validated (Section 4.2) against laboratory experiments (qualitative validation). The impact of VR

(quantitative validation) and heterogeneity on the onset instability and growth of fingers are numerically investigated in Section 4.3. Section 4.4 demonstrates the importance of an appropriate mesh resolution to adequately capture the initial stages of viscous fingers formation and development. Finally, flow pathway (channelling) is the focus of Section 4.5."

Now:

"(...) are both set to 2.

The MR expression (Eqn. 5) contains phase saturations, $S_w(x_i,t)$ and $S_{nw}(x_i,t)$ (Eqns. 6-7), however it is clear that during fluid displacement they change in time and space according to the global mass conservation equation. Assuming that phase viscosities remain invariant throughout the simulated fluid displacement, in this work, with no lack of generality, the MR can be replaced by the viscosity ratio (VR),

$$VR = \frac{\mu_i}{\mu_j},$$

which will be used in the parametrisation of the numerical simulations conducted in Section 4. Analysis performed in the following sections will make use of this simplified definition as phase saturation (S_α) is a time- and spatial-dependent field which is calculated along with pressure (p) and velocity ($\mathbf{u}_\alpha$).

In the next section, the numerical formulation used to simulate multi-fluid flow in porous media is briefly validated (Section 4.2) against laboratory experiments (qualitative validation). The impact of VR (quantitative validation) and heterogeneity on the onset of interface instability and growth of fingers are numerically investigated in Section 4.3. Section 4.4 demonstrates the importance of an appropriate mesh resolution to adequately capture the initial stages of viscous fingers formation and development. Finally, flow pathway (channelling) is the focus of Section 4.5."

R1.18: In section 4, again you mention that you don't use P1DGP1DG, but later on you do. What should I believe?

The authors acknowledge the confusion caused by the text in Section 4.1, thus

Before:

"In this work, all test-cases were performed in idealised geometries discretised with unstructured triangular and tetrahedral mesh using the P_1DG-P_2 and P_1DG-P_1 FE-pairs (Fig. 2) (...)"

Now:

"In this work, all test-cases were performed in idealised geometries discretised with unstructured triangular and tetrahedral mesh using the P_1DG-P_2, P_1DG-P_1 (Fig. 2, Sections 4.2-4.4) and P_1DG-P_1DG (Section 4.5) FE-pairs. (...)"

R1.19: In section 4.1 you mention that gravity is zero and the fluids are incompressible. What about Capillary pressure? It is important to mention if this is actually simulated or not, as it has a key effect in stabilising the growth of the fingering processes.

The authors are aware that capillary pressure may act as a diffusion term in the saturation equation (lumped with the Darcy equation) which can stabilise the system. The authors believe that it will have considerable impact in the formation & growth of fingers, in particular for anisotropic elements. However, in the simulations performed in this work, we assumed the flow as incompressible and neglected gravity and capillary pressure. Thus

Before:

"(...) Fluids are assumed incompressible and gravity was neglected. Initial (...)"

Now:

"(...) Fluids are assumed incompressible, also gravity and capillary pressure terms were neglected. Initial (...)"

R1.20: In 4.2 you present the sizes of the 2D domain in dimensionless units. If one unit is dimensionless then all of the parameters used should be dimensionless. Take a decision on this and stick with it, but currently the permeability values and times in seconds are meaningless.

Throughout the manuscript, units were corrected as suggested by the Reviewer.

R1.21: In 4.2 you mention that the pressure gradient is discontinuous across the interface. As far as I understand the pressure in porous media flow is never discontinuous. Moreover, in this case you are using a

formulation with continuous pressure, so the discretisation used would not allow to have a discontinuous pressure.
The authors acknowledge the confusion caused by the text in Section 4.2, thus
Before:

"(...) by a permeability value, Fig. 4(a). The spatial permeability distribution creates a rough pressure field, i.e., pressure gradient is discontinuous across the interface between different permeability zones, therefore the fluid flows to the larger permeability region.

A two-phase immiscible flow along homogeneous and parallel layers of contrasting petrophysical properties (i.e., permeability) is initially simulated. During the fluid displacement, crossflow between adjacent layers often occur due to viscous, capillary and/or gravitational forces that drive the flow. In this numerical simulation, crossflow is caused by viscous forces, which is commonly named viscous crossflow. Figures 4(b) and (c) show the continuous displacement of fluid 2 due to the injection of fluid 1. They also demonstrate the preferential flow path through high-permeability regions. Such flow behaviour, represented by the crossflow through the four regions, is in good qualitative agreement with experiments conducted by Dawe and Grattoni (2008)."

Now:

"(...) by a permeability value.

A two-phase immiscible flow along homogeneous and parallel layers of contrasting petrophysical properties (i.e., permeability) is initially simulated. During fluid displacement, crossflow between adjacent layers often occur due to viscous, capillary and/or gravitational forces in discontinuous spatial permeability distribution. Such preferential flows are shown in Figs. 3. In this case, crossflow behaviour from first to fourth quadrants of the domain is in good qualitative agreement with experiments conducted by Dawe and Grattoni (2008), Fig. 2(b)."

R1.22: Table 1. You are presenting here 6 columns of constant properties and the permeabilities are using a wrong format for the numbering as this is a document and not a computer code, the exponents should be written as 10 to the power of whichever number in superscript format. This should be reconsidered.
The table has been corrected according to the reviewer's suggestion.

R1.23: Figure 2 (A) is redundant; P1DGP2 is presented in better detail in Figure 1. I say Figure (A) because you mention triangle A and B while there are no letters below the figures. You should either add letters or reference to the different figures with right and left.
Authors thank the reviewer's suggestion and had merged Figs 1 and 2 and updated captions.

R1.24: In some figures you mention that some boundaries have no slip condition. In porous media flow you cannot have boundary conditions with slip condition so this information is unnecessary.
Authors acknowledge the problem. Mentions of no-slip boundary condition have been corrected throughout the manuscript's figures replacing "no slip condition" by "no-normal flux boundary condition".

R1.25: The scale in the figures should be with a white background, there is no need to waste ink! (In some figures you have done this already)
Authors thank the reviewer's suggestion and had updated all figures with coloured background.

Reviewer 2

Title: Numerical Investigation of Viscous Flow Instabilities in Multiphase Heterogeneous Porous Media
Authors: Christou et al.

This paper presents an interesting study on the numerical simulation of formation and growth of viscous fingers in heterogeneous porous media. The model is based on a control volume finite element method (CVFEM). 2D and 3D cases have been considered in this study and the dynamic mesh adaptivity is presented in the simulations. The paper is a potentially valuable work to be published in ADWR. However, I would like the authors to address the following concerns prior to acceptance for publication:
R2.1: The multiphase porous media model with CVFEM has been published in several papers by the authors and their collaborators. The authors should discuss the novelty of their paper and their contribution in this work. I found that most of the paper is about numerical simulation with different set up without detailed

discussion on the physics. As there is nothing new on the numerical aspect (as their method has been published in several papers), I would encourage the authors to focus on the physics of the problem here.
Authors thank the Reviewer's suggestion for highlighting the novelty of the work and to strength the focus on physics rather than numerical formulations. Authors have modified the Introduction (see **R1.1**) which outlined the novelty of this work. Also, changes in the text of Sections 3 and 4 (**R1.7** and **R1.16**) have strengthen the investigation of the physics aspect of viscous flow instabilities.

R2.2: For the model validation, there is a lack of quantitative comparison with any analytical solution or experiments.
Authors thank the Reviewer's suggestion, however model validation is a key aspect on this manuscript covering: (a) initial qualitative validation on preferential flows (Section 4.2); (b) quantitative validation of the model against semi-analytical solution for maximum finger width (Section 4.3). Also, in Section 4.3, growth and coalescence of dendritic finger branching obtained from the model is compared against analytical solutions (McLean and Saffman, 1981).

R2.3: There is no convergence study for fixed mesh in this paper. As the viscous finger is very sensitive to the mesh as pointed out by the authors, this should be included in the results and discussion.
Authors thank the Reviewer for pointing out that mesh convergence was missing in the manuscript. All simulations presented in this work were performed after solution mesh-independence assessments were conducted. Thus, in the second paragraph of Section 4.3,
Before:

> "(...) pressure gradient between source and sink regions was initially imposed to the system."

Now:

> "(...) pressure gradient between source and sink regions was initially imposed to the system. Solution mesh-independence was achieved through comparison of non-wetting phase saturation profiles along a line between source and sink regions (diagonal across the domain) with several mesh resolutions. Mesh convergence criterion was a maximum residual of 10^{-2} and it was achieved with approximately 3.5k elements (Fig. 7). Thus, all simulations shown in this section used mesh with resolution larger than 3546 P_1DG-P_2 triangular elements."

R2.4: The dynamic mesh adaptivity is used in the simulation. However, there is no discussion about the criterion for the mesh adaptivity.
Authors acknowledge that such information was missing in the original manuscript. We have included the criteria for dynamic mesh optimisation in the 3rd paragraph of Section 4.4, as outlined in **R1.7**.

R2.5: In section 4.5, the discussion mentioned that the simulation was using P1DG-P1DG element-pairs. However, the results for the continuous saturation P1DG-P1 are shown in the figures. The authors should include the results with the discontinuous formulation.
Authors apologise for the confusion, as a few figures of Section 4.5 were not properly uploaded during the submission. All simulations (fixed and adaptive mesh) shown in Section 4.5 were conducted using P_1DG-P_1DG elements.

R2.6: The symbol s_ua is in Eq. (3) is different from the symbol in Eq. (1).
The symbol has been corrected (**R1.2**).

R2.7: In Figure 13, looks like the figures (e) and (f) have been put in the wrong position.
Authors thank the Reviewer for pointing out this problem with the figures. Aforementioned pictures of Fig. 13 were swapped.

Reviewer 3

R3.1: The contribution of the paper is the use of a high order CVFEM technique with adaptive grids to solve unstable immiscible flow in heterogeneous porous media. I believe, the numerical method is excellent, but the physical problem solutions need improvement.
Authors thank the Reviewer's suggestion for strengthening the focus on the physics related to viscous flow instabilities. See **R2.1**.

R3.2: In standard nomenclature, Hele-Shaw cell is an open gap between two plates, not a porous medium between two plates. I believe, the authors are solving flow through a thin porous medium in most of the problem except in Section 4.5. These problems should not be called Hele-Shaw cell problems.

Authors thank the Reviewer note on the nature of Hele-Shaw cells. As indicated by the literature outlined in the manuscript (e.g., Saffman, 1959a; McLean & Saffman, 1959; Homsy, 1987; etc), single-/multi-phase flow in open-gap plates (HS cells) are excellent analogues for the study of 2D porous media flows governed by Darcy's law. Authors acknowledge that the unsteady HS cell problem (as stated by Crowdy (2006) [Quat. J. Mech. Appl. Math. 59:475-485] and Alimov (2006) [J. Appl. Math. Mech. 70:346-360]) was not the focus of this work and was not claimed to be. As such there is no mention of HS cell problem in the manuscript.

R3.3: Eq. 5 (Young-Laplace eq.) applies to open gap Hele-Shaw problems across interfaces. It applies to a single pore in porous media; but does not apply to macroscopic equations in porous media. In porous media, the saturations in macroscopic equations do not go sharply from phase 1 to 2. What we develop are diffused saturation fronts. Eq. 5 does not apply across these fronts.

Authors acknowledge the confusing text that precedes Young-Laplace equation (Eqn. 6) as the paragraph starts mentioning heterogeneous porous media domain and ends with Eqn. 6 relating capillary pressure and surface tension. As pointed out by the Reviewer, Eqn. 6 applies to interfaces between immiscible fluids in equilibrium and could not, in this format, be used to describe the pressure balance across the interface of fluids in motion through porous media. Most of Section 3 was rewritten (Equations 5 and 6 were removed) to ensure that text is more fluid and clearer as described in **R1.16-17**.

R3.4: For porous media, one needs relative permeability terms as well as macroscopic capillary pressure terms as a function of saturation. What capillary pressure function was used? Was it neglected?

The authors are aware that capillary pressure may act as a diffusion term in the saturation equation (lumped with the Darcy equation) which can stabilise the system. The authors believe that it will have considerable impact in the formation & growth of fingers, in particular for anisotropic elements. However, in the simulations performed in this work, we assumed the flow as incompressible and neglected gravity and capillary pressure. Thus, at the end of the second paragraph of Section 4.1,

Before:

> "(...) Fluids are assumed incompressible and gravity was neglected. Initial (...)"

Now:

> "(...) Fluids are assumed incompressible, also gravity and capillary pressure terms were neglected. Initial (...)"

R3.5: The application of no-slip equation on the side boundaries is incorrect for porous media flow.

Authors acknowledge the problem. Mentions of no-slip boundary condition have been corrected throughout the manuscript's figures replacing "no slip condition" by "no-normal flux boundary condition".

R3.6: Eq. 10: What is the value of b? Assuming b=1 mm, Nc' is too high. Nc >1 in the problems solved. Typical Nc in water flood of reservoirs is 10e-7. Velocity of the order of 1 cm/s is too high.

The authors acknowledge that 'b' was not included in the text, thus, in the first paragraph of page 11

Before:

> "(...) Here, numerical simulations performed with viscosity ratio of 10 and 150 indicated maximum fingers width of approximately 0.45-0.70 and 0.50-0.90 cm (Fig. 7 a and b), respectively (...)"

Now:

> "(...) Here, numerical simulations performed with viscosity ratio of 10 and 150 indicated maximum fingers width of approximately 0.45-0.70 and 0.50-0.90 cm (Fig. 8 a and b), respectively (assuming b = 1 mm) (...)"

In the cases analysed in Section 4.3, the authors focused on quantitative validation of the model formulation summarised in Section 2 (and the references within). Capillary number (N_c, as defined in Eqn. 5 of the original manuscript) is not a parameter currently used in the simulations (as it does not naturally appear in the solution of the differential equations, Eqns. 1-2). A brief calculation of N_c, assuming surface tension of the order 70 dynes/cm, leads to capillary number of the order 10^{-7}-10^{-6}, depending on the interface velocity (i.e., finger tip velocity) obtaining during the simulation. A range of the listed modified capillary number (defined in the fourth paragraph of Section 4.3), $N_c^{'}$ = [632,6.32x10^7] was used Guan & Pitchumani (2003) in the parametrisation of their numerical experiments. With respect to the order of magnitude of the inlet velocity used in the simulations reported in the manuscript, the authors acknowledge that the order is much higher than commonly used in reservoir simulation (of the order of 10^{-4}-10^{-3} cm/s). Values used in the simulations were

chosen to speed-up the results, however authors ensured that the flow regime was still Darcean (i.e., $Re=10^{-4}$-10^{-3}).

R3.7: Describe the term $S_{u\alpha}$, in Eq. 1.
The authors acknowledge that the term was not properly defined. Please see **R1.2** with the modifications introduced to address this issue.

R3.8: Fig. 4b looks OK, but Fig. 4c looks strange for an immiscible flood in porous media. Show a comparison with experimental figures of Dawe & Grattoni (2008).
Section 4.2 aims to validate the model against experiments. Immiscible displacements experiments from Dawe & Grattoni (2008) were used to qualitatively validate the model (i.e., flow behaviour during flow across regions of permeability gradient). The authors acknowledge that Figure 4c was not as appealing as Fig. 4b, as in this case the flow already evolved in time (t=3000 time-units). Flow displacement showed here was obtained from the model formulation (Section 2) with prescribed initial and boundary conditions (some of them are listed in Table 1). As the proposed numerical formulation is based on the two-fluids approach in which each phase/fluid is described by a set of (continuity and momentum) differential equations, with common terms appearing in each set, representing transfer of mass and/or momentum. In the simulations described in this manuscript, we assume that displacements are immiscible, i.e., there is no mass transfer across the interface of phases/fluids (and the simulation assumes no gravity forces or capillary pressure). Fluid displacement simulated in Section 4.2 assumes prescribed irreducible wetting and residual non-wetting phase saturations, as listed in Table 1. Thus, the smearing saturation shown in Figure 4c represents displacement but taking into account remaining fluids in the domain (described by irreducible wetting and residual non-wetting phase saturations). Changes in this section were reported in **R1.21**. In order to avoid such confusion wrt to simulated flow visualisation, authors changed Fig. 4 (now Figs. 2 and 3), in which Fig. 2 contains a sketch of the simulated domain and the set of saturation profile from Dawe & Grattoni (2008). Figure 3 contains snapshots of our simulations in which we have changed the saturation range limits for visualisation purposes.

R3.9: Consider adding the following references: -Doorwar, S. & Mohanty, K. K., "Extension for Dielectric Breakdown Model for Simulation of Viscous Fingering at Finite Viscosity Ratios," *Phys. Rev. E.*, 90 (1) (2014). DOI: 10.1103/PhysRevE.90.013028; Doorwar, S. and Mohanty, K. K., "Fingering Function for Unstable Immiscible Flows," SPE 173290-PA, SPE Journal, (July, 2016). http://dx.doi.org/10.2118/173290-PA.; Luo, H., Mohanty, K.K., Delshad, M., Pope, G. A. (2016). Modeling and Upscaling Unstable Water and Polymer Floods: Dynamic Characterization of the Effective Viscous Fingering. SPE 179648-PA, SPEREE, accepted October 6, 2016.
The authors thank the Reviewer for the excellent literature suggestions and some of them were introduced in the Introduction (see **R1.1**).