softmax

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0.1 This is the softmax workbook for ECE C147/C247 Assignment #2

Please follow the notebook linearly to implement a softmax classifier.

Please print out the workbook entirely when completed.

We thank Serena Yeung & Justin Johnson for permission to use code written for the CS 231n class (cs231n.stanford.edu). These are the functions in the cs231n folders and code in the jupyer notebook to preprocess and show the images. The classifiers used are based off of code prepared for CS 231n as well.

The goal of this workbook is to give you experience with training a softmax classifier.

```
In [1]: import random
        import numpy as np
        from cs231n.data_utils import load_CIFAR10
        import matplotlib.pyplot as plt

        %matplotlib inline
        %load_ext autoreload
        %autoreload 2
```

```
In [2]: def get_CIFAR10_data(num_training=49000, num_validation=1000, num_test=1000, num_dev=50
            """
            Load the CIFAR-10 dataset from disk and perform preprocessing to prepare
            it for the linear classifier. These are the same steps as we used for the
            SVM, but condensed to a single function.
            """
            # Load the raw CIFAR-10 data
            cifar10_dir = './cifar-10-batches-py' # You need to update this line
            X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

            # subsample the data
            mask = list(range(num_training, num_training + num_validation))
            X_val = X_train[mask]
            y_val = y_train[mask]
            mask = list(range(num_training))
            X_train = X_train[mask]
            y_train = y_train[mask]
            mask = list(range(num_test))
            X_test = X_test[mask]
```

```python
        y_test = y_test[mask]
        mask = np.random.choice(num_training, num_dev, replace=False)
        X_dev = X_train[mask]
        y_dev = y_train[mask]

        # Preprocessing: reshape the image data into rows
        X_train = np.reshape(X_train, (X_train.shape[0], -1))
        X_val = np.reshape(X_val, (X_val.shape[0], -1))
        X_test = np.reshape(X_test, (X_test.shape[0], -1))
        X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

        # Normalize the data: subtract the mean image
        mean_image = np.mean(X_train, axis = 0)
        X_train -= mean_image
        X_val -= mean_image
        X_test -= mean_image
        X_dev -= mean_image

        # add bias dimension and transform into columns
        X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
        X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
        X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
        X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

        return X_train, y_train, X_val, y_val, X_test, y_test, X_dev, y_dev


    # Invoke the above function to get our data.
    X_train, y_train, X_val, y_val, X_test, y_test, X_dev, y_dev = get_CIFAR10_data()
    print('Train data shape: ', X_train.shape)
    print('Train labels shape: ', y_train.shape)
    print('Validation data shape: ', X_val.shape)
    print('Validation labels shape: ', y_val.shape)
    print('Test data shape: ', X_test.shape)
    print('Test labels shape: ', y_test.shape)
    print('dev data shape: ', X_dev.shape)
    print('dev labels shape: ', y_dev.shape)
```

```
Train data shape:  (49000, 3073)
Train labels shape:  (49000,)
Validation data shape:  (1000, 3073)
Validation labels shape:  (1000,)
Test data shape:  (1000, 3073)
Test labels shape:  (1000,)
dev data shape:  (500, 3073)
dev labels shape:  (500,)
```

0.2 Training a softmax classifier.

The following cells will take you through building a softmax classifier. You will implement its loss function, then subsequently train it with gradient descent. Finally, you will choose the learning rate of gradient descent to optimize its classification performance.

```
In [3]: from nndl import Softmax
```

```
In [4]: # Declare an instance of the Softmax class.
        # Weights are initialized to a random value.
        # Note, to keep people's first solutions consistent, we are going to use a random seed

        np.random.seed(1)

        num_classes = len(np.unique(y_train))
        num_features = X_train.shape[1]

        softmax = Softmax(dims=[num_classes, num_features])
```

Softmax loss

```
In [5]: ## Implement the loss function of the softmax using a for loop over
        #  the number of examples

        loss = softmax.loss(X_train, y_train)
```

```
In [6]: print(loss)
```

```
2.3277607028048966
```

0.3 Question:

You'll notice the loss returned by the softmax is about 2.3 (if implemented correctly). Why does this make sense?

0.4 Answer:

Since our weights are random, we expect our model to guess the labels correctly 1/10 of the time. That means that the proportion in the softmax function will be ~1/10. Our loss function is $-log(softmax)$, so we get $-log(\frac{1}{10}) \approx 2.3$

Softmax gradient

```
In [7]: ## Calculate the gradient of the softmax loss in the Softmax class.
        # For convenience, we'll write one function that computes the loss
        #   and gradient together, softmax.loss_and_grad(X, y)
        # You may copy and paste your loss code from softmax.loss() here, and then
        #   use the appropriate intermediate values to calculate the gradient.
```

```
loss, grad = softmax.loss_and_grad(X_dev,y_dev)

# Compare your gradient to a gradient check we wrote.
# You should see relative gradient errors on the order of 1e-07 or less if you impleme
softmax.grad_check_sparse(X_dev, y_dev, grad)
```

```
numerical: 0.332009 analytic: 0.332009, relative error: 8.056897e-08
numerical: 0.543674 analytic: 0.543674, relative error: 3.675417e-08
numerical: -0.834213 analytic: -0.834213, relative error: 3.539436e-08
numerical: 2.460479 analytic: 2.460479, relative error: 5.566823e-09
numerical: -0.788769 analytic: -0.788769, relative error: 1.392953e-08
numerical: 0.832805 analytic: 0.832805, relative error: 2.331392e-08
numerical: 0.044043 analytic: 0.044043, relative error: 9.778745e-07
numerical: -1.186865 analytic: -1.186865, relative error: 2.435639e-09
numerical: 1.510502 analytic: 1.510502, relative error: 4.114666e-09
numerical: -2.932045 analytic: -2.932046, relative error: 1.727180e-08
```

0.5 A vectorized version of Softmax

To speed things up, we will vectorize the loss and gradient calculations. This will be helpful for stochastic gradient descent.

```
In [8]: import time
```

```
In [9]: ## Implement softmax.fast_loss_and_grad which calculates the loss and gradient
        #    WITHOUT using any for loops.

        # Standard loss and gradient
        tic = time.time()
        loss, grad = softmax.loss_and_grad(X_dev, y_dev)
        toc = time.time()
        print('Normal loss / grad_norm: {} / {} computed in {}s'.format(loss, np.linalg.norm(g

        tic = time.time()
        loss_vectorized, grad_vectorized = softmax.fast_loss_and_grad(X_dev, y_dev)
        toc = time.time()
        print('Vectorized loss / grad: {} / {} computed in {}s'.format(loss_vectorized, np.lina

        # The losses should match but your vectorized implementation should be much faster.
        print('difference in loss / grad: {} /{} '.format(loss - loss_vectorized, np.linalg.no

        # You should notice a speedup with the same output.
```

```
Normal loss / grad_norm: 2.350458377628373 / 312.42133095774625 computed in 0.2377328872680664
Vectorized loss / grad: 2.3504583776283714 / 312.4213309577462 computed in 0.0142881870269775
difference in loss / grad: 1.7763568394002505e-15 /2.8663620561962255e-13
```

0.6 Stochastic gradient descent

We now implement stochastic gradient descent. This uses the same principles of gradient descent we discussed in class, however, it calculates the gradient by only using examples from a subset of the training set (so each gradient calculation is faster).

0.7 Question:

How should the softmax gradient descent training step differ from the svm training step, if at all?

0.8 Answer:

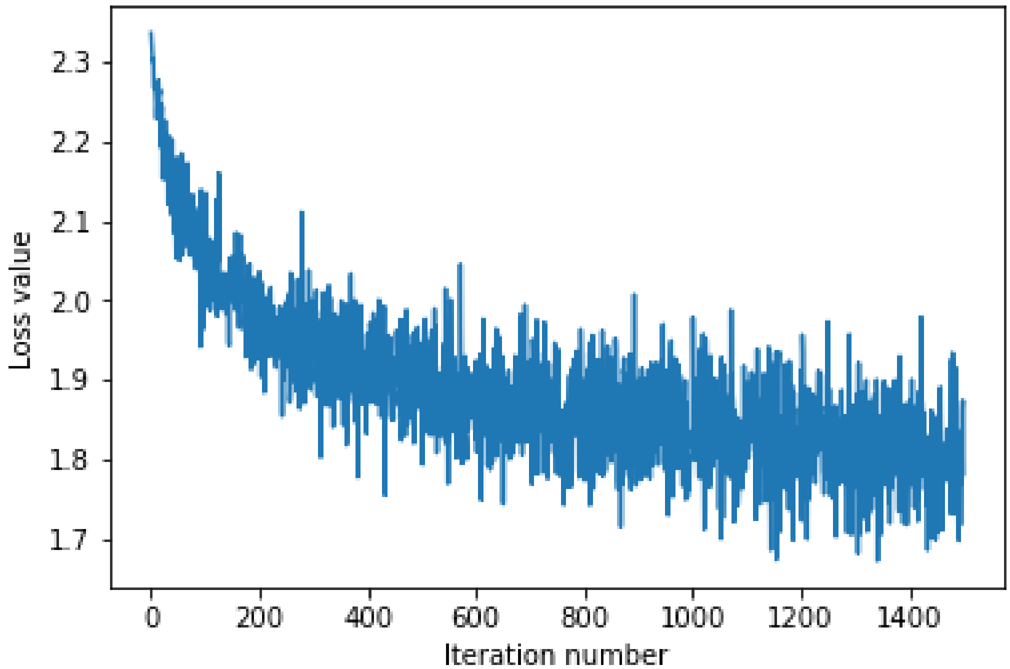
The softmax gradient descent training step should be identical to the svm training step.

In [10]:
```
# Implement softmax.train() by filling in the code to extract a batch of data
# and perform the gradient step.
import time


tic = time.time()
loss_hist = softmax.train(X_train, y_train, learning_rate=1e-7,
                          num_iters=1500, verbose=True)
toc = time.time()
print('That took {}s'.format(toc - tic))

plt.plot(loss_hist)
plt.xlabel('Iteration number')
plt.ylabel('Loss value')
plt.show()
```

```
iteration 0 / 1500: loss 2.3365926606637544
iteration 100 / 1500: loss 2.0557222613850827
iteration 200 / 1500: loss 2.0357745120662813
iteration 300 / 1500: loss 1.9813348165609888
iteration 400 / 1500: loss 1.9583142443981612
iteration 500 / 1500: loss 1.862265307354135
iteration 600 / 1500: loss 1.8532611454359382
iteration 700 / 1500: loss 1.8353062223725827
iteration 800 / 1500: loss 1.829389246882764
iteration 900 / 1500: loss 1.899215853035748
iteration 1000 / 1500: loss 1.97835035402523
iteration 1100 / 1500: loss 1.8470797913532633
iteration 1200 / 1500: loss 1.8411450268664082
iteration 1300 / 1500: loss 1.7910402495792102
iteration 1400 / 1500: loss 1.8705803029382257
That took 15.245999097824097s
```



0.8.1 Evaluate the performance of the trained softmax classifier on the validation data.

```
In [11]: ## Implement softmax.predict() and use it to compute the training and testing error.

         y_train_pred = softmax.predict(X_train)
         print('training accuracy: {}'.format(np.mean(np.equal(y_train,y_train_pred), )))
         y_val_pred = softmax.predict(X_val)
         print('validation accuracy: {}'.format(np.mean(np.equal(y_val, y_val_pred)), ))
```

```
training accuracy: 0.3811428571428571
validation accuracy: 0.398
```

0.9 Optimize the softmax classifier

You may copy and paste your optimization code from the SVM here.

```
In [12]: np.finfo(float).eps
```

```
Out[12]: 2.220446049250313e-16
```

```
In [13]: # ================================================================ #
         # YOUR CODE HERE:
         #   Train the Softmax classifier with different learning rates and
         #     evaluate on the validation data.
```

```python
#   Report:
#     - The best learning rate of the ones you tested.
#     - The best validation accuracy corresponding to the best validation error.
#
#   Select the SVM that achieved the best validation error and report
#     its error rate on the test set.
# ================================================================ #

learning_rates = [1e-4, 1e-3, 1e-2, 5e-2, 0.1, 0.25, 0.5]
val_accs = np.zeros(len(learning_rates))
for i in range(len(learning_rates)):
    softmax.train(X_train, y_train, learning_rate=learning_rates[i], num_iters=1500, v
    val_accs[i] = np.mean(np.equal(y_val, softmax.predict(X_val)))

best_learning_rate = learning_rates[np.argmax(val_accs)]
best_val_acc = np.max(val_accs)

print('best learning rate: {}'.format(best_learning_rate))
print('best validation accuracy: {}'.format(best_val_acc))

softmax.train(X_train, y_train, learning_rate=best_learning_rate, num_iters=1500, verb
test_pred = softmax.predict(X_test)
test_error = 1 - np.mean(np.equal(y_test, test_pred))

print('final test error rate: {}'.format(test_error))

# ================================================================ #
# END YOUR CODE HERE
# ================================================================ #
```

```
/Users/edwardzhang/Desktop/ece247/HW2/HW2-code/nndl/softmax.py:131: RuntimeWarning: divide by z
  loss = np.sum(np.log(np.sum(np.exp(a).T, axis=0)) - a[np.arange(num_train), y]) / num_train
/Users/edwardzhang/Desktop/ece247/HW2/HW2-code/nndl/softmax.py:135: RuntimeWarning: invalid val
  softmax = ea / sums[:, np.newaxis]


best learning rate: 0.0001
best validation accuracy: 0.297
final test error rate: 0.6719999999999999
```