Fifty years of the bottleneck model: A bibliometric review and future research directions

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A B S T R A C T

The bottleneck model introduced by Vickrey in 1969 has been recognized as a benchmark representation of the peak-period traffic congestion due to its ability to capture the essence of congestion dynamics in a simple and tractable way. This paper aims to provide a 50th anniversary review of the bottleneck model research since its inception. A bibliometric analysis approach is adopted for identifying the distribution of all journal publications, influential papers, top contributing authors, and leading topics in the past half century. The literature is classified according to recurring themes into travel behavior analysis, demand-side strategies, supply-side strategies, and joint strategies of demand and supply sides. For each theme, typical extended models developed to date are surveyed. Some potential directions for further studies are discussed.

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1. Introduction

The bottleneck model was first introduced by Vickrey in 1969, aiming at addressing the departure time choices of commuters on a bottleneck-constrained highway during the morning rush hours. In this model, all individuals are assumed to have an identical preferred time to arrive at their destination and incur a schedule delay cost proportional to the amount of time that they arrive early or late. Commuters choose their departure time to minimize their own travel cost based on a trade-off between bottleneck congestion delay cost and schedule delay cost of early or late arrival. This model is able to model the formation and dissipation of queuing behind the bottleneck in a simple and tractable way, thus making it a benchmark representation of the dynamics of traffic congestion in peak period.

The past 50 years (from 1969 to 2019) have witnessed significant progress in the bottleneck model research since the pioneering work of Vickrey (1969). A lot of insights into understanding the features of traffic congestion in peak period have been obtained via the bottleneck model. These insights cover various aspects, such as behavioral analysis (e.g., the nature of shifting peak, inefficiency of unpriced equilibria, behavioral difference of heterogeneous commuters, connection between morning and evening commutes, effects of commuter scheduling preferences), demand management (e.g., congestion / emission / parking pricing and tradable credit schemes, relationship between bottleneck congestion tolling and urban structure), and supply management (e.g., bottleneck / parking capacity expansion). The insights also play an important role

in deeply understanding the essence of commuters' travel behavior during morning/evening peak periods, and in evaluating and making reasonable transport policies for alleviating peak-period traffic congestion.

To date, there have been a few reviews on the topic of bottleneck models or their variations (e.g., Arnott et al., 1998; Lindsey and Verhoef, 2001; Small, 1992; Small, 2015). These early reviews appeared in different years, aiming to track the development of the bottleneck models on some selected specific topics. Because the research area is still growing and new disruptive trends of automation and sharing in mobility are emerging, it is timely to provide a state-of-the-art review of this area, particularly on the occasion of its 50th anniversary. This paper attempts to provide a systematic and critical review that differs from previous reviews in several aspects. First, it is meaningful to conduct a bibliometric study of the large body of literature to celebrate the 50th anniversary of Vickrey's bottleneck model. To do so, we carry out a literature review to analyze the research progress on the bottleneck model research throughout the past half century. The review tries to cover all relevant topics published in journals rather than some specific ones as covered in the previous review papers. Second, a bibliometric analysis approach is adopted that can trace the footprints underlying the scholarly publications by constructing network connections of the publications, journals, researchers, and keywords. With the aid of visualization technique (e.g., a software called VOSviewer), the bibliometric approach can map the landscape of the knowledge domain of the bottleneck model studies, allowing us to clearly identify the distribution of publications by journal, influential papers, top contributing authors, and leading topics. Third, based on the bibliometric analysis, a critical review on the previous relevant studies is provided, together with some discussions on the current research gaps and opportunities. It is noted that a bottleneck system consists of the following elements: users, the authority (or the government), and the bottleneck (i.e., transport infrastructure). From the perspectives of these elements, we categorize the literature into four classes: travel behavior analysis, demand-side strategies, supply-side strategies, and joint strategies of demand and supply sides. The travel behavior analysis from the users' perspective focuses on the equilibrium analysis of commuters' travel choice behavior, such as the choices of departure time, route, mode, and/or parking. The demand-side strategies from the government's perspective refer to the travel demand management strategies, such as congestion / emission / parking pricing and tradable credit schemes. The supply-side strategies from the transport infrastructure's perspective include such topics as bottleneck capacity expansion and parking capacity design. The joint strategies of demand and supply sides from both the government's and transport infrastructure's perspectives are a hybrid of demand-side and supply-side strategies. For each theme, typical models proposed in previous studies are reviewed.

The remainder of this paper is organized as follows. In the next section, a bibliometric study is conducted. Section 3 presents a literature review based on the literature categorization. In Section 4, some potential directions for further studies are discussed. Finally, Section 5 concludes the paper.

2. Bibliometric analysis

This section provides a general bibliometric analysis of various bottleneck model studies. The bibliometric analysis uses quantitative methods to classify bibliometric data and build up representative summaries. It has been recognized as a useful approach for analyzing the performances of journals, institutes and authors, as well as the characteristics of research fields or topics. With the aid of visualization technique (e.g., VOSviewer software), the bibliometric networks, such as co-citation network, co-authorship network, and keyword co-occurrence network, can be constructed and visually presented. To measure the influences of publications, authors and journals, various bibliometric indicators are considered, including the number of publications, total citations, and citations per paper.

In order to collect the publication data since 1969, we scout the three well-recognized journal databases or search engines, namely Web of Science Core Collection, Scopus, and Google Scholar, using such topics or keywords as bottleneck, bottleneck model(s), morning commute or commuting, and bottleneck congestion. We further retrieve literature by tracking the references cited by the papers searched from the three databases. In particular, we check all references citing the original work of Vickrey (1969), entitled "congestion theory and transport investment". After repeated sifting and checking, a total of 232 relevant papers during the period of 1969–2019 are finally retrieved.

Fig. 1 shows the distribution of the papers published during the period of 1969–2019 by time. It can be seen that little attention was paid to the bottleneck model during the first 20 years of 1969–1989, with only 15 relevant papers published. During the 20 years of 1990–2009, this topic received growing attention, with a total of 68 relevant papers published, and the number of relevant publications per five years exceeds 10, more than the total number of publications during the first 15 years (1969–1984). During the past 10 years from 2010 to 2019, this topic attracted further increasing interest, and a total of 149 relevant papers were published, accounting for 64.2% of total number of publications in the past 50 years. Particularly, the largest amount emerges in the most recent 5 years of 2015–2019, with 82 publications (about 35.3% of total number of publications). This continued growing tendency clearly shows that the bottleneck model is still an important and hot research topic in the field of transportation and this tendency is expected to continue in coming years.

Table 1 shows the top 15 journals (over a total of 38 journals) by the number of published related papers. It can be seen that these journals mainly belong to "transportation" and "economics" categories in terms of the journal categories in the JCR (Journal Citation Reports) published by Thomson Reuters. Transportation Research Part B (TR-B for short, a leading journal in transportation field) leads Table 1 with 75 papers (accounting for 32.3% of the total number of publications), followed by Journal of Urban Economics (JUE, a leading journal in urban economics field) with 27 papers (accounting for 11.6%). The total percentage of papers published in these two journals (a total of 102 papers) reaches nearly half of the total
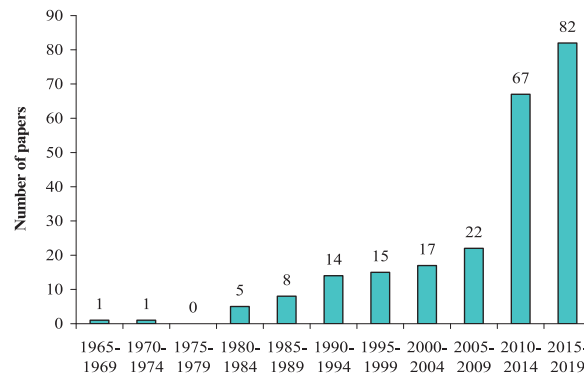
Fig. 1. The number of papers per five years during the period of 1969–2019.

Table 1

The top 15 journals by the number of published related papers.

Rank	Journal	Total number of related papers
1	Transportation Research Part B	75
2	Journal of Urban Economics	27
3	Transportation Science	14
4	Transportation Research Part A	13
5	Transportation Research Part C	10
6	Economics of Transportation	9
7	Transportation Research Record	9
8	Transportation Research Part E	8
9	Regional Science and Urban Economics	7
10	Journal of Transport Economics and Policy	6
11	American Economic Review	5
12	Transportmetrica A: Transport Science	5
13	Applied Economics	4
14	Journal of Public Economics	4
15	Transportmetrica B: Transport Dynamics	4

number of publications (about 44.0%). The number of papers published in each of Transportation Science (TS), Transportation Research Part A and Part C (TR-A, TR-C) reaches 10 or more. Notably, as a young journal founded in 2012, Economics of Transportation published 9 papers on this topic.

In order to look at the co-relation among journals publishing the 232 publications, a bibliographic coupling of the 38 journals is conducted, as shown in Fig. 2. The size of a solid circle (or a vertex) represents the number of publications related to the topic of the bottleneck model in a journal. The line between circles represents the co-citation relationship of journals. The color of the line represents the cluster of journals, such as the journal categories of economics or transportation. The width of the lines between circles represents the co-citation degree or intensity between journals (i.e., the total number of co-citations of the documents in the journals concerned). Specifically, a thick line means a strong co-citation degree between journals, and vice versa. It can be seen that the papers published in TR-B have highly been cited by those published in TR-A, TR-C, TR-E, JUE, TS, TRR (Transportation Research Record), Economics of Transportation, Transportmetrica A, JTEP (Journal of Transport Economics and Policy), Networks and Spatial Economics, and Journal of Public Economics. The papers published in JUE have a strong citation with those published in TR-B, RSUE (Regional Science and Urban Economics), and Economics of Transportation.

We now look at the most influential papers about the topic of the bottleneck model during the past five decades, which are determined according to total citations or average citations per year. It should be pointed out that in this paper, citation count is based on the SCI/SSCI citation databases. Here, SCI/SSCI means Science Citation Index Expanded and Social Science Citation Index in the Web of Science Core Collection. Table 2 shows the top 50 most influential papers, each having more than 50 citations. It can be noted that the top 3 most influential papers are Vickrey (1969), Small (1982) and ADL (1993a) in terms of the total number of citations. Here, "ADL" respectively refers to the first letters of the surnames of three scholars, i.e., Arnott R, de Palma A, and Lindsey R. All the 3 most cited papers are from American Economic Review (AER), which is a well-recognized top economics journal. Particularly, the pioneering work of Vickrey (1969) is the most influential paper with the highest total citations of 944, and the highest average citations of 18.51 per year. There are 18 papers, each having more than 100 citations, and 9 papers, each having an average of no less than 10 citations per year. It should be pointed out that the work of Fosgerau and Karlstrom (2010), entitled "the value of reliability", has had a total of 140 citations, and a second ranking in terms of average citations per year, regardless of its short publication history. 15 out of the top 50 papers are from TR-B, 8 from TS, 5 from JUE, and 5 from TR-A. In terms of total citations, 5 out of the top 10 most influential papers were

Table 2
The top 50 most cited papers.

Rank by TC	Journal	Total citations (TC)	Title	Author/s/Year	Citations per year (CPY)	Rank by CPY
1	American Economic Review	944	Congestion theory and transport investment	Vickrey (1969)	18.88	1
2	American Economic Review	570	The scheduling of consumer activities: work trips	Small (1982)	15.41	4
3	American Economic Review	402	A structural model of peak-period congestion: a traffic bottleneck with elastic demand	Arnott, de Palma, Lindsey (1993a)	15.46	3
4	Journal of Urban Economics	283	Economics of a bottleneck	Arnott, de Palma, Lindsey (1990a)	9.76	10
5	Transportation Research Part A	269	Dynamic network models and driver information systems	Ben-Akiva, de Palma, Kaysi (1991)	9.61	12
6	Transportation Research Record	184	Travel-time uncertainty, departure time choice, and the cost of morning commutes	Noland and Small, (1995)	7.67	17
7	Transportation Science	174	Schedule delay and departure time decisions in a deterministic model	Hendrickson, Kocur (1981)	4.58	28
8	Transportation Research Part B	167	Departure time and route choice for the morning commute	Arnott, de Palma, Lindsey (1990b)	5.76	23
9	Journal of Transport Economics and Policy	162	The welfare effects of congestion tolls with heterogeneous commuters	Arnott, de Palma, Lindsey (1994)	6.48	21
10	Transportation Research Part A	162	Does providing information to drivers reduce traffic congestion	Arnott, de Palma, Lindsey (1991a)	5.79	22
11	Transportation Research Part B	140	The value of reliability	Fosgerau, Karlstrom (2010)	15.56	2
12	Transportation Science	132	Existence of a time-dependent equilibrium distribution of arrivals at a single bottleneck	Smith (1984)	3.77	36
13	Transportation Science	126	Uniqueness of a time-dependent equilibrium distribution of arrivals at a single bottleneck	Daganzo (1985)	3.71	37
14	Econometrica	125	Congestion pricing and capacity of large hub airports – a bottleneck model with stochastic queues	Daniel (1995)	5.21	25
15	Transportation Science	119	Existence, uniqueness, and trip cost function properties of user equilibrium in the bottleneck model with multiple user classes	Lindsey (2004)	7.93	15
16	Journal of Public Economics	108	A temporal and spatial equilibrium analysis of commuter parking	Arnott, de Palma, Lindsey (1991b)	3.86	34
17	Transportation Science	108	Dynamic model of peak period traffic congestion with elastic arrival rates	Ben-Akiva, de Palma, Kanaroglou (1986)	3.27	38
18	Annals of Regional Science	101	Private toll roads: competition under various ownership regimes	de Palma, Lindsey (2000)	5.32	24
19	Transportation Research Part B	99	Analysis of the time-varying pricing of a bottleneck with elastic demand using optimal control theory	Yang, Huang (1997)	4.50	29
20	Journal of Urban Economics	98	Peak-load pricing of a transportation route with an unpriced substitute	Braid (1996)	4.26	31
21	Transportation Science	98	Morning commute for nonidentical travelers	Newell (1987)	3.06	40
22	Regional Science and Urban Economics	93	Simulating travel reliability	Noland, Small, Koskenoja, Chu (1998)	4.43	30
23	Transportation Research Part B	87	Value of time by time of day: a stated-preference study	Tseng, Verhoef (2008)	7.91	16

Table 2 (*continued*)

Rank by TC	Journal	Total citations (TC)	Title	Author/s/Year	Citations per year (CPY)	Rank by CPY
24	Transportation Research Part B	81	Integrated daily commuting patterns and optimal road tolls and parking fees in a linear city	Zhang, Huang, Zhang (2008)	7.36	19
25	Highway Research Record	81	Pricing, metering, and efficiently using urban transportation facilities	Vickrey (1973)	1.76	50
26	Transportation Research Part B	80	The value of travel time variance	Fosgerau, Engelson (2011)	10.00	8
27	Transportation Research Part A	80	Queueing at a bottleneck with single- and multi-step tolls	Laih (1994)	3.20	39
28	Transportation Research Part E	78	Fares and tolls in a competitive system with transit and highway: the case with two groups of commuters	Huang (2000)	4.11	32
29	Regional Science and Urban Economics	78	Route choice with heterogeneous drivers and group-specific congestion costs	Arnott, de Palma, Lindsey (1992)	2.89	41
30	European Economic Review	76	Information and time-of-usage decisions in the bottleneck model with stochastic capacity and demand	Arnott, de Palma, Lindsey (1999)	3.80	35
31	Journal of Urban Economics	74	Bottleneck congestion and modal split	Tabuchi (1993)	2.85	42
32	Transportation Research Part B	73	Managing rush hour travel choices with tradable credit scheme	Nie, Yin (2013)	12.17	5
33	Transportation Science	72	Stochastic equilibrium model of peak period traffic congestion	de Palma, Ben-Akiva, Lefevre, Litinas (1983)	2.00	45
34	Transportation Research Part B	71	Improving travel efficiency by parking permits distribution and trading	Zhang, Yang, Huang (2011)	8.88	14
35	Economics of Transportation	70	Valuation of travel time	Small (2012)	10.00	9
36	Transportation Research Part A	70	Integrated scheduling of daily work activities and morning-evening commutes with bottleneck congestion	Zhang, Yang, Huang, Zhang (2005)	5.00	26
37	Transportation Research Part B	68	Morning commute with competing modes and distributed demand: User equilibrium, system optimum, and pricing	Gonzales, Daganzo (2012)	9.71	11
38	Transportation Research Part B	66	Dynamic model of peak period congestion	Ben-Akiva, Cyna, de Palma (1984)	1.89	47
39	Transportation Research Part B	64	On the morning commute problem with bottleneck congestion and parking space constraints	Yang, Liu, Wang, Zhang (2013)	10.67	6
40	Journal of Urban Economics	64	Uniform versus peak-load pricing of a bottleneck with elastic demand	Braid (1989)	2.13	44
41	Transportation Research Part B	62	Managing bottleneck congestion with tradable credits	Xiao, Qian, Zhang (2013)	10.33	7
42	Transportation Research Record	62	Schedule delay and departure time decisions with heterogeneous commuters	Arnott, de Palma, Lindsey (1988)	2.00	46
43	International Journal of Transportation Economics	58	Commuter welfare under peak-period congestion tolls: who gains and who loses	Cohen (1987)	1.81	49
44	Transportation Research Part E	57	Tradable credit schemes for managing bottleneck congestion and modal split with heterogeneous users	Tian, Yang, Huang (2013)	9.50	13

Table 2 (continued)

Rank by TC	Journal	Total citations (TC)	Title	Author/s/Year	Citations per year (CPY)	Rank by CPY
45	Journal of Urban Economics	57	Endogenous trip scheduling: the Henderson approach reformulated and compared with the Vickrey approach	Chu, (1995)	2.38	43
46	Transportation Science	57	Traffic flow for the morning commute	Newell, (1988)	1.84	48
47	Transportation Research Part B	56	Congestion tolling in the bottleneck model with heterogeneous values of time	van den Berg, Verhoef (2011a)	7.00	20
48	Transportation Research Part A	56	Congestion pricing on a road network: a study using the dynamic equilibrium simulator METROPOLIS	de Palma, Kilani, Lindsey (2005)	4.00	33
49	Transportation Research Part B	55	Equilibrium properties of the morning peak-period commuting in a many-to-one mass transit system	Tian, Huang, Yang (2007)	4.58	27
50	Transportation Research Part B	53	Managing morning commute traffic with parking	Qian, Xiao, Zhang (2012)	7.57	18

Table 3
The top 23 leading authors with the number of publications ≥ 5.

Rank by TP	Full name	Institute and country/area	TP (total publications)	TC (total citations)	TC/TP	Rank by TC/TP
1	de Palma A.	Univ Paris Saclay, France	31	2185	70.48	4
2	Lindsey R.	Univ British Columbia, Canada	23	1969	85.61	3
3	Huang H.J.	Beihang Univ, China	23	703	30.57	10
4	Yang H.	Hong Kong Univ Sci & Technol, China	20	580	29.00	12
5	Fosgerau M.	Univ Copenhagen, Denmark	19	432	22.74	15
6	Arnott R.	Univ Calif Riverside, USA	17	1721	101.24	2
7	Verhoef E.T.	Vrije Univ Amsterdam, Netherlands	16	343	21.44	17
8	Zhang H.M.	Univ Calif Davis, USA	13	413	31.77	9
9	Liu W.	Univ New South Wales, Australia	13	241	18.54	19
10	van den Berg V.A.C.	Vrije Univ Amsterdam, Netherlands	11	227	20.64	18
11	Small K.A.	Univ Calif Irvine, USA	9	1031	114.56	1
12	Yin Y.	Univ Michigan, USA	9	217	24.11	13
13	Laih C.H.	Natl Taiwan Ocean Univ, Taiwan	8	147	18.38	20
14	Zhang X.	Tongji Univ, China	7	298	42.57	6
15	Nie Y.M.	Northwestern Univ, USA	7	151	21.57	16
16	Daniel J.I.	Univ Delaware, USA	6	224	37.33	7
17	Qian Z.	Carnegie Mellon Univ, USA	6	211	35.17	8
18	Xiao F.	Southwestern Univ Finance & Econ, China	6	183	30.50	11
19	Liu Y.	Natl Univ Singapore, Singapore	6	71	11.83	22
20	Zhang F.	Univ New South Wales, Australia	6	66	11.00	23
21	Daganzo C.F.	Univ Calif Berkeley, USA	5	285	57.00	5
22	Engelson L.	KTH Royal Inst Technol, Sweden	5	117	23.40	14
23	Gonzales E.J.	Univ Massachusetts, USA	5	84	16.80	21

written by ADL. In terms of average citations per year, 5 out of the top 10 most influential papers were published in TR-B, with publication years being between 2010 and 2013 and research focuses on tradable credit schemes (Nie and Yin, 2013; Xiao et al., 2013) and values of travel time and its variance (Fosgerau and Karlstrom, 2010; Fosgerau and Engelson, 2011).

In order to understand the co-citation relationship of authors in the bottleneck model research, Fig. 3 shows the bibliographic coupling network of authors, in which a solid circle represents a researcher and an edge represents the co-citation between a pair of researchers. The size of a solid circle represents the number of papers published by a researcher, and the width of an edge represents the co-citation intensity between studies by that pair of authors. It can be noted that as far as the total number of related publications is concerned, the authors, such as de Palma A, Lindsey R, Huang HJ, Yang H, Fosgerau M, Arnott R, Verhoef ET, Zhang HM, Liu W, and van den berg VAC, are the most productive, influential top 10 authors (see also Table 3) because they are associated with large circles.

Table 3 further shows the top 23 influential authors in terms of total number of publications (no less than 5 papers), together with total number of citations and average citations per paper. The research institute and country/area of the associated authors have also been indicated in this table. It can be seen that de Palma A leads the list in the total number of publications, with 31 publications. It is followed by Lindsey R and Huang HJ, each having 23 publications. 10 authors
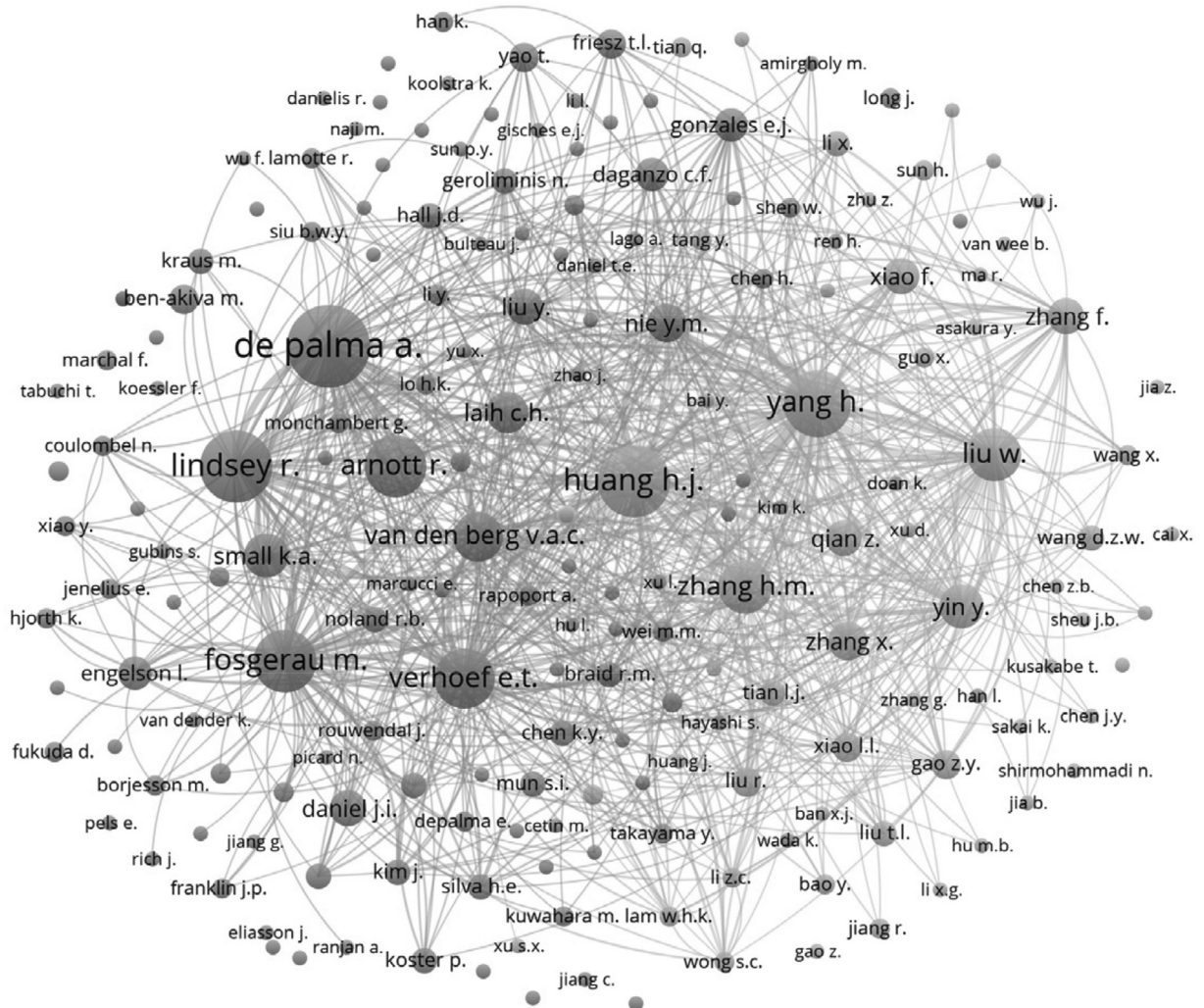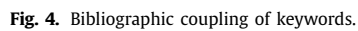
Fig. 2. Bibliographic coupling of 38 journals.

have more than 10 publications. Small KA, ADL, and Daganzo CF are the top 5 most influential authors in terms of average citations per year. de Palma A leads this table in the total number of citations (2185), and Small KA leads this list in the average citations per paper, reaching an average of 114 citations per paper. The other author, having more than 100 citations per paper, is Arnott R, reaching an average of 101 citations per paper.

In order to identify the research hotspots in the bottleneck model research, Fig. 4 shows the bibliographic coupling network of keywords, in which the size of the solid circle represents the number of occurrences of a keyword, and the width of the line represents the occurrence degree of the two keywords connected by that line. One can find some high-frequency keywords in Fig. 4, such as "traffic congestion", "bottleneck model", "transportation", "commuting", "morning commute", "travel time", "costs", "travel behavior", "traffic control", "numerical model", "traffic management", "scheduling", "departure time choice", "user equilibrium", "parking", "road pricing", "congestion pricing", "travel time variabilities", and "heterogeneity".

In order to make the review clearer, a cluster analysis of the 232 papers is conducted.[1] We categorize the 232 papers into four classes in terms of their research focuses: travel behavior analysis, travel demand management (i.e., demand-side strategies), infrastructure operations and management (i.e., supply-side strategies), and joint strategies of demand and supply sides. The travel behavior analysis mainly focuses on the analysis of the trip and/or activity scheduling behavior of travelers through building various travel choice behavior models, such as departure time / route / parking / mode choices, morning vs evening commutes, piecewise constant vs time-varying scheduling preferences, normal congestion vs hyper-congestion, homogeneous vs heterogeneous users, individual vs household, deterministic vs stochastic situations, single vs multiple bottlenecks, and analytical approach vs DTA (dynamic traffic assignment) approach. Travel demand management

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Fig. 3. Bibliographic coupling of authors.

focuses on a set of strategies and policies to reduce travel demand, or to redistribute the demand in space and/or time, including congestion / emission / parking pricing and their effects on urban system. Infrastructure operations and management means to determine the optimal capacity or service level of infrastructure elements (e.g., road bottleneck, parking lot, airport, port). Joint strategies are a hybrid of both demand-side and supply-side strategies. Among these modules, travel behavior analysis is a basis of the travel demand management studies and the infrastructure operations and management studies. The travel demand management strategies and the infrastructure operations and management strategies interplay through demand-supply interaction. The interrelationships among them are shown in Fig. 5. The shaded part in Fig. 5 represents the joint strategies of demand and supply management. In the following section, we will provide a systematic review of the bottleneck model studies published in the past half century based on the classification in Fig. 5.

3. Literature review

3.1. Travel behavior analysis

The classical Vickrey's bottleneck model aims to model the departure time choice behavior of commuters during the morning commute. For the convenience of readers, the detailed formulation of the classical bottleneck model is provided in Appendix. In this subsection, some basic assumptions underlying this model are presented. Various extensions to relax these assumptions are then reviewed. These extensions include considerations of other travel choice dimensions (e.g., route / parking / mode choices), morning-evening commutes, time-varying scheduling preferences, vehicle physical length in queue

Fig. 4. Bibliographic coupling of keywords.

and hypercongestion, heterogeneous users, household travel and carpooling, stochastic models and information, multiple bottlenecks, and DTA-approach bottlenecks.

3.1.1. The classical bottleneck model and its assumptions

The classical Vickrey's bottleneck model, as a stylized representation of the dynamics of traffic congestion, has been widely recognized as an important tool for modeling the formation and dissipation of queuing at a bottleneck in rush hours. In the model, it is assumed that homogeneous commuters travel from a single origin (home) to a single destination (workplace) along a single road that has a bottleneck with a fixed capacity during the morning rush hours. All commuters choose their departure time based on a trade-off between the bottleneck queuing delay and the schedule delay of arriving early or late. Equilibrium is reached when no individual has an incentive to alter his/her departure time.

The attractiveness of Vickrey's bottleneck model lies in its ability to derive closed-form solutions for equilibrium departure interval (i.e., the departure times of the first and last commuters from home), equilibrium departure rate, equilibrium queuing delay at the bottleneck, and equilibrium cumulative departures and arrivals. The derivations of these analytical solutions are built on some strong assumptions, stated as follows.

1) Departure time choice for morning commute. The classical bottleneck model involves only the departure time choice dimension for morning commute. The other travel choice dimensions, such as route, mode and parking choices, and the evening commute are not taken into account. In reality, commuters may also decide on their travel route and/or travel mode besides departure time, subject to parking capacity constraint. Moreover, their travel decisions are usually based on day-long schedules, but not on morning or evening activity schedule only. Some studies, e.g., de Palma and Lindsey (2002a); Zhang et al., (2005), and Li et al., (2014), showed that commuters' morning and evening departure-time decisions are interdependent under some conditions, and the morning and evening departure patterns for specific individuals are not symmetric. It is, thus, necessary to consider multiple travel choice dimensions of commuters on a whole day basis.
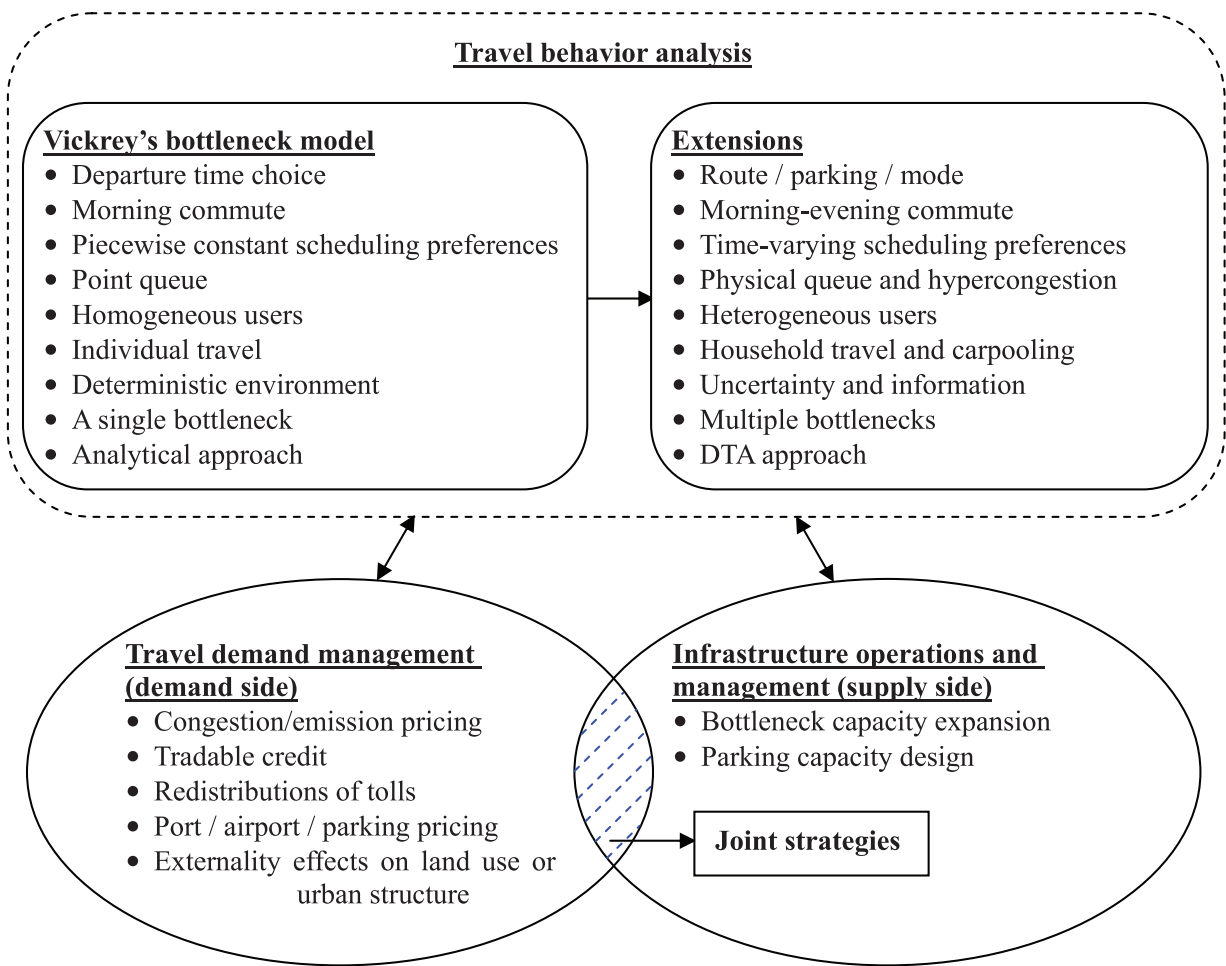
Fig. 5. Classification of research problems in bottleneck model studies.

2) Piecewise constant scheduling preferences. Vickrey's bottleneck model assumes that the value of travel time and the value of schedule delay of arriving early or late are constants, usually denoted by three parameters α, β and γ, respectively. However, some empirical studies have confirmed that the marginal utility of time for performing an activity at a certain location changes over time (see, e.g., Tseng and Verhoef, 2008; Jenelius et al., 2011; Hjorth et al., 2013; Peer and Verhoef, 2013; Peer et al., 2015). It is therefore meaningful to relax the assumption of piecewise constant scheduling preferences to develop a scheduling model with time-varying marginal activity utilities.

3) Normal congestion with point queue (or vertical queue). Vickrey's bottleneck model assumes that traffic flow does not fall under heavily congested conditions and flow increases with density, which is called "normal (or ordinary) congestion". However, in reality, a phenomenon of hypercongestion (i.e., flow decreases with density) may occur in the downtown areas of major cities during rush hours. On the other hand, the point queue or vertical queue assumes that any vehicle that has to queue before passing through a bottleneck is stacked in a vertical pile at the bottleneck, i.e., vehicles stack vertically and queues take place at a point. A vertical queue does not occupy any road space and has no influence on upstream approaching vehicles. However, in reality, vehicles have physical lengths, which influence the movements of vehicles at the bottleneck and thus the queuing delays, particularly the queue spillback may block upstream link.

4) Homogeneous individuals. The traditional bottleneck models mainly focus on individuals' travel choice behavior, and assume that the travel choice decision of an individual in a household is independent of that of other individuals. However, in reality, the interdependencies between household members (e.g., due to limited number of cars) indeed influence the activity schedules of household members. The classical bottleneck model also assumes that all commuters are homogeneous, i.e., they have the same desired arrival time and the same values of travel time and schedule delay. However, some studies have shown that there are big differences in the travel choice behavior of heterogeneous commuters due to their different travel preferences. There is thus a need to consider the heterogeneity of users.

5) Deterministic model. A bottleneck system is in general a dynamic and stochastic system. The dynamicity and stochasticity result from various random events, ranging from non-recurrent random incidents, such as traffic accident, vehicle

breakdown, signal failure, adverse weather and earthquake, to recurrent fluctuations in travel demand and capacity by time of day, day of week, and season. The travel time or queuing delay at the bottleneck is thus a stochastic variable. Furthermore, commuters may not have perfect information about the traffic condition, and thus cannot perceive the travel time accurately, leading them to make travel choice decisions somewhat haphazardly.

6) Only one bottleneck. The classical Vickrey's bottleneck model assumes that there is only one single bottleneck on the highway connecting commuters' home and workplace. However, in reality one commuter often traverses multiple bottlenecks on his/her way to work, e.g., a Y-shaped highway corridor with upstream and downstream bottlenecks. It is thus worthwhile to extend the single-bottleneck model to a multi-bottleneck case.

7) Analytical approach. The classical bottleneck model has well-defined analytical solutions, as shown in Appendix. This is because it tackles a single bottleneck only. In order to promote the realistic applicability of the model, it is necessary to extend the analytical bottleneck model to a general network with many links and many OD (origin-destination) pairs. To do this, a point-queue DTA approach (i.e., treating the queues on congested links in the network as a point bottleneck) and a traffic simulation technique may be adopted.

These aforementioned assumptions play a significant role in deriving the analytical solutions of the bottleneck model and in revealing the nature of congestion dynamics. However, they also restrict the model's explanatory power and applications for a general case due to ignorance of many realistic characteristics of traffic system. To strengthen the realism of the bottleneck model, these assumptions have been relaxed in the literature through various extensions, which are in turn reviewed as follows.

3.1.2. Incorporating route / parking / mode choice dimensions

The classical Vickrey's bottleneck model concerns only the departure time choice of commuters. In the literature, some extensions have been made to incorporate other travel choice dimensions, such as route / parking / mode choice dimensions.

In terms of route choice dimension, Arnott et al., (1990b) presented a simultaneous departure time and route choice model for a network with one OD pair and two parallel routes. It showed that at the no-toll equilibrium, the number of users on each route coincides with that in the social optimum. Optimal uniform and step tolls divert users towards longer routes, but only slightly. An optimal time-varying toll eliminates queuing without affecting route usage. Arnott et al., (1992) and Liu and Nie (2011) proposed multi-class departure time and route choice models for identifying the behavioral difference of different user classes. Siu and Lo (2014) and Liu et al., (2018) further addressed the simultaneous departure time and route choice problem in a bottleneck system with uncertain route travel time. Recently, Kim (2019) empirically estimated the social cost of traffic congestion in the US using a simultaneous departure time and route choice bottleneck model. It was shown that the annual cost of congestion borne by all US commuters is about 29 billion dollars. These aforementioned simultaneous departure time and route choice models have derived some insights into understanding the nature of commuters' route choice behavior during the morning commute. However, these studies usually considered only one OD pair and an auto-only bottleneck system. They also assumed that total travel demand was fixed (or inelastic) and the parking, as a source of congestion at the destination, was ignored.

Besides route choice dimension, parking dimension has also been incorporated in the bottleneck model studies, as shown in Table 4. It can be seen in Table 4 that most of the parking studies considered the morning commuting problems from home to work through a bottleneck-constrained route. Some exceptions are Zhang et al., (2008, 2019), who studied the integrated morning and evening commutes through one two-way route with one bottleneck each way. Some studies, such as Arnott et al., (1991b), Zhang et al., (2008, 2019), and Liu (2018), made a strong assumption about the parking order, i.e., commuters park outwards from (or inwards to) the CBD. Qian et al., (2011, 2012) divided the parking lots into two discrete classes: a closer and a farther parking cluster. Others, like Yang et al., (2013) and Liu et al., (2014a, 2016), considered parking reservation issues without/with expiration time and the effects of parking space constraints on the departure time and parking location choices. Liu and Geroliminis (2016) examined the effects of cruising-for-parking on commuters' departure time choices using MFD (macroscopic fundamental diagram) approach. However, all these studies did not concern the parking duration issue (Lam et al., 2006; Li et al., 2008), which directly affects the parking turnover and thus the real-time number of available parking spaces in a parking lot.

In addition, the bottleneck model has also been extended to consider mode choice dimension. For the convenience of readers, we summarize in Table 5 some principal contributions to the multi-modal bottleneck problems. It can be noted that most studies considered two physically separated modes (auto and rail), and thus cannot consider the congestion interaction between modes. Moreover, some studies, such as Tabuchi (1993), Danielis and Marcucci (2002), and Gonzales and Daganzo (2012), ignored the effects of passenger crowding discomfort in transit vehicles on commuters' travel choices. However, some studies, such as Huang (2000, 2002), Huang et al., (2007), Qian and Zhang (2011), and de Palma et al. (2017), showed that in-vehicle passenger crowding discomfort has a significant effect on passengers' travel choices. In order to achieve a social-optimum system, the in-vehicle passenger crowding in transit vehicles should be incorporated in the transit service optimization, together with passenger wait time at transit stops due to insufficient vehicle capacity.

3.1.3. Morning-evening commute

As previously stated, the standard bottleneck model focuses mainly on morning commuting problems, and little attention has been paid to evening or day-long commuting problems. This may be because the evening commuting is usually seen as a

Table 4
Incorporating parking dimension in bottleneck model studies.

Reference	Time period	Network structure	Travel mode	Attributes of parking spaces	Parking capacity constraint	Considering parking duration
Arnott et al. (1991b)	Morning commute	One route with one bottleneck	Auto	Continuous distribution outwards or inwards	×	×
Zhang et al. (2008)	Morning and evening commutes	One two-way route with one bottleneck each way	Auto	Continuous distribution outwards or inwards	×	×
Qian et al. (2011, 2012)	Morning commute	One route with one bottleneck	Auto	Two types: city central and peripheral	×	×
Yang et al. (2013)	Morning commute	One route with one bottleneck	Auto and rail	Reserved and unreserved parking spaces	√	×
Liu et al. (2014a)	Morning commute	One route with one bottleneck	Auto and rail	Reserved with expiration time and unreserved parking spaces	√	×
Liu et al. (2016)	Morning commute	Many-to-one network with one bottleneck each OD pair	Auto and rail	Reserved and unreserved parking spaces	√	×
Liu and Geroliminis (2016)	Morning commute	Urban road network	Auto	Curbside parking spaces	×	×
Liu (2018)	Morning commute	One route with one bottleneck	Autonomous vehicles	Continuous distribution outwards	×	×
Tian et al. (2019)	Morning commute	One route with one bottleneck	Regular and autonomous vehicles	Regular vehicles first occupy parking spaces	√	×
Zhang et al. (2019)	Morning and evening commutes	One two-way route with one bottleneck each way	Autonomous vehicles	Continuous distribution outwards	×	×

Note: " × " represents "no" and "√" represents "yes".

Table 5
Contributions to multi-modal bottleneck problems.

Reference	Decision variable(s)	Travel mode	Considering congestion interaction between modes	Considering crowding discomfort in transit vehicles	Internalizing in-vehicle crowding externality
Tabuchi (1993)	Modal split and road toll	Auto and rail	×	×	×
Huang and Yang (1999)	Rail fare and road toll	Auto and rail	×	×	×
Huang (2000)	Modal split, transit fare and road toll	Auto and rail	×	√	×
Huang (2002)	Road toll and rail fare	Auto and rail	×	√	√
Danielis and Marcucci (2002)	Modal split, congestion pricing and railway fare	Auto and rail	×	×	×
Kraus (2003)	Rail fare and train capacity	auto and rail	×	×	×
Huang et al., (2007)	Modal split	Auto and bus	√	√	×
Qian and Zhang (2011)	Modal split, rail fare, and auto toll	Auto (driving alone and carpool) and rail	×	√	×
Gonzales and Daganzo (2012)	Auto toll, transit headway and fare	Auto and bus (with dedicated lane)	×	×	×
Wu and Huang (2014)	Modal split and road toll level	Auto and rail	×	√	×

symmetric reverse process of the morning commuting. Some studies, such as Vickrey (1973), de Palma and Lindsey (2002a), Gonzales and Daganzo (2013), and Li et al., (2014), have shown that the morning and evening equilibrium departure patterns are not symmetric under some conditions, e.g., the bottleneck system has multiple alternative travel modes, or commuters are heterogeneous in terms of their preferred work start/end times and/or the values of travel time and schedule delay.

Although investigation of the morning and evening commuting problems in isolation may provide some important insights, in reality commuters usually make travel decisions based on their day-long activity schedules. To date, only a few published papers have involved the analysis of day-long commuting problems. For example, Zhang et al., (2008) presented an integrated day-long commuting model that links the morning and evening commuting trips via parking location choice. Gonzales and Daganzo (2013) incorporated mode choice dimension in the integrated morning and evening commuting problem. Daganzo (2013) further examined the two-mode day-long commuting problem when the wish times of arriving at and departing from workplace follow a continuous distribution. The day-long commuting models mentioned above adopted a trip-based modeling approach, and thus the time allocations of commuters for activities and travel during a day cannot be properly addressed.

Different from the trip-based morning-evening commuting models, Zhang et al., (2005) presented a day-long activity-travel scheduling model to address commuters' time allocations among activities and travel during a day. Their model connects the home-to-work commute in the morning and the work-to-home commute in the evening via work duration. Li et al., (2014) investigated the properties of the day-long activity-travel scheduling model. They presented a sufficient and necessary condition of interdependence between the morning and evening departure-time decisions, i.e., the marginal utility of work activity is not a constant, but depends on both the clock time of day and the work duration (implying a flexible work-hour scheme). Recently, Zhang et al., (2019) further investigated autonomous vehicles oriented morning-evening commuting and parking problems. These previous studies usually considered a simple activity chain, namely home-work-home chain. However, in reality commuters may engage in other activities before work (e.g., taking the kid to school) or after work (e.g., shopping or recreation). It is thus meaningful to incorporate other activity participations in the day-long activity-travel scheduling model.

3.1.4. Time-varying scheduling preferences

Vickrey's bottleneck model assumes that the value of travel time and the values of schedule delays of arriving early and late are constants α, β, and γ, respectively (see Eq. (A1) in Appendix). This assumption has been widely adopted in various extensions or variations of Vickrey's bottleneck model. However, some previous empirical studies have confirmed that the marginal activity utility varies in time and space.

Vickrey (1973) formulated the departure time choice model for the morning commuting problem, in which the utilities derived from time spent at home and at work are linear functions of time. Tseng and Verhoef (2008) estimated the scheduling model of the morning commuting problem, in which marginal utilities vary nonlinearly over time of a day. Jenelius et al., (2011) explored the effects of activity scheduling flexibility and interdependencies between different segments in a daily trip chain on delay cost and value of time. Hjorth et al., (2013) empirically estimated different types of activity scheduling preference functions (including const-step, const-affine or const-exp formulations) and compared them to a more general form (exp-exp formulation) with regard to model fit, based on the stated preference survey data collected from car commuters traveling in the morning peak in the city of Stockholm. Abegaz et al., (2017) used stated preference data to compare the valuation of travel time variability under a structural model where trip-timing preferences are defined in terms of time-dependent utility rates (i.e., a "slope model") against its reduced-form model where departure time is assumed to be optimally chosen. Fosgerau and Small (2017) presented a dynamic model of traffic congestion in which scheduling preferences are endogenously determined. This is different from the traditional activity scheduling models, in which the scheduling preferences are assumed to be exogenously given. Li et al., (2017) developed an activity-based bottleneck model for investigating the step tolling problem, in which the activity scheduling utilities of commuters at home and at work vary by the time of day. It showed that ignoring the preference heterogeneity of commuters would underestimate the efficacy of a step toll. Recently, Li and Huang (2018) investigated the user equilibrium problem of a single-entry traffic corridor with continuous scheduling preferences. The results showed that the introduction of continuous scheduling preferences makes inflow rate of early arrivals first increase and then decrease. Even though the introduction of continuous scheduling preferences can smooth the departure rate of commuters and make the user equilibrium flow pattern more stable, a series of shock waves still exist due to discontinuities in departure rates or sharply decreasing inflow rate at the entry point of the corridor.

These aforementioned studies mainly focused on evaluation or comparison of the effects of different forms of the activity scheduling preference functions. Other important factors were ignored. For example, the marginal utilities of commuters may change by gender, travel mode, income level, and so on. It is meaningful to reveal the effects of these heterogeneities on the activity scheduling preferences and to empirically calibrate the scheduling preference functions of various activities through field surveys.

3.1.5. Physical queue length and hypercongestion

The point-queue assumption in Vickrey's bottleneck model significantly facilitates the calculation of queuing delay at the bottleneck. However, it cannot account for the influence of vehicle queuing on upstream approaching vehicles due to ignoring the physical lengths of vehicles in queue. Lago and Daganzo (2007) investigated two important aspects: queue spillovers caused by insufficient road space, and merging interactions caused by the convergence of trips in a two-origin

and single-destination network with limited storage space. They obtained some unexpected findings, e.g., ramp metering is beneficial, and providing more freeway storages is counterproductive. Chen et al., (2019) explored the impact of queue-length-dependent capacity on travelers' departure time choices in the morning commute problem. It showed that multiple equilibria and even a continuum of equilibria may exist, and the equilibrium cost may be a locally decreasing function of the number of users.

The standard model for analyzing traffic congestion with vehicle queue length consideration usually incorporates a relationship between volume, speed and density of traffic flow. There is a well-defined inverse-U-shaped relationship between traffic volume and density. However, most of traffic flow models focus on the situation of "ordinary (or normal) congestion", in which traffic volume increases as traffic density increases (or travel speed decreases as traffic volume increases). This is because it is believed that traffic flow does not fall under heavily congested conditions. However, in reality, the phenomenon of hypercongestion may occur, especially in the downtown areas of major cities during rush hours. Hypercongestion refers to traffic jam situations where traffic volume decreases as traffic density increases.

Small and Chu (2003) presented tractable models for handling demand fluctuations for a straight uniform highway and for a dense street network located in a central business district (CBD). For the CBD model, they employed an empirical speed-density relationship for Dallas Texas to characterize hypercongested conditions. Arnott (2013) presented a bathtub model of downtown rush-hour traffic congestion that captures the hypercongestion phenomenon. It was shown that when demand is high relative to capacity, applying an optimal time-varying toll can generate benefits that may be considerably larger than those obtained from standard models and that exceed the toll revenue collected. Fosgerau and Small (2013) combined a variable-capacity bottleneck with $\alpha - \beta - \gamma$ scheduling preferences for a special case with only two possible levels of capacity. It showed that the marginal cost of adding a traveler is especially sensitive to the low level of capacity, and under hypercongestion the policies (an optimal toll, a coarse toll, and metering) can be designed so that travelers gain even without considering any toll revenue.

Fosgerau (2015) extended the bathtub model to assess the effects of the policies of road pricing, transit provision and traffic management under hypercongestion. It showed that the unregulated Nash equilibrium is also the social optimum among a wide range of potential outcomes and any reasonable road pricing scheme would be welfare decreasing when the speed of alternative transit mode is high enough such that hypercongestion does not occur in equilibrium. Large welfare gains can be achieved through road pricing when there is hypercongestion and travelers are heterogeneous. Gonzales (2015) further considered the hypercongestion issue in a multi-modal context. It showed that hypercongestion may arise when modes are not priced, a stable steady equilibrium state can emerge when cars and high-capacity transit are used simultaneously, and there always exist fixed coordinated prices (i.e., fixed difference of prices) for cars and transit to achieve a stable equilibrium state without hypercongestion. In order to derive a closed-form solution for no-toll equilibrium for hypercongestion, Arnott et al., (2016) proposed a special bathtub model through adapting the simplest bottleneck model to an isotropic downtown area where the congestion technology entails velocity being a negative linear function of traffic density. Liu and Geroliminis (2016) adopted an MFD approach to model the hypercongestion effects of cruising-for-parking in a congested downtown network. In addition to the hypercongestion issues, the traffic flow model that describes the relationship between velocity and density has also been extended for investigation of continuum corridor problems (see, e.g., Arnott and DePalma, 2011; DePalma and Arnott, 2012; Li and Huang, 2017; Lamotte and Geroliminis, 2018).

It should be mentioned that the tractability is a major challenge for the models with hypercongestion consideration. This is because the travel time of a traveler is determined by the decisions of other travelers throughout the duration of the trip, as pointed out in Fosgerau and Small (2013). Therefore, it is difficult to derive analytical solutions for a general case with general scheduling preferences, heterogeneous users, or travel time uncertainty. A simulation method has thus to be used.

3.1.6. Heterogeneity of users

The standard bottleneck model has a strong assumption that commuters are homogeneous, i.e., all commuters have the same preference for arriving early or late and have an identical value of time. This assumption has been relaxed in the literature to consider the heterogeneity of commuters, such as heterogeneities of travel preferences and work start time (e.g., flexible or staggered work hours). The heterogeneity may be represented in discrete or continuous form. The discrete type of heterogeneity means that all commuters are divided into several groups and the commuters in one group are assumed to have the same preference and work start time. The continuous type of heterogeneity assumes a continuous distribution for the preference and/or work start time. Considering user heterogeneity is important for achieving accurate estimates of welfare effects of various policy measures, such as congestion pricing, ramp metering, capacity investment, and flexible work schedules.

Table 6 provides a summary of bottleneck model studies involving heterogeneous users. It can be seen that the existing studies mainly focused on the case of piecewise constant scheduling preferences (i.e., $\alpha - \beta - \gamma$ preferences), and considered the users' heterogeneities in the following ways: (i) identical preferred arrival time and discretely / continuously distributed scheduling preference parameters; (ii) discretely distributed preferred arrival time and identical / discretely distributed scheduling preference parameters; and (iii) continuously (including uniformly) distributed preferred arrival time and identical / continuously distributed scheduling preference parameters. However, the cases of discretely (continuously) distributed preferred arrival time but continuously (discretely) distributed scheduling preference parameters are not investigated yet, which provide a research opportunity for further study. It can also be seen that most studies concerned the proportional heterogeneity (i.e., all commuters have the same ratios of β/α and γ/α), which helps derive the departure or-

Table 6
A summary of bottleneck model studies with heterogeneous users.

Reference	Heterogeneity Preferred arrival time	Values of time and schedule delay	Solution method
Vickrey (1973)	Identical	Discrete groups with different values of α and γ, but same α/β and γ/β	Analytical
Hendrickson and Kocur (1981)	Continuous	Same scheduling preference parameters	Analytical
Newell (1987)	Discrete	Discrete distributions for preference parameters	Analytical solutions only for some simplified special cases
Arnott et al., (1988)	Identical	Discrete groups with different values of α and β, but same γ/β	Analytical
Arnott et al., (1994)	Discrete	(i) Discrete groups with different values of α and β, but same γ/β (ii) Different values of γ	Analytical
van der Zijpp and Koolstra (2002)	Identical	Discrete groups with different utility/cost function parameters	Numerical (Nelder-Mead solution procedure)
Lindsey (2004)	Discrete	Discrete groups with different values of α, β and γ	Analyze existence and uniqueness, but not provide solution method
Mun and Yonekawa (2006)	Uniform	Same α, β and γ	Analytical
Leurent and Wagner (2009)	Continuous	Same α, β and γ	Numerical (heuristic algorithm)
Ramadurai et al., (2010)	Discrete	Discrete groups with different values of α, β and γ	Numerical (Lemke's algorithm)
van den Berg and Verhoef (2011a)	Identical	Continuously distributed α, and same β and γ	Analytical
van den Berg and Verhoef (2011b)	Identical	Continuously distributed α and μ ($= \alpha/\beta$), but same γ/β	Analytical
Doan et al., (2011)	Discrete	Discrete groups with different values of α, β and γ	Numerical (primal-dual algorithm)
Qian and Zhang (2013)	Identical	Continuously distributed β and γ but same γ/β	Analytical
Liu et al., (2015c)	Identical	Discrete groups with different values of α, β and γ	Numerical (diagonalization algorithm)
Wu and Huang (2015)	Identical	Uniformly distributed α and β, without allowing late arrival	Analytical
Takayama (2015)	Discrete	Same parameters for commuting cost function	Analytical
Amirgholy and Gonzales (2017)	Continuous	Uniformly and normally distributed schedule delay preferences	Analytical for uniformly distributed; Numerical for others
Silva et al., (2017)	Discrete	Same α, β and γ	Analytical
Zhu et al., (2018)	Discrete	Same parameters for marginal activity utility functions	Analytical
Lindsey et al., (2019)	Uniform	Two discrete groups with same γ/β	Analytical

der of different commuter groups and analytical equilibrium solutions. Some studies, such as Newell (1987), Lindsey (2004), Ramadurai et al., (2010), Doan et al., (2011), and Liu et al., (2015c), have relaxed this assumption to consider a general heterogeneity structure (i.e., α, β and γ are allowed to vary independently). Some properties of the model (e.g., the existence and uniqueness of solution) have been discussed. However, it is difficult to derive analytical solution of the model, which poses a challenge of designing an efficient solution algorithm. In addition, the marginal utility of an activity generally varies over time, as previously stated. It is thus necessary to relax the assumption of $\alpha - \beta - \gamma$ scheduling preferences to consider the case of time-varying scheduling preferences.

3.1.7. Household travel

Most of the previous bottleneck model studies focused on individual-based trips, and assumed that each household member makes activity-travel scheduling decisions independently. However, in reality, a large number of morning commute trips are indeed household-based travel, i.e., a multi-person trip among household members rather than a single-person trip. The interdependency between household members could influence the activity participation of each household member. Therefore, the intra-household interaction should be considered in the activity-travel scheduling models.

de Palma et al. (2015) proposed a variant of Vickrey's bottleneck model of the morning commute, in which individuals live as couples and value the time at home more when together than when alone. The results showed that the cost of congestion is higher for couples than for single individuals because the cost of arriving early rises proportionally more than the cost of arrival late decreases. The costs can be even higher if spouses collaborate with each other when choosing their departure times. Jia et al., (2016) explored the departure time choice problem of the household travel (commuter and children) in a home-school-work trip chain with two preferred arrival times (a school start time and a work start time). Liu et al., (2017b) further considered a hybrid of household travel (home-school-work trip chain) and individual travel (home-work trip). The findings showed that by appropriately coordinating the schedules of work and school, the traffic

congestion at highway bottleneck and thus the total travel cost can be reduced. Zhang et al., (2017) further investigated and compared the morning commuting equilibrium solutions in the "school near workplace" and "school near home" networks. It was shown that the dynamic commuting equilibrium solution is significantly affected by school locations, and in the "school near home" network, households always arrive at school no later than the desired school arrival time.

These abovementioned studies considered the morning trip-timing decisions of couples (de Palma et al., 2015) and of a parent and his/her children (Jia et al., 2016; Liu et al., 2017b; Zhang et al., 2017). However, for a family with two workers (husband and wife), couples must decide who takes the children to school in the morning and when, and who brings them back home in the evening. A parent has to trade-off not only his/her own schedule convenience with that of his/her spouse, but also the schedules of children. It is therefore meaningful to address the morning-evening activity scheduling issues with intra-household interaction consideration.

3.1.8. Carpooling

Carpooling or ridesharing refers to the case in which multiple persons travel together in an auto by sharing the cost. With carpooling, the seat capacity of an auto can be utilized more efficiently and the average individual travel costs, such as fuel cost, toll, and the stress of driving, are reduced. Carpooling is also recognized as a more environmentally friendly and sustainable way to commute by reducing vehicular carbon emissions as well as the need for parking spaces.

Recently, Xiao et al., (2016) incorporated carpooling behavior in morning commute problem with considering parking space constraint at destination. Three modes, namely solo-driving, carpooling, and transit, were considered. It was shown that the departure period of solo drivers covers the departure period of carpoolers, and as the number of parking spaces decreases, the number of solo-drivers decreases gradually, while the number of carpoolers first increases and then decreases. Liu and Li (2017) examined the morning commute problem in the presence of ridesharing program. Commuters simultaneously choose departure time from home and role in the program (solo driver, ridesharing driver and ridesharing rider). Ma and Zhang (2017) further explored the dynamic ridesharing problem on a highway with a single bottleneck together with parking. They designed the schemes with different ridesharing payments and shared parking prices. Recently, Wang et al., (2019) presented a variable-ratio charging-compensation scheme to investigate dynamic ridesharing problem using bottleneck model approach. Different objectives of the platform were considered, including minimization of system disutility, maximization of platform profit, and minimization of system disutility subject to zero platform profit. Yu et al., (2019) incorporated users' heterogeneities in the carpooling problem based on the traditional bottleneck model, and revealed the effects of heterogeneities on the efficiency of carpool subsidization.

All the aforementioned studies are based on a corridor with a single bottleneck, and thus cannot consider the interaction between flows of different links in the network (i.e., network effects). Extending the single-bottleneck model to a general network with multiple OD pairs and multiple bottlenecks could help deepen understanding of the effects of carpooling services on the urban transport system. The carpooling services can be implemented through mobile platforms in which passengers can call on riding services and drivers can respond to the service requests. The existing related studies considered a single carpooling platform. It is meaningful to examine the competition and/or collaboration among multiple carpooling platforms.

3.1.9. Stochastic models and information

Transportation systems are stochastic, dynamic and nonlinear systems due to various disturbance factors from supply side and/or demand side, such as traffic accident, bad weather, and within-day and/or day-to-day demand variations. Considering the impacts of stochastic factors on transportation systems has important implications for promoting the resilience and reliability of the systems.

Table 7 lists some major studies incorporating uncertainty effects in the bottleneck problems. It can be noted that most of previous studies focused on the supply uncertainty caused by travel time variation or capacity randomness. It is somehow surprising that no studies take into account the effects of the uncertainty in demand side in the bottleneck problems, though there are a few publications involving joint fluctuations in both supply and demand sides (e.g., Arnott et al., 1999; Fosgerau, 2010). In order to model the uncertainty effects of bottleneck system, a probability distribution function needs to be specified for the random variables concerned. In this regard, most studies adopted a general distribution, and a few studies adopted some specific distributions, such as uniform distribution (e.g., Xiao et al., 2014, 2015; Wang and Xu, 2016; Zhang et al., 2018), exponential distribution (Tian and Huang, 2015), uniform and exponential distributions (Noland et al., 1995, 1998), and Gumbel distribution (Xiao and Fukuda, 2015).

In terms of modeling method, expectation value model is usually adopted in stochastic optimization problems. However, in order to well capture the risky attitudes of travelers towards random fluctuations in demand and/or supply sides, some studies have also incorporated the effects of travel time variation in the objective functions of models, such as Fosgerau (2010); Borjesson et al., (2012); Engelson and Fosgerau (2016), and Xiao et al., (2017). It should be mentioned that in the field of travel time variability or reliability, Fosgerau and his collaborators have made a number of works by using a model of time-varying scheduling preferences. For instance, they presented a new measure of travel time variability (Engelson and Fosgerau, 2011) and explored the relationships among different measures of the cost of travel time variability (Engelson and Fosgerau, 2016). They also derived the value of travel time variability (Fosgerau and Engelson, 2011; Fosgerau and Fukuda, 2012), and revealed the relationship between the mean and variance of travel delay in dynamic queues

Table 7
Incorporating uncertainty effects in bottleneck model studies.

Reference	Source of uncertainty	Distribution of random variable(s)	Scheduling preferences	Objective function	Solution method
Noland et al. (1995, 1998)	Random travel time	Uniform and exponential	Piecewise constant	Min. expected travel cost	Analytical + simulation
Arnott et al. (1999)	Random capacity and demand	General	Piecewise constant	Min. expected travel cost	Provide model properties but no solution method
Fosgerau (2010)	Random capacity and demand	General	Consider bottleneck queue time but not schedule delay	Min. expected queue time and variance	Analytical
Fosgerau and Karlstrom (2010)	Random travel time	General	Piecewise constant	Max. expected utility	Analytical
Fosgerau and Engelson (2011)	Random travel time	General	Piecewise constant and linear time-varying	Max. expected utility	Analytical
Fosgerau and Fukuda (2012)	Random travel time	General	Piecewise constant	Min. expected cost.	Analytical + simulation
Borjesson et al., (2012)	Random travel time	General	Piecewise constant vs time-varying	Max. expected utility, mean + variance	Analytical + experiment
Jenelius (2012)	Random travel time	General	General case + piecewise constant vs linear time-varying	Max. expected utility	Analytical for piecewise constant & linear cases
Fosgerau and Lindsey (2013)	Random travel time	General	Time-varying	Max. expected utility	Analytical
Siu and Lo (2013)	Random capacity	General	Piecewise constant	Min. punctuality based expected cost	Simulation
Coulombel and de Palma (2014)	Random travel time	General + uniform & normal	Piecewise constant	Min. expected scheduling disutility	Analytical + simulation
Fosgerau et al., (2014)	Random travel time	General	Time-varying	Max. expected utility	Analytical
Xiao and Fukuda (2015)	Random travel time	General + Gumbel as an example	General case + piecewise constant vs linear time-varying	Min. rank-dependent expected travel cost	Analytical + simulation
Xiao et al., (2014, 2015)	Random capacity	uniform	Piecewise constant	Min. expected travel cost	Analytical
Koster et al., (2016)	Random air travel delay	General	Piecewise constant	Min. expected travel cost	Simulation + analytical for uniform distribution
Tian and Huang (2015)	Random auto travel time	Exponential	Piecewise constant	Min. expected travel cost	Simulation
Engelson and Fosgerau (2016)	Random travel time	General	Time-varying, piecewise constant	Min. expected cost, mean + standard deviation	Analytical
Wang and Xu (2016)	Random travel time	Uniform	Piecewise constant	Min. expected travel cost	Simulation
Xiao et al., (2017)	Random travel time	General	Piecewise constant, linear time-varying, constant-exponential	Min. mean + standard deviation of travel time	Analytical
Zhang et al., (2018)	Random capacity	Uniform	Piecewise constant	Min. expected travel cost	Analytical
Fosgerau and Jiang (2019)	Random travel time	General	Time-varying	Max. net payoff	Analytical + simulation

with random capacity and demand (Fosgerau, 2010). For a systematic review of the values of travel time and travel time reliability, the interested readers may refer to Small (2012).

It should be pointed out that these previous studies are mainly based on expectation value (risk-neutral) model or mean-variance (risk-averse) model. It is meaningful to consider other measures of risk, such as VaR (value at risk) and CVaR (conditional value at risk). This could lead to a difference in the value of travel time variability. Moreover, these previous studies assumed that the scheduling preferences are exogenously given. Incorporating endogenous scheduling preferences, as presented in Fosgerau and Small (2017), is also an important direction for further studies.

Obviously, variability or uncertainty in supply and/or demand sides involves lack of information about how a stochastic process is realized. Thereby, its analysis naturally invites considering the effects of information provision. Travelers may have only partial information about traffic conditions before or during a trip. With the aids of various information and communication technologies (e.g., global navigation satellite system, global positioning system), real-time traffic information can be collected and disseminated efficiently. Travelers can adjust their activity and travel schedules through day-to-day leaning and the traffic information guidance.

In this regard, Noland (1997) examined the congestion effects of providing commuters with pre-trip information and found that information provision does not necessarily bring benefits to the commuters using the information. Ziegelmeyer et al., (2008) investigated the impact of public information about past departure rates on congestion level and travel cost based on learning model and the ADL's bottleneck model. Liu et al., (2017a) considered the effect of travelers' inertia in the day-to-day behavioral adjustment due to traffic information updating. Khan and Amin (2018) studied the effects of heterogeneous information (market penetration and accuracy) on traffic congestion. Liu and Liu (2018) explored the impact of the cost of information provision on information quality provision strategy. Zhu et al., (2019) examined the day-to-day departure time adjustment process of travelers with bounded rationality based on long-term historical knowledge (or short-term travel experience) and real-time information provision. However, all the aforementioned studies only considered a simplified case of one single route, which may not be able to capture the full impact of information on traffic congestion. It is thus necessary to extend it to a general network with multiple routes in a further study.

3.1.10. Multiple bottlenecks

Some studies have relaxed the assumption that commuters pass through only one bottleneck during commuting peak periods to consider the case of passing through multiple bottlenecks during a trip. Kuwahara (1990) analyzed the equilibrium queuing patterns at a two-tandem bottleneck on one freeway for which some commuters may pass through both bottlenecks. Arnott et al., (1993b) studied a Y-shaped highway corridor with two upstream bottlenecks and one downstream bottleneck. They found that expanding the capacity of one of the upstream bottlenecks can raise total travel cost (i.e., a paradox occurs), and metering access to reduce effective upstream capacity can improve efficiency. Optimal capacity for an upstream bottleneck is equal to, or smaller than, the optimal capacity for the downstream bottleneck. Kim (1999) further analyzed the dynamic equilibrium queuing patterns for a two-tandem bottleneck with two origins and one destination. It was found that in some cases, a queue does not occur at the upstream since the departure rate is always equal to its capacity at equilibrium. In order to avoid traffic congestion at a two-tandem bottleneck, the downstream bottleneck should be enlarged prior to the upstream bottleneck. Daniel et al., (2009) further demonstrated the bottleneck paradox phenomenon by an experimental method in a Y-shaped bottleneck network with two groups of commuters. The commuters in group one pass through only the downstream bottleneck, whereas the commuters in group two must pass through both upstream and downstream bottlenecks. They found that the observed departure times at aggregate level are in close agreement with the equilibrium solution. Akamatsu et al., (2015) discussed the existence and uniqueness of the solution of departure-time choice equilibrium for a corridor with multiple discrete bottlenecks and heterogeneous users. These previous related studies mainly focused on a specific occasion (e.g., a two-tandem or Y-shaped bottleneck structure), and thus the results obtained might not be applicable to a general network. Therefore, further investigations on general bottleneck networks are needed.

3.1.11. DTA-approach bottlenecks

The bottleneck models presented in the aforementioned literature usually adopted analytical approaches because they treated only some simple cases with one or two routes. In order to apply the bottleneck models to real large-scale networks, a DTA-based bottleneck modeling approach was presented, which was inspired by Vickrey's bottleneck model. In this approach, the usual components of Vickrey's bottleneck model are applied separately to the links in the network. In this regard, de Palma and his colleagues have developed a dynamic network model, called METROPOLIS, in which the travel mode, departure time and route choices can be endogenously determined. METROPOLIS has been implemented both with a vertical queue for each link (i.e., the physical length of a queue is not considered), and with a horizontal queue which means the queue on one link can affect other links (i.e., queue spillback effects). The model is solved using microsimulation, and has been applied to evaluate various policies, such as congestion pricing (see, de Palma and Lindsey, 2006; de Palma et al., 2005, de Palma et al., 2008). For more details of METROPOLIS, please refer to de Palma et al. (1997) and de Palma and Marchal (2002).

3.1.12. Others

Besides the aforementioned various topics, some other topics related to the bottleneck model have also been studied, summarized as follows.

i. Properties of equilibrium solution. Smith (1984) showed the existence of user equilibrium solution for a single-bottleneck model with homogeneous users. Daganzo (1985) proved the uniqueness of the user equilibrium solution. Newell (1987) extended the analysis to the case of heterogeneous linear schedule delay functions. An elastic demand version of the bottleneck model was analyzed in Arnott et al., (1993a).

ii. Variations of model formulation and solution algorithm. de Palma et al. (1983) proposed a stochastic departure time choice logit model to consider the commuters' perception errors of utility. Han et al., 2013a, Han et al., 2013b) reformulated Vickrey's bottleneck model as a partial differential equation formulation. Otsubo and Rapoport (2008) presented a discrete version of Vickrey's bottleneck model and a solution algorithm for computing the equilibrium solution. Nie and Zhang (2009) proposed numerical solution procedures for the morning commute problem. Guo and Sun (2019) considered personal perception in the travel cost function, aiming to incorporate the commuters' psychological tastes towards early arrival at the workplace in the bottleneck model.

iii. Doubly dynamic adjustments (day-to-day and within-day dynamics). Ben-Akiva et al., (1984, 1986) presented a dynamic simulation model to describe the evolution of queues and delays from day to day. Ben-Akiva et al., (1991) further presented a framework for evaluating the effects of traffic information systems based on the doubly dynamic adjustment model incorporating the drivers' information acquisition and integration. Guo et al., (2018) considered the bounded rationality factor due to individual's limited cognitive level and imperfect information in the doubly dynamic bottleneck model.

iv. Time varying bottleneck capacity. The classical bottleneck model usually assumes a constant or invariant bottleneck capacity. Using optimal control theory, Yang and Huang (1997) presented a bottleneck model with queue-dependent capacity and elastic demand for design of time-varying toll schemes, and found that queues must not be eliminated in the optimal state of the system. Zhang et al., (2010) presented another bottleneck model in which the bottleneck capacity varies exogenously over time, in discrete steps. They derived user equilibrium and system optimal traffic patterns with (exogenously) time-varying capacities and the optimal tolls leading to the system optimum pattern.

v. Ramp metering. Arnott et al., (1993b) suggested an optimal metering policy to improve the efficiency of a Y-shaped bottleneck system. O'Dea (1999) found that in the bottleneck model, metering can produce a sizable benefit and should not be regarded as a substitute for congestion pricing. Shen and Zhang (2010) designed a Pareto-improving metering strategy for a multi-ramp linear freeway based on an analysis of priority order of the ramps. These studies mainly focused on a simple bottleneck system or a freeway. We can extend it to a general network in a further study.

3.2. Demand-side strategies

3.2.1. Congestion pricing

Queuing delay is a pure deadweight loss for the society and results in inefficient use of transportation infrastructure. In order to make efficient use of transportation resources, congestion pricing has been widely suggested as a viable measure to internalize externalities caused by queuing at the bottleneck so as to relieve peak-period traffic congestion. Congestion pricing scheme is generally based on the economic theory of marginal cost pricing and is a mechanism to improve social benefit (Yang and Huang, 2005). For comprehensive reviews of congestion pricing, readers can refer to Lindsey et al., (2012), van den Berg (2012, van den Berg, 2014), and Fosgerau and van Dender (2013).

A substantial stream of research has been conducted on bottleneck congestion pricing. Table 8 provides a summary of the bottleneck congestion pricing studies. It should be pointed out that multi-modal bottleneck tolling studies are not shown in Table 8, and readers can refer to Table 5. It can be seen in Table 8 that many of the existing studies focused on the topics of step tolling, users' heterogeneity, and tradable credit scheme. Based on different assumptions, the step bottleneck tolling studies can be classified into three main categories: the ADL model of Arnott et al., (1990a, 1993a, 1998), the Laih model of Laih (1994, Laih, 2004), and the braking model of Lindsey et al., (2012) and Xiao et al., (2012). The Laih model implicitly assumed that separate queues exist for tolled users and untolled users who arrive before the toll is turned off. Despite this strong assumption, the Laih model is useful for estimating the approximate efficiency of a multi-step toll scheme. The ADL model assumed that a mass of commuters departs just after the toll is lifted. The braking model considered that as the end of the tolling period approaches, drivers have an incentive to stop before reaching the tolling point and wait until the toll is switched off.

The congestion pricing studies shown in Table 8 usually fall into the family of the piecewise constant $\alpha - \beta - \gamma$ preferences, and focus on the case of normally recurrent traffic congestion. Li et al., (2017) further investigated the congestion tolling problems in a framework of time-varying scheduling preferences. They compared the single-step and multi-step toll schemes with linear time-varying and piecewise constant marginal activity utilities. Arnott (2013) and Fosgerau (2015) incorporated the hypercongestion phenomenon in the congestion tolling problems using the bathtub model.

3.2.2. Emission pricing

Vehicular use also causes environmental externality, besides congestion externality. In order to control vehicle pollution emissions and improve air quality, emission tax policy has been suggested. Bulteau (2016) proposed a microeconomic model of urban toll system to internalize the negative externality effects (congestion and pollution) generated by vehicular use. In the proposed model, two modes of transportation (i.e., cars and public transport) were taken into account, and the vehicle emission rate was implicitly assumed to be a constant. Based on the bottleneck congestion model of Arnott et al., (1990a,

Table 8

A summary of bottleneck congestion pricing research.

Topic concerned	References	Remarks
Time-varying congestion tolling with fixed travel demand	de Palma and Arnott (1986), Bernstein and Muller (1993), Mun (1999), Daganzo and Garcia (2000)	Social optimum toll to totally eliminate bottleneck queue during morning peak Pareto-improving time-varying toll in a time window Minimize total social cost or user cost
Elastic travel demand	Braid (1989), Arnott et al., (1993a), Yang and Huang (1997)	Consider the elasticity of travel demand to travel cost Maximize total social surplus
Step tolling	Laih (1994, 2004), Arnott et al., (1990a, 1993a), Fosgerau (2011), Xiao et al., (2011, 2012), Lindsey et al., (2012), van den Berg (2012, 2014), Gonzales and Christofa (2014), Knockaert et al., (2016), Ren et al., (2016), Bao et al., (2017), Xu et al., (2019)	Single-step or multi-step tolling scheme as a substitute for the time-varying tolling scheme Typical models: ADL model, Laih model, Lindsey et al. braking model
Route or lane substitute	Braid (1996), Hall (2018)	Two routes: one tolled and one free A portion of lanes are tolled
Heterogeneous travelers	Cohen (1987), Arnott et al., (1992, 1994), Xiao et al., (2011), van den Berg and Verhoef (2011a,b, van den Berg and Verhoef, 2014, van den Berg and Verhoef, 2016), Yao et al., (2012), Chen et al., (2015a, Chen et al., 2015b), Braid (2018)	Differentiate users by value of time and scheduling preferences Behavioral difference in choices of departure time, travel route etc. Who gains and who loses
Tradable credit scheme	Zhang et al., (2011), Nie and Yin (2013), Xiao et al., (2013), Tian et al., (2013), Liu et al., (2014a,b), Nie (2015), Bao et al., (2019), Xiao et al., (2019)	A substitute for congestion tolling scheme Parking permits and trading
Reward scheme	Rouwendal et al., (2012), Yang and Tang (2018)	A reward scheme means a subsidy instead of a penalty of tolling Fare-reward scheme for transit users
Regulatory regime of bottleneck	de Palma and Lindsey (2002b, 2008), Fu et al., (2018)	Private regime (profit maximization) Public regime (welfare maximization) Mixed regime
Stochastic environments	Yao et al., (2010), Xiao et al., (2015), Hall and Savage (2019)	Stochastic toll Stochastic capacity

1993a), three alternative toll schemes were compared: a fine toll (time-varying toll), a coarse toll (varies according to the peak period and off-peak period), and a uniform toll (constant over time). The policy of redistributing the gains from urban tax to public transport was also evaluated. Liu et al., (2015b) presented a variable speed limit scheme to reduce total traffic emissions and travel costs based on Vickrey's bottleneck model and constant vehicle emission rate assumption, and evaluated its effectiveness in improving traffic flow efficiency of the bottleneck system. These studies only considered one OD pair with one route and assumed a constant vehicle emission rate. It is meaningful to extend these studies to incorporate the network effects and the changing vehicle emissions by vehicle type and speed.

3.2.3. Tradable credit

Tradable credit scheme has recently been advocated as a useful tool for regulating the externalities caused by vehicular use and as a promising substitution for congestion pricing scheme. This is because such scheme does not involve money transfer from the public to the government, which can significantly increase the public acceptability towards the scheme (Yang and Wang, 2011). Zhang et al., (2011) studied various parking permit schemes in a many-to-one network, in which each origin is connected to a single destination by a bottleneck-constrained highway and a parallel transit line. They compared three parking permit distribution schemes for commuters living in different origins: uniform, Pareto improving, and system optimum schemes. Liu et al., (2014a,b) further developed tradable parking permit schemes to realize parking reservations for homogeneous or heterogeneous commuters in terms of their values of time. Nie and Yin (2013) proposed a general analytical framework for design of system optimal tradable credit scheme and analysis of the efficiency of tradable credit scheme for a two-route system. Their results showed that the tradable credit scheme could provide substantial efficiency gains for a wide range of scenarios. Tian et al., (2013) examined the efficiency of a tradable credit scheme in a competitive highway/transit network with continuous heterogeneity in terms of individuals' values of time. Xiao et al., (2013) explored the efficiency and effectiveness of a tradable credit system with identical and non-identical commuters. Credits are tradable between the commuters, and the credit price is determined by a competitive market. The credit system consists of a time-varying credit charged at the bottleneck and an initial credit distribution to the commuters.

Nie (2015) proposed a market-based tradable credit scheme for managing traffic congestion at critical bottlenecks (e.g., bridges and tunnels). It was assumed that users who avoid traveling in the peak-time window will be rewarded with mobility credits and those who do not will pay a congestion toll in the form of credits or cash. The travelers may trade their credits with each other. It was shown that the best choice of the rewarding-charging ratio is 1, i.e., each peak-time user is charged one credit and each off-peak user is awarded one credit. Shirmohammadi and Yin (2016) designed a tradable credit scheme to maintain the queue length of the bottleneck to be less than a queue length threshold specified by the authority. Sakai et al., (2017) proposed a model for designing Pareto-improving pricing scheme with bottleneck permits for a V-shaped two-to-one merging bottleneck. They showed that the first-best pricing scheme for this V-shaped network does

not always achieve a Pareto improvement, because the cost of one group of drivers is increased due to the permit pricing. Xiao et al., (2019) presented two tradable parking permit schemes for a corridor system with three alternative travel modes, i.e., transit, driving alone and carpool, when the parking supply at destination is insufficient. It was found that the prices of parking permits, regardless of whether the trip is completed as a carpool or not, decrease with the parking supply, and the price that a solo driver should pay is higher than that a carpooler should pay. The tradable uniform parking permit scheme is more efficient than the tradable differentiated parking permit scheme for solo-driving and carpooling travelers.

It can be noted that these abovementioned studies mainly focused on a single OD pair with one or two routes, and the transaction costs for trading the credits were usually ignored. It is thus important to look at the impacts of network effects and transaction costs on the effectiveness of the tradable credit schemes. In reality, markets always create speculators. It is also meaningful to look at the effects of collusive behavior among credit purchasers. In addition, when the parking supply is insufficient at destination, one may first park his/her car at a park-and-ride lot and then transfers to a transit vehicle to reach final destination. It is therefore necessary to extend the existing models to incorporate the park-and-ride services in a further investigation.

3.2.4. Redistribution of toll revenue

The redistribution of toll revenue is an important factor influencing the public acceptability of the toll schemes and thus practical implementation. Adler and Cetin (2001) developed an analytical model for a two-node two-route network, aiming to explore a direct redistribution approach in which money collected from the drivers on a more desirable route is directly transferred to the users on a less desirable route. It was shown that this model about toll collection and subsidization would reduce the travel cost for all travelers and totally eliminate the wait time in the queue. Compared with the social optimal solution, the direct redistribution model yields almost identical results. Mirabel and Reymond (2011) analyzed the impact of toll redistribution on total cost and on modal split between railroad and road based on the two-mode model of Tabuchi (1993). In their model, it was assumed that toll revenue from road was redistributed to public transport. Two kinds of road toll regimes were considered, i.e., a fine toll and a uniform toll. It was shown that a toll policy is more efficient as long as toll revenue is directed towards public transport when the railroad fare is equal to average cost. These previous studies mainly focused on the redistribution of toll revenue for public transport improvement purpose. It will be meaningful to take into account other use purposes, such as transportation infrastructure investment and fiscal revenue, and to determine the optimal redistribution proportion among different uses.

3.2.5. Parking pricing

Although the first-best time-varying toll may eliminate queuing completely, congestion toll scheme may not be politically feasible. Parking charging can be considered as a possible substitute for the congestion tolling because parking charges seem to be much easier to implement than congestion tolls. Similar to congestion tolls, parking charges may be used to disperse demand over time so as to reduce congestion and gain efficiency. Zhang et al., (2008) presented a morning-evening commuting model to determine a location-dependent parking fee scheme that optimizes the commuter morning / evening commuting pattern. Qian et al., (2011) analyzed the regulatory schemes of parking market: price-ceiling and quantity tax/subsidy schemes. It was shown that both price-ceiling and quantity tax/subsidy regulations can efficiently reduce system cost and commuter cost under certain conditions, and help ensure the stability of the parking market. Fosgerau and de Palma (2013) determined the optimal parking charges and evaluated the benefits of parking pricing as an alternative to congestion tolls. Zhang and van Wee (2011) proposed a duration-dependent parking fee scheme, and compared it with three other pricing regimes: no charging, optimal time-varying road tolls, and a combination of optimal time-varying road tolls and location-dependent parking fees. Ma and Zhang (2017) derived dynamic parking charges for a bottleneck system with ridesharing, in which all travelers were assumed to participate in the ridesharing program, i.e., a traveler was either a driver or a passenger. As a substitute for parking pricing, parking permit schemes have also been studied in the literature (Liu et al., 2014b; Zhang et al., 2011; Xiao et al., 2019), as presented in SubSection 3.2.3.

The aforementioned studies did not consider commuters' time spent on searching for available parking spaces. The search for parking spaces comprises a wasteful commuting component that contributes to traffic congestion, and thus should be considered in commuting cost. On the other hand, parking facilities are usually supplied by both private firms and public sector. It will be interesting to examine this mixed market and compare it with the extreme cases of either private-only or public-only parking provision regime. In addition, a mixed market consisting of solo-driving and ridesharing should be investigated for analyzing the effects of parking pricing on the market.

3.2.6. Port pricing

Ship queuing and waiting at a general anchorage to enter the berth under the port congestion are similar to the auto queuing and waiting at the road bottleneck. The congestion pricing concept for a road bottleneck has been extended to address the port congestion pricing issues. In this regard, Laih and his collaborators have undertaken a number of studies (see, Laih and Hung, 2004; Laih et al., 2007, Laih et al., 2015; Laih and Chen, 2008, Laih and Chen, 2009; Laih and Sun, 2013). They derived optimal time-varying and/or step toll schemes to eliminate or decrease the port congestion. By levying port congestion tolls, the departure schedules of container ships can be rationally changed, and thus the arrival times of container ships at the busy port can be smoothed or dispersed. As a result, the queuing delays of container ships for port

entry decrease. They also derived the resultant changes of container ships' departure schedules after levying port conges-tion tolls. However, they did not consider the redistribution of port congestion charges, which can help promote the public acceptability of port congestion charging scheme.

3.2.7. Airport pricing

In the literature, there are some studies about airport congestion pricing issues. For example, Daniel (1995) proposed an airport runway congestion pricing model (i.e., a bottleneck model with time-dependent stochastic queuing) for estimating congestion prices and capacities for large hub airports. The proposed stochastic bottleneck model combines stochastic queu-ing, time-varying traffic rates, and intertemporal adjustment of traffic in response to queuing delay and fees. Daniel and Pahwa (2000) showed that the stochastic bottleneck model of Daniel (1995) can generate more realistic traffic patterns than earlier models, such as deterministic bottleneck model of Vickrey (1969). Daniel and Harback (2008, Daniel and Har-back, 2009) adopted the stochastic bottleneck model to address the airport congestion pricing issues for 27 major US hub airports. Daniel (2011) further determined the equilibrium congestion pricing schedules, traffic rates, queuing delays, lay-over times, and connection times by time of day for four Canadian airports (Toronto, Vancouver, Calgary, and Montreal). Daniel (2014) examined the efficiency and practicality of airport slot constraints using a deterministic bottleneck model of landing and takeoff queues. It was shown that slot constraints at US airports would be ineffective, and effective slot con-straints require many narrow slot windows. Silva et al. (2014) studied airlines' interactions and scheduling behavior, together with airport pricing, using a combination of a deterministic bottleneck model and a vertical structure model that explicitly considers the roles of airlines and passengers. Wan et al. (2015) treated terminal congestion and runway congestion sep-arately, and studied its implication for design of optimal airport charges and/or terminal capacity investment. To capture the difference between these two types of congestion, they adopted a deterministic bottleneck model for the terminal and a conventional congestion model for the runways. They showed that the welfare-optimal uniform airfares do not yield the first-best outcome. The first-best fares charged to the business passengers are higher than the leisure passengers' fare if and only if the relative schedule-delay cost of business passengers is higher than that of leisure passengers. These airport pricing studies usually focused on a single airport, and did not consider the effects of airport pricing on the competition and collaboration among regional airports (e.g., the airports of Hong Kong, Guangzhou, Shenzhen, Zhuhai, and Macao in the Greater Bay Area of China), which deserves a further study.

3.2.8. Urban spatial structure

Queuing delays at the bottleneck during the morning and evening commutes may be an important factor influencing household residential location choice, which shapes urban spatial structure of a city (Mun et al., 2005). Arnott (1998) in-corporated the departure time choice into a model of urban spatial structure by using Vickrey's bottleneck model. It was shown that in contrast to the standard static model (without time dimension), congestion tolling in the bottleneck model can cause urban form to become less concentrated, and thus may have less pronounced effects on urban spatial structure than was previously thought. Fosgerau and de Palma (2012) introduced spatial heterogeneity into the bottleneck model via considering dynamic congestion in an urban setting where trip origins are spatially distributed. It was shown that at equi-librium, travelers sort according to their distances to the destination; the queue is always unimodal regardless of the spatial distribution of trip origins; and the travelers located beyond a critical distance from the CBD tend to gain from tolling, even when toll revenue is not redistributed, while nearby travelers lose. Gubins and Verhoef (2014) considered a monocentric city with a traffic bottleneck located at the entrance to the CBD. The commuters' departure times, household residential locations, and lot sizes are all endogenously determined. They showed that road pricing may lead to urban sprawl, even when the collected toll revenue is not redistributed back to the city inhabitants. Takayama and Kuwahara (2017) further developed a model considering commuters' heterogeneity, departure time and residential location choices in a monocentric city with a single bottleneck. The results showed that commuters sort themselves temporally and spatially according to their values of time and schedule delay flexibility. Imposing a congestion toll without redistributing toll revenue causes the physical expansion of the city, which is opposite to the results of traditional location models. Franco (2017) examined the effects of change in downtown parking supply on urban welfare, mode choice and urban spatial structure using a general spatial equilibrium model of a closed monocentric city with two transport modes, endogenous residential parking supply and bottleneck congestion at the CBD.

Xu et al., (2018) presented an integrated model of urban spatial structure and traffic congestion for a two-zone monocen-tric city in which the two zones are connected by a congested highway and a crowded railway. The commuters' departure time and mode choices are governed by a bottleneck model, and the endogenous interactions between travel and residential relocation choices are analyzed. Fosgerau et al., (2018) presented a unified model of the bottleneck model and the mono-centric city model. The model generates a number of new insights regarding the interaction between congestion dynamics and urban spatial equilibrium. Unlike the traditional static congested city models, their model leads to an optimal city that is less dense in the center and denser in the suburb than the city at the laissez-faire equilibrium. This result is similar to that in Gubins and Verhoef (2014). Vandyck and Rutherford (2018) developed a spatial general equilibrium model to study economy-wide and distributional implications of congestion pricing in the presence of agglomeration externalities and un-employment. Fosgerau and Kim (2019) presented a new monocentric city framework that combines a discrete urban space with multiple Vickrey-type bottlenecks. They confirmed empirically the relationship between residential location choice and trip-timing choice, i.e., commuters traveling a longer distance tend to arrive at work early or late (i.e., at off-peak times)

while commuters with a shorter distance tend to arrive at the peak time. These aforementioned studies considered the role of households' residential location decisions in shaping urban spatial structure, but ignored the role of firms' location decisions. In a further study, the effects of both households' and firms' location decisions should be simultaneously considered in the analysis of urban spatial equilibrium.

3.2.9. Public transit services

The classical Vickrey's bottleneck model has also been employed to address transit passenger travel choice behavior and transit system optimization issues. Kraus and Yoshida (2002) incorporated the commuter's time-of-use decision into a model of transit pricing and transit service optimization, in which waiting time at a transit stop was treated analogously to queuing time at the highway bottleneck. It was shown that increased ridership leads to higher average user cost, and the relationship between service frequency and ridership does not conform to the well-known square root principle. Yoshida (2008) further studied the effects of passengers' queuing rules at transit stops (including the first-in-first-out and the random-access queuing) on the mass-transit policies, such as the number of trains and runs, scheduling, and pricing. The results showed that when the shadow value of a unit of waiting time exceeds that of a unit of time late for work, the passengers' queuing discipline does not have any effect on the optimal or second-best mass-transit policy. Otherwise, the aggregate travel cost with random-access queuing is lower than that with first-in-first-out. Tian et al., (2007) analyzed the equilibrium properties of commuters' trip timing during the morning commute on a many-to-one linear corridor transit system with considering in-vehicle passenger crowding effect and schedule delay cost.

Monchambert and de Palma (2014) considered a bi-modal competitive system, consisting of a public transport mode (bus), which may be unreliable, and an alternative mode (taxi). The results showed that the public transport service reliability at the competitive equilibrium increases with the taxi fare, and the public transport service reliability and thus patronage at equilibrium are lower than those at the first-best social optimum. de Palma et al. (2017) investigated trip-timing decisions of rail transit users who trade off in-vehicle passenger crowding costs and disutility from traveling early or late. Three fare regimes, namely no fare, an optimal uniform fare, and an optimal time-dependent fare, were studied and compared, together with determination of the optimal long-run number and capacities of trains. Wang et al., (2017) designed the policies of transit subsidies (including cost and passenger subsidies) from either government funding or road toll revenue to circumvent the Downs-Thomson Paradox appearing in a competitive highway/transit system. Yang and Tang (2018) proposed a fare-reward scheme for managing rail transit peak-hour congestion with homogeneous commuters, in which a commuter is rewarded with one free trip during pre-specified shoulder periods after taking a certain number of paid trips during the peak hours. Such a fare-reward scheme aims to shift commuters' departure time to reduce their queuing at stations in an incentive-compatible manner while keeping the transit operator's revenue intact. Tang et al., (2019) further considered the heterogeneous commuters, in terms of commuters' scheduling flexibility (i.e., arrival time flexibility interval), and proposed an incentive-based hybrid fare scheme, which combines the fare-reward scheme with a non-rewarding uniform fare scheme. It was shown that the hybrid fare scheme can create a revenue-preserving win-win-win situation for the transit operator, flexible commuters and non-flexible commuters.

These previous studies have provided many insights into understanding the travel choice behavior of transit passengers, operations and scheduling of transit services, and the effects of various transit policies, such as transit service pricing and subsidies. However, they usually consider transit mode only or two physically isolated modes (e.g., auto and rail). In reality, auto and bus share the same roadway, and thus the interaction between them cannot be ignored. The congestion externality caused by intermodal interaction should be considered in the transit fare pricing, together with the in-vehicle crowding externality in transit vehicles.

3.3. Supply-side strategies

3.3.1. Bottleneck capacity design and capacity allocation

Arnott et al., (1993a) concerned the capacity expansion issue of a road bottleneck with homogeneous commuters. It was shown that the self-financing result (i.e., toll revenue exactly covers its capital cost) holds even when the variation of the toll by time of day is constrained (e.g., a coarse toll). Arnott and Kraus (1995) investigated under what circumstances the first-best pricing and investment rules (i.e., first-best self-financing rule, or trip price equals marginal cost) for a congestible bottleneck facility apply when both the time variation of the congestion charge is constrained and users are different in unobservable characteristics so that the same congestion charge must be applied to heterogeneous users, in terms of work start time or value of time. Their findings indicated that the first-best self-financing rule holds if the congestion externality is anonymous, independent of user type. Thereby, marginal cost pricing of a congestible facility is feasible even if users differ in observationally indistinguishable ways, when a completely flexible toll is employed. But, when there are constraints on the time variation of the toll (e.g., uniform toll), marginal cost pricing is infeasible and a variant of Ramsey pricing is (second-best) optimal. Liu et al., (2015a) designed a highway use reservation system to allocate highway space to potential users at different time intervals. They also evaluated the efficiency of the reservation system. Lamotte et al., (2017) addressed the capacity allocation issue of a road between two vehicle types (i.e., conventional and bookable autonomous vehicles), using a variant of the bottleneck model. These studies usually assumed a fixed total travel demand, a single travel mode and a deterministic environment. In further studies, these assumptions can be relaxed to consider elastic demand, multiple travel modes and/or stochastic situation.

3.3.2. Parking capacity design

Qian et al., (2011, 2012) investigated the design problems of parking capacity, parking fee, and access time when all parking lots in the parking market are operated by multiple profit-driven private operators or by a welfare-driven social planner. Franco (2017) examined how the changes in CBD parking supply affect residential land rents, residential parking supply, mode choice, welfare, air pollution, share of auto users, population densities and city size, and whether the self-financing theorem holds in the context of the urban spatial model. Liu (2018) presented an equilibrium model of departure time and parking location choices for optimizing the parking supply that minimizes the total system cost (i.e., the sum of travel cost and social cost of parking supply) under either user equilibrium or system optimum pattern. He found that the optimal planning of parking with autonomous vehicles is significantly different from that without autonomous vehicles. Zhang et al., (2019) further analyzed the optimal parking supply strategy for autonomous vehicles to minimize the total system cost based on an integrated morning-evening commuting model. These previous studies did not concern the competition of different parking types (e.g., on-street and off-street) and the parking facility ownership issues (private and public), which can be considered in further studies.

3.4. Joint strategies of supply and demand sides

In the literature, there are a few studies involving joint strategies of capacity investment and demand management. For instance, Arnott et al., (1994) explored the welfare effects of a toll-financed capacity expansion (i.e., toll revenues are used to finance transport investment) using a bottleneck model with user heterogeneity consideration. It was shown that if initial capacity is sufficiently small, a toll-financed expansion leaves all drivers better off. Xiao et al., (2012) studied the feasibility of expanding bottleneck capacity by toll revenue. The results showed that if the revenue generated by the optimal flat toll is used to finance the capacity expansion, the trip cost of each commuter is reduced in the long run. However, the revenue from the optimal flat toll can never cover the capital cost of constructing the optimal capacity for minimizing the total system cost under constant returns. Qian et al., (2012) derived the optimal parking capacity, fee, and access time which altogether yield the minimum total social cost. Wan et al., (2015) investigated the joint impacts of airport terminal capacity expansion and time-varying terminal fine toll on passenger demand (including business and leisure passengers) and airport system. These previous studies usually assumed a constant returns to scale and piecewise constant scheduling preferences, which can be extended to consider other returns to scale (e.g., increasing) and time-varying scheduling preferences.

3.5. Limitations of existing related studies

In the previous subsections, we have reviewed the literature about bottleneck model studies from the perspectives of travel behavior analysis, demand-side strategies, supply-side strategies, and joint strategies of both demand and supply sides. In spite of broad extensions conducted since the pioneering work of Vickrey (1969), there are still some limitations in the existing related studies, summarized as follows.

(i) As shown in Section 3.1, various strong assumptions are often made in the related studies, aiming to simplify the model and derive analytical solutions. Such simplicity may lead to a large deviation of the model results from the actual values, and thus restricts explanatory power and real applications of the model. In order to model more realism, it is necessary to relax these assumptions in further studies.

(ii) The existing studies have mainly focused on the topics of travel behavior analysis and demand-side strategies (particularly on congestion tolling). However, only limited attention has been paid to the topics of supply-side strategies (e.g., financing mode for capacity expansion due to fiscal deficit) and joint strategies (e.g., using congestion tolls to finance capacity expansion). The disposition of toll revenue also lacks adequate research. These topics provide potential research opportunities for further studies.

(iii) Driving effects of information technology innovation on social development, such as sharing economy and smart mobility, are seldom incorporated in the previous related studies. As such, rapid development of new technologies has been bringing about significant social reform, which is changing people's behavior and reshaping urban development. By incorporating these factors causing social changes, the bottleneck model could continue to provide new theoretical insights.

4. Future research directions

According to the literature review and analysis of the limitations of existing related studies presented in the previous section, one can identify some new and important gaps and opportunities for further studies, presented as follows.

4.1. Financing mode for bottleneck capacity expansion

One solution to the bottleneck congestion is to expand the capacity of the bottleneck. Such expansion needs a huge capital cost, which imposes a heavy financial burden on local authority. In order to broaden the range of fiscal sources for bottleneck capacity expansion, various franchising programs, such as build-operate-transfer (BOT) or public-private partnership (PPP) projects, have been implemented in practice to encourage private sectors to invest in massive transit projects. In

a BOT contract, the private investor negotiates with the government to finance, design, construct, and operate transportation infrastructure for a certain period (i.e., a concession period). Upon the expiration of the concession period, the government will take over the infrastructure.

A PPP contract, as another procurement model of public projects or services, implies a collaborative agreement between private sectors and government targeted at financing, designing, implementing and operating infrastructure and services. Partnerships between private sectors and government provide advantages to both parties. The technology and innovation of private sectors can help provide better public services through improved operational efficiency. The government provides the private sector with incentives to deliver projects on time and within budget. The PPP contract specifies the rights and obligations of each party, embodying risk and revenue allocations between the parties. It is important to address the BOT or PPP contract design issues of the bottleneck capacity expansion, particularly under a situation of the shortage of funds.

4.2. Redistribution of externality-based charging

Congestion pricing schemes have been operating for years in a few countries and regions, such as Singapore, London, Stockholm, and Milan. Such schemes are not worldwide implemented yet due to low public acceptance, which is caused by the following factors: privacy, equity, complexity, and uncertainty (Gu et al., 2018). The privacy issue means that the itineraries of travelers are recorded by the charging facilities at different locations. The equity issue implies that congestion pricing hurts the poor from using road facilities and makes the road resources become a privilege of the rich. The complexity issue concerns the desire for a simple and well-understood proposal for calculation of congestion charges. The uncertainty issue includes the uncertainty in the effectiveness of the proposed scheme, and the uncertainty in revenue allocation. In order to improve public acceptance towards congestion pricing policy, the redistribution of toll revenue from congestion pricing is a critical issue. The government should make a reasonable allocation scheme of toll revenue to improve people's livelihood, such as expanding road capacity, improving public transit services, and reducing taxation. To achieve strong public support, the details of the use of toll revenue should be publicized to the society.

It is well known that the main economic principle behind congestion pricing is to internalize the congestion externality caused by transportation. Transportation contributes to environmental externality due to vehicular pollution emissions, besides congestion externality. In order to control air pollution level and improve air quality, clean air action programs have been launched in some large Chinese cities, such as Beijing and Shanghai. The measures adopted in the program include subsidizing use of clean energy (e.g., electric or natural gas vehicles), retrofit of old motorized vehicles, and purification of vehicular pollutant emissions (e.g., free provision of vehicular exhaust purifier). To achieve financial sustainability, it is proposed to levy the emission taxes and redistribute part of the emission taxes to fund the aforementioned programs. Therefore, further studies can be focused on how to redistribute the emission tax revenue, which will affect the practical implementation of emission pricing scheme and the public acceptance towards this scheme.

4.3. Auto ownership rationing in an era of sharing economy

Auto sharing or ridesharing, as an emerging hot topic in the filed of transportation, may have a significant effect on the auto ownership rationing. It is expected that implementation of ridesharing has a potential to reduce the maximum number of autos and parking spaces required in the transportation system, which affects the traffic congestion level and the residential location choice and thus spatial distribution of residents in the urban system. In the ridesharing service system, the platform for ridesharing (e.g., Didi or Uber) plays an important role in matching shared autos and passengers (Wang and Yang, 2019), and the fleet size, service price or subsidy for ridesharing can help adjust the shared auto utilization rate, balance the modal split, and thus relieve the traffic congestion level of the system. Further studies can, therefore, be made to consider the relationships among ridesharing, auto ownership rationing, and urban spatial structure, and to investigate the fleet size, pricing or subsidizing problem of ridesharing in a competitive multi-modal transportation system. The competition and collaboration between different ridesharing platforms and between ridesharing platform and public transit are also an important direction for future study.

4.4. Modeling effects of new technology revolution

It is widely recognized that the rapid developments of information and communication technologies have significantly changed people's learning, work and life styles. For example, telecommuting or teleworking, as an alternative work arrangement, becomes a growing trend in the information age. Telecommuting will drive people away from workplaces, and thus save office space in urban areas and change the household residential location choice farther from the workplaces, leading to a more spread-out city. It will also reduce the number of work trips and thus the demand for ground transportation, leading to reduction in energy consumption, traffic congestion and air pollution. However, telecommuting reduces the chance and time for teamwork and face-to-face communications. As a result, team productivity may actually suffer, which hurts the productivity of individual's firm and the urban economy. Regardless of its two sides, the telecommuting has recently become a major working mode of various professions due to outbreak of COVID-19 across the globe, making people more aware of its importance.

On the other hand, rapid developments of new technologies also change the mobility of people and goods. It is believed that the emerging 5 G and self-driving technologies will revolutionize the transportation industry. The 5 G technology will enable road users and transportation infrastructure to communicate with everything else on the road. The self-driving technology can drive vehicles automatically, and thus the car users do not need to carry out the driving task and thus they can spend in-vehicle time in the autonomous cars on work or leisure activities, yielding extra activity utility. The end-to-end connectivity across the city with the 5 G technology allows autonomous vehicles to drive close to each other through cooperating and platooning technologies, thus leading to increased network capacity and decreased traffic congestion in the peak period. The 5 G technology can also alert autonomous vehicles of change of traffic conditions, such as collisions, weather and traffic accidents, through direct and real-time communication from vehicle to vehicle, causing increased safety and reliability on the road. It is naturally needed to investigate the effects of new technology revolution on the movement behavior of people and goods and to design an efficient and sustainable urban system.

5. Conclusion

The goal of this paper is to undertake a broad literature review of the bottleneck model research over the past half century. The review undertaken in this paper uses a bibliometric analysis approach, in which the literature data of a total of 232 relevant papers are extracted from three well-recognized journal databases or search engines, namely Web of Science Core Collection, Scopus, and Google Scholar. This analysis identifies the leading topics, top contributing authors, influential papers, and distributions of publications by journal, allowing readers to track how and where the literature has evolved. The literature is classified in terms of recurring themes into four main categories: travel behavior analysis, demand-side strategies, supply-side strategies, and joint strategies of demand and supply sides. For each theme, typical models proposed in previous studies are reviewed. Based on a systematic review, we have identified some main gaps and opportunities in the bottleneck model research, which provides potential avenues for future research in this important and exciting area. By incorporating technological progress in the new digital era, the bottleneck model research keeps pace with the times and thus to contribute to new theoretical development.

Acknowledgments

Appendix. Basic formulation of the bottleneck model

The classical bottleneck model describes the departure time choice of commuters during morning commute. Every morning, N homogeneous commuters travel from home to work along a highway containing a bottleneck with a capacity s. To simplify the analysis, all commuters want to reach the workplace at an identical preferred arrival time t^*.[2] Without loss of generality, the free-flow travel time from home to work is assumed to be zero. Thus, a commuter arrives at the bottleneck immediately after leaving home and arrives at the workplace immediately after leaving the bottleneck. When the arrival rate at the bottleneck exceeds the bottleneck's capacity, a queue develops. Those who arrive early or late face a schedule delay cost. Commuters choose their departure times based on a trade-off between the bottleneck congestion and the schedule delay cost.

Let $C(t)$ denote the travel cost of commuters departing from home to work at time t. It is composed of queuing delay cost at the bottleneck and schedule delay cost of arriving early or late. Let $T(t)$ be the queuing delay time at the bottleneck at time t. $C(t)$ is then given as

$$C(t) = \alpha T(t) + \beta \max(0, t^* - t - T(t)) + \gamma \max(0, t + T(t) - t^*), \tag{A1}$$

where α is the unit cost of travel time, β is the unit cost of arriving early, and γ is the unit cost of arriving late. According to the empirical study of Small (1982), the relationship $\gamma > \alpha > \beta$ should hold.

The queuing delay time $T(t)$ equals the queue length $D(t)$ divided by the bottleneck capacity s, i.e., $T(t) = D(t)/s$, where $D(t)$ is the difference between the cumulative arrivals and cumulative departures by that time, i.e.,

$$D(t) = \int_{t_q}^{t} r(t)\mathrm{d}t - s(t - t_q), \tag{A2}$$

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where $r(t)$ is the departure rate of commuters from home at time t and t_q is the time at which the queue begins.

At the equilibrium, all commuters have the same travel cost $C(t)$ regardless of their departure time. This means $dC(t)/dt = 0, \forall t \in (t_q, t_{q'})$, where $t_{q'}$ is the time when the queue ends. One can thus derive the equilibrium departure rate $r(t)$ as

$$r(t) = \begin{cases} \frac{\alpha}{\alpha - \beta} s, \forall t \in (t_q, \tilde{t}), \\ \frac{\alpha}{\alpha + \gamma} s, \forall t \in (\tilde{t}, t_{q'}), \end{cases} \tag{A3}$$

where $\tilde{t}$ is the departure time from home at which a commuter can arrive at workplace punctually, i.e., $\tilde{t} + T(\tilde{t}) = t^*$. Eq. (A3) shows that the equilibrium departure rate curve is piecewise constant.

In the morning peak period $(t_q, t_{q'})$, the capacity of the bottleneck is fully utilized, and thus $t_{q'} - t_q = N/s$ holds. At the equilibrium, the first and last commuters do not face a queue, their queuing delays are zero, and their schedule delay costs must thus be equal, expressed as

$$\beta (t^* - t_q) = \gamma \left(t_{q'} - t^* \right). \tag{A4}$$

From Eq. (A4), $t_{q'} - t_q = N/s$ and $\tilde{t} + T(\tilde{t}) = t^*$, one obtains

$$t_q = t^* - \frac{\gamma}{\beta + \gamma} \frac{N}{s}, \quad t_{q'} = t^* + \frac{\beta}{\beta + \gamma} \frac{N}{s} \text{ and } \tilde{t} = t^* - \frac{\beta \gamma}{\alpha (\beta + \gamma)} \frac{N}{s} \tag{A5}$$

The resultant equilibrium travel cost is $\bar{C} = (\beta \gamma / (\beta + \gamma))(N/s)$. From equilibrium condition $C(t) = C(t_q) = C(t_{q'})$ and Eqs. (A1) and (A4), one can derive the queuing delay time as

$$T(t) = \begin{cases} \frac{\beta}{\alpha - \beta} (t - t_q), \forall t \in \left[t_q, \tilde{t} \right], \\ \frac{\gamma}{\alpha + \gamma} (t_{q'} - t), \forall t \in \left[\tilde{t}, t_{q'} \right]. \end{cases} \tag{A6}$$

Eq. (A6) shows that a queue builds up linearly from t_q to $\tilde{t}$ and then dissipates linearly until it disappears at $t_{q'}$. This means that the queuing delay curve is piecewise linear.

References

Abegaz, D., Hjorth, K., Rich, J., 2017. Testing the slope model of scheduling preferences on stated preference data. Transportation Research Part B 104, 409–436.

Adler, J.L., Cetin, M., 2001. A direct redistribution model of congestion pricing. Transportation Research Part B 35 (5), 447–460.

Akamatsu, T., Wada, K., Hayashi, S., 2015. The corridor problem with discrete multiple bottlenecks. Transportation Research Part B 81, 808–829.

Amirgholy, M., Gonzales, E.J., 2017. Analytical equilibrium of bicriterion choices with heterogeneous user preferences: application to the morning commute problem. Transportmetrica B 5 (4), 455–487.

Arnott, R., 1998. Congestion tolling and urban spatial structure. Journal of Regional Science 38 (3), 495–504.

Arnott, R., 2013. A bathtub model of downtown traffic congestion. Journal of Urban Economics 76 (1), 110–121.

Arnott, R., de Palma, A., Lindsey, R., 1988. Schedule delay and departure time decisions with heterogeneous commuters. Transportation Research Record 1197, 56–67.

Arnott, R., de Palma, A., Lindsey, R., 1990a. Economics of a bottleneck. Journal of Urban Economics 27 (1), 111–130.

Arnott, R., de Palma, A., Lindsey, R., 1990b. Departure time and route choice for the morning commute. Transportation Research Part B 24 (3), 209–228.

Arnott, R., de Palma, A., Lindsey, R., 1991a. Does providing information to drivers reduce traffic congestion? Transportation Research Part A 25 (5), 309–318.

Arnott, R., de Palma, A., Lindsey, R., 1991b. A temporal and spatial equilibrium analysis of commuter parking. Journal of Public Economics 45 (3), 301–335.

Arnott, R., de Palma, A., Lindsey, R., 1992. Route choice with heterogeneous drivers and group-specific congestion costs. Regional Science and Urban Economics 22 (1), 71–102.

Arnott, R., de Palma, A., Lindsey, R., 1993a. A structural model of peak-period congestion: a traffic bottleneck with elastic demand. American Economic Review 83 (1), 161–179.

Arnott, R., de Palma, A., Lindsey, R., 1993b. Properties of dynamic traffic equilibrium involving bottlenecks, including a paradox and metering. Transportation Science 27 (2), 148–160.

Arnott, R., de Palma, A., Lindsey, R., 1994. The welfare effects of congestion tolls with heterogeneous commuters. Journal of Transport Economics and Policy 28 (2), 139–161.

Arnott, R., de Palma, A., Lindsey, R., 1998. Recent developments in the bottleneck model. In: Button, K.J., Verhoef, E.T. (Eds.), Road Pricing, Traffic Congestion and the Environment: Issues of Efficiency and Social Feasibility. Edward Elgar, Aldershot, pp. 79–110.

Arnott, R., de Palma, A., Lindsey, R., 1999. Information and time-of-usage decisions in the bottleneck model with stochastic capacity and demand. European Economic Review 43 (3), 525–548.

Arnott, R., DePalma, E., 2011. The corridor problem: preliminary results on the no-toll equilibrium. Transportation Research Part B 45 (5), 743–768.

Arnott, R., Kokoza, A., Naji, M., 2016. Equilibrium traffic dynamics in a bathtub model: a special case. Economics of Transportation 7–8, 38–52.

Arnott, R., Kraus, M., 1995. Financing capacity in the bottleneck model. Journal of Urban Economics 38 (3), 272–290.

Bao, Y., Verhoef, E.T., Koster, P., 2019. Regulating dynamic congestion externalities with tradable credit schemes: does a unique equilibrium exist? Transportation Research Part B 127, 225–236.

Bao, Y., Xiao, F., Gao, Z., Gao, Z.Y., 2017. Investigation of the traffic congestion during public holiday and the impact of the toll-exemption policy. Transportation Research Part B 104, 58–81.

Ben-Akiva, M., Cyna, M., de Palma, A., 1984. Dynamic model of peak period congestion. Transportation Research Part B 18, 339–355.

Ben-Akiva, M., de Palma, A., Kanaroglou, P., 1986. Dynamic model of peak period traffic congestion with elastic arrival rates. Transportation Science 20, 164–181.

Ben-Akiva, M., de Palma, A., Kaysi, I., 1991. Dynamic network models and driver information systems. Transportation Research Part A 25, 251–266.

Bernstein, D., Muller, J., 1993. Understanding the competing short-run objectives of peak period road pricing. Transportation Research Record 1395, 122–128.

Borjesson, M., Eliasson, J., Franklin, J.P., 2012. Valuations of travel time variability in scheduling versus mean-variance models. Transportation Research Part B 46 (7), 855–873.

Braid, R.M., 1989. Uniform versus peak-load pricing of a bottleneck with elastic demand. Journal of Urban Economics 26 (3), 320–327.

Braid, R.M., 1996. Peak-load pricing of a transportation route with an unpriced substitute. Journal of Urban Economics 40 (2), 179–197.

Braid, R.M., 2018. Partial peak-load pricing of a transportation bottleneck with homogeneous and heterogeneous values of time. Economics of Transportation 16, 29–41.

Bulteau, J., 2016. Revisiting the bottleneck congestion model by considering environmental costs and a modal policy. International Journal of Sustainable Transportation 10 (3), 180–192.

Chen, H., Liu, Y., Nie, Y.M., 2015a. Solving the step-tolled bottleneck model with general user heterogeneity. Transportation Research Part B 81, 210–229.

Chen, H., Nie, Y.M., Yin, Y., 2015b. Optimal multi-step toll design under general user heterogeneity. Transportation Research Part B 81, 775–793.

Chen, J.Y., Jiang, R., Li, X.G., Hu, M.B., Jia, B., Gao, Z.Y., 2019. Morning commute problem with queue-length-dependent bottleneck capacity. Transportation Research Part B 121, 184–215.

Chu, X., 1995. Endogenous trip scheduling: the Henderson approach reformulated and compared with the Vickrey approach. Journal of Urban Economics 37 (3), 324–343.

Cohen, Y., 1987. Commuter welfare under peak-period congestion tolls: who gains and who loses? International Journal of Transportation Economics 14, 239–266.

Coulombel, N., de Palma, A., 2014. The marginal social cost of travel time variability. Transportation Research Part C 47, 47–60.

Daganzo, C.F., 1985. The uniqueness of a time-dependent equilibrium distribution of arrivals at a single bottleneck. Transportation Science 19 (1), 29–37.

Daganzo, C.F., 2013. System optimum and pricing for the day-long commute with distributed demand, autos and transit. Transportation Research Part B 55, 98–117.

Daganzo, C.F., Garcia, R.C., 2000. A Pareto improving strategy for the time-dependent morning commute problem. Transportation Science 34 (3), 303–311.

Daniel, J.I., 1995. Congestion pricing and capacity of large hub airports: a bottleneck model with stochastic queues. Econometrica 63 (2), 327–370.

Daniel, J.I., 2011. Congestion pricing of Canadian airports. Canadian Journal of Economics 44 (1), 290–324.

Daniel, J.I., 2014. The untold problems with airport slot constraints. Economics of Transportation 3 (1), 16–28.

Daniel, J.I., Harback, K.T., 2008. (When) Do hub airlines internalize their self-imposed congestion delays? Journal of Urban Economics 63 (2), 583–612.

Daniel, J.I., Harback, K.T., 2009. Pricing the major US hub airports. Journal of Urban Economics 66 (1), 33–56.

Daniel, J.I., Pahwa, M., 2000. Comparison of three empirical models of airport congestion pricing. Journal of Urban Economics 47 (1), 1–38.

Daniel, T.E., Gisches, E.J., Rapoport, A., 2009. Departure times in Y-shaped traffic networks with multiple bottlenecks. American Economic Review 99 (5), 2149–2176.

Danielis, R., Marcucci, E., 2002. Bottleneck road congestion pricing with a competing railroad service. Transportation Research Part E 38 (5), 379–388.

de Palma, A., Arnott, R., 1986. Usage-dependent peak-load pricing. Economics Letters 20 (2), 101–105.

de Palma, A., Ben-Akiva, M., Lefevre, C., Litinas, N., 1983. Stochastic equilibrium model of peak period traffic congestion. Transportation Science 17, 430–453.

de Palma, A., Kilani, M., Lindsey, R., 2005. Congestion pricing on a road network: a study using the dynamic equilibrium simulator METROPOLIS. Transportation Research Part A 39, 588–611.

de Palma, A., Lindsey, R., 2000. Private toll roads: competition under various ownership regimes. Annals of Regional Science 34 (1), 13–35.

de Palma, A., Lindsey, R., 2002a. Comparison of morning and evening commutes in the Vickrey bottleneck model. Transportation Research Record 1807, 26–33.

de Palma, A., Lindsey, R., 2002b. Private roads, competition, and incentives to adopt time-based congestion tolling. Journal of Urban Economics 52 (2), 217–241.

de Palma, A., Lindsey, R., 2006. Modelling and evaluation of road pricing in Paris. Transport Policy 13 (2), 115–126.

de Palma, A., Lindsey, R., Monchambert, G., 2017. The economics of crowding in rail transit. Journal of Urban Economics 101, 106–122.

de Palma, A., Lindsey, R., Picard, N., 2015. Trip-timing decisions and congestion with household scheduling preferences. Economics of Transportation 4, 118–131.

de Palma, A., Lindsey, R., Wu, F., 2008. Private operators and time-of-day tolling on a congested road network. Journal of Transport Economics and Policy 42 (3), 397–433.

de Palma, A., Marchal, F., 2002. Real cases applications of the fully dynamic METROPOLIS tool-box: an advocacy for large-scale mesoscopic transportation systems. Networks and Spatial Economics 2, 347–369.

de Palma, A., Marchal, F., Nesterov, Y., 1997. METROPOLIS: modular system for dynamic traffic simulation. Transportation Research Record 1607, 178–184.

DePalma, E., Arnott, R., 2012. Morning commute in a single-entry traffic corridor with no late arrivals. Transportation Research Part B 46 (1), 1–29.

Doan, K., Ukkusuri, S.V., Han, L., 2011. On the existence of pricing strategies in the discrete time heterogeneous single bottleneck model. Transportation Research Part B 45 (9), 1483–1500.

Engelson, L., Fosgerau, M., 2011. Additive measures of travel time variability. Transportation Research Part B 45 (10), 1560–1571.

Engelson, L., Fosgerau, M., 2016. The cost of travel time variability: three measures with properties. Transportation Research Part B 91, 555–564.

Fosgerau, M., 2010. On the relation between the mean and variance of delay in dynamic queues with random capacity and demand. Journal of Economic Dynamics and Control 34 (4), 598–603.

Fosgerau, M., 2011. How a fast lane may replace a congestion toll. Transportation Research Part B 45 (6), 845–851.

Fosgerau, M., 2015. Congestion in the bathtub. Economics of Transportation 4 (4), 241–255.

Fosgerau, M., de Palma, A., 2012. Congestion in a city with a central bottleneck. Journal of Urban Economics 71 (3), 269–277.

Fosgerau, M., de Palma, A., 2013. The dynamics of urban traffic congestion and the price of parking. Journal of Public Economics 105, 106–115.

Fosgerau, M., Engelson, L., 2011. The value of travel time variance. Transportation Research Part B 45 (1), 1–8.

Fosgerau, M., Engelson, L., Franklin, J.P., 2014. Commuting for meetings. Journal of Urban Economics 81, 104–113.

Fosgerau, M., Fukuda, D., 2012. Valuing travel time variability: characteristics of the travel time distribution on an urban road. Transportation Research Part C 24, 83–101.

Fosgerau, M., Jiang, G., 2019. Travel time variability and rational inattention. Transportation Research Part B 120, 1–14.

Fosgerau, M., Karlstrom, A., 2010. The value of reliability. Transportation Research Part B 44 (1), 38–49.

Fosgerau, M., Kim, J., 2019. Commuting and land use in a city with bottlenecks: theory and evidence. Regional Science and Urban Economics 77, 182–204.

Fosgerau, M., Kim, J., Ranjan, A., 2018. Vickrey meets Alonso: commute scheduling and congestion in a monocentric city. Journal of Urban Economics 105, 40–53.

Fosgerau, M., Lindsey, R., 2013. Trip-timing decisions with traffic incidents. Regional Science and Urban Economics 43 (5), 764–782.

Fosgerau, M., Small, K.A., 2013. Hypercongestion in downtown metropolis. Journal of Urban Economics 76 (1), 122–134.

Fosgerau, M., Small, K.A., 2017. Endogenous scheduling preferences and congestion. International Economic Review 58 (2), 585–615.

Fosgerau, M., van Dender, K., 2013. Road pricing with complications. Transportation 40 (3), 479–503.

Franco, S.F., 2017. Downtown parking supply, work-trip mode choice and urban spatial structure. Transportation Research Part B 101, 107–122.

Fu, X., van den Berg, V.A.C., Verhoef, E.T., 2018. Private road supply in networks with heterogeneous users. Transportation Research Part A 118, 430–443.

Gonzales, E.J., 2015. Coordinated pricing for cars and transit in cities with hypercongestion. Economics of Transportation 4 (1–2), 64–81.

Gonzales, E.J., Christofa, E., 2014. Empirical assessment of bottleneck congestion with a constant and peak toll: san Francisco–Oakland Bay Bridge. EURO Journal on Transportation and Logistics 3 (3–4), 267–288.

Gonzales, E.J., Daganzo, C.F., 2012. Morning commute with competing modes and distributed demand: user equilibrium, system optimum, and pricing. Transportation Research Part B 46 (10), 1519–1534.

Gonzales, E.J., Daganzo, C.F., 2013. The evening commute with cars and transit: duality results and user equilibrium for the combined morning and evening peaks. Transportation Research Part B 57, 286–299.

Gu, Z., Liu, Z., Cheng, Q., Saberi, M., 2018. Congestion pricing practices and public acceptance: a review of evidence. Case Studies on Transport Policy 6 (1), 94–101.

Gubins, S., Verhoef, E.T., 2014. Dynamic bottleneck congestion and residential land use in the monocentric city. Journal of Urban Economics 80, 51–61.

Guo, R.Y., Yang, H., Huang, H.J., Li, X., 2018. Day-to-day departure time choice under bounded rationality in the bottleneck model. Transportation Research Part B 117, 832–849.

Guo, X., Sun, H., 2019. Modeling the morning commute problem in a bottleneck model based on personal perception. Journal of Advanced Transportation 2019, https://doi.org/10.1155/2019/6184827.

Hall, J.D., 2018. Pareto Improvements from Lexus Lanes: the effects of pricing a portion of the lanes on congested highways. Journal of Public Economics 158, 113–125.

Hall, J.D., Savage, I., 2019. Tolling roads to improve reliability. Journal of Urban Economics 113. doi:10.1016/j.jue.2019.103187.

Han, K., Friesz, T.L., Yao, T., 2013a. A partial differential equation formulation of Vickrey's bottleneck model, part I: methodology and theoretical analysis. Transportation Research Part B 49, 55–74.

Han, K., Friesz, T.L., Yao, T., 2013b. A partial differential equation formulation of Vickrey's bottleneck model, part II: numerical analysis and computation. Transportation Research Part B 49, 75–93.

Hendrickson, D., Kocur, G., 1981. Schedule delay and departure time decisions in a deterministic model. Transportation Science 15, 62–77.

Hjorth, K., Borjesson, M., Engelson, L., Fosgerau, M., 2013. Estimating exponential scheduling preferences. Transportation Research Part B 81, 230–251.

Huang, H.J., 2000. Fares and tolls in a competitive system with transit and highway: the case with two groups of commuters. Transportation Research Part E 36 (4), 267–284.

Huang, H.J., 2002. Pricing and logit-based mode choice models of a transit and highway system with elastic demand. European Journal of Operational Research 140 (3), 562–570.

Huang, H.J., Tian, Q., Yang, H., Gao, Z.Y., 2007. Modal split and commuting pattern on a bottleneck-constrained highway. Transportation Research Part E 43 (5), 578–590.

Huang, H.J., Yang, H., 1999. Optimal utilization of a transport system with auto/transit parallel modes. Optimal Control Applications and Methods 20, 297–313.

Jenelius, E., 2012. The value of travel time variability with trip chains, flexible scheduling and correlated travel times. Transportation Research Part B 46 (6), 762–780.

Jenelius, E., Mattsson, L.G., Levinson, D., 2011. Traveler delay costs and value of time with trip chains, flexible activity scheduling and information. Transportation Research Part B 45 (5), 789–807.

Jia, Z., Wang, D.Z.W., Cai, X., 2016. Traffic managements for household travels in congested morning commute. Transportation Research Part E 91, 173–189.

Khan, Z., Amin, S., 2018. Bottleneck model with heterogeneous information. Transportation Research Part B 112, 157–190.

Kim, J., 2019. Estimating the social cost of congestion using the bottleneck model. Economics of Transportation 19. doi:10.1016/j.ecotra.2019.100119.

Kim, K., 1999. Traffic congestion at a two-tandem bottleneck. International Journal of Urban Sciences 3 (1), 54–71.

Knockaert, J., Verhoef, E.T., Rouwendal, J., 2016. Bottleneck congestion: differentiating the coarse charge. Transportation Research Part B 83, 59–73.

Koster, P., Pels, E., Verhoef, E.T., 2016. The user costs of air travel delay variability. Transportation Science 50 (1), 120–131.

Kraus, M., 2003. A new look at the two-mode problem. Journal of Urban Economics 54 (3), 511–530.

Kraus, M., Yoshida, Y., 2002. The commuter's time-of-use decision and optimal pricing and service in urban mass transit. Journal of Urban Economics 51 (1), 170–195.

Kuwahara, M., 1990. Equilibrium queueing patterns at a two-tandem bottleneck during the morning peak. Transportation Science 24 (3), 217–229.

Lago, A., Daganzo, C.F., 2007. Spillovers, merging traffic and the morning commute. Transportation Research Part B 41 (6), 670–683.

Laih, C.H., 1994. Queueing at a bottleneck with single- and multi-step tolls. Transportation Research Part A 28 (3), 197–208.

Laih, C.H., 2004. Effects of the optimal step toll scheme on equilibrium commuter behaviour. Applied Economics 36 (1), 59–81.

Laih, C.H., Chen, K.Y., 2008. Economics on the optimal n-step toll scheme for a queuing port. Applied Economics 40 (2), 209–228.

Laih, C.H., Chen, K.Y., 2009. Economics on the optimal port queuing pricing to bulk ships. Applied Economics 41 (22), 2817–2825.

Laih, C.H., Hung, K.C., 2004. The optimal step toll scheme for heavily congested ports. Journal of Marine Science and Technology 12 (1), 16–24.

Laih, C.H., Lin, B., Chen, K.Y., 2007. Effects of the optimal port queuing pricing on arrival decisions for container ships. Applied Economics 39 (14), 1855–1865.

Laih, C.H., Sun, P.Y., 2013. Effects of the optimal n-step toll scheme on bulk carriers queuing for multiple berths at a busy port. Transport Policy 28, 42–50.

Laih, C.H., Tsai, Y.C., Chen, Z.B., 2015. Optimal non-queuing pricing for the Suez Canal. Maritime Economics and Logistics 17 (3), 359–370.

Lam, W.H.K., Li, Z.C., Huang, H.J., Wong, S.C., 2006. Modeling time-dependent travel choice problems in road networks with multiple user classes and multiple parking facilities. Transportation Research Part B 40, 368–395.

Lamotte, R., de Palma, A., Geroliminis, N., 2017. On the use of reservation-based autonomous vehicles for demand management. Transportation Research Part B 99, 205–227.

Lamotte, R., Geroliminis, N., 2018. The morning commute in urban areas with heterogeneous trip lengths. Transportation Research Part B 117, 794–810.

Leurent, F., Wagner, N., 2009. User equilibrium in a bottleneck under multipeak distribution of preferred arrival time. Transportation Research Record 2091, 31–39.

Li, C.Y., Huang, H.J., 2017. Morning commute in a single-entry traffic corridor with early and late arrivals. Transportation Research Part B 97, 23–49.

Li, C.Y., Huang, H.J., 2018. User equilibrium of a single-entry traffic corridor with continuous scheduling preference. Transportation Research Part B 108, 21–38.

Li, Z.C., Lam, W.H.K., Wong, S.C., 2014. Bottleneck model revisited: an activity-based perspective. Transportation Research Part B 68, 262–287.

Li, Z.C., Lam, W.H.K., Wong, S.C., 2017. Step tolling in an activity-based bottleneck model. Transportation Research Part B 101, 306–334.

Li, Z.C., Lam, W.H.K., Wong, S.C., Huang, H.J., Zhu, D.L., 2008. Reliability evaluation for stochastic and time-dependent networks with multiple parking facilities. Networks and Spatial Economics 8, 355–381.

Lindsey, R., 2004. Existence, uniqueness, and trip cost function properties of user equilibrium in the bottleneck model with multiple user classes. Transportation Science 38 (3), 293–314.

Lindsey, R., de Palma, A., Silva, H.E., 2019. Equilibrium in a dynamic model of congestion with large and small users. Transportation Research Part B 124, 82–107.

Lindsey, R., van den Berg, V.A.C., Verhoef, E.T., 2012. Step tolling with bottleneck queuing congestion. Journal of Urban Economics 72 (1), 46–59.

Lindsey, R., Verhoef, E.T., 2001. Traffic congestion and congestion pricing. In: Button, K.J., Hensher, D.A. (Eds.), Handbook of Transport Systems and Traffic Control. Pergamon,, Amsterdam, pp. 77–105.

Liu, P., Liu, Y., 2018. Optimal information provision at bottleneck equilibrium with risk-averse travelers. Transportation Research Record 2672, 69–78.

Liu, W., 2018. An equilibrium analysis of commuter parking in the era of autonomous vehicles. Transportation Research Part C 92, 191–207.

Liu, W., Geroliminis, N., 2016. Modeling the morning commute for urban networks with cruising-for-parking: an MFD approach. Transportation Research Part B 93, 470–494.

Liu, W., Li, X., Zhang, F., Yang, H., 2017a. Interactive travel choices and traffic forecast in a doubly dynamical system with user inertia and information provision. Transportation Research Part C 85, 711–731.

Liu, W., Yang, H., Yin, Y., 2014a. Expirable parking reservations for managing morning commute with parking space constraints. Transportation Research Part C 44, 185–201.

Liu, W., Yang, H., Yin, Y., 2015a. Efficiency of a highway use reservation system for morning commute. Transportation Research Part C 56, 293–308.

Liu, W., Yang, H., Yin, Y., Zhang, F., 2014b. A novel permit scheme for managing parking competition and bottleneck congestion. Transportation Research Part C 44, 265–281.

Liu, W., Yin, Y., Yang, H., 2015b. Effectiveness of variable speed limits considering commuters' long-term response. Transportation Research Part B 81, 498–519.

Liu, W., Zhang, F., Yang, H., 2016. Managing morning commute with parking space constraints in the case of a bi-modal many-to-one network. Transport-metrica A 12 (2), 116–141.

Liu, W., Zhang, F., Yang, H., 2017b. Modeling and managing morning commute with both household and individual travels. Transportation Research Part B 103, 227–247.

Liu, Y., Li, Y., 2017. Pricing scheme design of ridesharing program in morning commute problem. Transportation Research Part C 79, 156–177.

Liu, Y., Li, Y., Hu, L., 2018. Departure time and route choices in bottleneck equilibrium under risk and ambiguity. Transportation Research Part B 117, 774–793.

Liu, Y., Nie, Y.M., 2011. Morning commute problem considering route choice, user heterogeneity and alternative system optima. Transportation Research Part B 45 (4), 619–642.

Liu, Y., Nie, Y.M., Hall, J.D., 2015c. A semi-analytical approach for solving the bottleneck model with general user heterogeneity. Transportation Research Part B 71, 56–70.

Ma, R., Zhang, H.M., 2017. The morning commute problem with ridesharing and dynamic parking charges. Transportation Research Part B 106, 345–374.

Mirabel, F., Reymond, M., 2011. Bottleneck congestion pricing and modal split: redistribution of toll revenue. Transportation Research Part A 45 (1), 18–30.

Monchambert, G., de Palma, A., 2014. Public transport reliability and commuter strategy. Journal of Urban Economics 81, 14–29.

Mun, S.I., 1999. Peak-load pricing of a bottleneck with traffic jam. Journal of Urban Economics 46 (3), 323–349.

Mun, S.I., Konishi, K.J., Yoshikawa, K., 2005. Optimal cordon pricing in a non-monocentric city. Transportation Research Part A 39 (7–9), 723–736.

Mun, S.I., Yonekawa, M., 2006. Flextime, traffic congestion and urban productivity. Journal of Transport Economics and Policy 40 (3), 329–358.

Newell, G.F., 1987. The morning commute for nonidentical travelers. Transportation Science 21 (2), 63–129.

Newell, G.F., 1988. Traffic flow for the morning commute. Transportation Science 22 (1), 47–58.

Nie, Y.M., 2015. A new tradable credit scheme for the morning commute problem. Networks and Spatial Economics 15 (3), 719–741.

Nie, Y.M., Yin, Y., 2013. Managing rush hour travel choices with tradable credit scheme. Transportation Research Part B 50, 1–19.

Nie, Y.M., Zhang, H.M., 2009. Numerical solution procedures for the morning commute problem. Mathematical and Computer Modelling 49 (7–8), 1295–1310.

Noland, R.B., 1997. Commuter responses to travel time uncertainty under congested conditions: expected costs and the provision of information. Journal of Urban Economics 41 (3), 377–406.

Noland, R.B., Small, K.A., 1995. Travel-time uncertainty, departure time choice, and the cost of morning commutes. Transportation Research Record 1493, 150–158.

Noland, R.B., Small, K.A., Koskenoja, P.M., Chu, X., 1998. Simulating travel reliability. Regional Science and Urban Economics 28 (5), 535–564.

O'Dea, W.P., 1999. Optimal metering in the bottleneck congestion model. Journal of Urban Economics 45 (1), 125–142.

Otsubo, H., Rapoport, A., 2008. Vickrey's model of traffic congestion discretized. Transportation Research Part B 42 (10), 873–889.

Peer, S., Verhoef, E.T., 2013. Equilibrium at a bottleneck when long-run and short-run scheduling preferences diverge. Transportation Research Part B 57, 12–27.

Peer, S., Verhoef, E.T., Knockaert, J., Koster, P., Tseng, Y.Y., 2015. Long-run versus short-run perspectives on consumer scheduling: evidence from a re-vealed-preference experiment among peak-hour road commuters. International Economic Review 56 (1), 303–323.

Qian, Z., Xiao, F., Zhang, H.M., 2011. The economics of parking provision for the morning commute. Transportation Research Part A 45 (9), 861–879.

Qian, Z., Xiao, F., Zhang, H.M., 2012. Managing morning commute traffic with parking. Transportation Research Part B 46 (7), 894–916.

Qian, Z., Zhang, H.M., 2011. Modeling multi-modal morning commute in a one-to-one corridor network. Transportation Research Part C 19 (2), 254–269.

Qian, Z., Zhang, H.M., 2013. The morning commute problem with heterogeneous travellers: the case of continuously distributed parameters. Transportmet-rica A 9 (2), 178–203.

Ramadurai, G., Ukkusuri, S.V., Zhao, J., Pang, J.S., 2010. Linear complementarity formulation for single bottleneck model with heterogeneous commuters. Transportation Research Part B 44 (2), 193–214.

Ren, H., Xue, Y., Long, J., Gao, Z.Y., 2016. A single-step-toll equilibrium for the bottleneck model with dropped capacity. Transportmetrica B 4 (2), 92–110.

Rouwendal, J., Verhoef, E.T., Knockaert, J., 2012. Give or take? Rewards versus charges for a congested bottleneck. Regional Science and Urban Economics 42 (1–2), 166–176.

Sakai, K., Liu, R., Kusakabe, T., Asakura, Y., 2017. Pareto-improving social optimal pricing schemes based on bottleneck permits for managing congestion at a merging section. International Journal of Sustainable Transportation 11 (10), 737–748.

Shen, W., Zhang, H.M., 2010. Pareto-improving ramp metering strategies for reducing congestion in the morning commute. Transportation Research Part A 44 (9), 676–696.

Shirmohammadi, N., Yin, Y., 2016. Tradable credit scheme to control bottleneck queue length. Transportation Research Record 2561, 53–63.

Silva, H.E., Lindsey, R., de Palma, A., van den Berg, V.A.C., 2017. On the existence and uniqueness of equilibrium in the bottleneck model with atomic users. Transportation Science 51 (3), 863–881.

Silva, H.E., Verhoef, E.T., van den Berg, V.A.C., 2014. Airlines' strategic interactions and airport pricing in a dynamic bottleneck model of congestion. Journal of Urban Economics 80, 13–27.

Siu, B.W.Y., Lo, H.K., 2013. Punctuality-based departure time scheduling under stochastic bottleneck capacity: formulation and equilibrium. Transportmetrica B 1 (3), 195–225.

Siu, B.W.Y., Lo, H.K., 2014. Punctuality-based route and departure time choice. Transportmetrica A 10 (7), 585–621.

Small, K.A., 1982. The scheduling of consumer activities: work trips. American Economic Review 72 (3), 467–479.

Small, K.A., 1992. Trip scheduling in urban transportation analysis. American Economic Review 82 (2), 482–486.

Small, K.A., 2012. Valuation of travel time. Economics of Transportation 1, 2–14.

Small, K.A., 2015. The bottleneck model: an assessment and interpretation. Economics of Transportation 4, 110–117.

Small, K.A., Chu, X., 2003. Hypercongestion. Journal of Transport Economics and Policy 37 (3), 319–352.

Smith, M.J., 1984. The existence of a time-dependent equilibrium distribution of arrivals at a single bottleneck. Transportation Science 18, 385–394.

Tabuchi, T., 1993. Bottleneck congestion and modal split. Journal of Urban Economics 34 (3), 414–431.

Takayama, Y., 2015. Bottleneck congestion and distribution of work start times: the economics of staggered work hours revisited. Transportation Research Part B 81, 830–847.

Takayama, Y., Kuwahara, M., 2017. Bottleneck congestion and residential location of heterogeneous commuters. Journal of Urban Economics 100, 65–79.

Tang, Y., Yang, H., Wang, B., Huang, J., Bai, Y., 2019. A Pareto-improving and revenue-neutral scheme to manage mass transit congestion with heterogeneous commuters. Transportation Research Part C, in press, https://doi.org/10.1016/j.trc.2019.05.016.

Tian, L.J., Huang, H.J., 2015. Modeling the modal split and trip scheduling with commuters' uncertainty expectation. European Journal of Operational Re-search 244 (3), 815–822.

Tian, L.J., Sheu, J.B., Huang, H.J., 2019. The morning commute problem with endogenous shared autonomous vehicle penetration and parking space con-straint. Transportation Research Part B 123, 258–278.

Tian, L.J., Yang, H., Huang, H.J., 2013. Tradable credit schemes for managing bottleneck congestion and modal split with heterogeneous users. Transportation Research Part E 54, 1–13.

Tian, Q., Huang, H.J., Yang, H., 2007. Equilibrium properties of the morning peak-period commuting in a many-to-one mass transit system. Transportation Research Part B 41 (6), 616–631.

Tseng, Y.Y., Verhoef, E.T., 2008. Value of time by time of day: a stated-preference study. Transportation Research Part B 42 (7–8), 607–618.

van den Berg, V.A.C., 2012. Step-tolling with price-sensitive demand: why more steps in the toll make the consumer better off. Transportation Research Part A 46 (10), 1608–1622.

van den Berg, V.A.C., 2014. Coarse tolling with heterogeneous preferences. Transportation Research Part B 64, 1–23.

van den Berg, V.A.C., Verhoef, E.T., 2011a. Congestion tolling in the bottleneck model with heterogeneous values of time. Transportation Research Part B 45 (1), 60–78.

van den Berg, V.A.C., Verhoef, E.T., 2011b. Winning or losing from dynamic bottleneck congestion pricing? The distributional effects of road pricing with heterogeneity in values of time and schedule delay. Journal of Public Economics 95 (7–8), 983–992.

van den Berg, V.A.C., Verhoef, E.T., 2014. Congestion pricing in a road and rail network with heterogeneous values of time and schedule delay. Transportmetrica A 10 (5), 377–400.

van den Berg, V.A.C., Verhoef, E.T., 2016. Autonomous cars and dynamic bottleneck congestion: the effects on capacity, value of time and preference heterogeneity. Transportation Research Part B 94, 43–60.

van der Zijpp, N., Koolstra, K., 2002. Multiclass continuous-time equilibrium model for departure time choice on single-bottleneck network. Transportation Research Record 1783, 134–141.

Vandyck, T., Rutherford, T.F., 2018. Regional labor markets, commuting, and the economic impact of road pricing. Regional Science and Urban Economics 73, 217–236.

Van Eck, N.J., Waltman, L., 2014. Visualizing Bibliometric Networks. In Ding, Y., Rousseau, R., Wolfram, D. (Eds.), Measuring Scholarly Impact: Methods and Practice (pp. 285–320). Springer.

Vickrey, W.S., 1969. Congestion theory and transport investment. American Economic Review 59 (2), 251–260.

Vickrey, W.S., 1973. Pricing, metering, and efficiently using urban transportation facilities. Highway Research Record 476, 36–48.

Waltman, L., Van Eck, N.J., 2013. A smart local moving algorithm for large-scale modularity-based community detection. European Physical Journal B 86, 471.

Wan, Y., Jiang, C., Zhang, A., 2015. Airport congestion pricing and terminal investment: effects of terminal congestion, passenger types, and concessions. Transportation Research Part B 82, 91–113.

Wang, D.Z.W., Xu, L., 2016. Equilibrium trip scheduling in single bottleneck traffic flows considering multi-class travellers and uncertainty: a complementarity formulation. Transportmetrica A 12 (4), 297–312.

Wang, H., Yang, H., 2019. E-hailing ride-sourcing systems: a framework and review. Transportation Research Part B 129, 122–155.

Wang, J.P., Ban, X.J., Huang, H.J., 2019. Dynamic ridesharing with variable-ratio charging-compensation scheme for morning commute. Transportation Research Part B 122, 390–415.

Wang, W.W., Wang, D.Z.W., Zhang, F., Sun, H., Zhang, W., Wu, J., 2017. Overcoming the Downs-Thomson Paradox by transit subsidy policies. Transportation Research Part A 95, 126–147.

Wu, W.X., Huang, H.J., 2014. Equilibrium and modal split in a competitive highway/transit system under different road-use pricing strategies. Journal of Transport Economics and Policy 48, 153–169.

Wu, W.X., Huang, H.J., 2015. An ordinary differential equation formulation of the bottleneck model with user heterogeneity. Transportation Research Part B 81, 34–58.

Xiao, F., Qian, Z., Zhang, H.M., 2011. The morning commute problem with coarse toll and nonidentical commuters. Networks and Spatial Economics 11 (2), 343–369.

Xiao, F., Qian, Z., Zhang, H.M., 2013. Managing bottleneck congestion with tradable credits. Transportation Research Part B 56, 1–14.

Xiao, F., Shen, W., Zhang, H.M., 2012. The morning commute under flat toll and tactical waiting. Transportation Research Part B 46 (10), 1346–1359.

Xiao, L.L., Huang, H.J., Liu, R., 2015. Congestion behavior and tolls in a bottleneck model with stochastic capacity. Transportation Science 49 (1), 47–65.

Xiao, L.L., Liu, R., Huang, H.J., 2014. Stochastic bottleneck capacity, merging traffic and morning commute. Transportation Research Part E 64, 48–70.

Xiao, L.L., Liu, T.L., Huang, H.J., 2016. On the morning commute problem with carpooling behavior under parking space constraint. Transportation Research Part B 91, 383–407.

Xiao, L.L., Liu, T.L., Huang, H.J., 2019. Tradable permit schemes for managing morning commute with carpool under parking space constraint. Transportation, https://doi.org/10.1007/s11116-019-09982-w, available online.

Xiao, Y., Coulombel, N., de Palma, A., 2017. The valuation of travel time reliability: does congestion matter? Transportation Research Part B 97, 113–141.

Xiao, Y., Fukuda, D., 2015. On the cost of misperceived travel time variability. Transportation Research Part A 75, 96–112.

Xu, D., Guo, X., Zhang, G., 2019. Constrained optimization for bottleneck coarse tolling. Transportation Research Part B 128, 1–22.

Xu, S.X., Liu, R., Liu, T.L., Huang, H.J., 2018. Pareto-improving policies for an idealized two-zone city served by two congestible modes. Transportation Research Part B 117, 876–891.

Yang, H., Huang, H.J., 1997. Analysis of the time-varying pricing of a bottleneck with elastic demand using optimal control theory. Transportation Research Part B 31 (6), 425–440.

Yang, H., Huang, H.J., 2005. Mathematical and Economic Theory of Road Pricing. Elsevier, Oxford UK.

Yang, H., Liu, W., Wang, X., Zhang, X., 2013. On the morning commute problem with bottleneck congestion and parking space constraints. Transportation Research Part B 58, 106–118.

Yang, H., Wang, X.L., 2011. Managing network mobility with tradable credits. Transportation Research Part B 45 (3), 580–594.

Yang, H., Tang, Y., 2018. Managing rail transit peak-hour congestion with a fare-reward scheme. Transportation Research Part B 110, 122–136.

Yao, T., Friesz, T.L., Wei, M.M., Yin, Y., 2010. Congestion derivatives for a traffic bottleneck. Transportation Research Part B 44 (10), 1149–1165.

Yao, T., Wei, M.M., Zhang, B., Friesz, T.L., 2012. Congestion derivatives for a traffic bottleneck with heterogeneous commuters. Transportation Research Part B 46 (10), 1454–1473.

Yoshida, Y., 2008. Commuter arrivals and optimal service in mass transit: does queuing behavior at transit stops matter? Regional Science and Urban Economics 38 (3), 228–251.

Yu, X., van den Berg, V.A.C., Verhoef, E.T., 2019. Carpooling with heterogeneous users in the bottleneck model. Transportation Research Part B 127, 178–200.

Zhang, F., Liu, W., Wang, X., Yang, H., 2017. A new look at the morning commute with household shared-ride: how does school location play a role? Transportation Research Part E 103, 198–217.

Zhang, W.W., Zhao, H., Jiang, R., 2018. Impact of capacity drop on commuting systems under uncertainty. Journal of Advanced Transportation 2018, Article ID 6809304, https://doi.org/10.1155/2018/6809304.

Zhang, X., Huang, H.J., Zhang, H.M., 2008. Integrated daily commuting patterns and optimal road tolls and parking fees in a linear city. Transportation Research Part B 42 (1), 38–56.

Zhang, X., Liu, W., Waller, S.T., Yin, Y., 2019. Modelling and managing the integrated morning-evening commuting and parking patterns under the fully autonomous vehicle environment. Transportation Research Part B 128, 380–407.

Zhang, X., van Wee, B., 2011. Efficiency comparison of various parking charge schemes considering daily travel cost in a linear city. European Journal of Transport and Infrastructure Research 11 (2), 234–255.

Zhang, X., Yang, H., Huang, H.J., 2011. Improving travel efficiency by parking permits distribution and trading. Transportation Research Part B 45 (7), 1018–1034.

Zhang, X., Yang, H., Huang, H.J., Zhang, H.M., 2005. Integrated scheduling of daily work activities and morning-evening commutes with bottleneck congestion. Transportation Research Part A 39 (1), 41–60.

Zhang, X., Zhang, H.M., Li, L., 2010. Analysis of user equilibrium traffic patterns on bottlenecks with time-varying capacities and their applications. International Journal of Sustainable Transportation 4 (1), 56–74.

Zhu, T., Long, J., Liu, H., 2018. Optimal official work start times in activity-based bottleneck models with staggered work hours. Transportmetrica B 7, 657–683.

Zhu, Z., Li, X., Liu, W., Yang, H., 2019. Day-to-day evolution of departure time choice in stochastic capacity bottleneck models with bounded rationality and various information perceptions. Transportation Research Part E 131, 168–192.

Ziegelmeyer, A., Koessler, F., My, K.B., Denant-Boemont, L., 2008. Road traffic congestion and public information: an experimental investigation. Journal of Transport Economics and Policy 42 (1), 43–82.