Persistent_Homology_Features_retina_DiffMap+TDA_report

November 5, 2021

0.1 Apply TDA on retina data

The following code includes:

1. dimensionality reduction using diffusion maps

- diffusion map: pydiffmap

https://pydiffmap.readthedocs.io/en/master/usage.html

2. generate both persistence barcodes and persistence diagrams from the resulting 3D point clouds for six stimuli types

- persistence diagram: ripser (https://ripser.scikit-tda.org/en/latest/index.html)
- persistence barcode: persim
  - https://persim.scikit-tda.org/en/latest/index.html
  - https://github.com/scikit-tda/persim/pull/24
  - https://github.com/scikit-tda/persim/blob/4702251c22d4fffbb1c29409f466745c6b6c26c5/persim/visual

3. Compute the pairwise Wasserstein distance between the persistence diagrams:

- package used: gudhi hera
- useful reference: https://github.com/giotto-ai/giotto-tda/issues/603

[1]:
```
import persim
```

[2]:
```
# Basic imports
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.gridspec as gridspec
import networkx as nx
# from IPython.display import Video

# scikit-tda imports..... Install all with -> pip install scikit-tda
#--- this is the main persistence computation workhorse
import ripser
# from persim import plot_diagrams
import persim
# import persim.plot
```

```python
# teaspoon imports...... Install with -> pip install teaspoon
#---these are for generating data and some drawing tools
import teaspoon.MakeData.PointCloud as makePtCloud
import teaspoon.TDA.Draw as Draw

#---these are for generating time series network examples
from teaspoon.SP.network import ordinal_partition_graph
from teaspoon.TDA.PHN import PH_network
from teaspoon.SP.network_tools import make_network
from teaspoon.parameter_selection.MsPE import MsPE_tau
import teaspoon.MakeData.DynSysLib.DynSysLib as DSL
```

[3]:
```python
import kmapper as km
from sklearn.cluster import AgglomerativeClustering
from sklearn.feature_extraction.text import TfidfVectorizer
from sklearn.decomposition import TruncatedSVD
from sklearn.manifold import Isomap
from sklearn.preprocessing import MinMaxScaler
import matplotlib.pyplot as plt
import numpy as np
from kmapper import jupyter
import pydiffmap
from pydiffmap import diffusion_map as dm
from pydiffmap.visualization import embedding_plot, data_plot
```

[4]:
```python
# import dionysus as ds
import networkx as nx
import numpy as np
import matplotlib.pyplot as plt

import pylab as pl
from matplotlib import collections  as mc
from sklearn.datasets import load_digits
from skimage.morphology import skeletonize
import math
import sys
```

[5]:
```python
from scipy.io import loadmat
retina_original_data = loadmat('retina-201205_bgonlyb50RelNormconsecDel142.
↪mat')['X']
retina_original_data.shape
```

[5]: (698, 6, 264)

0.2 Draw barcode diagrmas:

[6]:
```
class Barcode:
    __doc__ = """
        Barcode visualisation made easy!
        Note that this convenience class requires instantiation as the number
        of subplots produced depends on the dimension of the data.
        """

    def __init__(self, diagrams, verbose=False):
        """
        Parameters
        ===========
        diagrams: list-like
            typically the output of ripser(nodes)['dgms']
        verbose: bool
            Execute print statemens for extra information; currently only echoes
            number of bars in each dimension (Default=False).
        Examples
        ===========
        >>> n = 300
        >>> t = np.linspace(0, 2 * np.pi, n)
        >>> noise = np.random.normal(0, 0.1, size=n)
        >>> data = np.vstack([((3+d) * np.cos(t[i]+d), (3+d) * np.sin(t[i]+d))␣
↪for i, d in enumerate(noise)])
        >>> diagrams = ripser(data)
        >>> bc = Barcode(diagrams['dgms'])
        >>> bc.plot_barcode()
        """
        if not isinstance(diagrams, list):
            diagrams = [diagrams]

        self.diagrams = diagrams
        self._verbose = verbose
        self._dim = len(diagrams)

    def plot_barcode(self, figsize=None, show=True, export_png=False, dpi=100,␣
↪**kwargs):
        """Wrapper method to produce barcode plot
        Parameters
        ===========
        figsize: tuple
            figure size, default=(6,6) if H0+H1 only, (6,4) otherwise
        show: boolean
            show the figure via plt.show()
        export_png: boolean
            write image to png data, returned as io.BytesIO() instance,
```

```
            default=False
        **kwargs: artist paramters for the barcodes, defaults:
            c='grey'
            linestyle='-'
            linewidth=0.5
            dpi=100 (for png export)
        Returns
        ===========
        out: list or None
            list of png exports if export_png=True, otherwise None
        """
        if self._dim == 2:
            if figsize is None:
                figsize = (6, 6)

            return self._plot_H0_H1(
                figsize=figsize,
                show=show,
                export_png=export_png,
                dpi=dpi,
                **kwargs
            )

        else:
            if figsize is None:
                figsize = (6, 4)

            return self._plot_Hn(
                figsize=figsize,
                show=show,
                export_png=export_png,
                dpi=dpi,
                **kwargs
            )

    def _plot_H0_H1(self, *, figsize, show, export_png, dpi, **kwargs):
        out = []

        fig, ax = plt.subplots(2, 1, figsize=figsize)

        for dim, diagram in enumerate(self.diagrams):
            self._plot_many_bars(dim, diagram, dim, ax, **kwargs)

        if export_png:
            fp = io.BytesIO()
            plt.savefig(fp, dpi=dpi)
            fp.seek(0)
```

```
            out += [fp]

    if show:
        plt.show()
    else:
        plt.close()

    if any(out):
        return out

def _plot_Hn(self, *, figsize, show, export_png, dpi, **kwargs):
    out = []

    for dim, diagram in enumerate(self.diagrams):
        fig, ax = plt.subplots(1, 1, figsize=figsize)

        self._plot_many_bars(dim, diagram, 0, [ax], **kwargs)

        if export_png:
            fp = io.BytesIO()
            plt.savefig(fp, dpi=dpi)
            fp.seek(0)

            out += [fp]

        if show:
            plt.show()
        else:
            plt.close()

    if any(out):
        return out

def _plot_many_bars(self, dim, diagram, idx, ax, **kwargs):
    number_of_bars = len(diagram)
    if self._verbose:
        print("Number of bars in dimension %d: %d" % (dim, number_of_bars))

    if number_of_bars > 0:
        births = np.vstack([(elem[0], i) for i, elem in enumerate(diagram)])
        deaths = np.vstack([(elem[1], i) for i, elem in enumerate(diagram)])

        inf_bars = np.where(np.isinf(deaths))[0]
        max_death = deaths[np.isfinite(deaths[:, 0]), 0].max()

        number_of_bars_fin = births.shape[0] - inf_bars.shape[0]
```

```
            number_of_bars_inf = inf_bars.shape[0]

            _ = [self._plot_a_bar(ax[idx], birth, deaths[i], max_death,␣
↪**kwargs) for i, birth in enumerate(births)]

        # the line below is to plot a vertical red line showing the maximal␣
↪finite bar length
        ax[idx].plot(
            [max_death, max_death],
            [0, number_of_bars - 1],
            c='r',
            linestyle='--',
            linewidth=0.7
        )

        title = "H%d barcode: %d finite, %d infinite" % (dim,␣
↪number_of_bars_fin, number_of_bars_inf)
        ax[idx].set_title(title, fontsize=9)
        ax[idx].set_yticks([])

        for loc in ('right', 'left', 'top'):
            ax[idx].spines[loc].set_visible(False)

    @staticmethod
    def _plot_a_bar(ax, birth, death, max_death, c='gray', linestyle='-',␣
↪linewidth=0.5):
        if np.isinf(death[0]):
            death[0] = 1.05 * max_death
            ax.plot(
                death[0],
                death[1],
                c=c,
                markersize=4,
                marker='>',
            )

        ax.plot(
            [birth[0], death[0]],
            [birth[1], death[1]],
            c=c,
            linestyle=linestyle,
            linewidth=linewidth,
        )
```

0.3 Draw persistent diagrams:

[7]:
```
## My code from here:

def drawTDAtutorial(P,diagrams, R):
    fig, axes = plt.subplots(nrows=1, ncols=3, figsize = (20,5))

    # Draw diagrams
    plt.sca(axes[0])
    plt.title('0-dim Diagram')
    Draw.drawDgm(diagrams[0])
#     R = max(diagrams[0][1])
    plt.axis([0,R,0,R])

    plt.sca(axes[1])
    plt.title('1-dim Diagram')
    Draw.drawDgm(diagrams[1])
#     R = max(diagrams[1][1])
    plt.axis([0,R,0,R])

    plt.sca(axes[2])
    plt.title('2-dim Diagram')
    Draw.drawDgm(diagrams[2])
#     R = max(diagrams[2][1])
    plt.axis([0,R,0,R])
```

0.4 Dimensionality reduction and TDA:

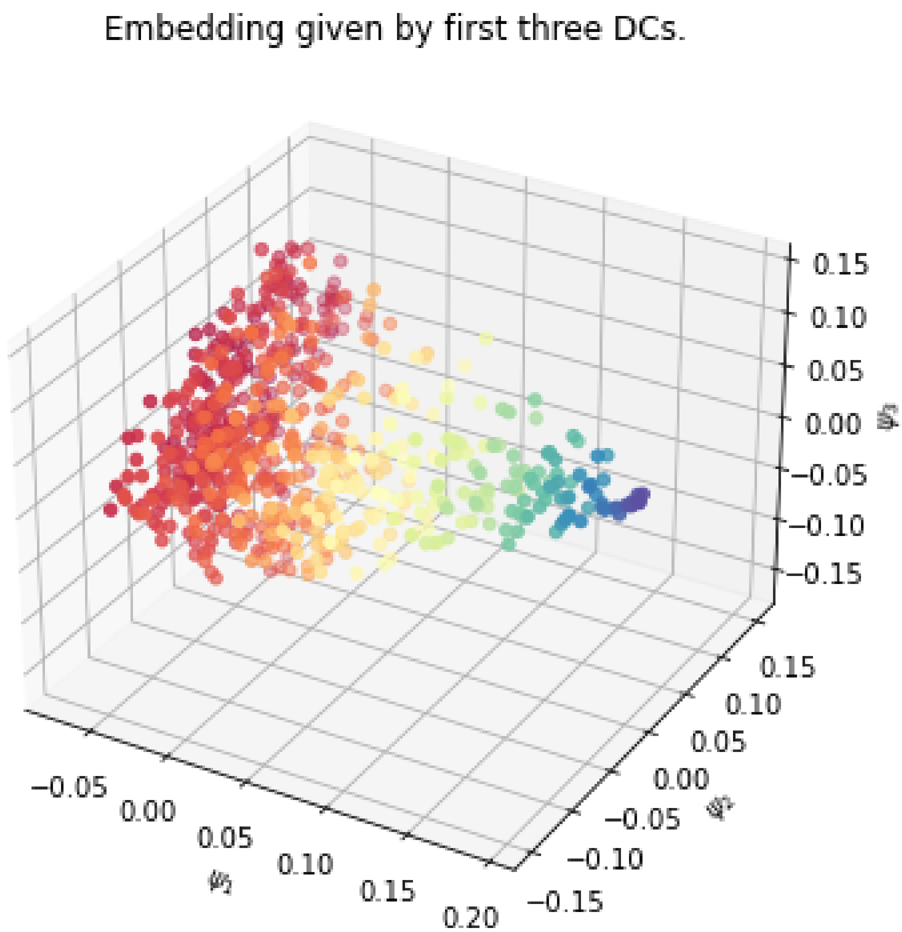
pydiffmap: n_evecs is the number of eigenvectors that are computed, epsilon is a scale parameter used to rescale distances between data points, alpha is a normalization parameter (typically between 0.0 and 1.0) that influences the effect of the sampling density, and k is the number of nearest neighbors considered when the kernel is computed. A larger k means increased accuracy but larger computation time

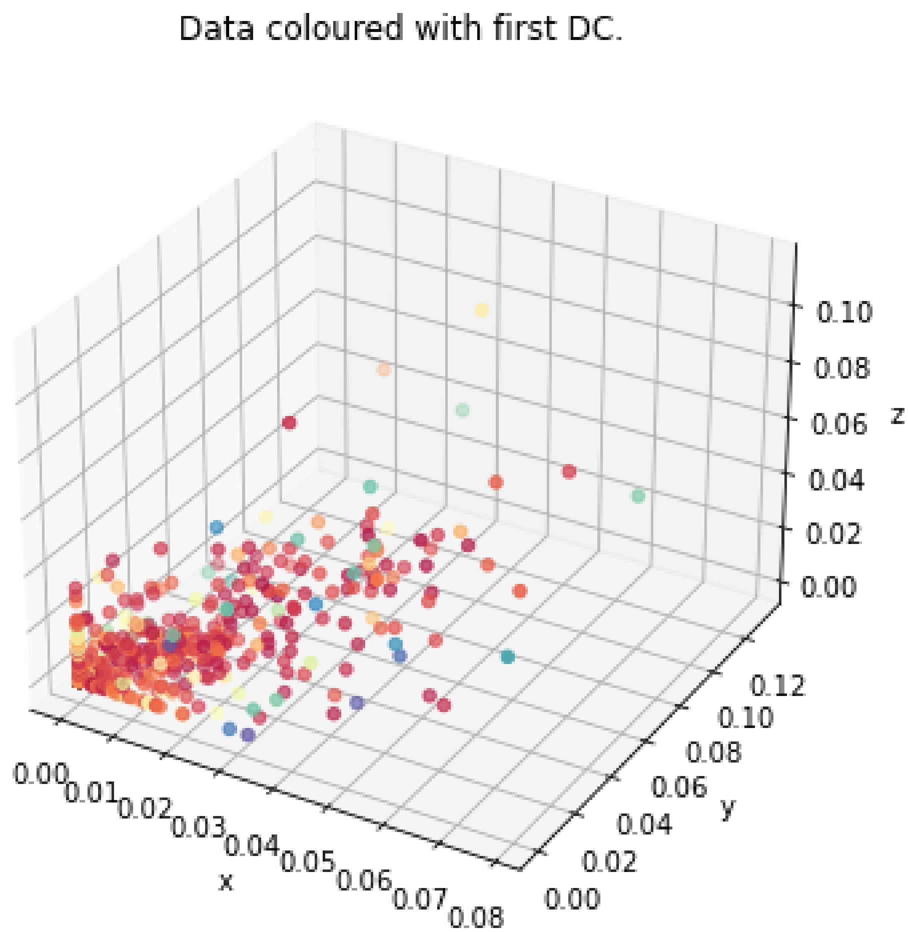
[8]:
```
def diffusion_tda(X):
  ## diffusion map with automatic epsilon detection:
    mydmap = dm.DiffusionMap.from_sklearn(n_evecs = 3, alpha = 1, epsilon =  1.
 ↪0 , k=100)

  # Fit to and transform the data
    X_dmap = mydmap.fit_transform(X)

    embedding_plot(mydmap, dim=3, scatter_kwargs = {'c': X_dmap[:,0], 'cmap':␣
 ↪'Spectral'})
    data_plot(mydmap, dim=3, scatter_kwargs = {'cmap': 'Spectral'})
    plt.show()
    print("SHAPE",X_dmap.shape)
```

```
    ax = plt.axes(projection ="3d")
    ax.scatter3D(X_dmap[:,0], X_dmap[:,1], X_dmap[:,2])
    plt.show()

#     plt.scatter(X_dmap[:,0], X_dmap[:,1])
#     plt.show()

    X_diagrams = ripser.ripser(X_dmap, maxdim = 2)['dgms']

  ## draw persistence diagrams
    drawTDAtutorial(X_dmap,X_diagrams,R=0.1)

  ## draw persistence barcodes
    Barcode(X_diagrams).plot_barcode()

    return X_diagrams, Barcode(X_diagrams)
```

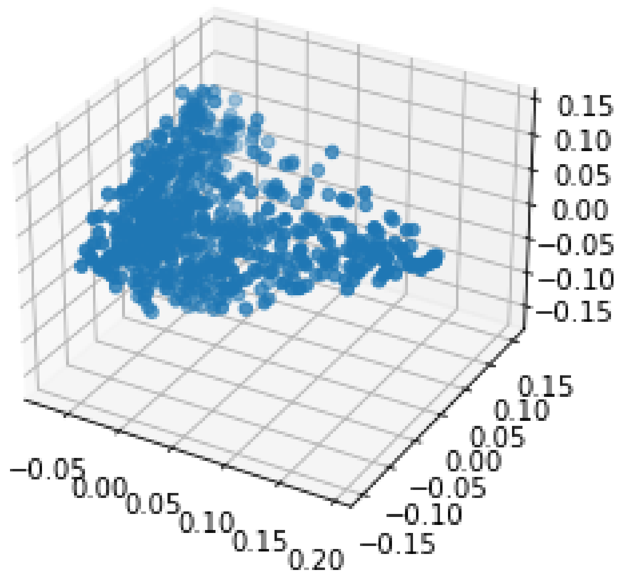
[9]:

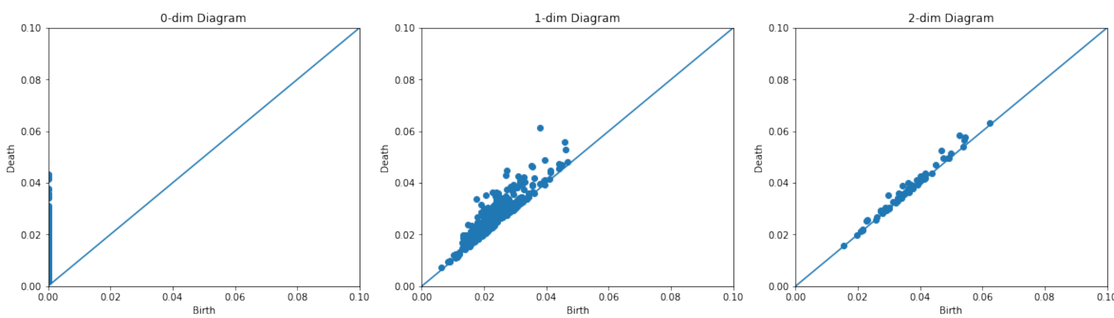
```
X1 = retina_original_data[:,0,:]
X1.shape
X1_diagrams, X1_barcode = diffusion_tda(X1)

## isomap what info is lost and the specific distance measure
## possible loss of info: analyze and say something
## show the barcode


## compare with the traditional method: directly compute the distance between
## metric space: 698 points
# dsitance between two sequence of images
# two distance between images
# compare some of the distances
# goromov hausdorff distance between barcodes - carlsson review
# observe the diff compared to traditional distance
```

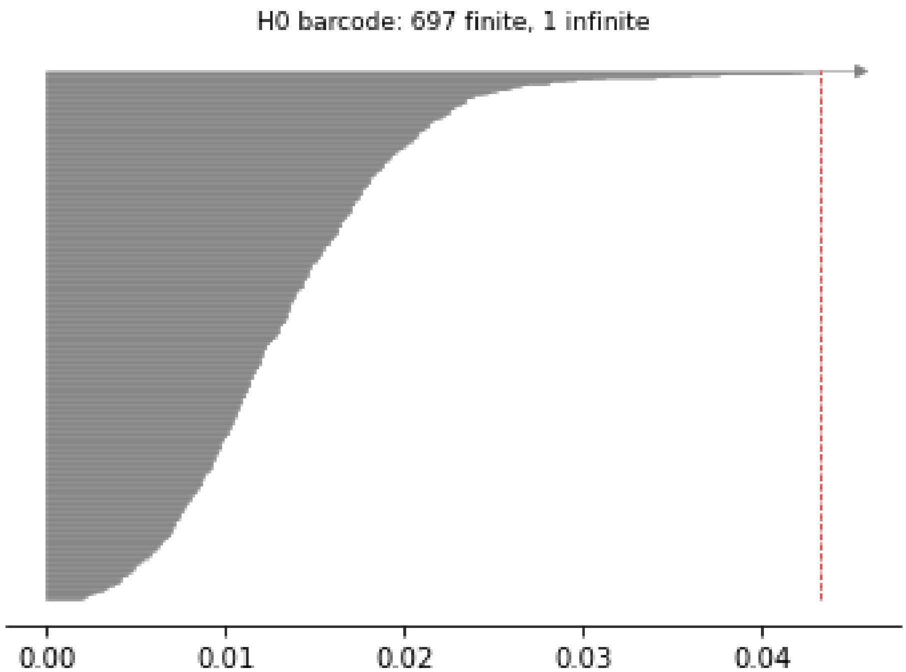

Embedding given by first three DCs.
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.05
0.00
0.05
0.10
0.15
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15

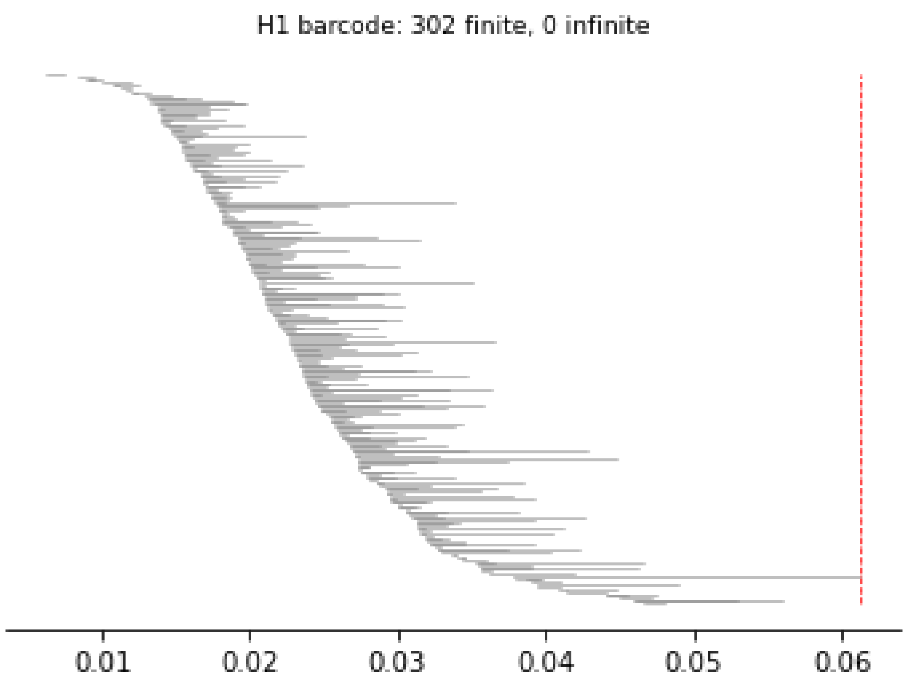

Data coloured with first DC.
0.10
0.08
0.06
0.04
0.02
0.00
z
0.12
0.10
0.08
0.06
0.04
0.02
0.00
y
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
x

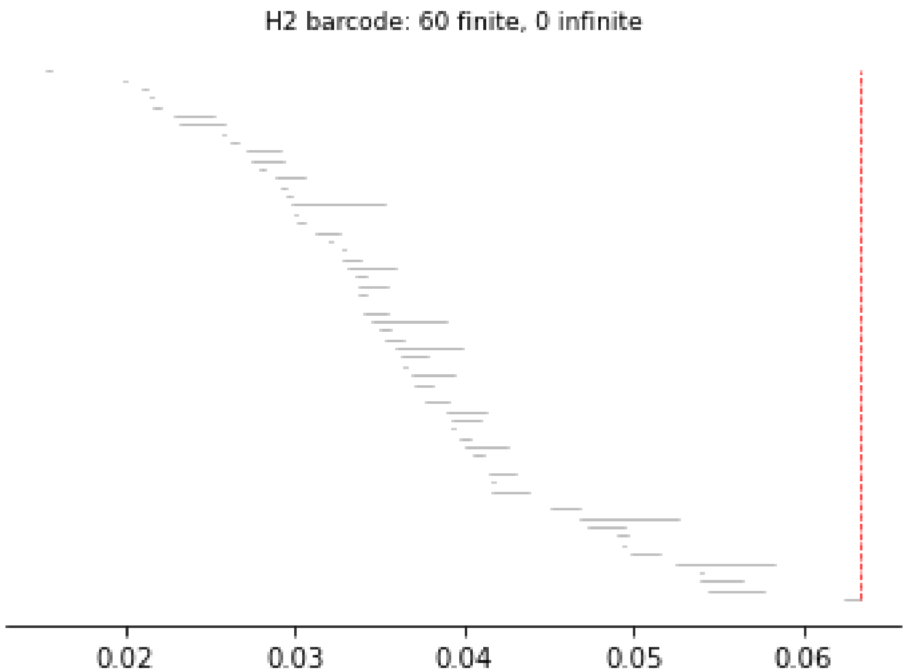
```
SHAPE (698, 3)
```


0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.05
0.00
0.05
0.10
0.15
0.20

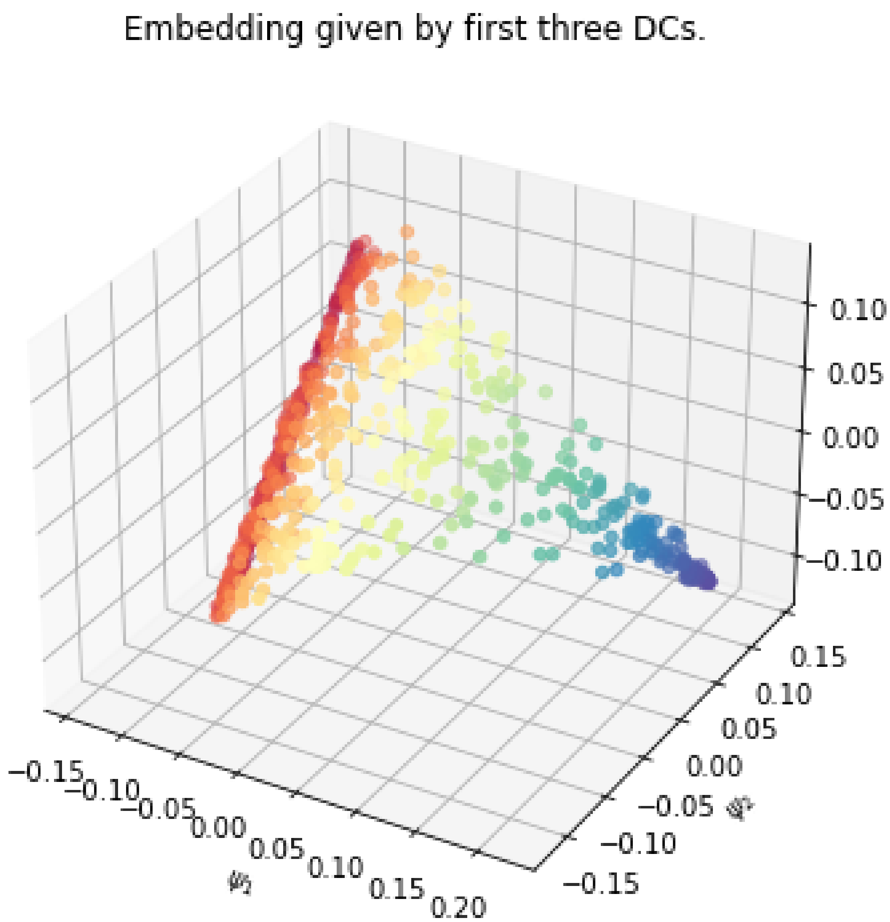

0-dim Diagram
1-dim Diagram
2-dim Diagram
Birth
Death
0.00
0.02
0.04
0.06
0.08
0.10

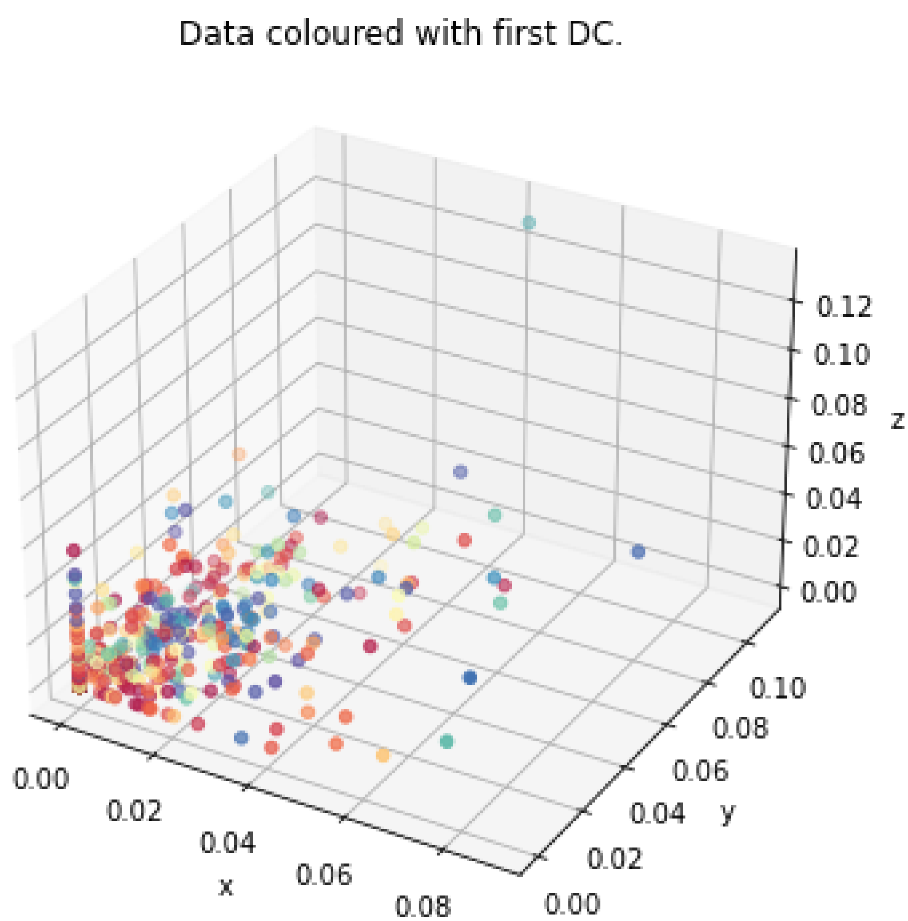

H0 barcode: 697 finite, 1 infinite
0.00
0.01
0.02
0.03
0.04


H1 barcode: 302 finite, 0 infinite
0.01
0.02
0.03
0.04
0.05
0.06

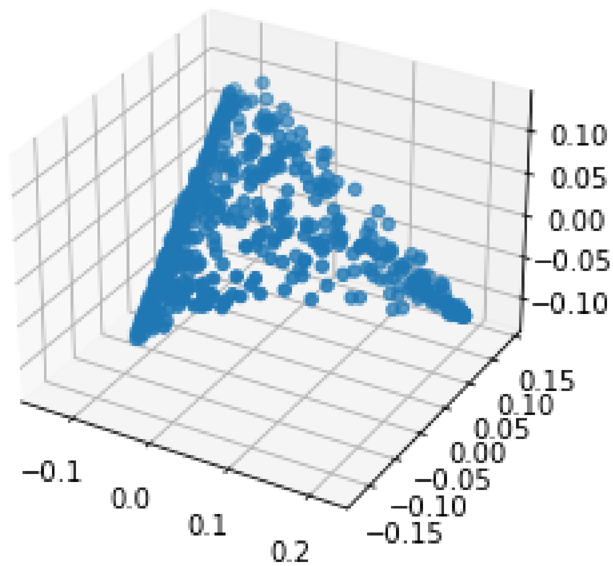


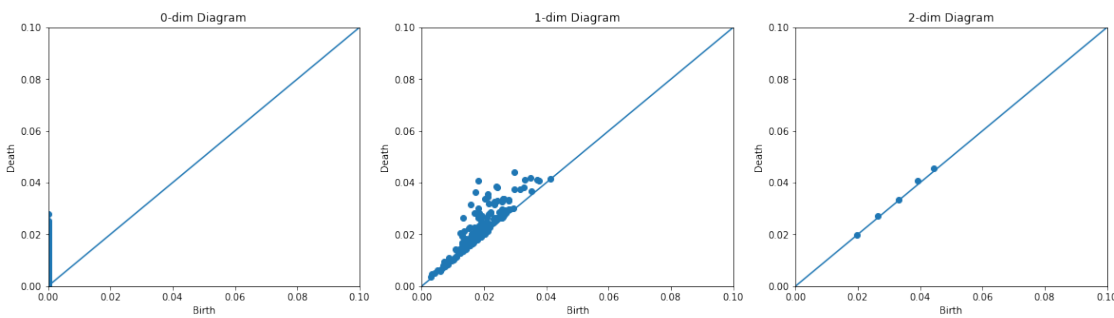
[10]:
```
X2 = retina_original_data[:,1,:]
X2_diagrams, X2_barcode = diffusion_tda(X2)
```

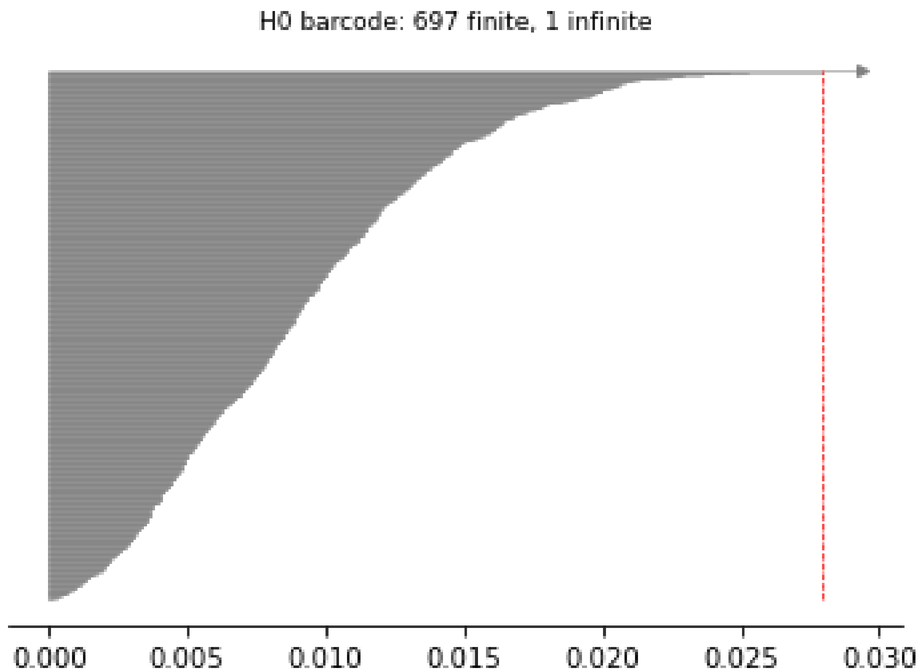

Embedding given by first three DCs.
0.10
0.05
0.00
−0.05
−0.10
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20

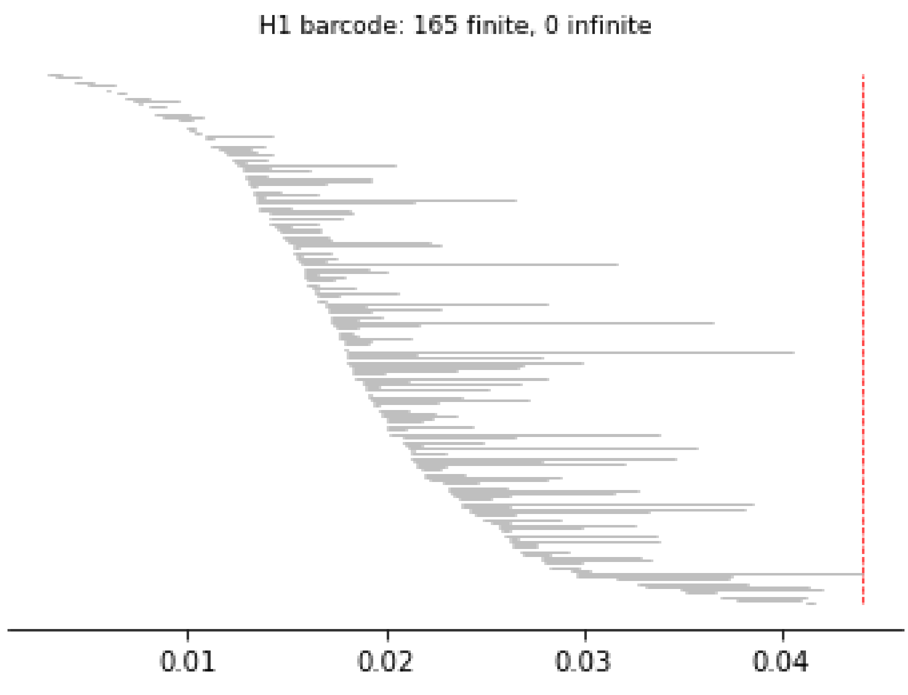


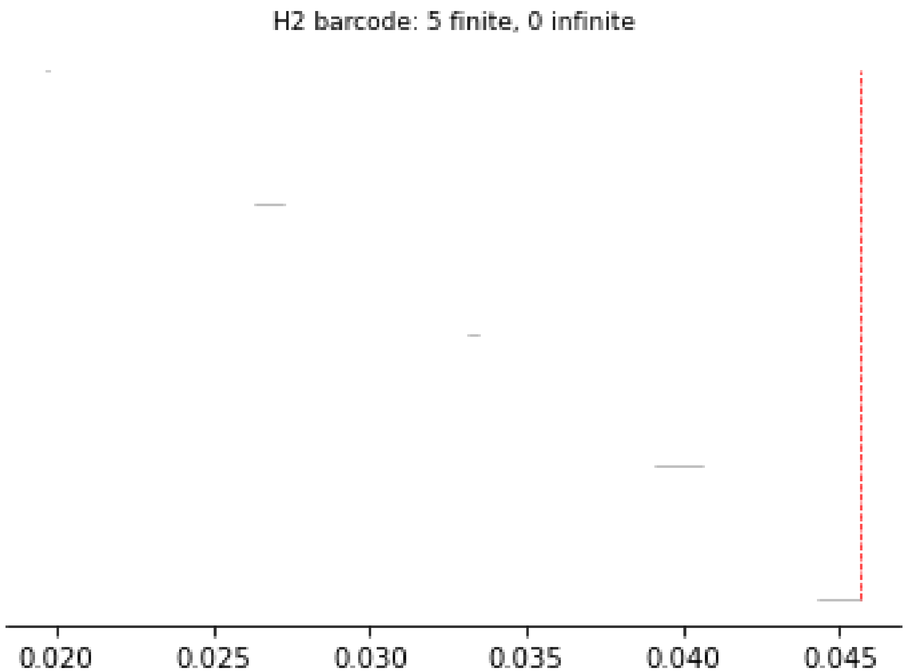

```
SHAPE (698, 3)
```


0.10
0.05
0.00
−0.05
−0.10
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.1
0.0
0.1
0.2

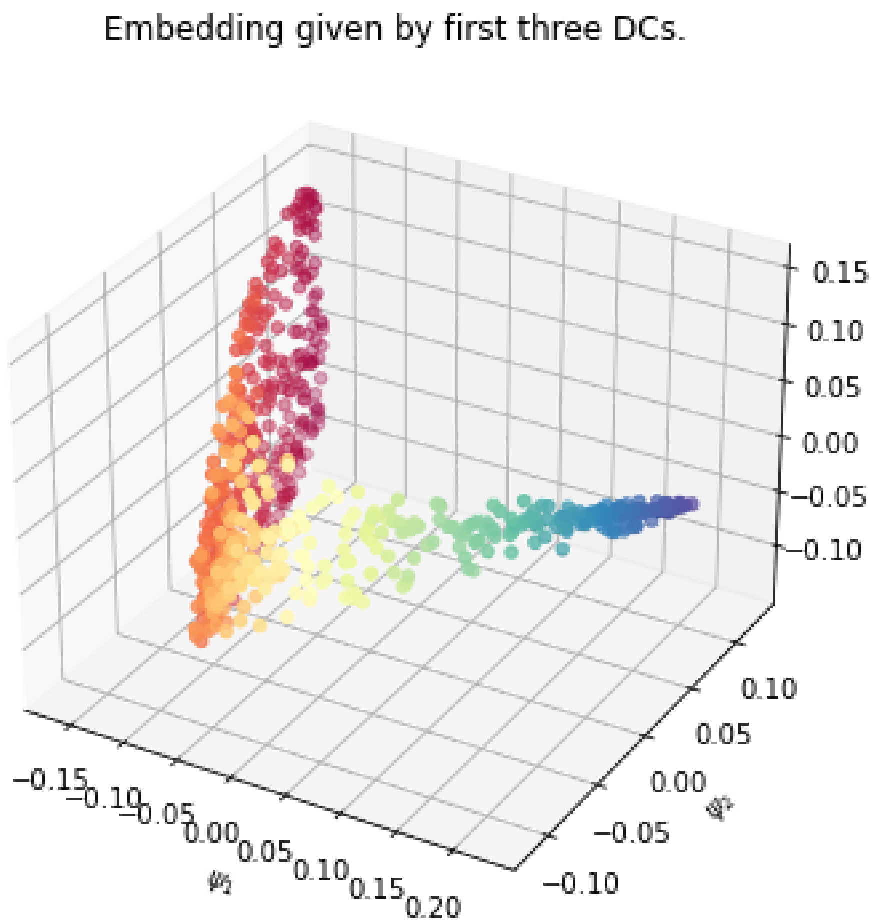

0-dim Diagram
1-dim Diagram
2-dim Diagram
Birth
Death


H0 barcode: 697 finite, 1 infinite
0.000
0.005
0.010
0.015
0.020
0.025
0.030

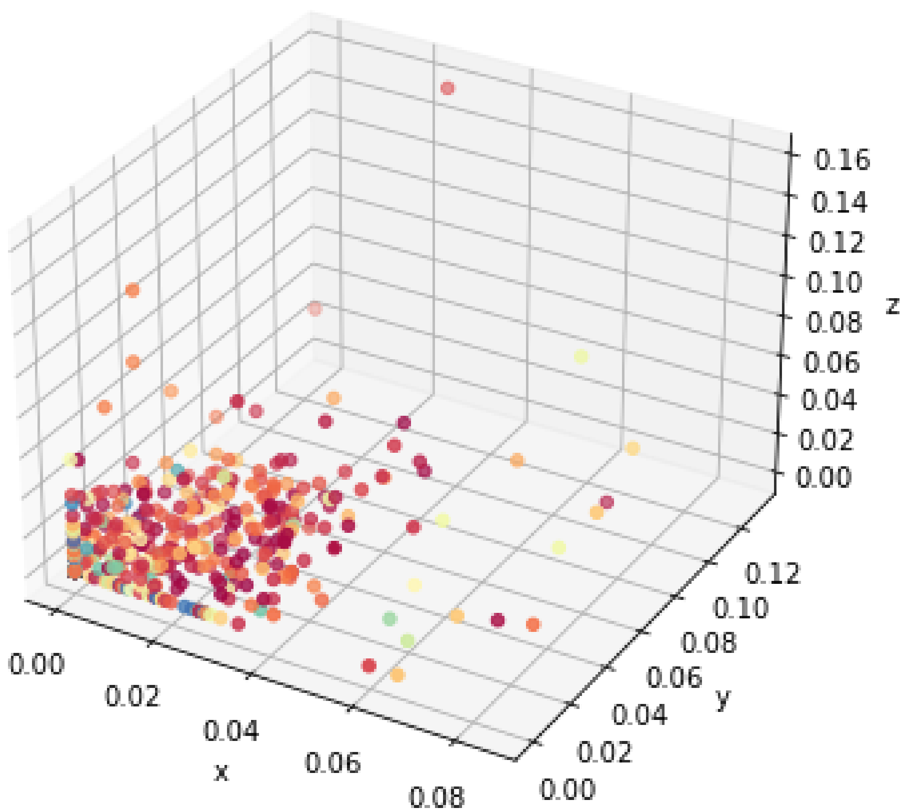

H1 barcode: 165 finite, 0 infinite
0.01
0.02
0.03
0.04



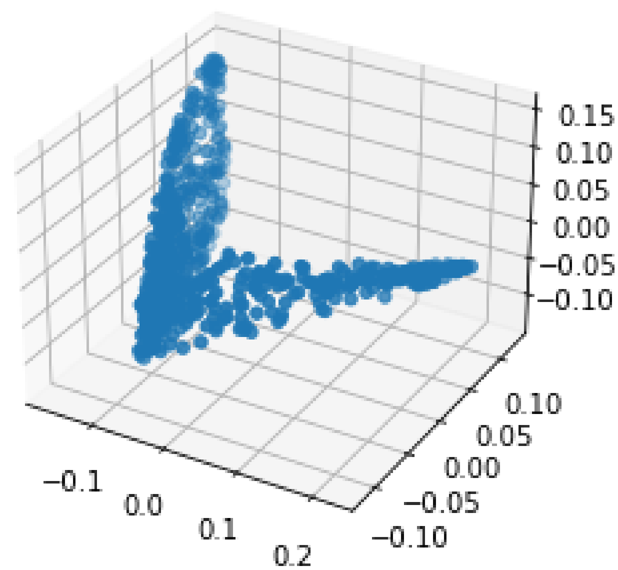
[11]:
```
X3 = retina_original_data[:,2,:]
X3_diagrams, X3_barcode = diffusion_tda(X3)
```

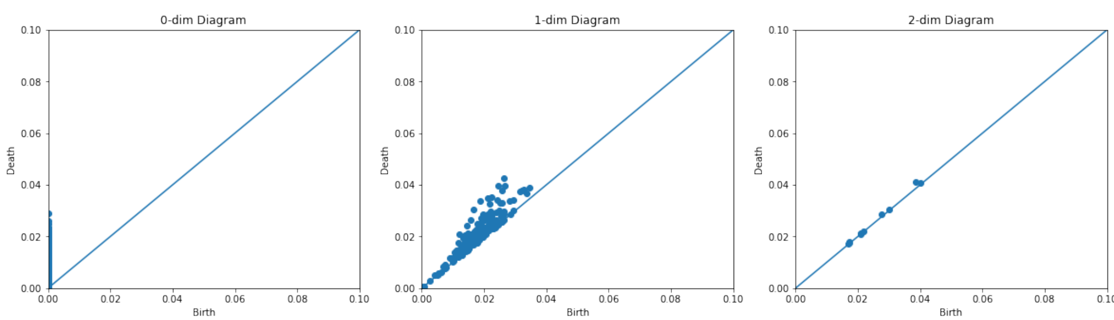

Embedding given by first three DCs.
0.15
0.10
0.05
0.00
−0.05
−0.10
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20

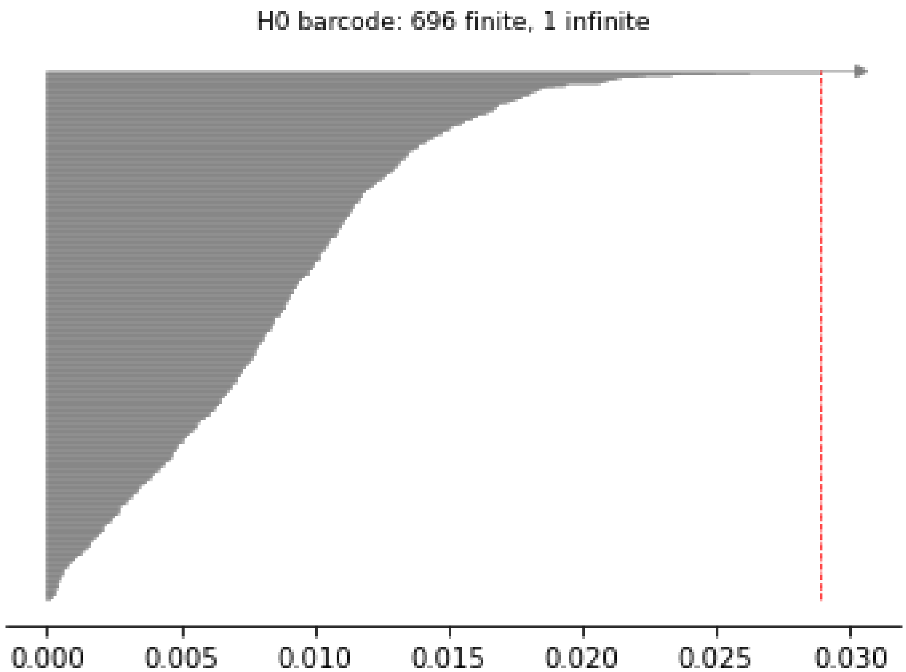
Data coloured with first DC.

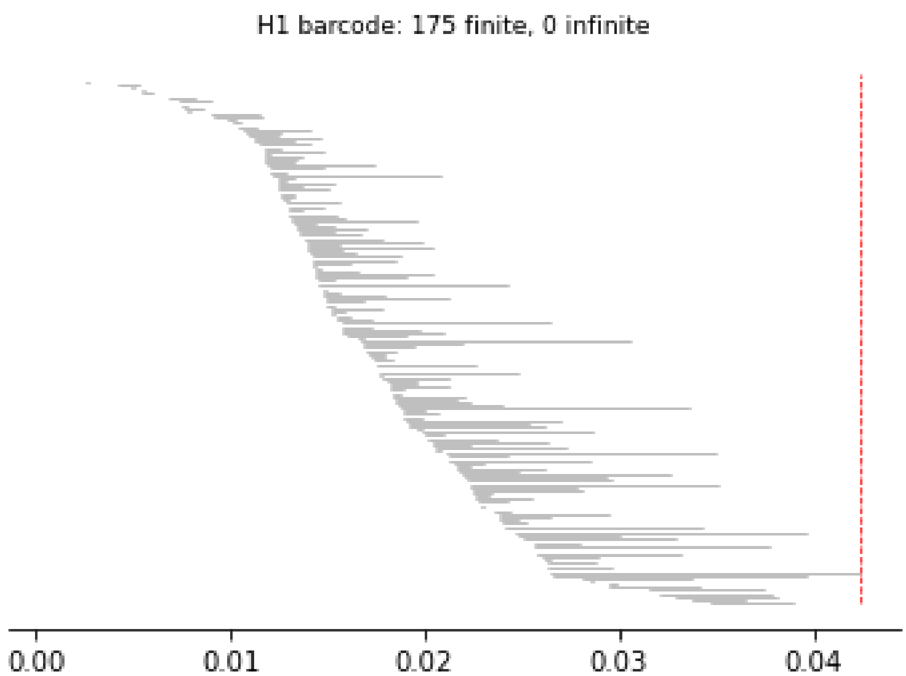


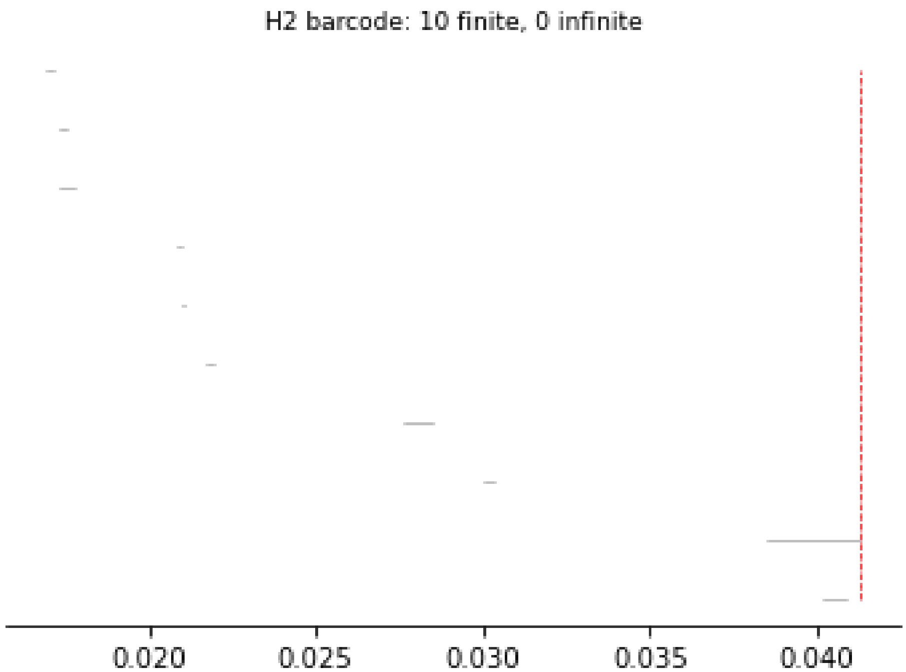
SHAPE (698, 3)


0.15
0.10
0.05
0.00
−0.05
−0.10
0.10
0.05
0.00
−0.05
−0.10
−0.1
0.0
0.1
0.2

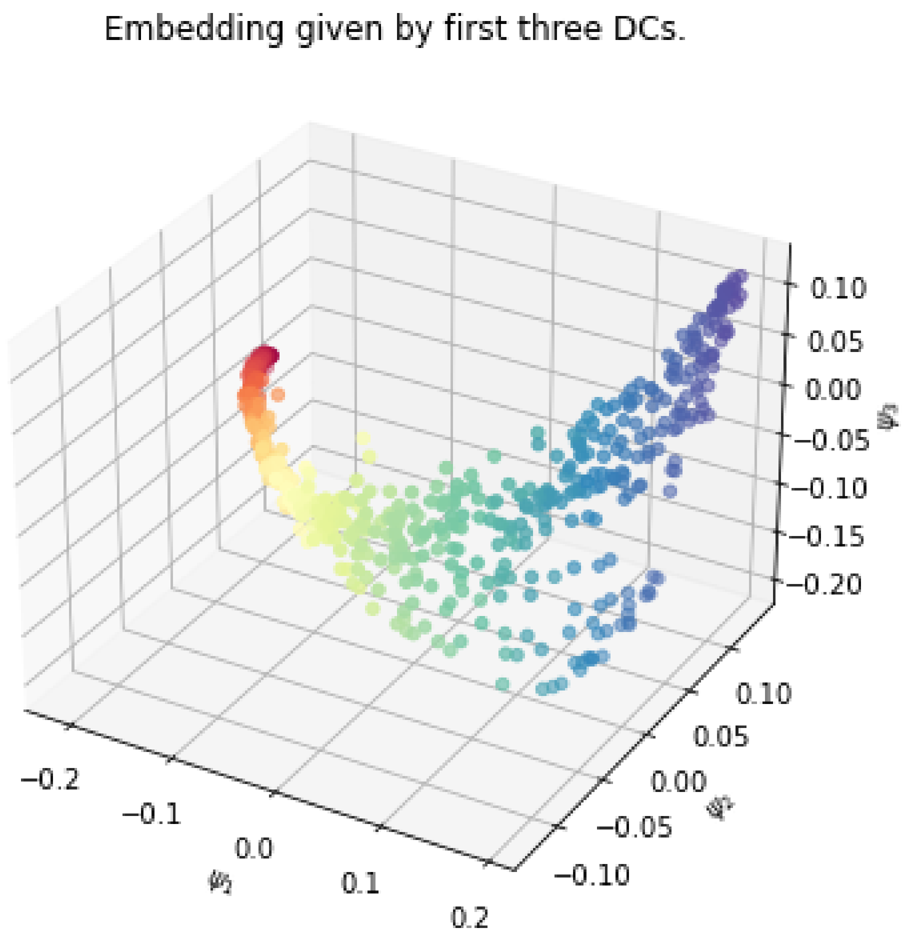

0-dim Diagram
1-dim Diagram
2-dim Diagram
Birth
Death
0.00
0.02
0.04
0.06
0.08
0.10

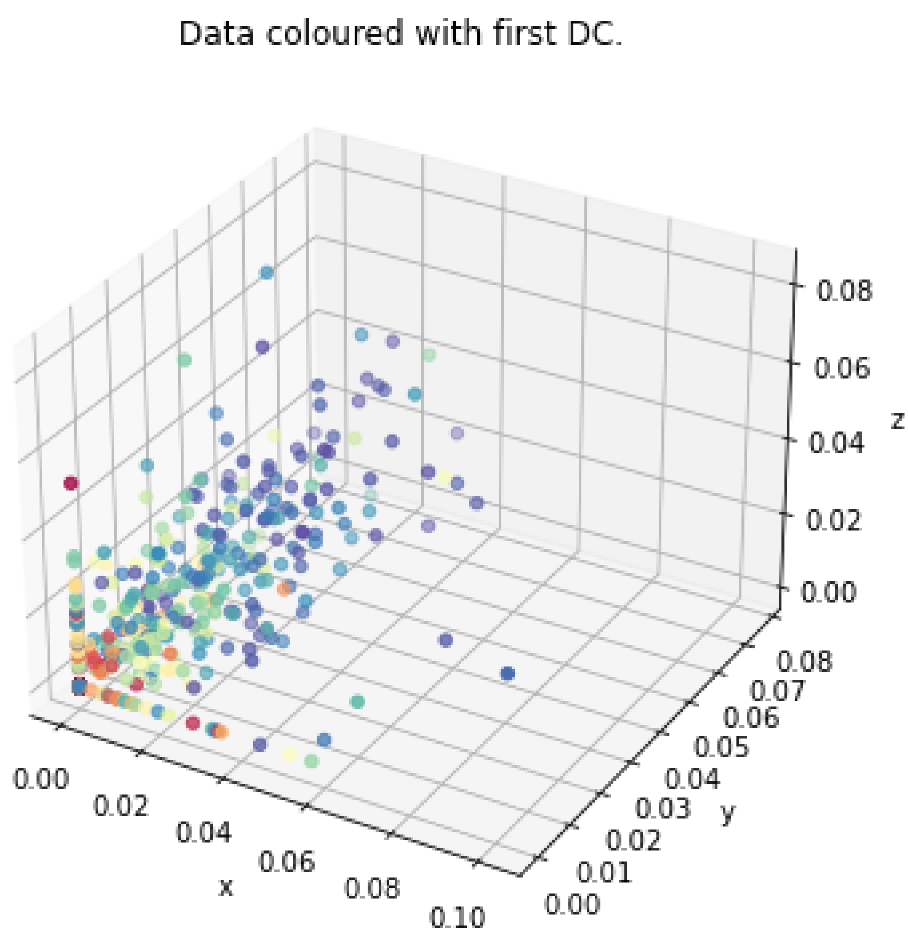

H0 barcode: 696 finite, 1 infinite
0.000
0.005
0.010
0.015
0.020
0.025
0.030


H1 barcode: 175 finite, 0 infinite
0.00
0.01
0.02
0.03
0.04

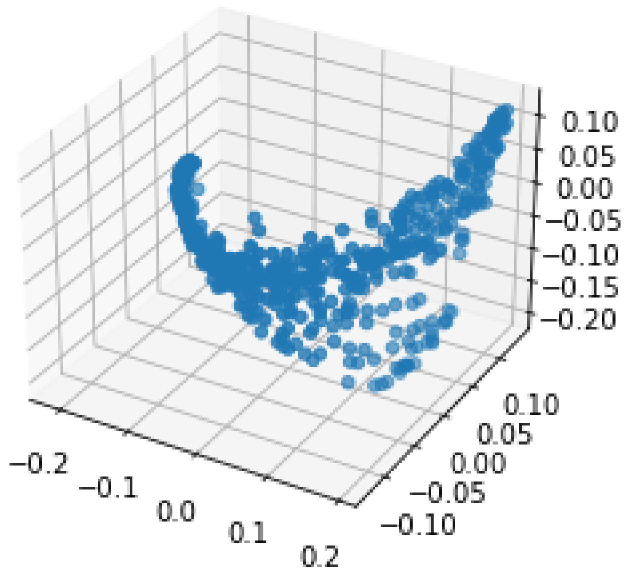


[12]:
```
X4 = retina_original_data[:,3,:]

X4_diagrams, X4_barcode = diffusion_tda(X4)
```

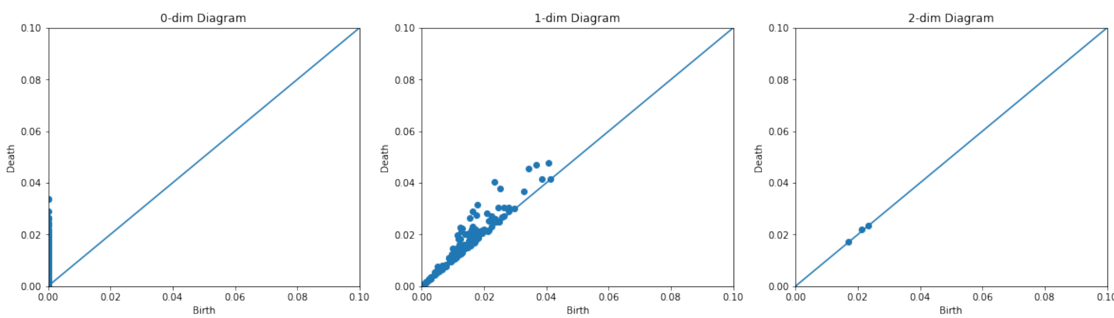

Embedding given by first three DCs.
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
0.10
0.05
0.00
−0.05
−0.10
−0.2
−0.1
0.0
0.1
0.2


Data coloured with first DC.
z
0.08
0.06
0.04
0.02
0.00
y
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
x
0.00
0.02
0.04
0.06
0.08
0.10

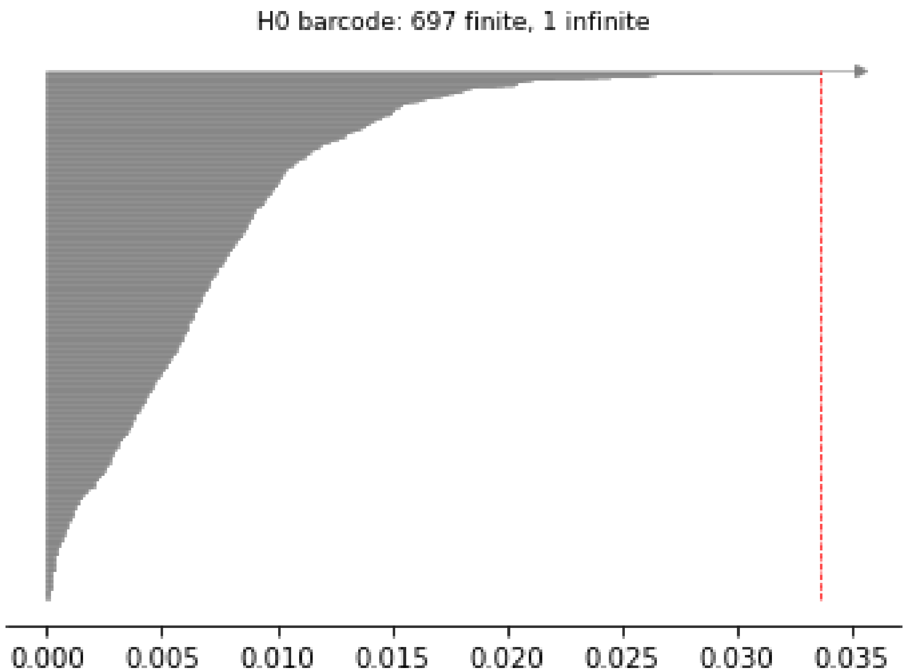
SHAPE (698, 3)



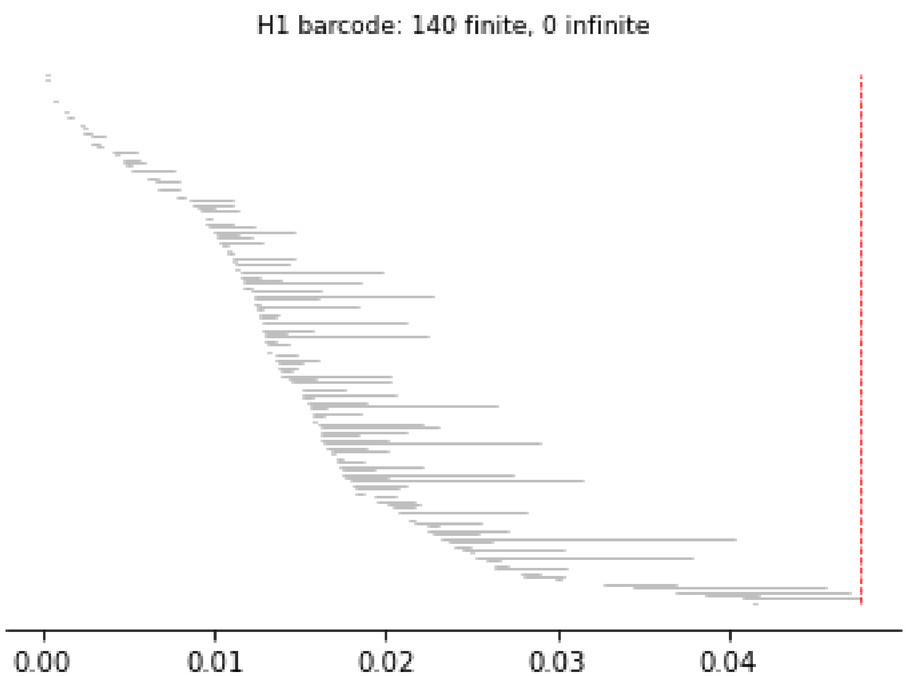
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
0.10
0.05
0.00
−0.05
−0.10
−0.2
−0.1
0.0
0.1
0.2



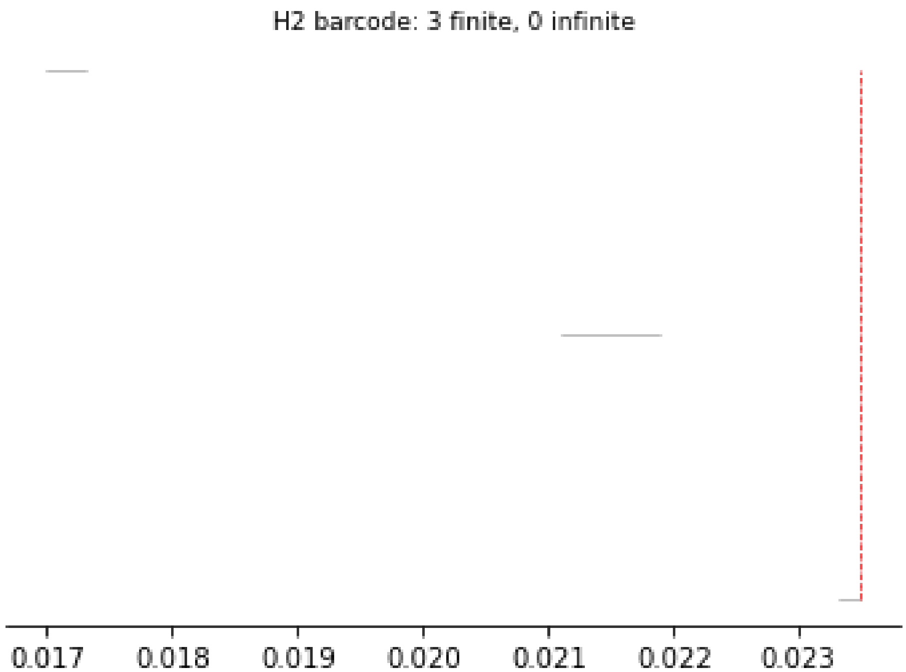
0-dim Diagram
1-dim Diagram
2-dim Diagram
Death
Birth
0.00
0.02
0.04
0.06
0.08
0.10



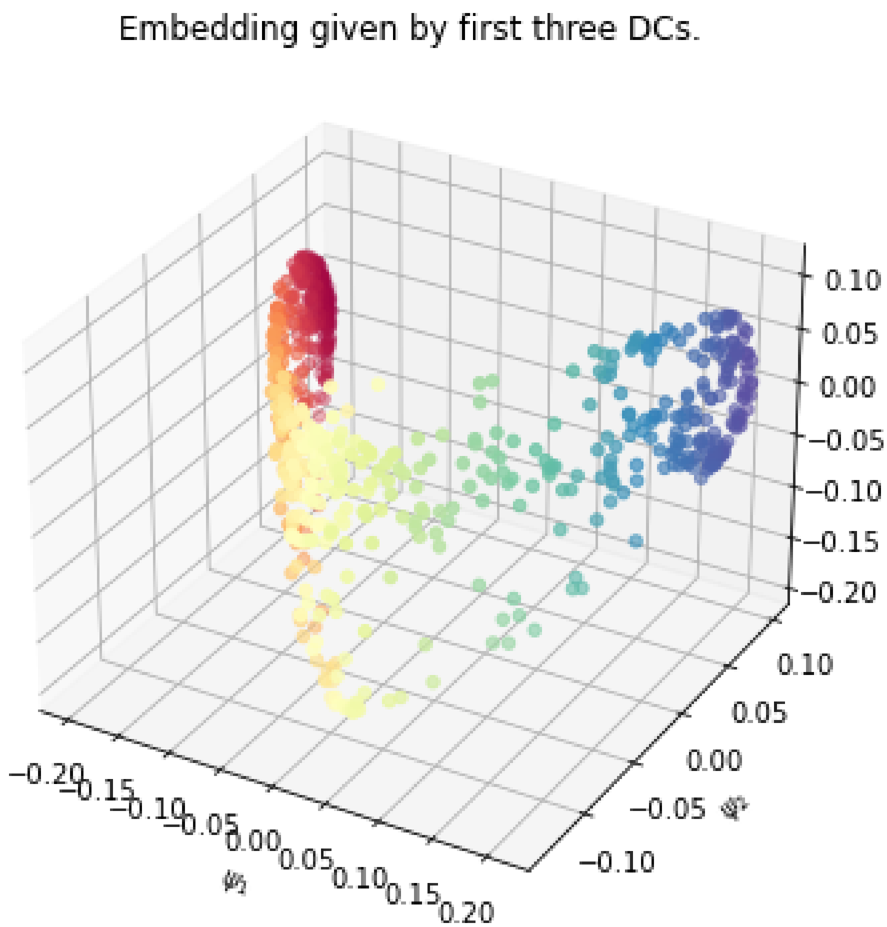
H0 barcode: 697 finite, 1 infinite
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035



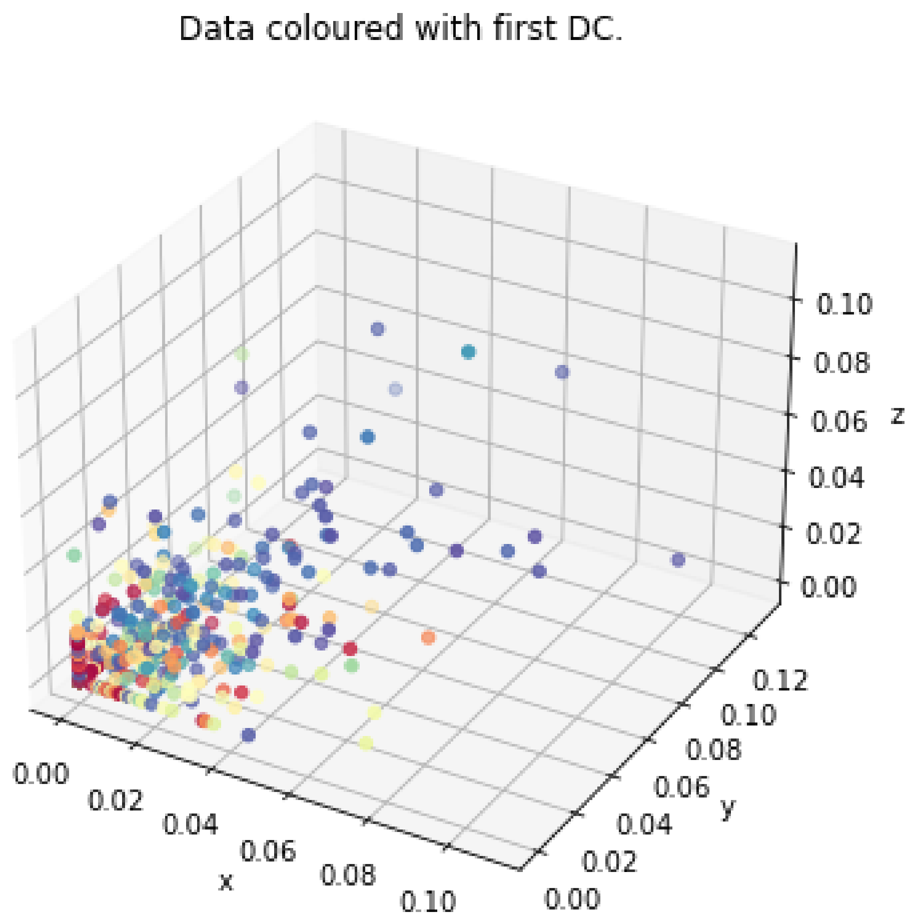
H1 barcode: 140 finite, 0 infinite
0.00
0.01
0.02
0.03
0.04



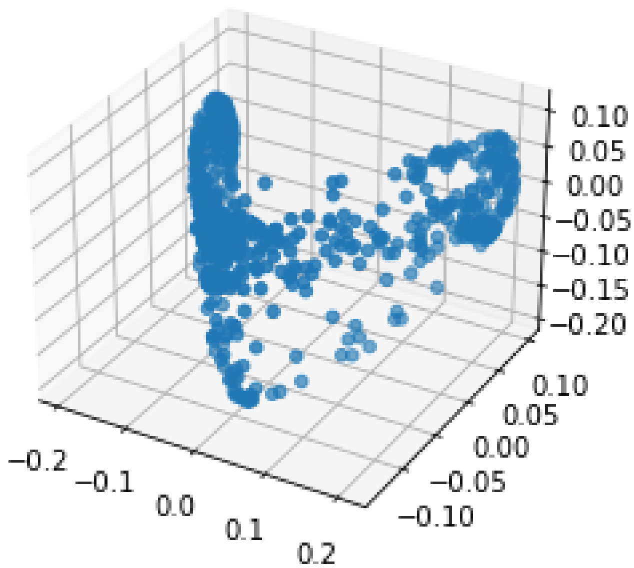
[13]:
```
X5 = retina_original_data[:,4,:]


X5_diagrams, X5_barcode = diffusion_tda(X5)
```



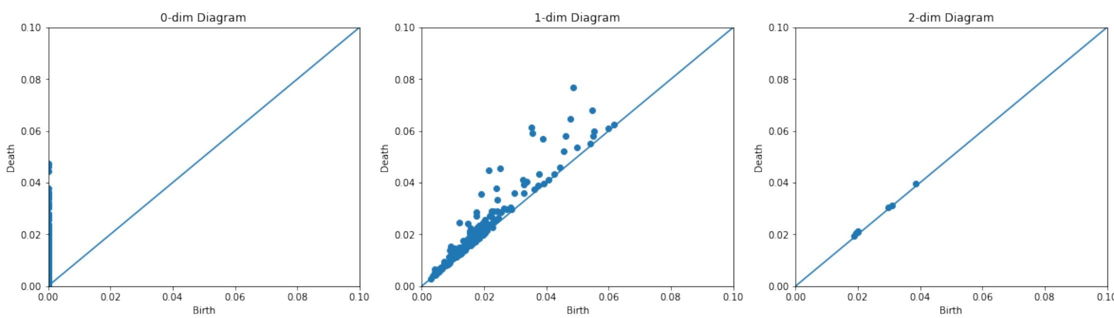
Embedding given by first three DCs.
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
0.10
0.05
0.00
−0.05
−0.10
−0.20
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20


Data coloured with first DC.
0.10
0.08
0.06
0.04
0.02
0.00
z
0.12
0.10
0.08
0.06
0.04
0.02
0.00
y
0.00
0.02
0.04
0.06
0.08
0.10
x

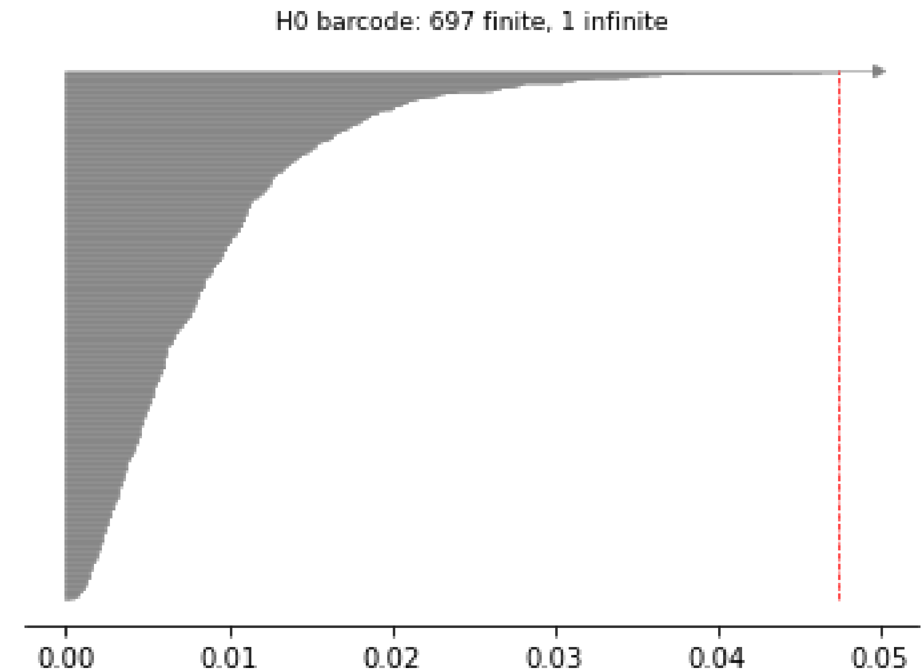
```
SHAPE (698, 3)
```

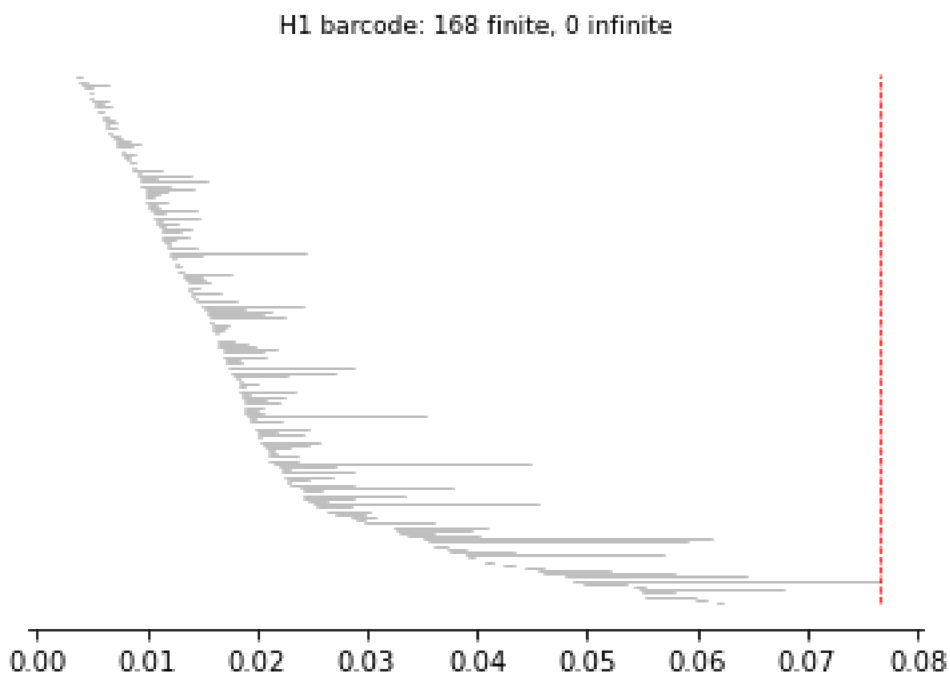


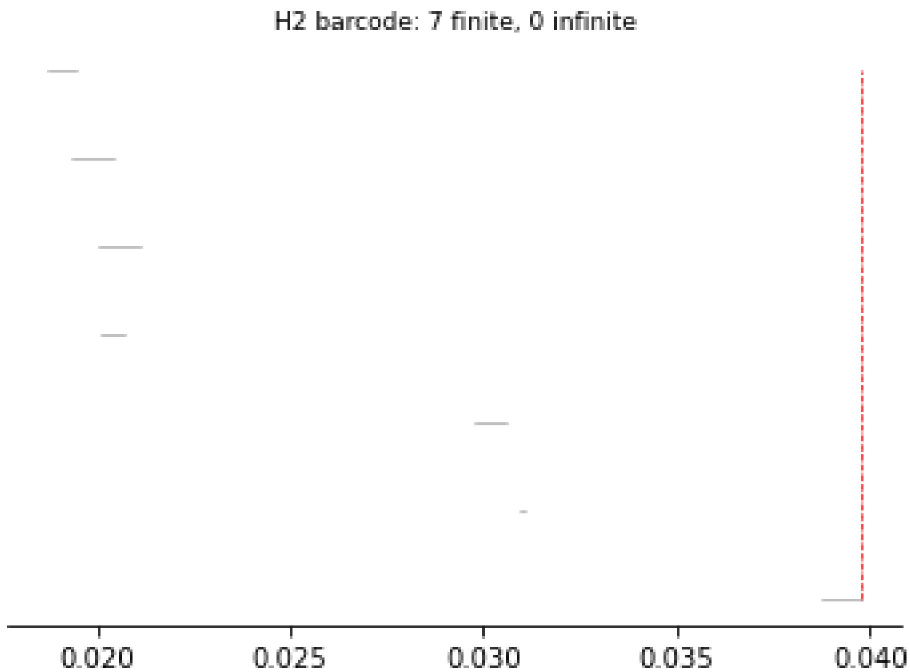
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
0.10
0.05
0.00
−0.05
−0.10
−0.2
−0.1
0.0
0.1
0.2



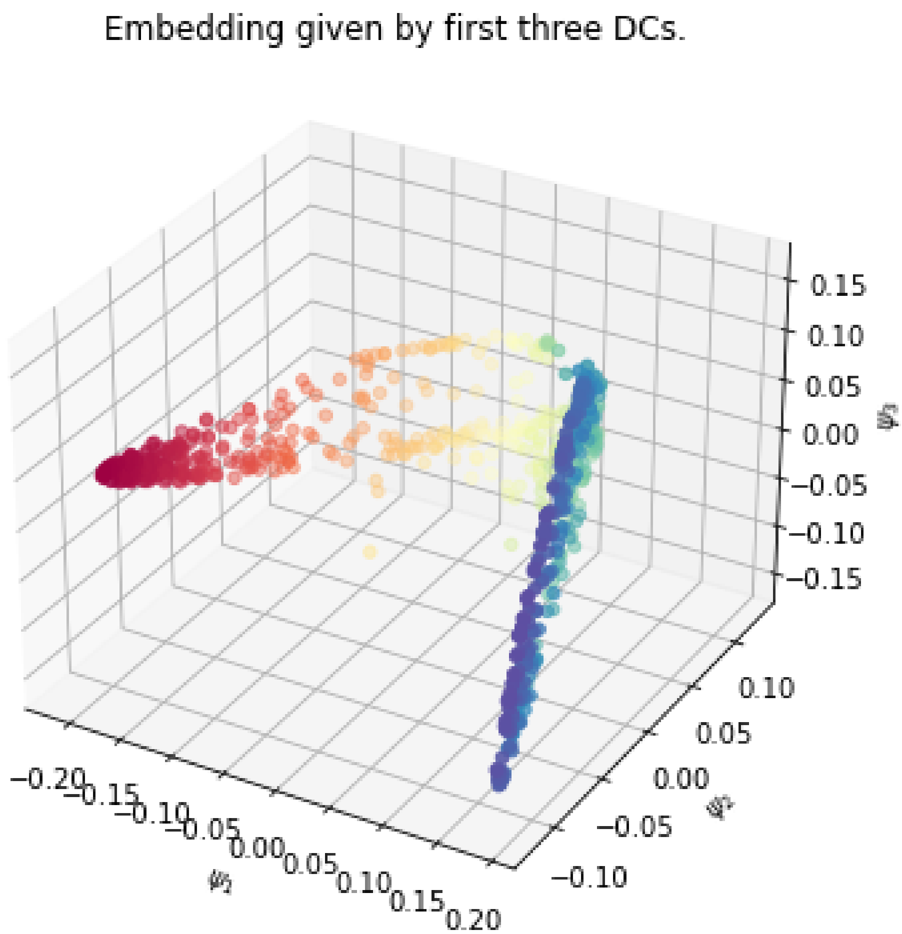
0-dim Diagram
1-dim Diagram
2-dim Diagram
Death
Birth
0.00
0.02
0.04
0.06
0.08
0.10

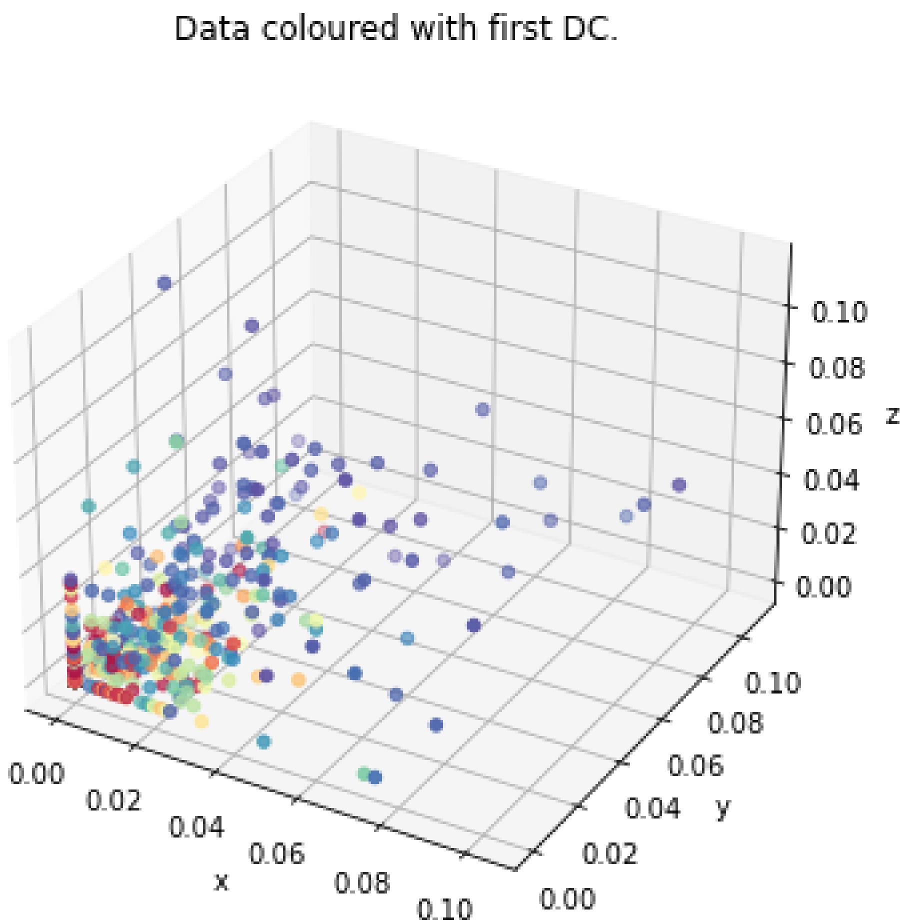

H0 barcode: 697 finite, 1 infinite
0.00
0.01
0.02
0.03
0.04
0.05


H1 barcode: 168 finite, 0 infinite
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08

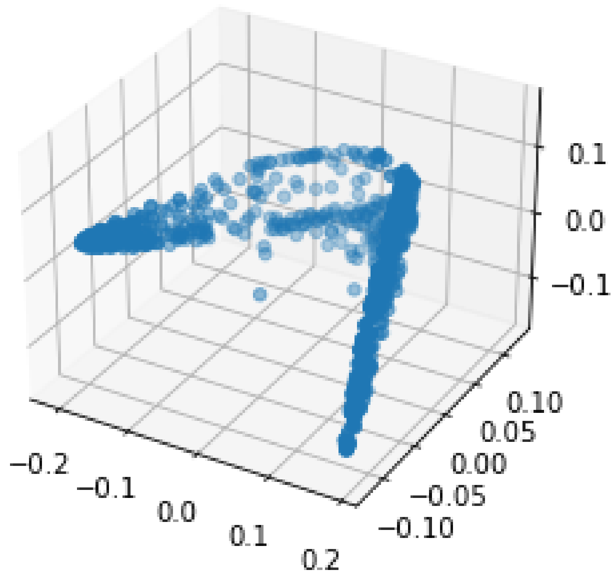


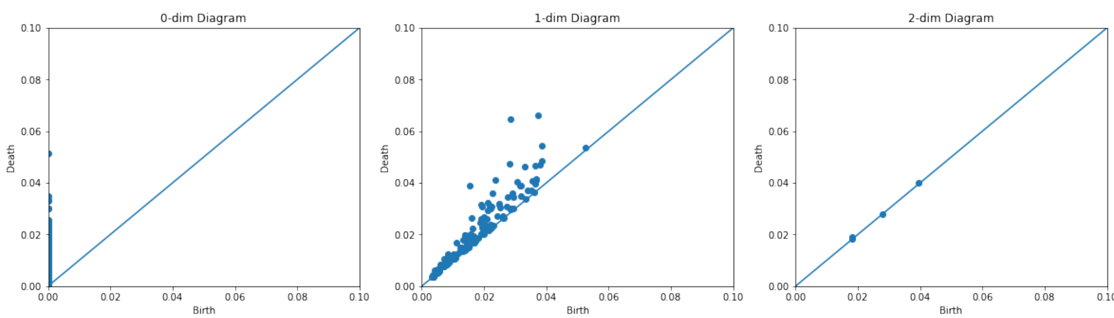
[14]:
```
X6 = retina_original_data[:,5,:]

X6_diagrams, X6_barcode = diffusion_tda(X6)
```

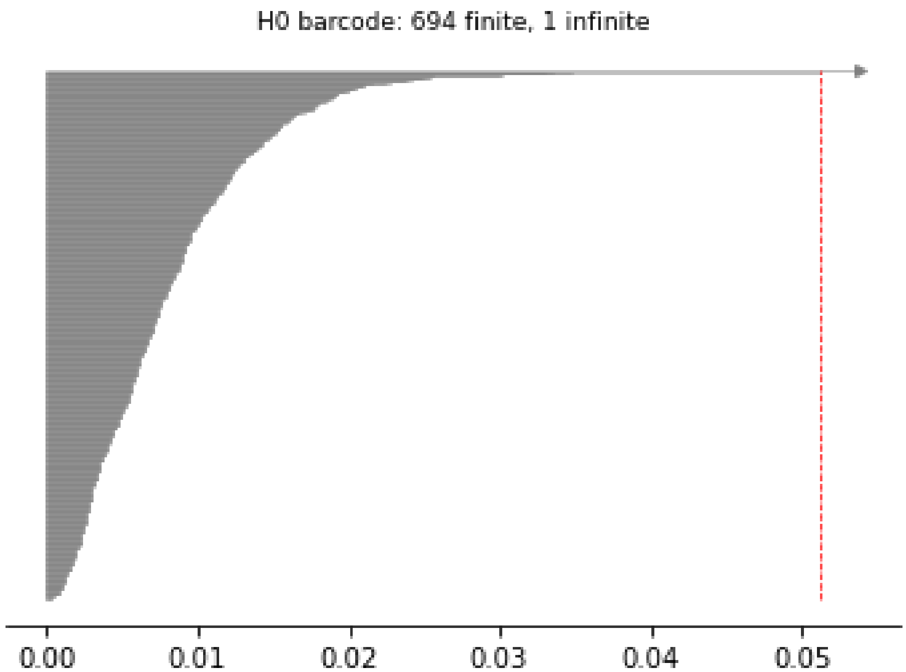

Embedding given by first three DCs.
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
0.10
0.05
0.00
−0.05
−0.10
−0.20
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20

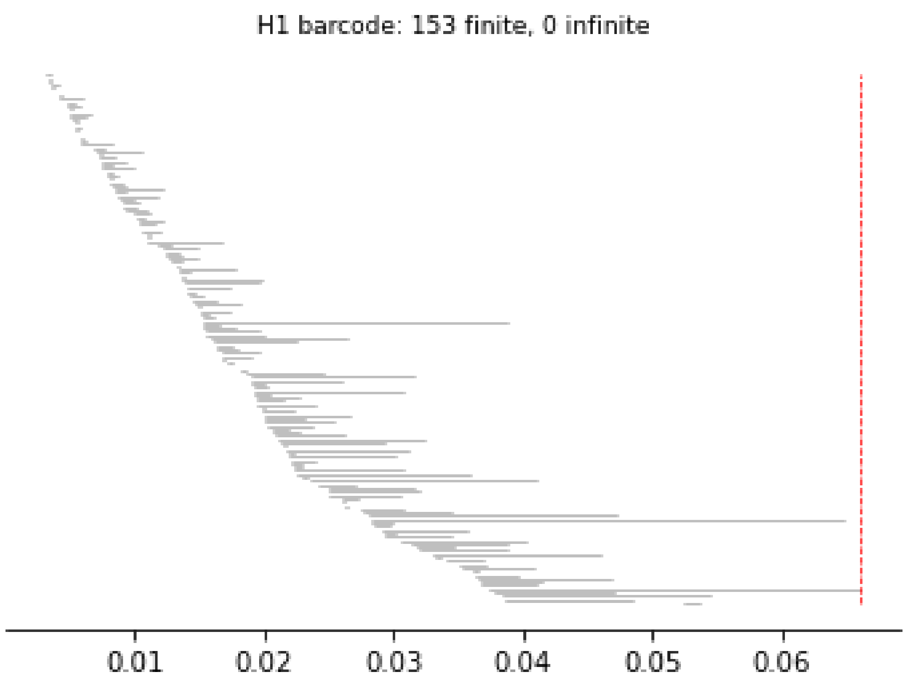

Data coloured with first DC.
0.10
0.08
0.06
z
0.04
0.02
0.00
0.10
0.08
0.06
0.04
y
0.02
0.00
0.00
0.02
0.04
0.06
x
0.08
0.10

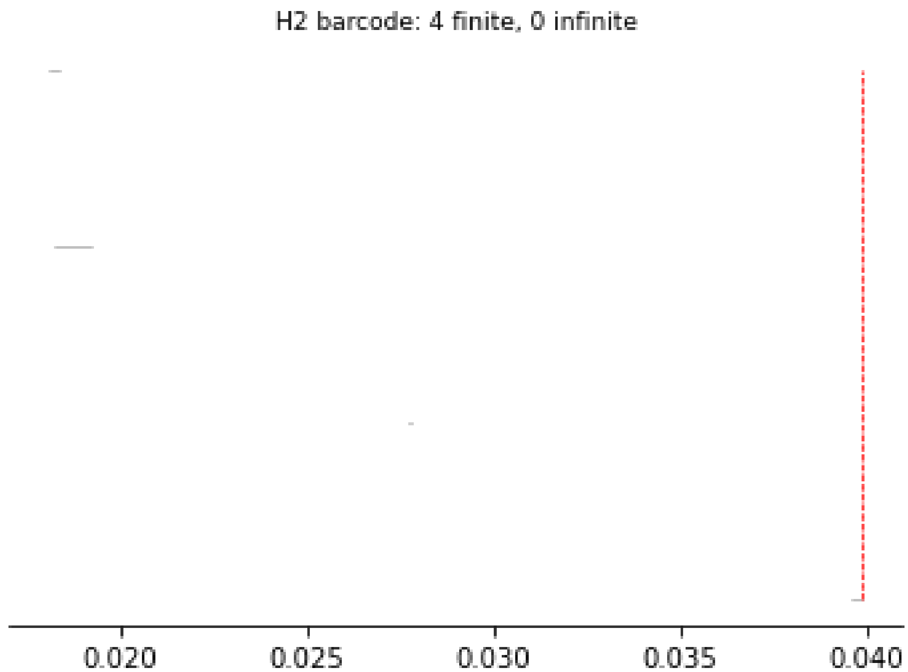
```
SHAPE (698, 3)
```


0.1
0.0
−0.1
0.10
0.05
0.00
−0.05
−0.10
−0.2
−0.1
0.0
0.1
0.2


0-dim Diagram
1-dim Diagram
2-dim Diagram
Death
Birth
0.00
0.02
0.04
0.06
0.08
0.10


H0 barcode: 694 finite, 1 infinite
0.00
0.01
0.02
0.03
0.04
0.05


H1 barcode: 153 finite, 0 infinite
0.01
0.02
0.03
0.04
0.05
0.06



0.5 Compute the pairwise Wasserstein distance:

useful resource: https://github.com/giotto-ai/giotto-tda/issues/603

0.5.1 first check that Wasserstein distance computed by two packages are the same:

[15]:
```
from gtda.diagrams import PairwiseDistance
pd = PairwiseDistance(['wasserstein'])
```

[16]:
```
pd_X1_X2_persim = persim.wasserstein(X1_diagrams[0], X2_diagrams[0])
```

```
C:\Users\ifisa\anaconda3\lib\site-packages\persim\wasserstein.py:51:
UserWarning: dgm1 has points with non-finite death times;ignoring those points
  warnings.warn(
C:\Users\ifisa\anaconda3\lib\site-packages\persim\wasserstein.py:61:
UserWarning: dgm2 has points with non-finite death times;ignoring those points
  warnings.warn(
```

[17]:
```
pd_X1_X2_persim
```

[17]: 2.7718896795122

[18]:
```
## check that the result is consistent with the persim package:
from gudhi.wasserstein import wasserstein_distance
from gudhi.hera import wasserstein_distance as hera
```

```
pd_X1_X2_gudhi = hera(X1_diagrams[0], X2_diagrams[0],internal_p=2)
pd_X1_X2_gudhi
```

```
POT (Python Optimal Transport) package is not installed. Try to run $ conda
install -c conda-forge pot ; or $ pip install POT
```

[18]:

```
2.7719287214918436
```

0.5.2 Compute all pairwise distance between the six point clouds:

[95]:

```
from gudhi.wasserstein import wasserstein_distance
from gudhi.hera import wasserstein_distance as hera
X_diagrams = []
X_diagrams.append(X1_diagrams)
X_diagrams.append(X2_diagrams)
X_diagrams.append(X3_diagrams)
X_diagrams.append(X4_diagrams)
X_diagrams.append(X5_diagrams)
X_diagrams.append(X6_diagrams)

pd_gudhi = np.zeros((6,6,3))
for dim in range(3):
    print(dim)
    print("-------------------")
    for i in range(6):
        print("$X" + "_" + str(i+1) + '$ &')
        for j in range(6):
#             if i != j:
            pd_gudhi[i,j,dim] = hera(X_diagrams[i][dim],␣
 ↪X_diagrams[j][dim],internal_p=2)
#                 print('Wasserstein distance between persistence diagrams for'␣
 ↪+ str(i+1) +' and '+ str(j+1) + ' (H' + str(dim) + ') is:')
            print(str(float("{0:.2f}".format(pd_gudhi[i,j,dim])))+'&')
        print("\\" + "\\")
        print("\hline")
```

```
0
-------------------
$X_1$ &
0.0&
2.77&
3.05&
3.95&
3.08&
3.42&
\\
\hline
```

```
$X_2$ &
2.77&
0.0&
0.43&
1.35&
1.0&
0.84&
\\
\hline
$X_3$ &
3.05&
0.43&
0.0&
1.03&
0.94&
0.66&
\\
\hline
$X_4$ &
3.95&
1.35&
1.03&
0.0&
1.11&
0.72&
\\
\hline
$X_5$ &
3.08&
1.0&
0.94&
1.11&
0.0&
0.51&
\\
\hline
$X_6$ &
3.42&
0.84&
0.66&
0.72&
0.51&
0.0&
\\
\hline
1
-------------------
$X_1$ &
```

```
0.0&
0.46&
0.5&
0.64&
0.64&
0.55&
\\
\hline
$X_2$ &
0.46&
0.0&
0.17&
0.29&
0.36&
0.3&
\\
\hline
$X_3$ &
0.5&
0.17&
0.0&
0.25&
0.33&
0.3&
\\
\hline
$X_4$ &
0.64&
0.29&
0.25&
0.0&
0.27&
0.29&
\\
\hline
$X_5$ &
0.64&
0.36&
0.33&
0.27&
0.0&
0.26&
\\
\hline
$X_6$ &
0.55&
0.3&
0.3&
```

```
0.29&
0.26&
0.0&
\\
\hline
2
-------------------
$X_1$ &
0.0&
0.06&
0.06&
0.06&
0.06&
0.06&
\\
\hline
$X_2$ &
0.06&
0.0&
0.01&
0.0&
0.01&
0.0&
\\
\hline
$X_3$ &
0.06&
0.01&
0.0&
0.0&
0.01&
0.0&
\\
\hline
$X_4$ &
0.06&
0.0&
0.0&
0.0&
0.0&
0.0&
\\
\hline
$X_5$ &
0.06&
0.01&
0.01&
0.0&
```

```
0.0&
0.0&
\\
\hline
$X_6$ &
0.06&
0.0&
0.0&
0.0&
0.0&
0.0&
\\
\hline
```

Pairwise Wasserstein distance between persistence diagrams for homology group H_0:

[20]:
```
print(pd_gudhi[:,:,0])
```

```
[[0.         2.77192872 3.04745634 3.95276261 3.07892065 3.41508927]
 [2.77192524 0.         0.42930521 1.34856909 0.99806364 0.84377773]
 [3.04747962 0.42930766 0.         1.03143331 0.94296253 0.65615236]
 [3.95283538 1.34850632 1.0315476  0.         1.10848366 0.72082808]
 [3.07892922 0.99806358 0.94295643 1.10848369 0.         0.51151537]
 [3.41507195 0.84376872 0.65615035 0.72081878 0.51151656 0.        ]]
```

Pairwise Wasserstein distance between persistence diagrams for homology group H_1:

[21]:
```
print(pd_gudhi[:,:,1])
```

```
[[0.         0.45973786 0.50093082 0.64066787 0.63832201 0.54722478]
 [0.45974146 0.         0.16690206 0.29471953 0.36368453 0.30304126]
 [0.50092305 0.1668978  0.         0.24557967 0.33290296 0.29892867]
 [0.64068023 0.29472335 0.24558021 0.         0.26870174 0.28557107]
 [0.63833715 0.36368295 0.33290654 0.26869569 0.         0.26458892]
 [0.54724428 0.30303734 0.29893539 0.28556935 0.26458771 0.        ]]
```

Pairwise Wasserstein distance between persistence diagrams for homology group H_2:

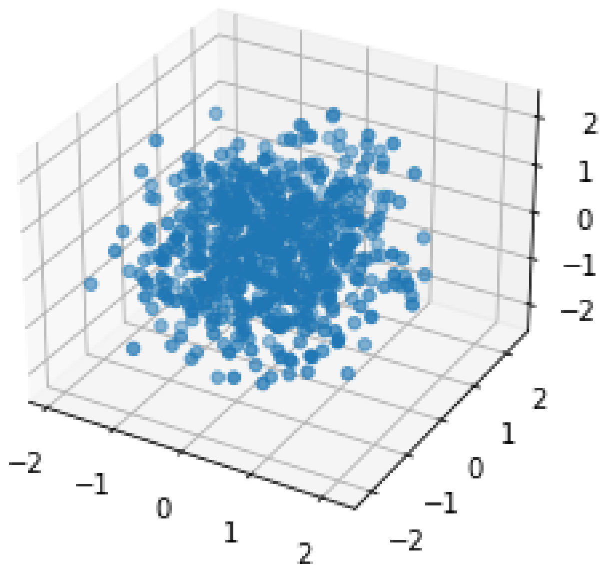
[22]:
```
print(pd_gudhi[:,:,2])
```

```
[[0.         0.05800773 0.05728896 0.0593703  0.06085189 0.0596126 ]
 [0.05800773 0.         0.00500477 0.00380341 0.00596403 0.0036489 ]
 [0.05728896 0.00500477 0.         0.00485482 0.00647532 0.00494322]
 [0.0593703  0.00380341 0.00485482 0.         0.00479106 0.00202047]
 [0.06085189 0.00596403 0.00647532 0.00479106 0.         0.00432514]
 [0.0596126  0.0036489  0.00494322 0.00202047 0.00432514 0.        ]]
```

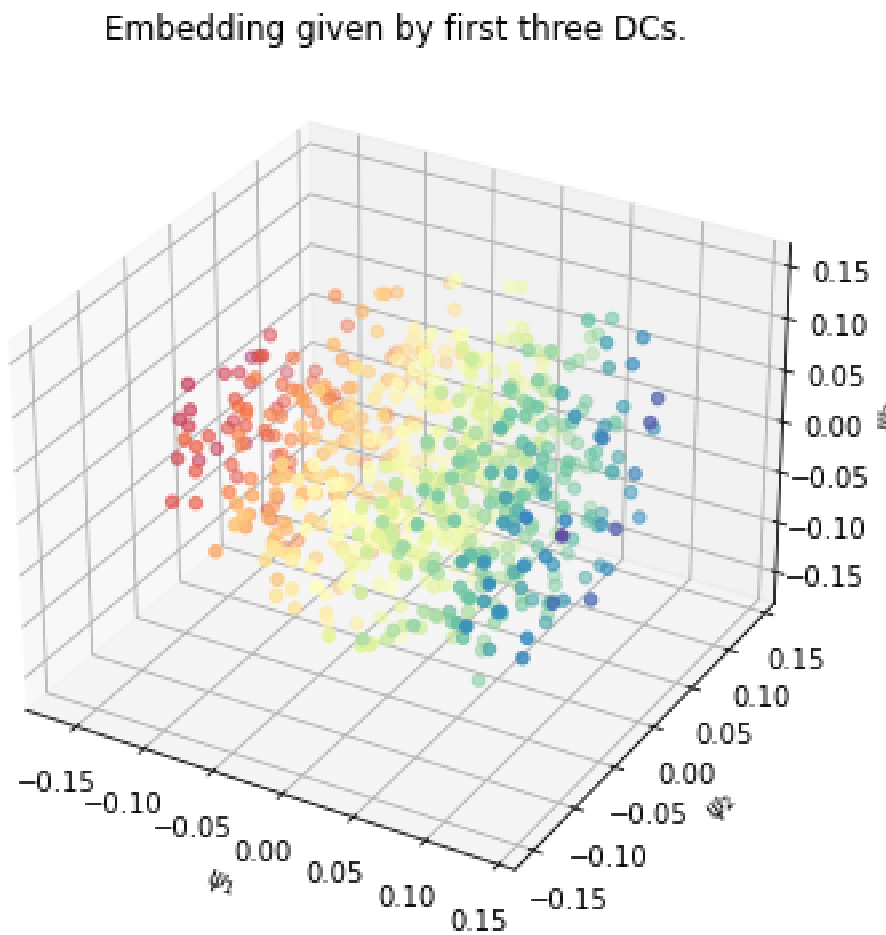
0.5.3 compare with known shape, eg a circle:

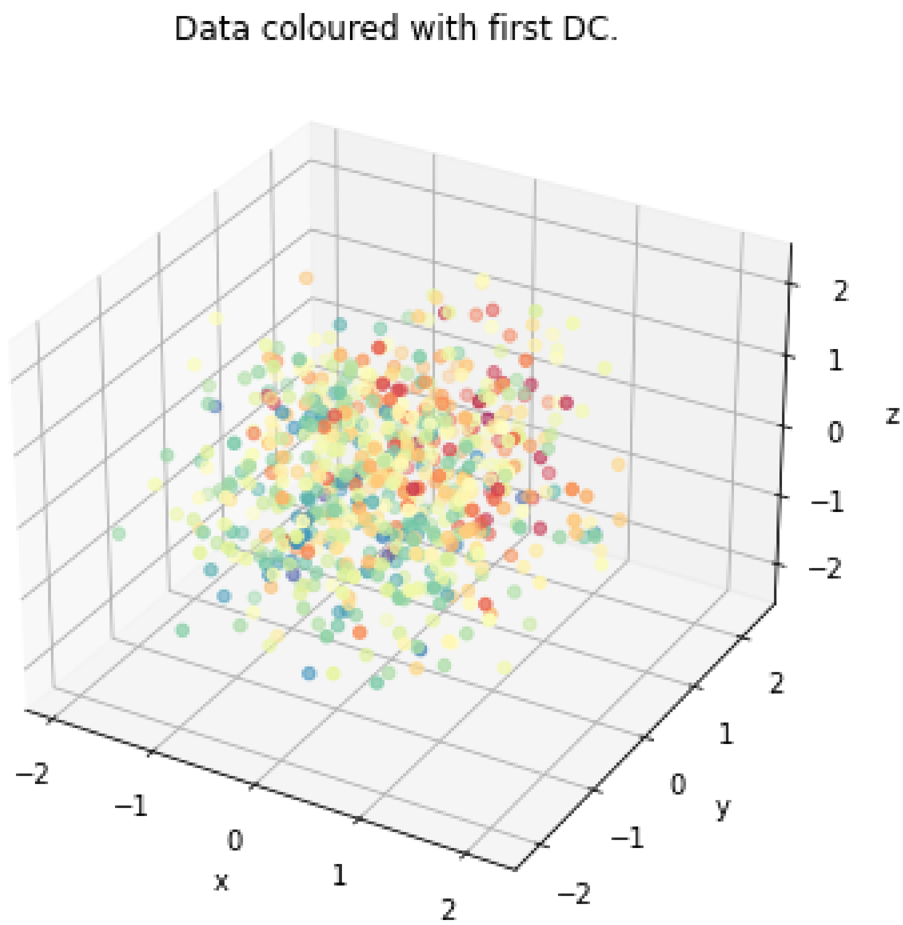
[23]:
```
import tadasets
```

[24]:
```
dsphere = tadasets.dsphere(n=698, d=12, r=3.14, ambient=14, noise=0.14)
ax = plt.axes(projection ="3d")
ax.scatter3D(dsphere[:,0], dsphere[:,1], dsphere[:,2])
plt.show()
```

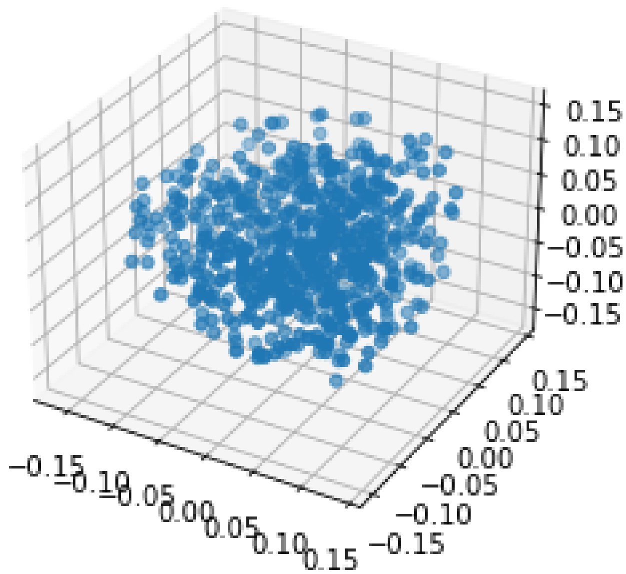


[25]:
```
dsphere_diagrams, dsphere_barcode = diffusion_tda(dsphere)
```

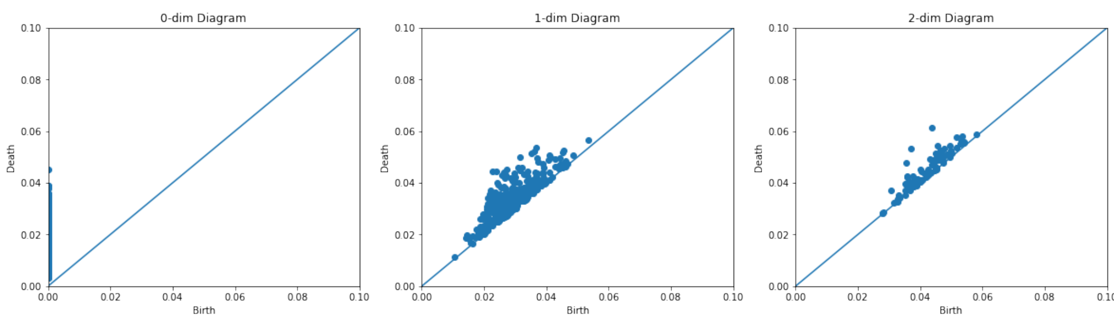

Embedding given by first three DCs.
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15



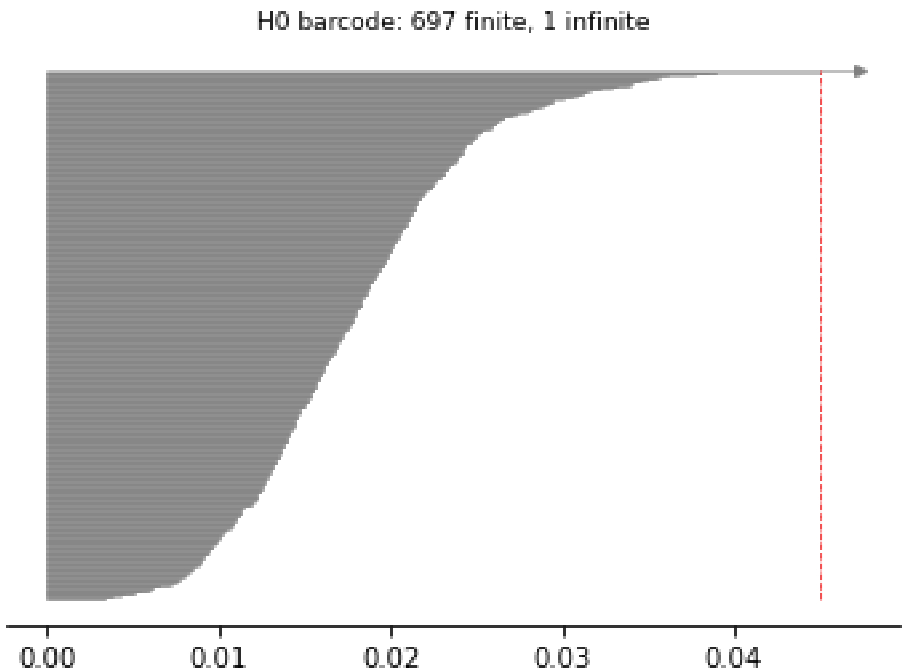
SHAPE (698, 3)

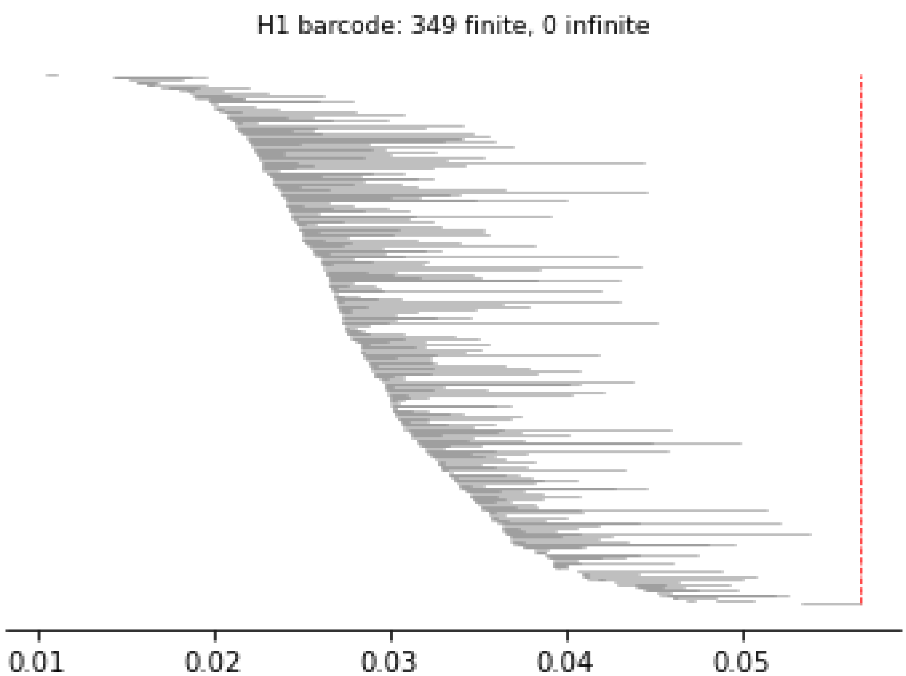


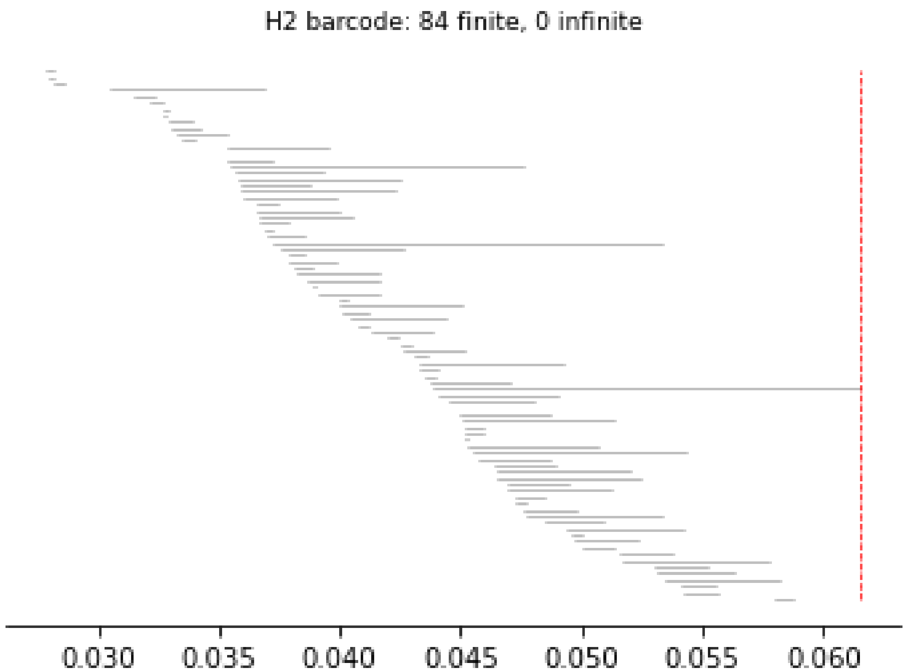
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15



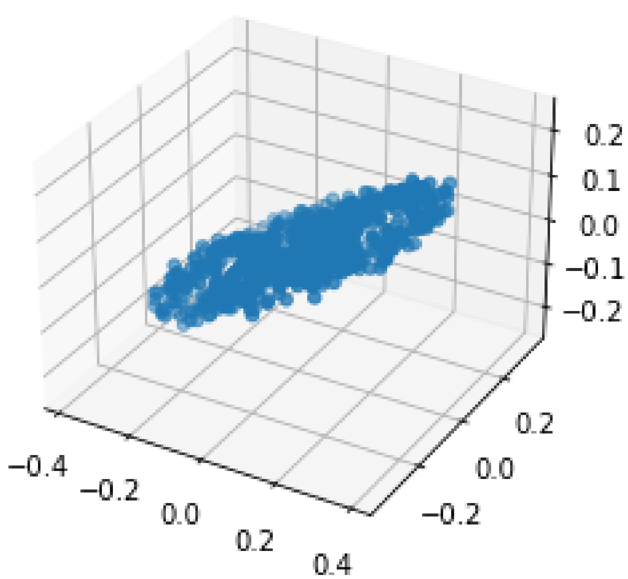
0-dim Diagram
1-dim Diagram
2-dim Diagram
Death
Birth
0.00
0.02
0.04
0.06
0.08
0.10


H0 barcode: 697 finite, 1 infinite
0.00
0.01
0.02
0.03
0.04


H1 barcode: 349 finite, 0 infinite
0.01
0.02
0.03
0.04
0.05



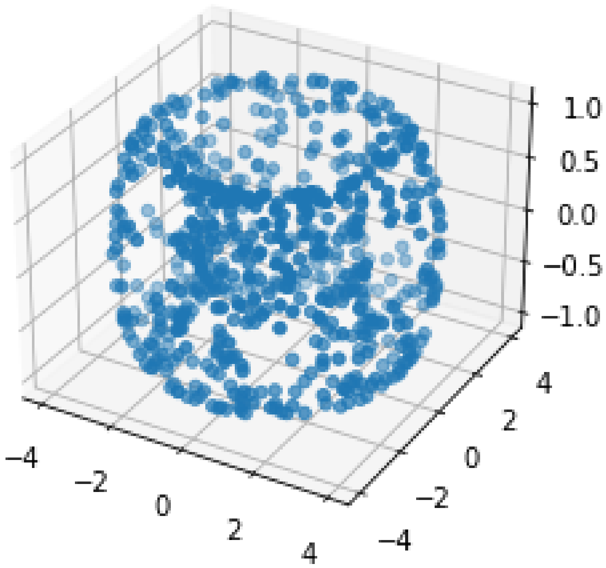
[26]:
```
torus = tadasets.torus(n=698, c=2, a=1, ambient=200, noise=0.2)
ax = plt.axes(projection ="3d")
ax.scatter3D(torus[:,0], torus[:,1], torus[:,2])
plt.show()
```



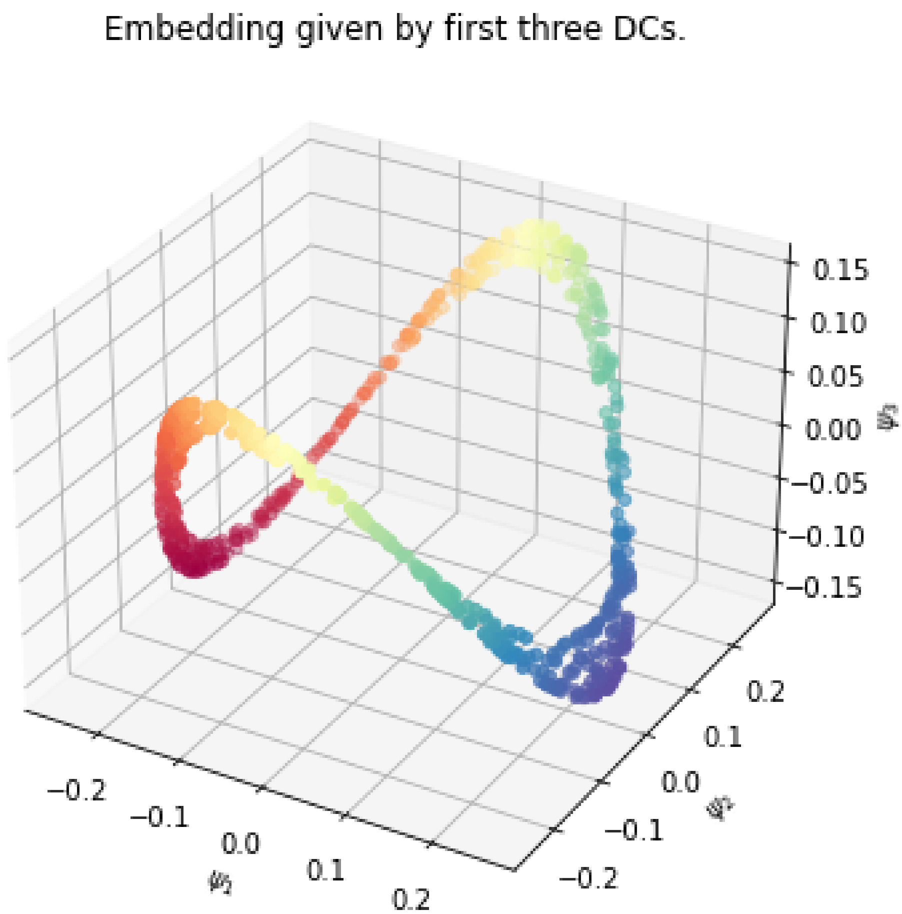
[42]:
```python
class TorusSampler():

    def __init__(self, r=1, R=3, x0=0, y0=0, z0=0):
        self._r = r
        self._R = R
        self._x0 = x0
        self._y0 = y0
        self._z0 = z0

    '''
    This sampling may not be uniform
    '''
    def Sample(self, n):
        u = 2 * np.pi * np.random.rand(n)
        v = 2 * np.pi * np.random.rand(n)

        cosu = np.cos(u)
        sinu = np.sin(u)

        cosv = np.cos(v)
        sinv = np.sin(v)

        x = self._x0 + (self._R + self._r * cosu) * cosv
        y = self._y0 + (self._R + self._r * cosu) * sinv
        z = self._z0 + self._r * sinu

        return np.array([x, y, z]).T
```

[77]:
```python
torus = TorusSampler(1, 3).Sample(698)
```

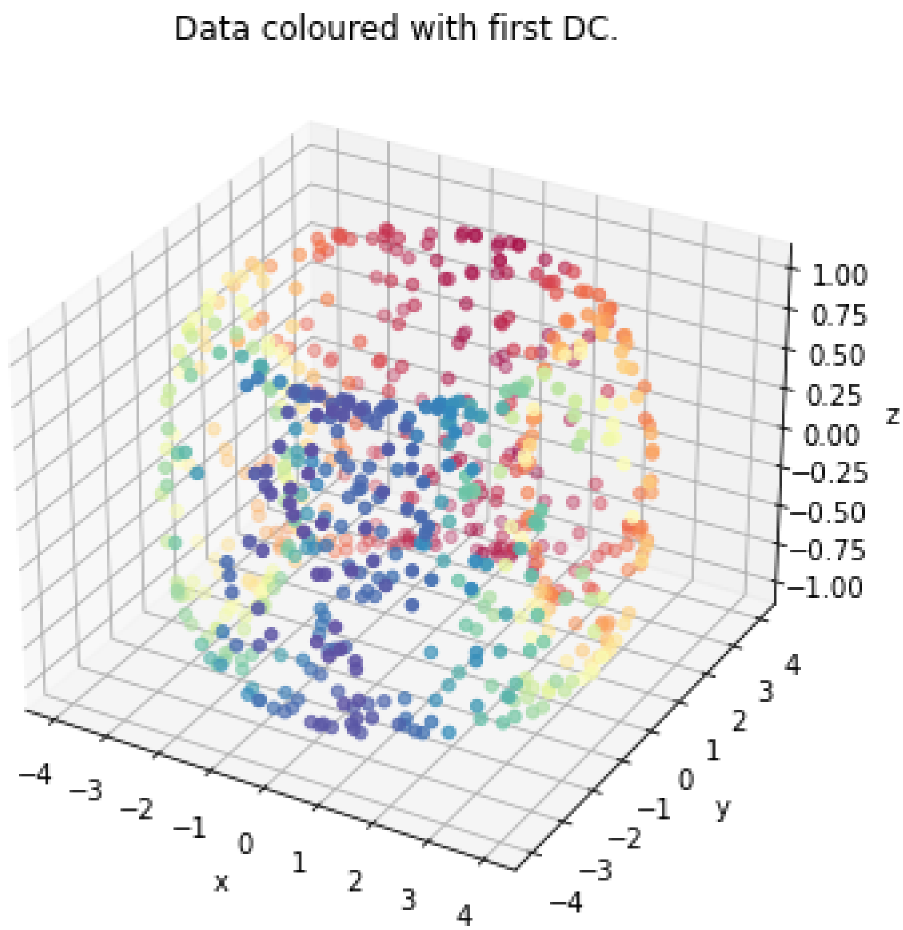
[78]:
```python
ax = plt.axes(projection ="3d")
ax.scatter3D(torus[:,0], torus[:,1], torus[:,2])
plt.show()
```



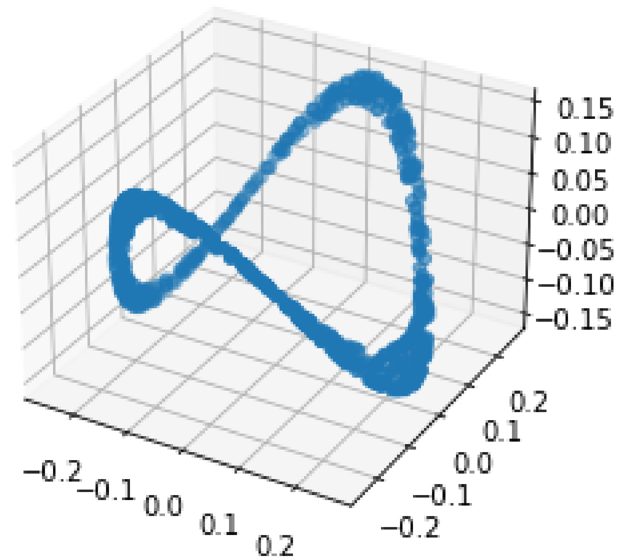
[79]:
```
torus_diagrams, torus_barcode = diffusion_tda(torus)
```

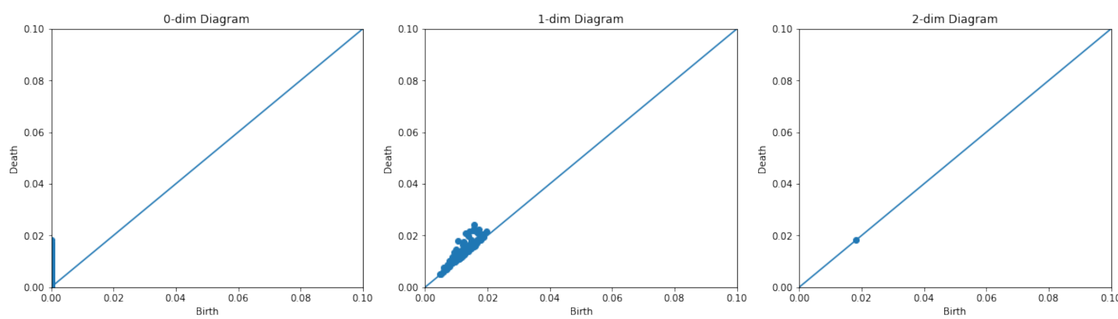


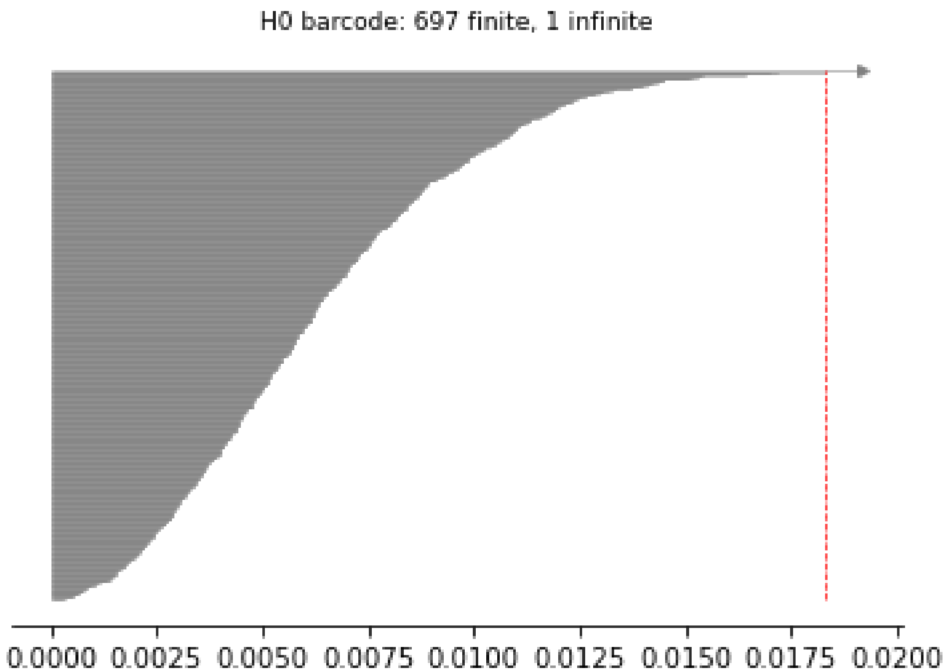
Embedding given by first three DCs.
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
0.2
0.1
0.0
−0.1
−0.2
−0.2
−0.1
0.0
0.1
0.2

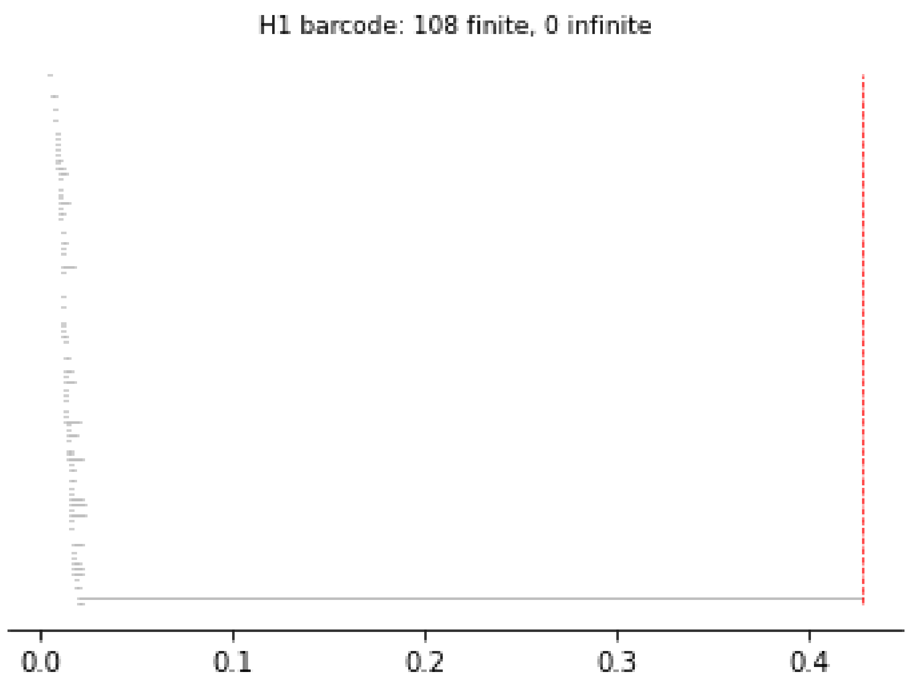

Data coloured with first DC.
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
z
−4
−3
−2
−1
0
1
2
3
4
x
4
3
2
1
0
−1
−2
−3
−4
y

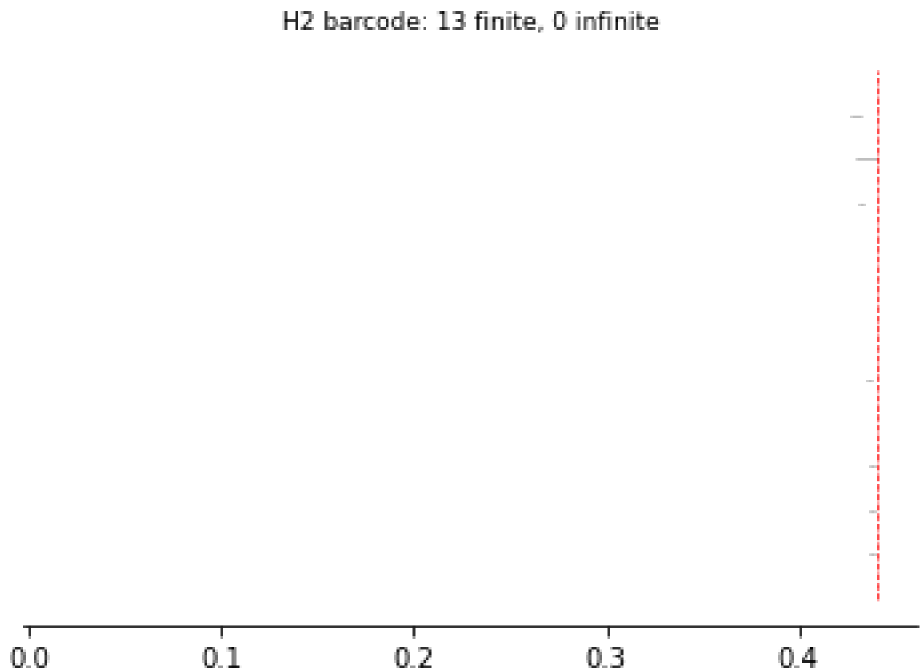
```
SHAPE (698, 3)
```


0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
0.2
0.1
0.0
−0.1
−0.2
−0.2
−0.1
0.0
0.1
0.2


0-dim Diagram
1-dim Diagram
2-dim Diagram
Death
Birth
0.00
0.02
0.04
0.06
0.08
0.10


H0 barcode: 697 finite, 1 infinite
0.0000 0.0025 0.0050 0.0075 0.0100 0.0125 0.0150 0.0175 0.0200


H1 barcode: 108 finite, 0 infinite
0.0 0.1 0.2 0.3 0.4



[76]:
```
from gudhi.wasserstein import wasserstein_distance
from gudhi.hera import wasserstein_distance as hera
X_diagrams = []
X_diagrams.append(X1_diagrams)
X_diagrams.append(X2_diagrams)
X_diagrams.append(X3_diagrams)
X_diagrams.append(X4_diagrams)
X_diagrams.append(X5_diagrams)
X_diagrams.append(X6_diagrams)

pd_dsphere_gudhi = np.zeros((6,3))
for dim in range(3):
    for i in range(6):
#           print(X_diagrams[i][dim])
#           print(dsphere_diagrams[dim])
        pd_dsphere_gudhi[i,dim] = hera(X_diagrams[i][dim],␣
 ↪dsphere_diagrams[dim])
        print('Wasserstein distance between persistence diagrams for circle and␣
 ↪point cloud ' + str(i+1) + ' (H' + str(dim) + ') is:')
        print(pd_dsphere_gudhi[i,dim])
```

```
Wasserstein distance between persistence diagrams for circle and point cloud 1
(H0) is:
2.4809560721041635
Wasserstein distance between persistence diagrams for circle and point cloud 2
```

```
(H0) is:
4.463884397916445
Wasserstein distance between persistence diagrams for circle and point cloud 3
(H0) is:
4.641338810155503
Wasserstein distance between persistence diagrams for circle and point cloud 4
(H0) is:
5.17980453916789
Wasserstein distance between persistence diagrams for circle and point cloud 5
(H0) is:
4.531922676127579
Wasserstein distance between persistence diagrams for circle and point cloud 6
(H0) is:
4.827321461983956
Wasserstein distance between persistence diagrams for circle and point cloud 1
(H1) is:
0.6241310045588762
Wasserstein distance between persistence diagrams for circle and point cloud 2
(H1) is:
0.7671945061301813
Wasserstein distance between persistence diagrams for circle and point cloud 3
(H1) is:
0.7996520071465056
Wasserstein distance between persistence diagrams for circle and point cloud 4
(H1) is:
0.866591851816338
Wasserstein distance between persistence diagrams for circle and point cloud 5
(H1) is:
0.8496004594489932
Wasserstein distance between persistence diagrams for circle and point cloud 6
(H1) is:
0.7646189627703279
Wasserstein distance between persistence diagrams for circle and point cloud 1
(H2) is:
0.10986399976536632
Wasserstein distance between persistence diagrams for circle and point cloud 2
(H2) is:
0.12334374524652958
Wasserstein distance between persistence diagrams for circle and point cloud 3
(H2) is:
0.12340327631682158
Wasserstein distance between persistence diagrams for circle and point cloud 4
(H2) is:
0.12458728905767202
Wasserstein distance between persistence diagrams for circle and point cloud 5
(H2) is:
0.12582713179290295
Wasserstein distance between persistence diagrams for circle and point cloud 6
```

```
(H2) is:
0.12466537859290838
```

[80]:
```python
from gudhi.wasserstein import wasserstein_distance
from gudhi.hera import wasserstein_distance as hera
X_diagrams = []
X_diagrams.append(X1_diagrams)
X_diagrams.append(X2_diagrams)
X_diagrams.append(X3_diagrams)
X_diagrams.append(X4_diagrams)
X_diagrams.append(X5_diagrams)
X_diagrams.append(X6_diagrams)

pd_torus_gudhi = np.zeros((6,3))
for dim in range(3):
    for i in range(6):
#         print(X_diagrams[i][dim])
#         print(dsphere_diagrams[dim])
        pd_torus_gudhi[i,dim] = hera(X_diagrams[i][dim], torus_diagrams[dim])
        print('Wasserstein distance between persistence diagrams for circle and␣
↪point cloud ' + str(i+1) + ' (H' + str(dim) + ') is:')
        print(pd_torus_gudhi[i,dim])
```

```
Wasserstein distance between persistence diagrams for circle and point cloud 1
(H0) is:
3.6310234030825086
Wasserstein distance between persistence diagrams for circle and point cloud 2
(H0) is:
1.4639444853646637
Wasserstein distance between persistence diagrams for circle and point cloud 3
(H0) is:
1.3077519815615233
Wasserstein distance between persistence diagrams for circle and point cloud 4
(H0) is:
0.7306229655132483
Wasserstein distance between persistence diagrams for circle and point cloud 5
(H0) is:
1.3777969512157142
Wasserstein distance between persistence diagrams for circle and point cloud 6
(H0) is:
1.0552870598621666
Wasserstein distance between persistence diagrams for circle and point cloud 1
(H1) is:
0.7361079549882561
Wasserstein distance between persistence diagrams for circle and point cloud 2
(H1) is:
0.48583426291588694
Wasserstein distance between persistence diagrams for circle and point cloud 3
```

```
(H1) is:
0.4428885536908638
Wasserstein distance between persistence diagrams for circle and point cloud 4
(H1) is:
0.3395286252562073
Wasserstein distance between persistence diagrams for circle and point cloud 5
(H1) is:
0.45408774679526687
Wasserstein distance between persistence diagrams for circle and point cloud 6
(H1) is:
0.468785529024899
Wasserstein distance between persistence diagrams for circle and point cloud 1
(H2) is:
0.05699422350153327
Wasserstein distance between persistence diagrams for circle and point cloud 2
(H2) is:
0.017472262494266033
Wasserstein distance between persistence diagrams for circle and point cloud 3
(H2) is:
0.018393791280686855
Wasserstein distance between persistence diagrams for circle and point cloud 4
(H2) is:
0.016043948009610176
Wasserstein distance between persistence diagrams for circle and point cloud 5
(H2) is:
0.018177962861955166
Wasserstein distance between persistence diagrams for circle and point cloud 6
(H2) is:
0.01596159115433693
```

[97]:
```python
print("Comparing six point-clouds with 3-sphere (H0):")
print(pd_dsphere_gudhi[:,0])
print("Comparing six point-clouds with torus (H0):")
print(pd_torus_gudhi[:,0])
```

```
Comparing six point-clouds with 3-sphere (H0):
[2.48095607 4.4638844  4.64133881 5.17980454 4.53192268 4.82732146]
Comparing six point-clouds with torus (H0):
[3.6310234  1.46394449 1.30775198 0.73062297 1.37779695 1.05528706]
```

[82]:
```python
print("Comparing six point-clouds with 3-sphere (H1):")
print(pd_dsphere_gudhi[:,1])
print("Comparing six point-clouds with torus (H1):")
print(pd_torus_gudhi[:,1])
```

```
Comparing six point-clouds with 3-sphere (H1):
[0.624131   0.76719451 0.79965201 0.86659185 0.84960046 0.76461896]
Comparing six point-clouds with torus (H1):
```

```
[0.73610795 0.48583426 0.44288855 0.33952863 0.45408775 0.46878553]
```

[83]:

```python
print("Comparing six point-clouds with 3-sphere (H2):")
print(pd_dsphere_gudhi[:,2])
print("Comparing six point-clouds with torus (H2):")
print(pd_torus_gudhi[:,2])
```

```
Comparing six point-clouds with 3-sphere (H2):
[0.109864   0.12334375 0.12340328 0.12458729 0.12582713 0.12466538]
Comparing six point-clouds with torus (H2):
[0.05699422 0.01747226 0.01839379 0.01604395 0.01817796 0.01596159]
```