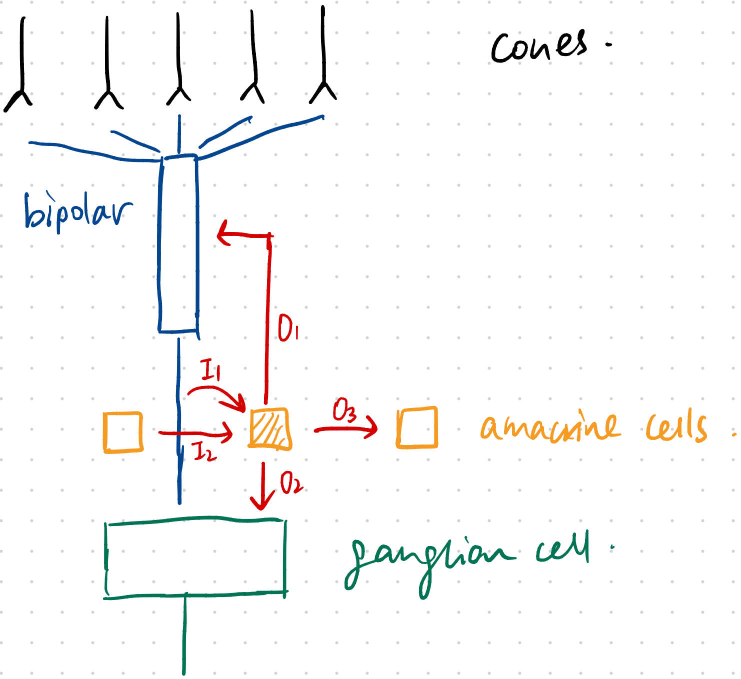
Missing from the previous lecture: amacrine cell.

Block diagram of amacrine cell pathways:



① Amacrine cells receive input from the bipolar cells and other amacrine cells. I_1, I_2

② Amacrine cells produce outputs back to bipolar cells, to ganglion cells, and to other amacrine cells. O_1, O_2, O_3.

Functions of amacrine cells:

- feedback inhibition (via O_1)
- feed-forward inhibition (via O_3)
- lateral inhibition (via O_2)

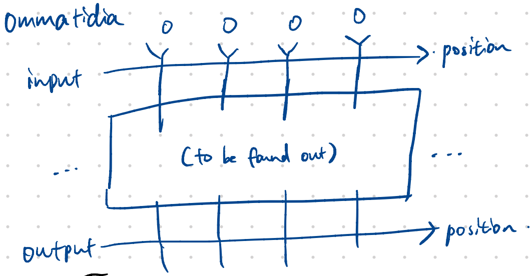
studied in the next lecture (lateral inhibition network)
the center-surround mechanism.

Week 3: The Lateral Inhibition Network.

(Hartline, Grantt, Wald)

Experiments on Limulus Polythemus and its compound eye system.

The Lateral Plexus:



follows the equation from last lecture:

$$C_m \frac{dV}{dt} = \sum_{\text{sources}} I_{\text{source}} = \boxed{\sum_{\text{synapses}} I_{\text{synapse}}}$$

with simplified assumptions

integrate and fire

$$(\text{output firing rate})_i \propto \sum_j \underbrace{\rho_{i,j}}_{\text{synaptic coefficient between neuron } i \text{ and neuron } j} \cdot (\text{input firing rate})_j$$

methods to map the stimulus parameter to neural activity:

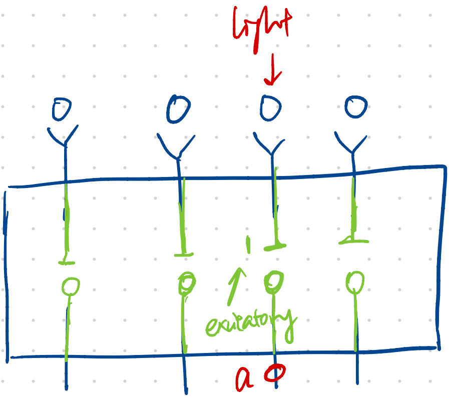
- rate code.
- time code
- population code

(*) during the processing itself, the eye is already "selecting" input and hence the input information from the external, real world

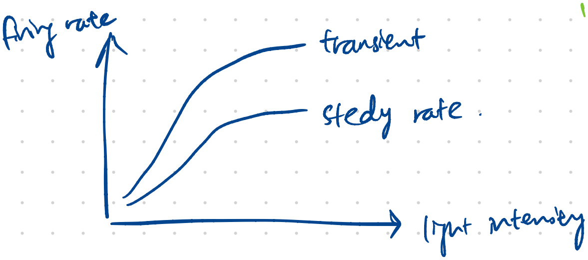
What happens between the input & the output?

3 Experiments:

(1) Illuminate light on one ommatidia.



① check at one position a



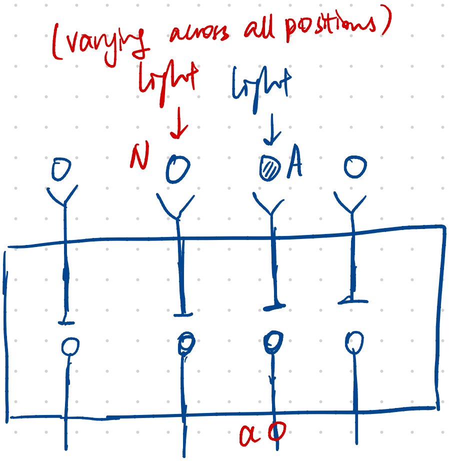
implies

⇒ presence of excitatory synapse at this position

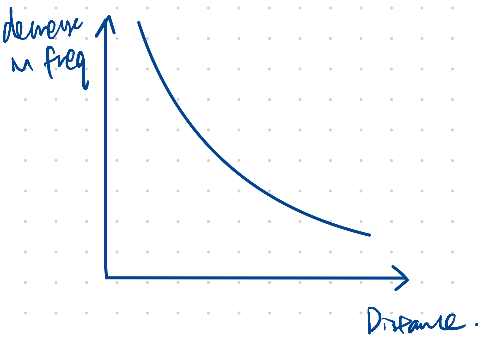
② check at all positions:

⇒ no interaction across positions.

(2) Illuminate light on two ommatidia.



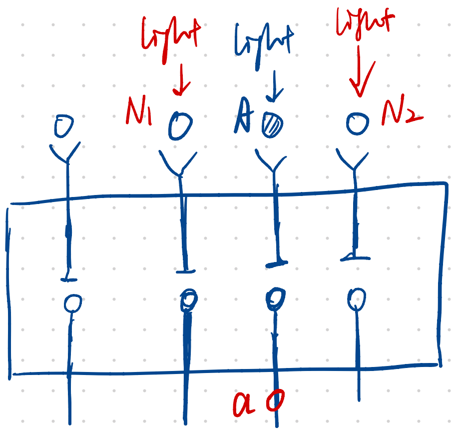
Check position a while varying the light



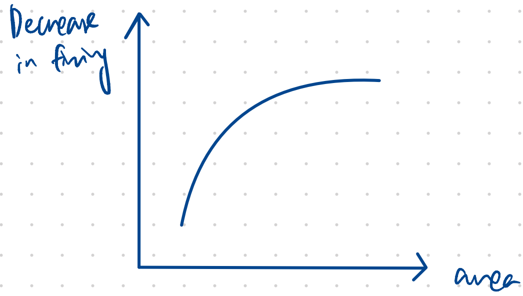
implies $\Rightarrow$ ① The surrounding ommatidia has inhibitory effect.

② This inhibitory effect decreases as distance from the central ommatidia increases.

(2) Illuminate light on three ommatidia.

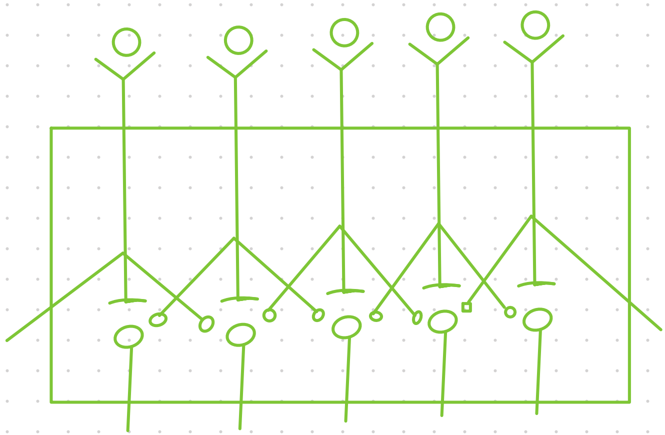


Check position a :



⇓

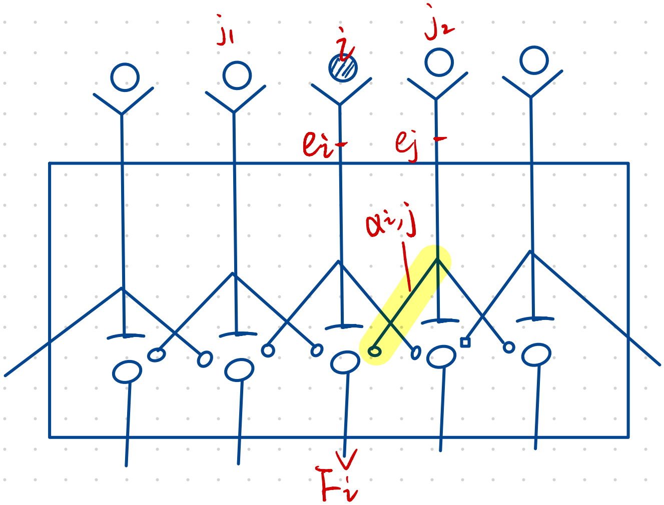
A simplified model:



Assumptions : ① only nearest neighbor
② no threshold.

Summary & Algebraic formulation:

- feed-forward network
- excitatory & inhibitory connections.



$$F_i = e_i - \sum_{j \in neigh(i)} \alpha_{i,j}\, e_j \,. \quad (\alpha_{i,j} \in \mathbb{R}^+)$$

More realistically, (but not needed for our purpose)

(input activity to neuron at position i)

$$F_i = e_i - \sum_{j \in neigh(i)} \boxed{a_{i,j}} \lfloor e_j - \theta \rfloor ,$$

(firing rate of neuron at position i)

(inhibitory synaptic coefficient from j to i).

where $\lfloor e_j - \theta \rfloor = \begin{cases} e_j - \theta & \text{if } e_j \geq \theta . \\ 0 & \text{if } e_j < \theta . \end{cases}$

(threshold function)

Mach bands:

visual illusion

perceived brightness $\neq$ physical brightness.

"sensation surface" "light surface"

curvature of light.

brightness

$$e = a \log \left\{ i/b \pm k \frac{\frac{d^2 i}{dx^2}}{i} \right\}$$

→ Laplacian operation.

↓

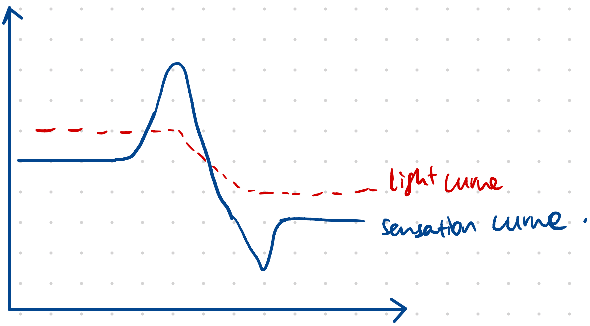
a filtering operation.

Recurrent networks:

$$F_i = e_i - \sum_{j \in neigh(i)} a_{i,j} \lfloor \underline{F_j} - \theta \rfloor$$

(use the output instead of the input at position j)

More on Mach bands:



$\alpha_{ij} = -0.2.$

$100 - 0.2\times100 - 0.2\times50 = 70$

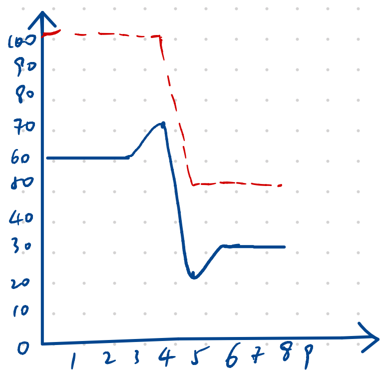
$100 - 0.2\times100 - 0.2\times100 = 60$

$50 - 0.2\times100 - 0.2\times50 = 20$

$50 - 0.2\times50 - 0.2\times50 = 30$

e.g.:

input	100	100	100	100	50	50	50	50
output	60	60	60	70	20	30	30	30.



Chapter 6: Limulus Linear's. An Image Processing Machine.

image (input) $\xrightarrow{\text{network}}$ image (output).

<u>Def 1: Linearity</u>: Let L be a time/space-invariant linear system with input $I(x)$, output $O(x)$.

① additive. $L(I_1 + I_2) = L(I_1) + L(I_2)$

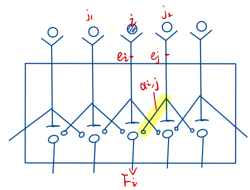
② homogeneous. $L(\alpha I) = \alpha L(I), \alpha \in \mathbb{R}$.

→ principle of superposition:

$$L(\alpha I_1 + \beta I_2) = \alpha L(I_1) + \beta L(I_2), \quad \alpha, \beta \in \mathbb{R}.$$

The lateral inhibition network is an example of linear system:

$$I \longrightarrow \boxed{L} \longrightarrow O.$$

$I \longrightarrow$  $\longrightarrow O$.

<u>Def 2</u>: unit step function

$$u(x) := \begin{cases} 0 & (\text{if } x < 0) \\ 1 & (\text{if } x > 0) \\ \text{undefined.} & (\text{if } x = 0). \end{cases}$$

applied to balanced lateral inhibition network: suppose $\alpha_{ij} = 0.2$

input at x	...	100	100	100	50	50	50	...
shifted input at $x+1$	...	100	100	100	100	50	50	...
output at x	...	60	60	70	20	30	30	...
shifted output at $x+1$	...	60	60	60	70	20	30	...
diff in output	...	0	0	10	-50	10	0	...

Def 3 : Dirac delta function (to model the unit impulse).

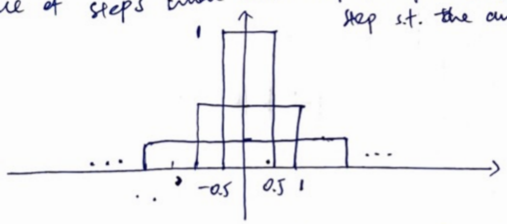
Given the unit step function $u(x)$ (in def 2), the Dirac delta function:

$$\boxed{\delta(x) = \frac{d\, u(x)}{dx}}$$

The Dirac delta function is not really a "function". It's a distribution which is well-defined only when it appears under an integral sign!

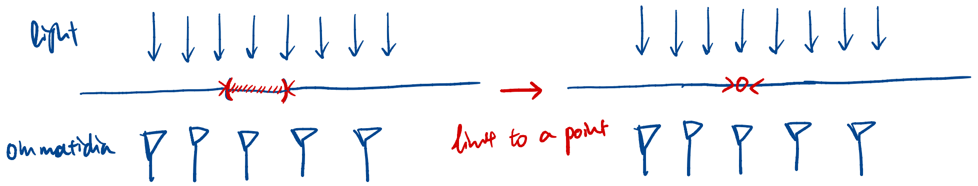
- $\delta(x) = \begin{cases} 0 & (\text{if } x \neq 0) . \\ \infty & (\text{if } x = 0) . \end{cases}$
- $\int_{-\infty}^{+\infty} \delta(x)\, dx = 1$
- $x\, \delta(x) = 0$.

The Dirac Delta function can be thought of as the limit of a sequence of steps where each step is higher and narrower than the previous step s.t. the area is always 1 .



The Dirac delta function is used to model the density of an idealized point charge .

Intuition: Given a fixed amount of light (=1).



total amount of light on the shaded area =1

- as the area goes to a point, the amount of light on the area goes to infinity;
- the sum of light is still 1 .

Def 5: Impulse response: the response of a linear system to an impulse function as input:

$h(x) = L[\delta(x)]$

Def 6: step response: the response to a unit step function

(the integral of impulse function).

$$S(x) = L(u(x)).$$

$$= L \int_{-\infty}^{x} \delta(\lambda) d\lambda$$

$$= \int_{-\infty}^{x} L[\delta(\lambda)] d\lambda$$

$$= \int_{-\infty}^{x} h(\lambda) d\lambda \quad \text{——— (1)}$$

(λ is a "dummy" variable).

Approximate an arbitrary input as a sequence of step functions:

(using area under curve)

Approximate the signal $I(x)$ as:

$$I(x) \approx \hat{I}(x) = I(0)u(x) + [I(\Delta) - I(0)]u(x-\Delta) + \dots$$

$$= I(0)u(x) + \sum_{k=1}^{\infty} \{I(k\Delta) - I((k-1)\Delta)\} u(x-k\Delta)$$

Then, $$\hat{O}(x) = I(0)S(x) + \sum_{k=1}^{\infty} \left\{\frac{I(k\Delta) - I((k-1)\Delta)}{\Delta}\right\} S(x-k\Delta)\Delta$$

(by linearity)

$$I(x) \approx \hat{I}(x) = I(0)\,u(x) + \sum_{k=1}^{\infty} \{ I(k\Delta) - I((k-1)\Delta) \}\, u(x-k\Delta)$$

$$\Rightarrow \quad \hat{O}(x) = I(0)\,S(x) + \sum_{k=1}^{\infty} \left\{ \frac{I(k\Delta) - I((k-1)\Delta)}{\Delta} \right\} S(x-k\Delta)\,\Delta$$

$$\Big\downarrow \; \Delta \to d\tau,\ k\Delta \to \tau \qquad\qquad \Big\downarrow \; \Delta \to d\tau,\ k\Delta \to \tau$$

(τ is a dummy variable here)

$$O(x) = I(0)\,S(x) + \int_{0^+}^{\infty} \frac{dI(\tau)}{d\tau}\, S(x-\tau)\, d\tau$$

since $I(x)$ may be discontinuous at $x=0$.

Now, suppose $I(x)$ starts from $\chi > -\infty$ (instead of starting from $x=0$ as assumed above). Then,

$$I(x) \approx \hat{I}(x) = \sum_{k=\chi}^{\infty} \{ I(k\Delta) - I(k-1)\Delta) \}\, u(x-k\Delta).$$

$$\Rightarrow \quad \hat{O}(x) = \sum_{k=\chi}^{\infty} \left\{ \frac{I(k\Delta) - I(k-1)\Delta}{\Delta} \right\} S(x-k\Delta)\,\Delta.$$

$$\Big\downarrow \; \chi \to -\infty,\ I(\chi)\,S(x-\chi) \to 0 \qquad\qquad \Big\downarrow \; \chi \to -\infty,\ I(\chi)\,S(x-\chi) \to 0.$$

$$O(x) = \int_{-\infty}^{\infty} \frac{dI(\tau)}{d\tau}\, S(x-\tau)\, d\tau.$$

$$= \int_{-\infty}^{\infty} I(\tau)\, \frac{d\,S(x-\tau)}{d\tau}\, d\tau$$

Convolution!

$$= \int_{-\infty}^{\infty} I(\tau)\, h(x-\tau)\, d\tau. \quad (\text{by } ①).$$

$$= I(x) * h(x)$$

Conclusion: the output of a linear network can be computed as the convolution of the impulse response with the input.

i.e. $O(x) = I(x) * h(x) = h(x) * I(x)$

$$= \int_{-\infty}^{\infty} I(\tau)\, h(x-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, I(x-\tau)\, d\tau.$$