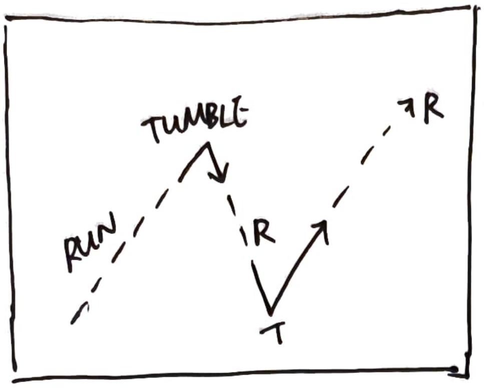
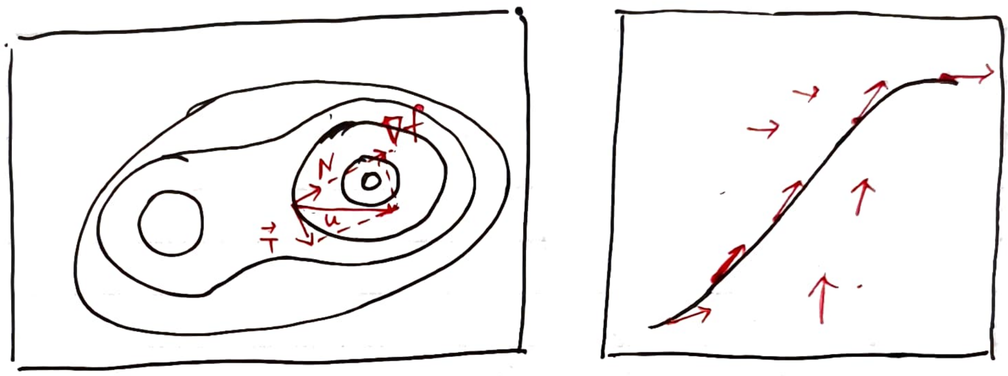
Lecture 1 : A. Coli

High-level summary :

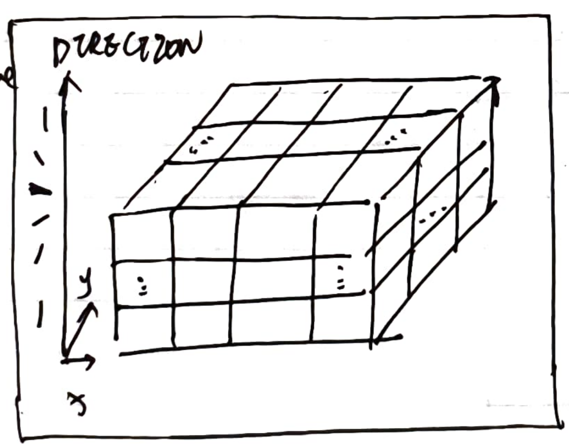
A. coli movements



Gradient Ascent



(T, N) coordinate frame



Model of E. coli → A. coli.

"artificial"

Suppose A. coli is pointing in (the internal variable) DIRECTION.

- at each individual step:

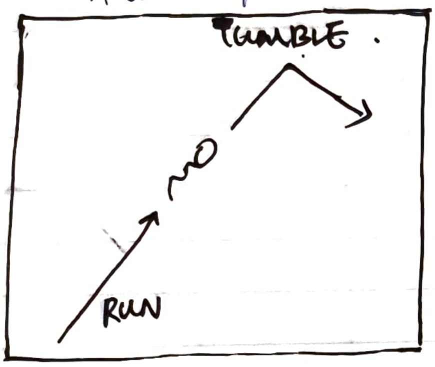
  A. coli alternates between 2 STATES:

  ① RUN. +2 units in DIRECTION.

  ② TUMBLE. rotate/update DIRECTION.*

  (* E. coli can only ↻ and not ↺

  → A. coli step ② rotation angle ~ unif([20°, 90°]).

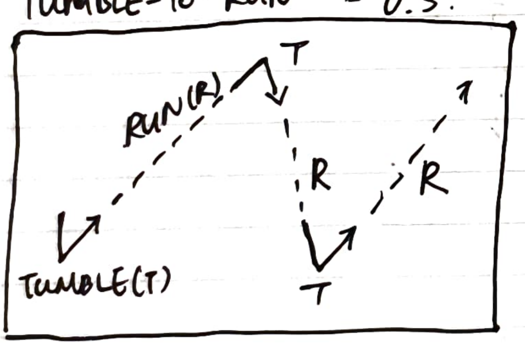


- composition of steps.

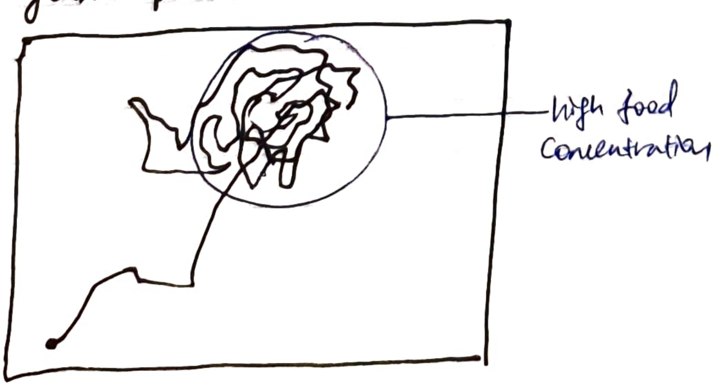
  switch between states follow the probabilities:

  RUN-TO-TUMBLE = 0.1

  TUMBLE-TO-RUN = 0.3.



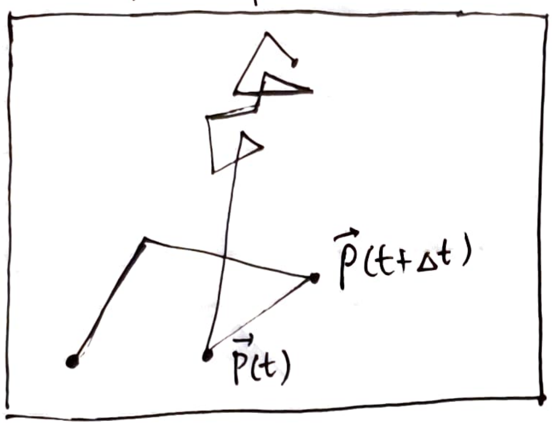
- global path.



A. coli measures a positive difference, i.e., when the food conc. is ↑, the angle of TUMBLE is smaller.

Model of finding highest conc. of food. → Gradient Ascent.

- Geometrical description of A-coli movements:



~ a dynamical systems problem!

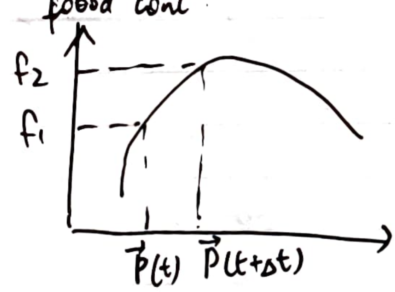
unit vectors.

$$\vec{p}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}. \quad \text{or eqv. } \vec{p}(t) = x(t)\,\hat{i} + y(t)\,\hat{j}.$$

$$\downarrow {+\Delta t}$$

$$\vec{p}(t+\Delta t) = \begin{pmatrix} x(t+\Delta t) \\ y(t+\Delta t) \end{pmatrix}.$$

- Food function:



Derivative $f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}.$

$$f(x+\Delta x) - f(x) = \Delta x\, f'(x) + \underline{\underline{\varepsilon\, \Delta x}}$$

approx error.

(goes to 0 when Δx is inf. small).

↳ derivative is a local linear approx to 1D curve.

partial derivative in the direction of $\vec{u}$:

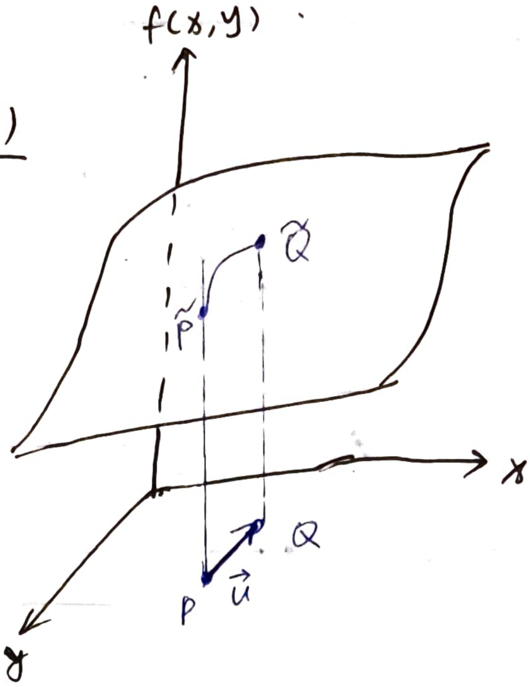
$$D_{\vec{u}} f(\vec{x}) = \lim_{h \to 0} \frac{f(\vec{x}+h\vec{u}) - f(\vec{x})}{h}.$$

$$= \vec{u} \cdot \nabla f$$

where ∇f is the gradient vector:

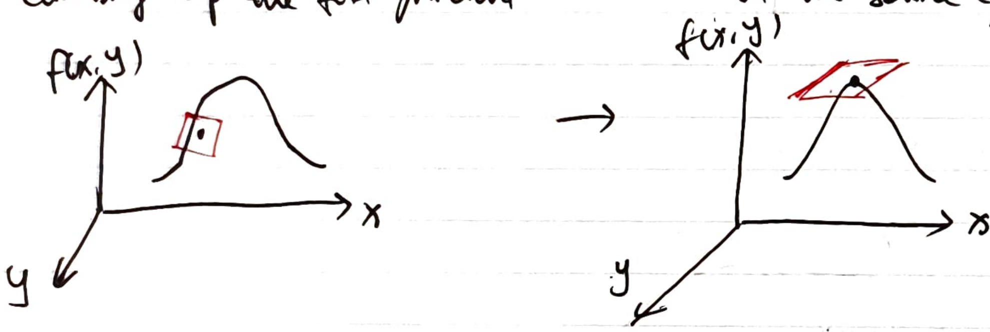
$$\nabla f := \left(\frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j}\right).$$

points uphill.

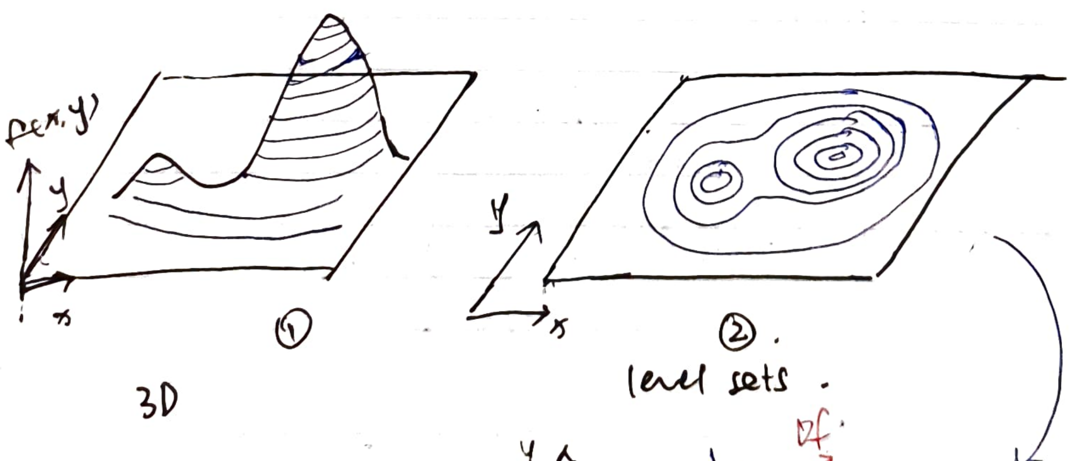


Analog for 2D surface: tangent surface is the best local approx of 2D surface.

climbing up the food gradient → at the source (global optimum)



Two equ. ways of visualizing the food func:

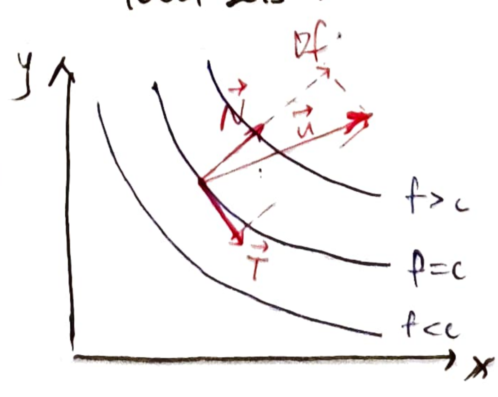


① 3D

② level sets. zoom in

$\vec{u} = \vec{u}_{\parallel} + \vec{u}_{\perp}$

tangent normal



Gradient ascent* algorithm :

- <u>Initialization</u>: Start at point $p^0 = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \in$ domain of f.
- <u>Back propagation</u> update rule from p^n to p^{n+1}.

$$p^n = \begin{pmatrix} x_n \\ y_n \end{pmatrix}, \quad \Delta P := \nabla f(x, y).$$

$$\Rightarrow p^{n+1} = p^n + h \Delta p^n.$$

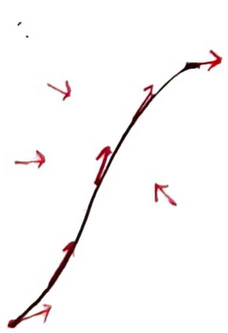
$$= \begin{pmatrix} x_n \\ y_n \end{pmatrix} + h \nabla f \begin{pmatrix} x_n \\ y_n \end{pmatrix}.$$

- <u>Convergence</u>: Stop when $\| p^{n+1} - p^n \| \leq \varepsilon \approx 0$.

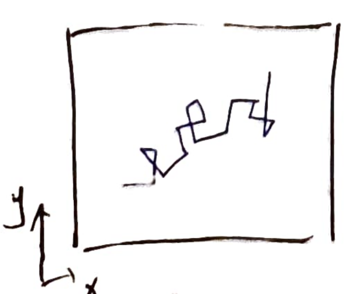
* : "ascent" here because we want to <u>maximize</u> the food func.
In ML, gradient descent because we want to <u>minimize</u> the loss func.

- <u>vector field</u>:

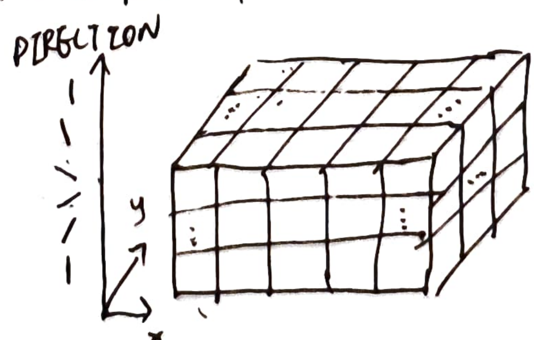
We have an integral trajectory $\gamma(t)$ of a vector field $a(x)$ iff the velocity vector $\frac{d\gamma(t)}{dt} = a(\gamma(t))$



- How to represent all possible paths of movements of A. coli?



path



(T, N) coordinate frame.