Our Goals
The Method
The Results
References

Manifold Structure of High-Dimensional Data in Artificial and Biological Neural Networks

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Our goals (neurobiological): modeling the visual system

- Primary visual cortex (V1)

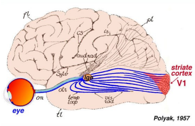


Figure 1: Visual input goes from the eye to primary visual cortex (V1). (Adapted from Polyak (1957))

- How is the structure of neurobiological networks in V1 similar/different from artificial neural networks (ANNs)?
- In the end, the goal is to come up with more accurate models of the visual system.

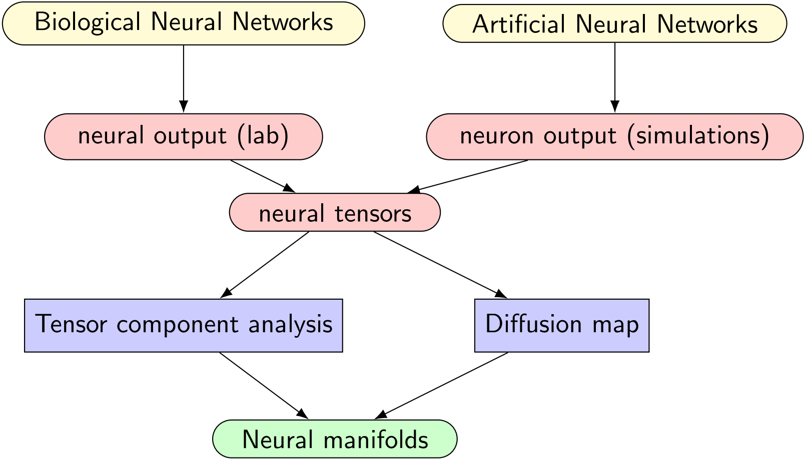
Our goals (computational/mathematical): learning the neural manifolds

- *Neural manifolds*: (informal) clusters of neurons grouped by their firing patterns in response to a given visual stimulus.
- Compare the neural manifolds for biological neural networks vs artificial neural networks:
  CNN, RNN and the recent Transformer and Perceiver networks.

Significance

- Artificial neural networks have proved capable of many vision tasks at a level competitive to biological systems.
- However, whether artificial and biological neural networks use the same computational strategies **remains an open question.** [3]
- Modeling the visual system is an important task:
  - for **neuroscience**: scientific understanding of the brain
  - for **AI**: reverse-engineering, e.g., LeCun, Bengio, & Hinton: "ConvNets have their roots in the neocognitron" which was one of the earliest computational models of the visual system. [5]

Summary of the method



Data: from lab experiments

- Visual stimuli of artificial gratings are flashed in front of the mouse.
- Each visual stimuli are shifting in 8 directions over time.
- Neuron output is recorded with electrodes and encoded in peristimulus (PSTH) diagrams.
- Each PSTH diagram shows the firing rate of one neuron over time for 8 directions.

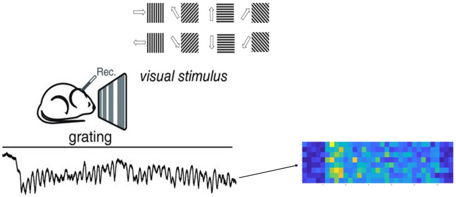


Figure 2: Visualising neural data from lab experiments.

Data: from computational simulations

- Input natural images of different objects (e.g. cars, cats, and dogs) to Artificial Neural Networks (ANNs).
- Compute output (numerical values) from neuron units.

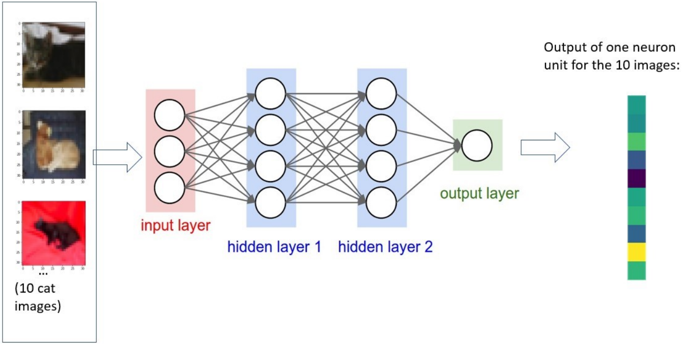


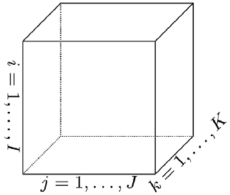
Figure 3: Computing output of one neuron unit in the ANNs given 10 input cat images.

Tensors

Definition (Tensors)

An N-way tensor is an element of the tensor product of N vector spaces.

- *1-way tensor = vector,* $v = [v_1 \quad v_2 \quad \dots \quad v_n]^T$.
- *2-way tensor = matrix,* $A = \begin{pmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ A_{m1} & A_{m2} & \dots & A_{mn} \end{pmatrix}$.
- *3-way tensor of dimension I-by-J-by-K:*



Neural tensors

Definition (Neural population response)

Suppose $\mathcal{S}$ is a set of S visual stimuli (e.g., images) $\mathcal{S} = \{s_1, s_2, \ldots, s_S\}$, each having T number of transformations (translations or rotations). The neural population response of a set of N neurons to a stimulus m_i over all transformations is $\mathcal{N} = \{\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N\}$, where $\vec{n}_i \in \mathbb{R}^T$.

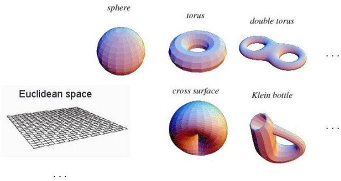
Definition (Neural tensors)

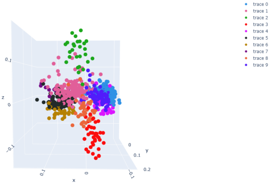
Each neural tensor encodes the neural population response of a set of neurons to a set of stimulus over all transformations. It is thus a 3-way tensor of dimension N-by-S-by-T.

Neural manifolds

Definition (Manifold)

A <u>manifold</u> is a topological space that “locally” resembles Euclidean space.





Definition (Neural manifolds)

Clusters of neurons grouped by their firing patterns in response to a given visual stimulus. The distance defined on the neural manifolds indicate how similarly the neurons respond.

Dimensionaliy Reduction

Assumption (The Manifold Hypothesis)

Real-world high-dimensional data lie on low-dimensional manifolds embedded within the high-dimensional space. [1]

- **Image data** are high-dimensional (the number of pixels), but can be reparameterized with much smaller number of variables (feature extraction).
- **Neural data** are high-dimensional, but the neural connections constrain the possible neural firing patterns to a low-dimensional manifold spanned by a few independent patterns. [2]

Linear dimensionality reduction: tensor component analysis + k-means clustering

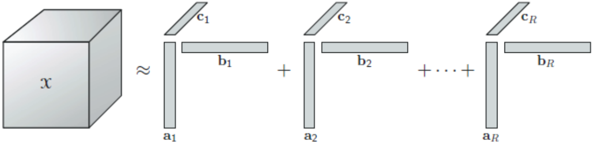


Figure 4: Intuition for tensor component analysis. (Adapted from [4].)

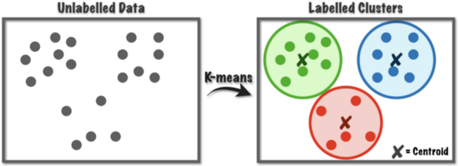


Figure 5: Intuition for k-means clustering.

Results: first five principal components from biological neural tensor

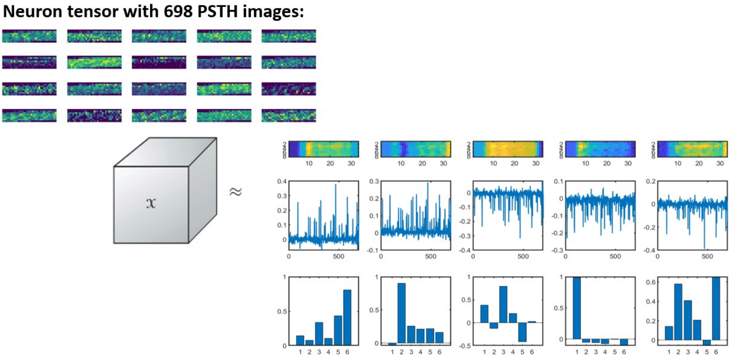


Figure 6: First 5 tensor factors for neural data.

Results: visualize neural manifolds from biological neural tensor

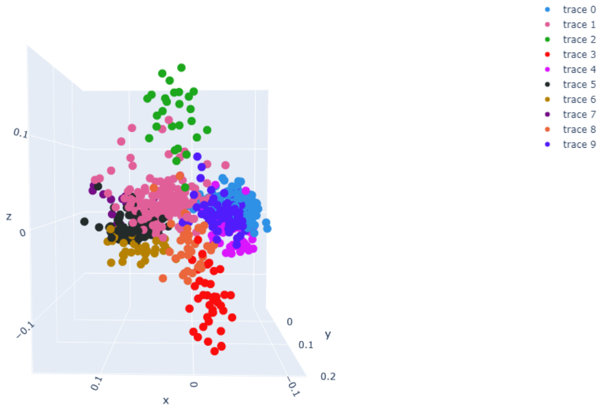
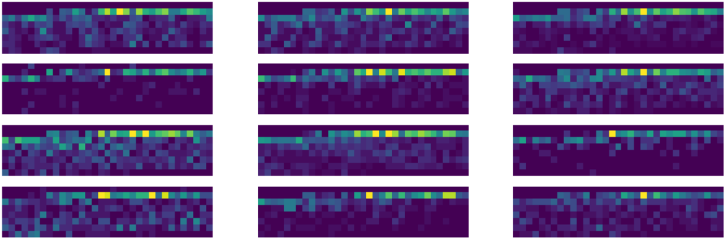
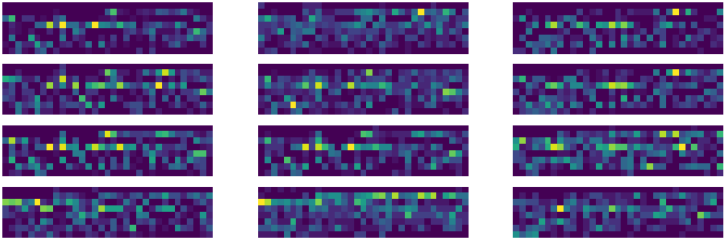


Figure 7: Neural manifolds: clusters of neurons grouped by firing patterns, each point represent a neuron.

Results: look inside the neural manifolds



(a) PSTH diagrams showing responses of neurons within some arbitrary cluster to stimuli of type 1.



(b) PSTH diagrams showing responses of neurons within a different cluster to stimuli of type 1.

Results: first five principal components from artificial neural tensor

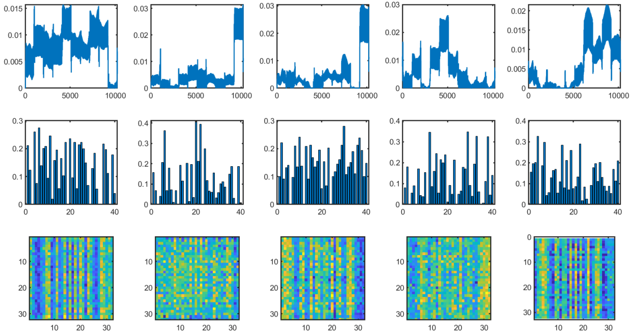


Figure 9: First 5 tensor factors for artificial neural data.

Results: visualize neural manifolds from artificial neural tensor

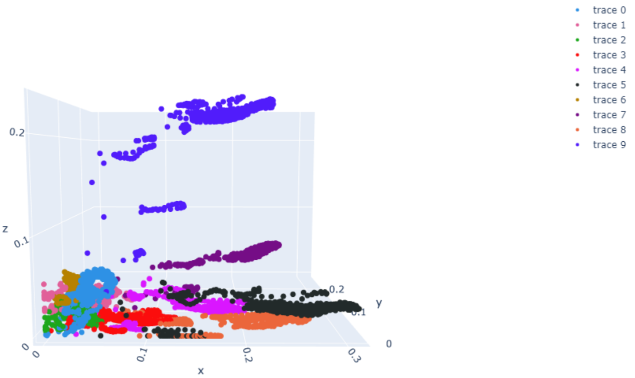


Figure 10: Neural manifolds.

Non-linear dimensionality reduction: diffusion map

Applying diffusion map to synthetic spiral data and MNIST handwritten digits images data:

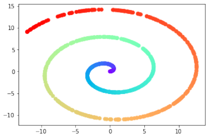


Figure 11: Visualising the original spiral data.

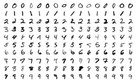


Figure 13: Sample data from the MNIST.

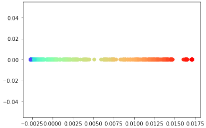


Figure 12: First non-trivial coordinate function.

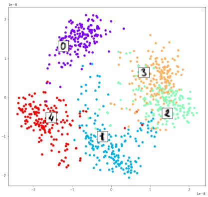
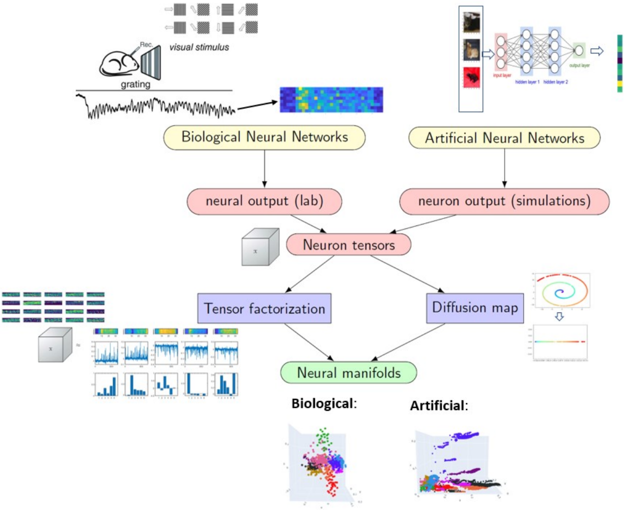


Figure 14: First two non-trivial coordinate functions.

Putting it all together and next steps:



Yale NUS College

Bonus Slide:)

Look for the bare necessities
The simple bare necessities
Forget about your worries and your strife
I mean the bare necessities
Old Mother Nature's recipes
That bring the bare necessities of life

— Baloo's song [The Jungle Book]

Acknowledgements

I would like to thank my supervisor Prof. Francesca Spagnuolo and co-supervisor and mentor Prof. Steven W. Zucker and Dr. Luciano Dyballa for their generous guidance and advice. They have made possible the many serendipitous moments in this project.

Artificial neural tensors

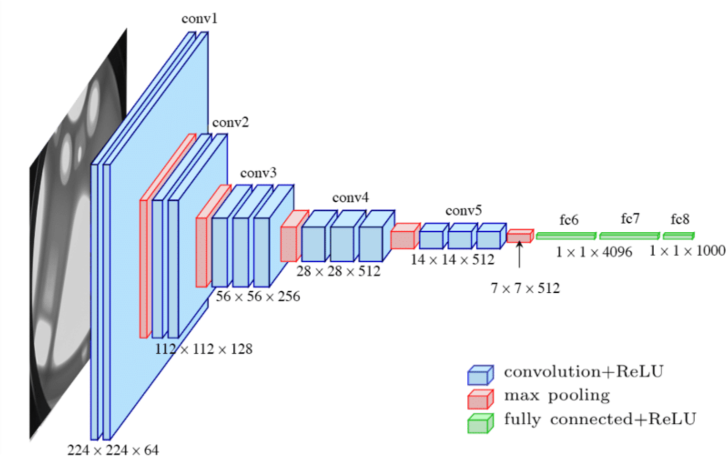


Figure 15: VGG-16

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