## Homework 4 Extension: Tensor Decomposition for Faces Data

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## 1 Normalization by dividing the max

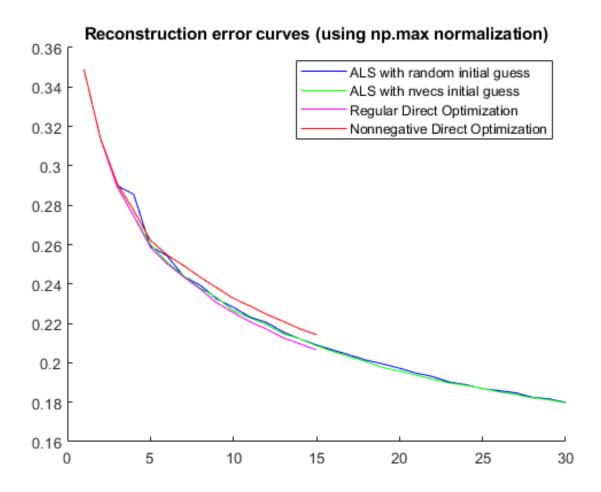


Figure 1: Comparing the reconstruction error of different tensor decomposition methods.

From the graph above, we observe that after 10-15 factors, the error no longer decreases as fast as it did earlier. Thus the optimal number of factors to decompose the original tensor into is approximately 10-15.

Further, we observe that the initially the ALS algorithm performs better when using the leading mode-n singular vectors as the initial guess (using the nvecs option). After the number of factors increases to around 10, using the random initial guess and the nvecs option yields approximately the same performance.

Another observation is that the direct optimization method performs consistently better without the non-negativity constraint. However, non-negative direct optimization provides faster computation, which means we need to make some trade-off between the computational time and the performance.

We now visualize the first five factors obtained from the different methods as images:

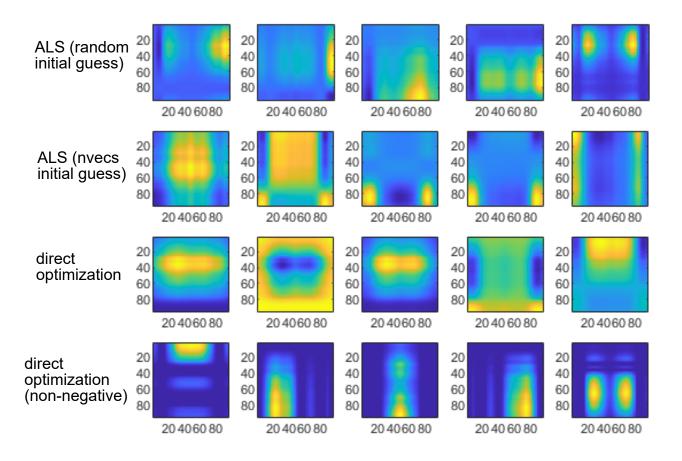


Figure 2: Visualizing the factors.

## 2 Normalization by dividing the norm

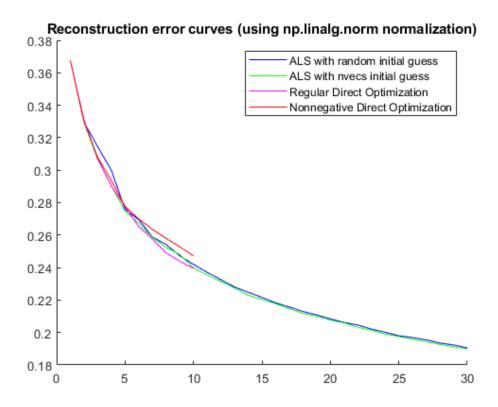


Figure 3: Comparing the reconstruction error of different tensor decomposition methods.

The same observations under the max normalization still hold, with some additional remarks:

- (i) Normalizing by dividing the maximum yields the best result with regular direct optimization (around 0.21 error rate as compared to around 0.24 error rate).
- (ii) When normalizing the data by dividing the norm throughout, the ALS with random initial guess has more stable changes as the factors increases at the beginning.

As with the previous section, we visualize the factors as images and compare these images with the previous method:

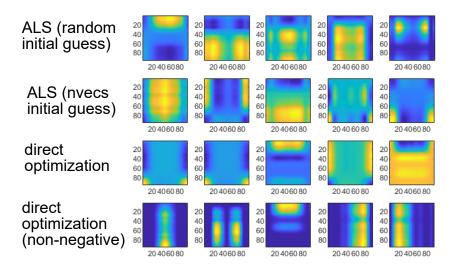


Figure 4: Visualizing the factors: normalization by dividing the max

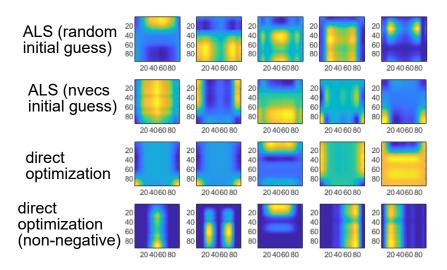


Figure 5: Visualizing the factors: normalization by dividing the norm