Week 2's Exercises Geometric Algorithms — Spring 2025 Computer Science Department VU Amsterdam, 1081HV Amsterdam

April 08, 2025

This text contains 11 exercises covering lectures 3 and 4, that is, 2D convex hulls and line-segment intersection. Here, BCKO refers to the following textbook: Computational Geometry: Theory and Applications (3rd edition), and written by Mark de Berg, Otfried Cheong, Marc van Kreveld, and Mark Overmars.

Note. The answers to these exercises will not be published on Canvas. Instead, we will discuss the answers to these exercises during our exercise class this Friday, April 11.

EXERCISE 1. Prove that the intersection of two convex sets in the plane is convex. ¹

EXERCISE 2. Prove that the smallest-perimeter polygon \mathcal{P} containing a set P of points in the plane is convex. ²

EXERCISE 3. Let E be an unsorted set of n segments that are the edges of a convex polygon. Describe an $O(n \log n)$ -time algorithm that computes from E a list containing all vertices of the polygon, sorted in clockwise order.

EXERCISE 4. Let S be a set of n line segments in the plane. Prove that the convex hull of S is exactly the same as the convex hull of the 2n endpoints of the segments. 4

EXERCISE 5. Solve parts a and d of Exercise 1.7 in BCKO.

EXERCISE 6. Design a divide-and-conquer algorithm for computing the convex hull of any given set of n points in the plane. Do not forget to analyze the running time of your algorithm. ⁵

EXERCISE 7. Solve Exercise 2.1 in BCKO.

EXERCISE 8. Solve Exercise 2.2 in BCKO.

EXERCISE 9. Solve Exercise 2.11 in BCKO.

EXERCISE 10. Solve Exercise 2.14 in BCKO.

¹This is Exercise 1.1(a) in BCKO.

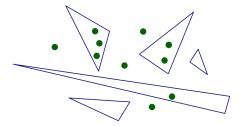
²This is Exercise 1.1(b) in BCKO.

³This is Exercise 1.3 in BCKO and also partially discussed during lecture 3.

⁴This is Exercise 1.6(a) in BCKO.

⁵This is Exercise 1.8 in BCKO.

EXERCISE 11. Let S be a set of n disjoint triangles in the plane, and let P be a set of m points in the plane. Design an efficient algorithm to decide, for each point p of P, which triangles from S contains p, if any. What is the running time of your algorithm? ⁶



 $^{^6}$ This exercise is similar to exercises 2.10 and 2.12 of BCKO.