Graph Optimization
Lab session 6 - exercise

Consider a network design problem defined over a directed graph $G = (N, A)$. A set of unsplittable demands K must be routed on the graph. Each demand is described by three parameters, a source node $S_k \in N$, a destination node $D_k \in N$, and an amount of flow F_k that must be sent from the source node to the destination node. The subset of active arcs must be selected: if the arc (i, j) is activated it provides a capacity λ at a cost c_{ij}. The problem consists in deciding the routing of each demand and the subset of arcs to activate with the aim of minimizing the sum of the costs.

1. Write the arc-based formulation of the problem and its continuous relaxation.

2. Consider the instances described in files `networkDesign_1.dat, networkDesign_2.dat` and `networkDesign_3.dat`. Solve the integer problem and the continuous relaxation (LB), compute the percentage gap and fill the table assigning two different values to λ: 50 and 10 for `networkDesign_1.dat` and `networkDesign_2.dat` and 500 and 100 for the last instance.

instance	λ	integer optimum	LB	gap
`networkDesign_1.dat`	50			
`networkDesign_1.dat`	10			
`networkDesign_2.dat`	50			
`networkDesign_2.dat`	10			
`networkDesign_3.dat`	500			
`networkDesign_3.dat`	100			

   Does the quality of the bound vary for the different values of λ?

3. Add to the formulation all the *single-node* cutset based inequalities, i.e. the cutset based inequalities associated with a partition of the nodes where one subset contains a single node, and compute the corresponding continuous relaxation (SN LB).

4. Add to the formulation all the *two-nodes* cutset based inequalities, i.e. the cutset based inequalities associated with a partition of the nodes where one subset contains two nodes, and compute the corresponding continuous relaxation (TN LB).

5. Add to the formulation all the *single-node* and *two-nodes* cutset based inequalities and compute the corresponding continuous relaxation (STN LB). Fill the table, computing the percentage gaps.

instance	λ	SN LB	gap	TN LB	gap	STN LB	gap
`networkDesign_1.dat`	50						
`networkDesign_1.dat`	10						
`networkDesign_2.dat`	50						
`networkDesign_2.dat`	10						
`networkDesign_3.dat`	500						
`networkDesign_3.dat`	100						