Graph Optimization
Lab session 3

Exercize 1. In wireless networks, the main installation costs are given by the devices that must be installed at the relay nodes. A basic optimization problem in wireless network design is known as *Minimum Relay Problem* (MRP).

Let us consider an undirected graph $G = (N, E)$, where each node represents a possible relay location, and each edge represents a communication link that must be served/covered by the wireless network. The cost c_i of installing a relay at location $i \in N$ is given. The *Minimum Relay Problem* consists in selecting a set of locations with overall minimum installation cost in such a way that every communication link is covered by at least one installed relay.

The problem can be modeled as follows:

$$\min \sum_{i \in N} c_i x_i \tag{1}$$

$$x_i + x_j \geq 1, \quad \forall e = \{i, j\} \in E \tag{2}$$

$$x_i \in \{0, 1\} \tag{3}$$

A feasible solution can be computed by applying a ***greedy*** procedure, such as:

1. greedy 1 chooses the node that covers the highest number of uncovered edges
2. greedy 2 chooses the node that provides the minimum ratio cost-number of covered edges among those uncovered so far
3. greedy 3 chooses the minimum cost node

A feasible solution of the problem can be computed by applying a ***randomized rounding*** procedure:

- Until the set of edges E is entirely covered by $C \subseteq N$ (that is every edge has at least one endpoint in C):

1. Solve the Linear Programming relaxation of the problem and obtain the fractional solution x^*.
2. Build a set of vertices C as follows:
   (a) Put in C all vertices $i \in N$ with $x_i^* = 1$
   (b) Select randomly a vertex v with $0 < x_v^* < 1$, and add v to C with probability x_v^* (generate a random parameter $0 \leq r \leq 1$ and add v if $r \leq x_v^*$).

Compare the approaches on the five instances.