Graph Optimization
Lab session 2 - exercise

Exercise 1. Consider the bin packing problem. A set of N items and a set of N bins, each with capacity B, are given. The weight w_i of each item $i \in \{1, \ldots, N\}$ is given, as well. The problem is to assign each item to a bin, while guaranteeing that the sum of the weights of the items assigned to each bin does not exceed the bin capacity. The aim of the problem is to minimize the number of used bins.

Let $\mathcal{S}$ be the set of subsets of items whose sum of weights does not exceed the bin capacity. Consider the following formulation of the problem:

$$\min \sum_{s \in \mathcal{S}} \lambda_s \tag{1}$$

$$\sum_{s | i \in s} \lambda_s = 1 \qquad \forall i = 1, \ldots, N \tag{2}$$

$$\lambda_s \in \{0, 1\} \qquad \forall s \tag{3}$$

where

$$\lambda_s = \begin{cases} 1 & \text{if the subset } s \text{ is assigned to a bin in the solution} \\ 0 & \text{otherwise.} \end{cases}$$

The continuous relaxation of the problem, where $\lambda_i \geq 0$, can be solved through column generation. The dual constraint associated with each primal λ_s variable is :

$$\sum_{i \in s} \pi_i \leq 1, \forall s \in \mathcal{S}$$

where π_i is the dual variable associated with (2). The pricing problem can be formulated as:

$$\max \sum_{i=1}^{N} \pi_i u_i$$

$$\sum_{i=1}^{N} w_i u_i \leq B$$

$$u_i \in \{0, 1\}, \forall i = 1, \ldots, N$$

1. Download files `BinPacking_CG.mod` and `BinPacking_CG_scheme.run`. Starting from the scheme, write a script file implementing the column generation procedure.

2. Dowload the five bin packing instances (`instanceBP_*.dat`) and compute their continuous relaxation through column generation, filling the first two columns of the table below. As initial solution assign each item to a different bin.

instance	continuous relaxation(CG)	# columns	continuous relaxation
instanceBP_1			
instanceBP_2			
instanceBP_3			
instanceBP_4			
instanceBP_5			

3. Consider the following formulation of the bin packing problem with a polynomial number of variables and write it in a .mod file to solve its continuous relaxation and fill the third column of the table:

$$
\begin{aligned}
\min \sum_{j=1}^{N} y_j & \\
\sum_{j=1}^{N} x_{ij} = 1, & \qquad \forall i = 1, \dots, N \\
\sum_{i=1}^{N} w_i x_{ij} \leq B y_j, & \qquad \forall j = 1, \dots, N \\
y_j \in \{0, 1\}, & \qquad \forall i = 1, \dots, N \\
x_{ij} \in \{0, 1\}, & \qquad \forall i = 1, \dots, N, j = 1, \dots, N
\end{aligned}
$$

4. Compare the solutions provided by the column generation and by the relaxation of the model in 3. Is the column generation providing a better bound? Why?