# Graph Optimization
# Lab session 5

**Exercize 1.** Consider the following problem. A set of items $I = 1 \dots n$ is given. For each item $i \in I$ profit $p_i$, weight $w_i$, volume $q_i$ and cost $c_i$ are given. Two different capacities, a maximum weight $B_1$ and a maximum volume $B_2$, are given together with a budget $B_3$. A subset of items must be selected such that their total profit is maximum and the three capacity constraints, limiting total weight, total volume and total cost, are satisfied.

1. Write in AMPL a procedure to compute the Lagrangian relaxation obtained relaxing all the three capacity constraints with multipliers $\mu$:
   - use the normalized subgradient $\left(\mu^{k+1} = \max\left\{0, \mu^k - t_k \frac{g^k}{||g^k||}\right\}\right)$;
   - the step at iteration $k$ $t_k$ is equal to $0.995 t_{k-1}$;
   - stop the procedure after 10 iterations;
   - save the best upper bound obtained as $UB_{LR}$;
   - use the feasible solutions found by the Lagrangian relaxation as lower bounds $LB_{LR}$ of the problem;
   - apply the Lagrangian relaxation procedure to instances `exTest12.dat` and `exTest123.dat` and fill the Table 1.

| instance | $LB_{LR}$ | $UB_{LR}$ | time | $LB_{SR}$ | $UB_{SR}$ | time |
|---|---|---|---|---|---|---|
| exTest12 | | | | | | |
| exTest123 | | | | | | |

Table 1: Relaxation comparison

2. Write in AMPL a procedure to compute the surrogate relaxation obtained combining the three capacity constraints with multipliers $m$:
   - update multipliers $m$ as follows: $\left(m^{k+1} = \max\left\{0, m^k - t_k g^k\right\}\right)$;
   - the step at iteration $k$ $t_k$ is equal to $0.995 t_{k-1}$;
   - stop the procedure after 10 iterations;
   - save the best upper bound obtained as $UB_{SR}$;
   - use the feasible solutions found by the surrogate relaxation as lower bounds $LB_{SR}$ of the problem;
   - apply the surrogate relaxation to instances `exTest12.dat` and `exTest123.dat` and fill the table.
3. Write in AMPL a greedy procedure for the problem, apply it to instances `exTest12.dat` and `exTest123.dat` and fill the table.
4. Write in AMPL a k-opt neighborhood procedure for the problem, apply it to instances `exTest12.dat` and `exTest123.dat` and fill the Table 1.

| instance | greedy | time | $k-$ opt | time | iterations |
|---|---|---|---|---|---|
| exTest12 | | | | | |
| exTest123 | | | | | |

Table 2: Heuristic comparison