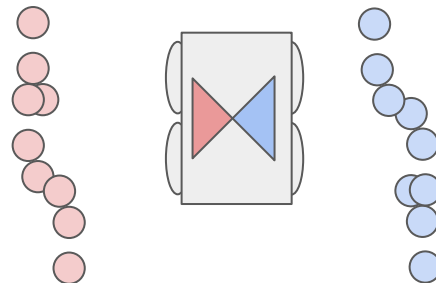
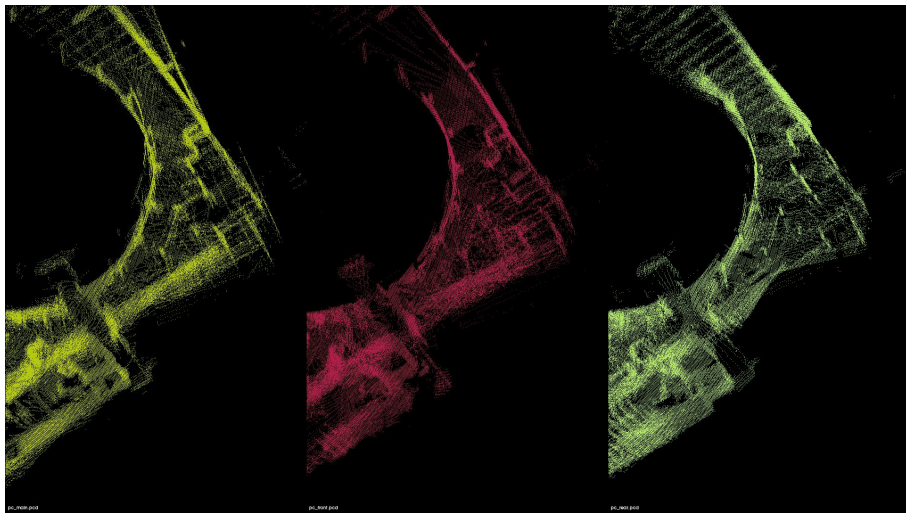


# DARPA Sub-T: Lidar-Lidar Calibration

Dakota Wenberg, Aaron Ray, Ryan Sander  
VNAV Final Project

# Problem Overview

- Build a consistent global point cloud
- If individual scans do not share keypoints, must **find lidar-lidar transform**



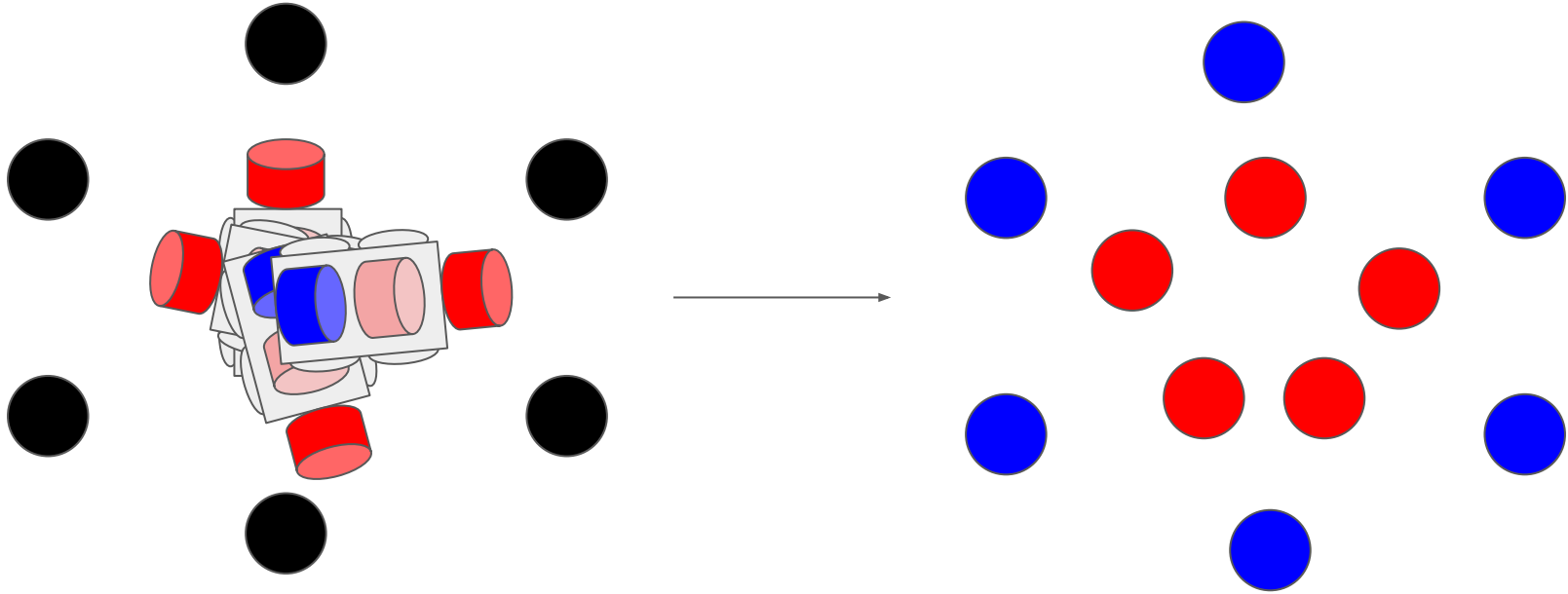
Clouds for each lidar may be disjoint

# Preview

- How do we calibrate the Lidars? **Hand-Eye Calibration**
- **Solving** the Hand-Eye Calibration problem
- **Evaluating** calibration quality
- Next steps

# Bad calibration example

- What happens to the global point cloud if the calibration is off?
  - Not just a rigid transformation!

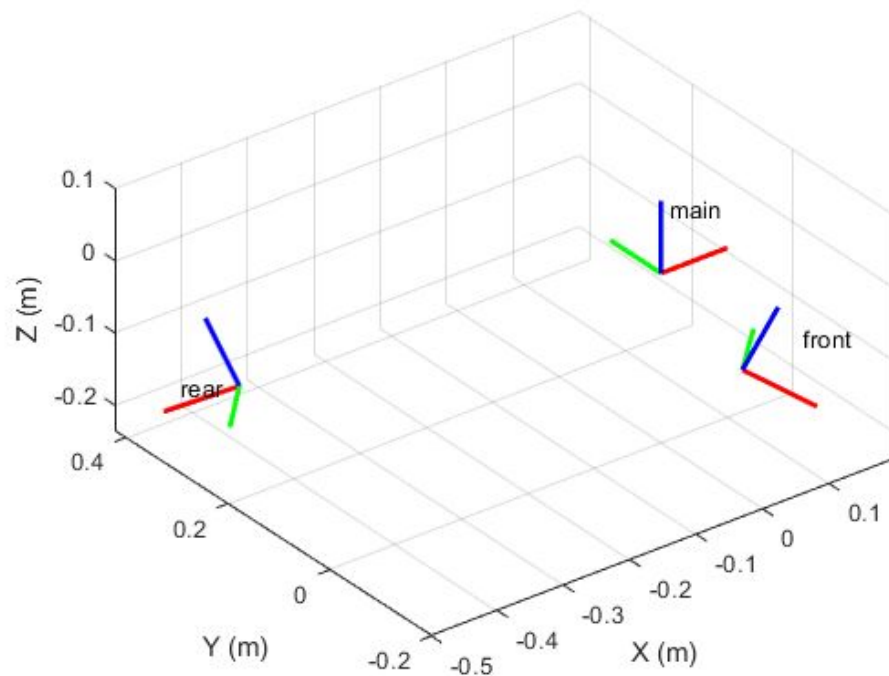
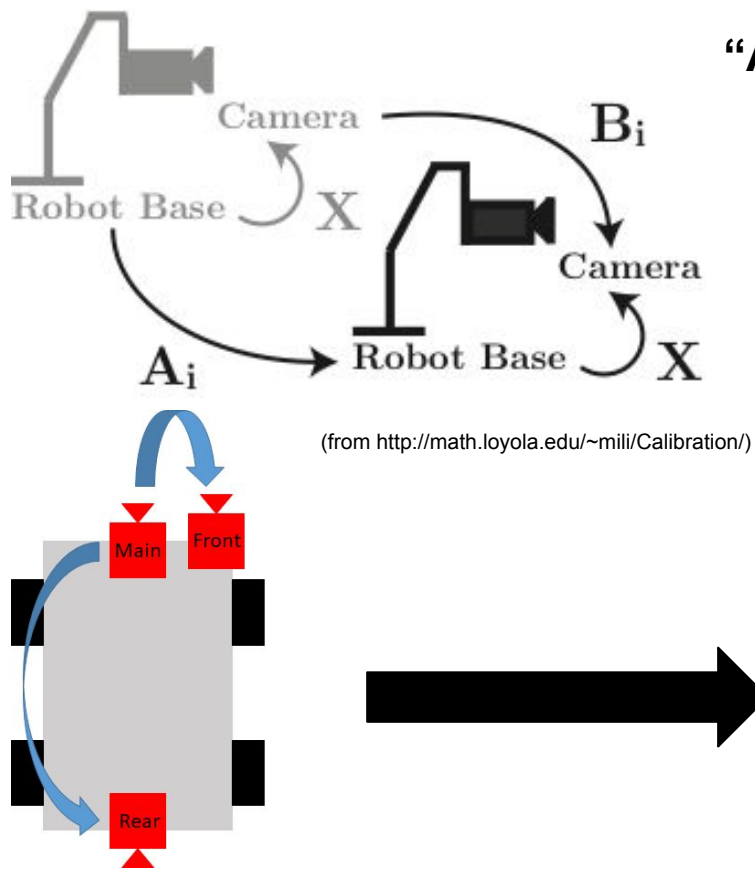


Robot thinks red lidar is at  but really it is at 

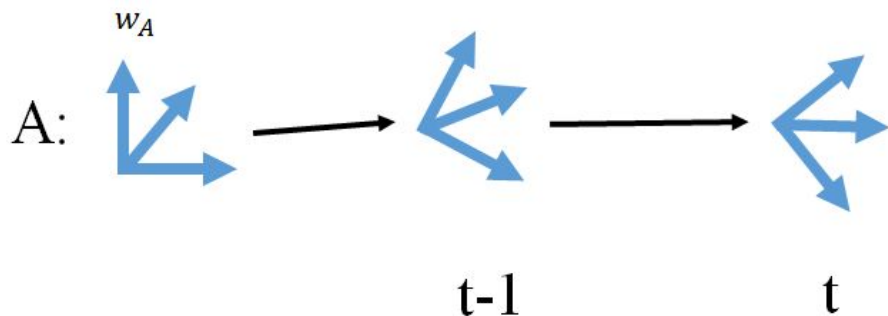
Resulting point clouds don't match

# Our Hand-Eye Setup

$$AX = XB$$



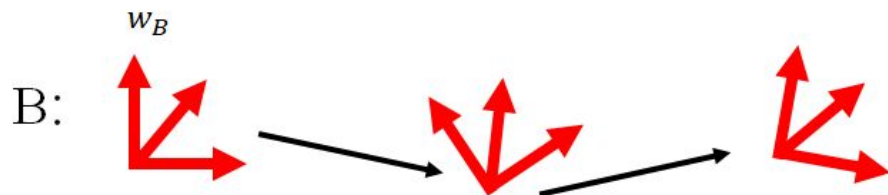
# Hand-Eye Calibration



- Given: Two arbitrary trajectories

$$T_{A_t}^{A_{t-1}} = \left( T_{A_{t-1}}^{w_A} \right)^{-1} T_{A_t}^{w_A}$$

- Same for B



$$T_{B_t}^{B_{t-1}} = T_A^B T_{A_t}^{A_{t-1}} T_B^A$$

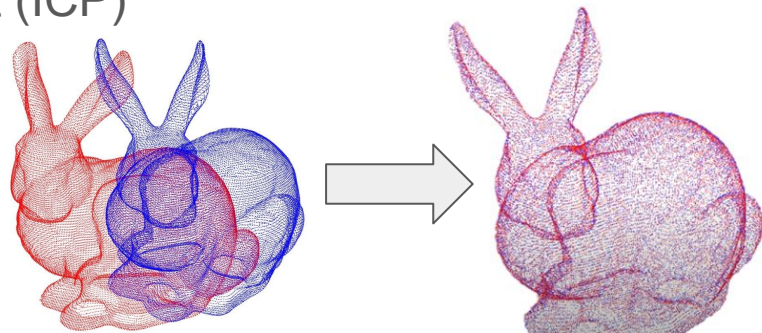
$$T_A^B = \left( T_B^A \right)^{-1}$$

$$\boxed{T_{B_t}^{B_{t-1}}} = \boxed{T_A^B} \boxed{T_{A_t}^{A_{t-1}}} \boxed{(T_A^B)^{-1}}$$

 -Known  -Unknown

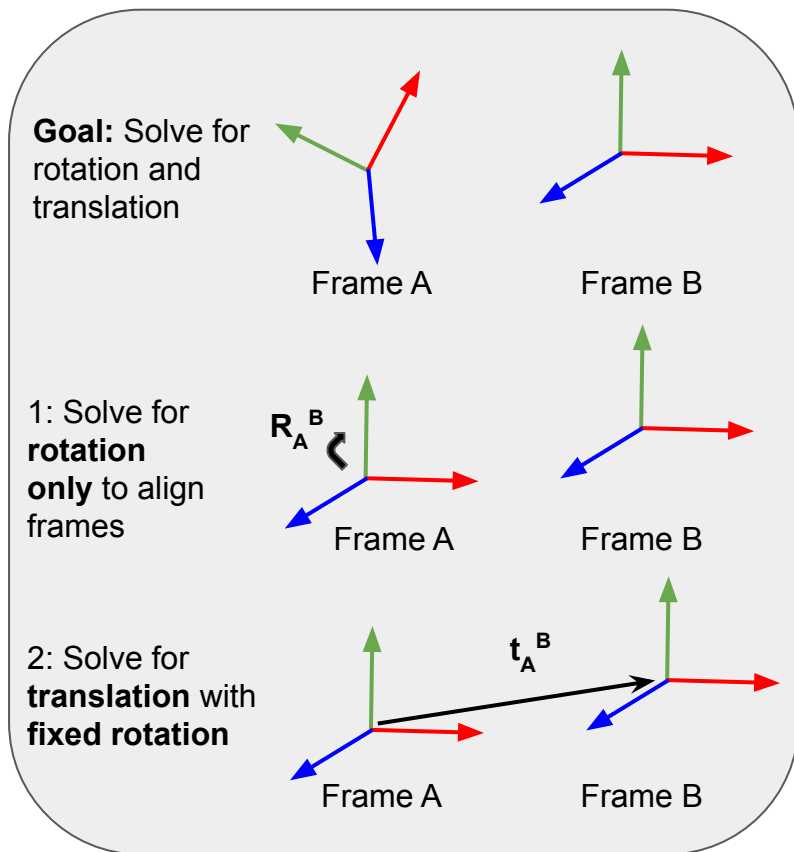
# ICP For Motion Estimation

- $AX = XB$  assumes we have measurements of motion (A and B). In practice, how do we get this? → Iterative Closest Point (ICP)
- Two point clouds are “best matched”
  - Point to point
- Rotation and translation is estimated
  - Minimizing point to point RMSE
- From this we can compute covariance
- **How do we use ICP?** We use the **variance** (diagonal) elements of the 6 x 6 ICP covariance matrix to eliminate certain data points whose variance exceeds a threshold amount.



# Closed Form Solution-Based Methods

- Problem solved in closed form in two stages, as in Shiu and Ahmad [1] and Tsai and Lenz [2].
- **Advantages of these approaches:**
  - Does not require an iterative optimization procedure.
  - Closed-form solution may be easier to interpret intuitively.
- **Disadvantages of this approach:**
  - Many of these approaches fix rotation to solve for translation. This leads to errors in **rotation** affecting **errors** in translation.





# Optimization-Based Solutions

- Ransac- esque sample consensus to find initial guess
- Nonlinear optimization to refine estimate
  - In our implementation, we focus on the local nonlinear optimization, because we have a good initial guess
- These optimization problems can be carried out using nonlinear techniques such as **Gauss-Newton** or **Levenberg-Marquadt**.

# Unweighted Pose Estimation

- **Idea:** Minimize the **unweighted** sum of squared errors between our **pose estimate** and the “**round-trip**” (**chained**) **pose estimate**:

$$\begin{aligned}\hat{\mathbf{T}}_A^B &= \arg \min_{\mathbf{T}_A^B \in \mathbf{SE}(3)} \frac{1}{N} \sum_{i=1}^N \|\mathbf{B}_i \mathbf{T}_A^B \mathbf{A}_i^{-1} - \mathbf{T}_A^B\|_F^2 \\ &= \arg \min_{\mathbf{T}_A^B \in \mathbf{SE}(3)} \frac{1}{N} \sum_{i=1}^N \text{tr}(\mathbf{M}_i \mathbf{M}_i^T), \quad \mathbf{M}_i = \mathbf{B}_i \mathbf{T}_A^B \mathbf{A}_i^{-1} - \mathbf{T}_A^B\end{aligned}$$

# Weighted Relative Pose Estimation (MLE Estimate)

- **Idea:** Some samples have more uncertainty than others in **rotation** and **translation**. To account for this uncertainty, we can assign less weight to our uncertain estimates using **rotation** and translation **variance estimates**.

$$\begin{aligned}\hat{\mathbf{T}}_A^B &= \arg \min_{\mathbf{T}_A^B \in \mathbf{SE}(3)} \frac{1}{N} \sum_{i=1}^N r_i \|\mathbf{B}_i \mathbf{T}_A^B \mathbf{A}_i^{-1} - \mathbf{T}_A^B\|_{\Omega_i}^2 \\ &= \arg \min_{\mathbf{T}_A^B \in \mathbf{SE}(3)} \frac{1}{N} \sum_{i=1}^N r_i \text{tr}(\mathbf{M}_i \boldsymbol{\Omega}_i \mathbf{M}_i^T),\end{aligned}$$

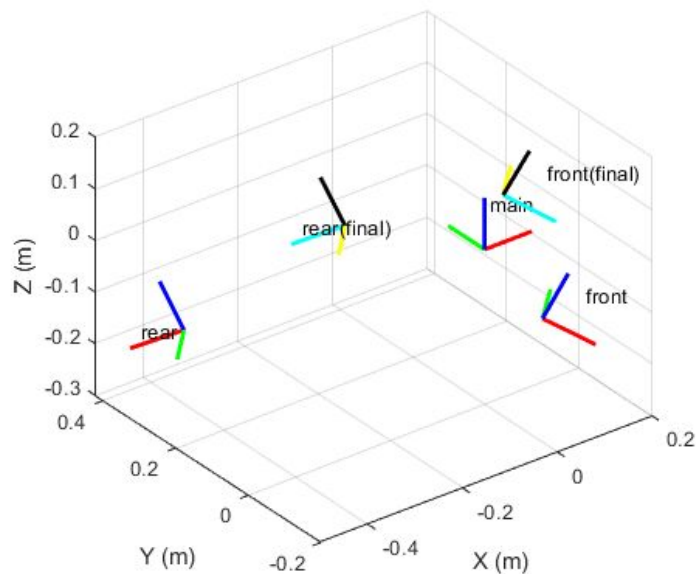
$$\text{Where } \mathbf{M}_i = \mathbf{B}_i \mathbf{T}_A^B \mathbf{A}_i^{-1} - \mathbf{T}_A^B, \quad \boldsymbol{\Omega}_i = \begin{bmatrix} \omega_i \mathbf{I}_3 & \mathbf{0}_3 \\ \mathbf{0}_3^T & \rho_i \end{bmatrix}$$

For some  $\omega_i, \rho_i \in \mathbb{R}, r_i \in \{0, 1\}$

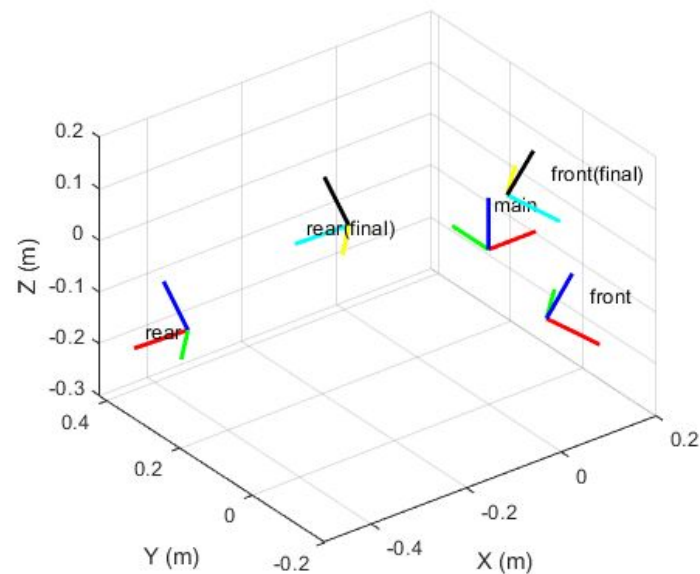
# Dirty Details

- Time aligning
- Covariance
- ICP Covariance
- One of the lidars has occlusion on robot (can we tell numerically that the covariance of these ICP measurements is higher?)

# Unweighted vs Weighted Calibration



Unweighted



Weighted

# Evaluating Calibration Quality

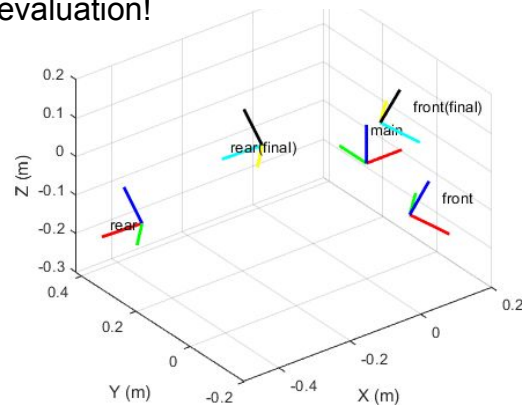
Can look at transform loop consistency over all data:

$$\hat{\mathbf{T}}_A^B = \arg \min_{\mathbf{T}_A^B \in \mathbf{SE}(3)} \frac{1}{N} \sum_{i=1}^N \|\mathbf{B}_i \mathbf{T}_A^B \mathbf{A}_i^{-1} - \mathbf{T}_A^B\|_{\Omega_i}^2$$

$$T_{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^N \|\mathbf{B}_i \hat{\mathbf{T}}_A^B \mathbf{A}_i^{-1} - \hat{\mathbf{T}}_A^B\|_{\Omega_i}^2}$$

Estimate	RMSE <b>R</b> , Weighted	RMSE <b>t</b> , Weighted	RMSE <b>R</b> , Unweighted	RMSE <b>t</b> , Unweighted
Initial ( $\mathbf{T}_0$ )	2.7793e-02	3.6588e-01	2.2885e-02	2.8327e-02
Final ( $\mathbf{T}_F$ )	2.7779e-02	3.6278e-01	2.2873e-02	2.8087e-02

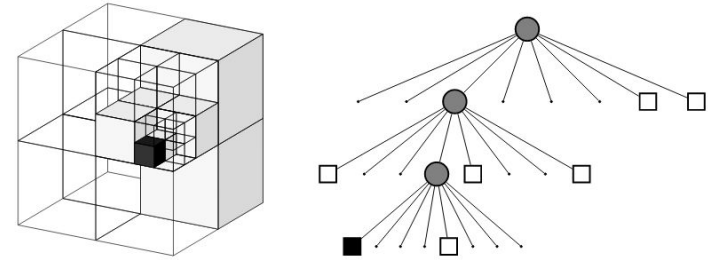
Choices about data weights  
affect optimization and  
evaluation!



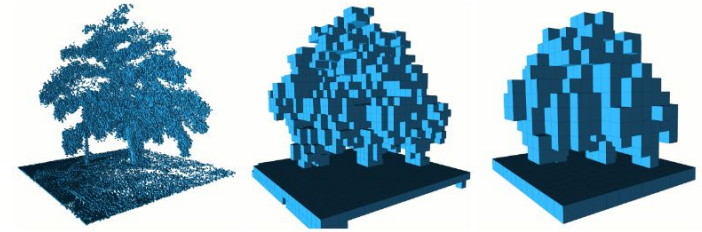
Want an “external” method with separate assumptions from how we optimize

- Evaluate resulting point cloud

# Building Point Clouds



**Fig. 2** Example of an octree storing free (shaded white) and occupied (black) cells. The volumetric model is shown on the left and the corresponding tree representation on the right.

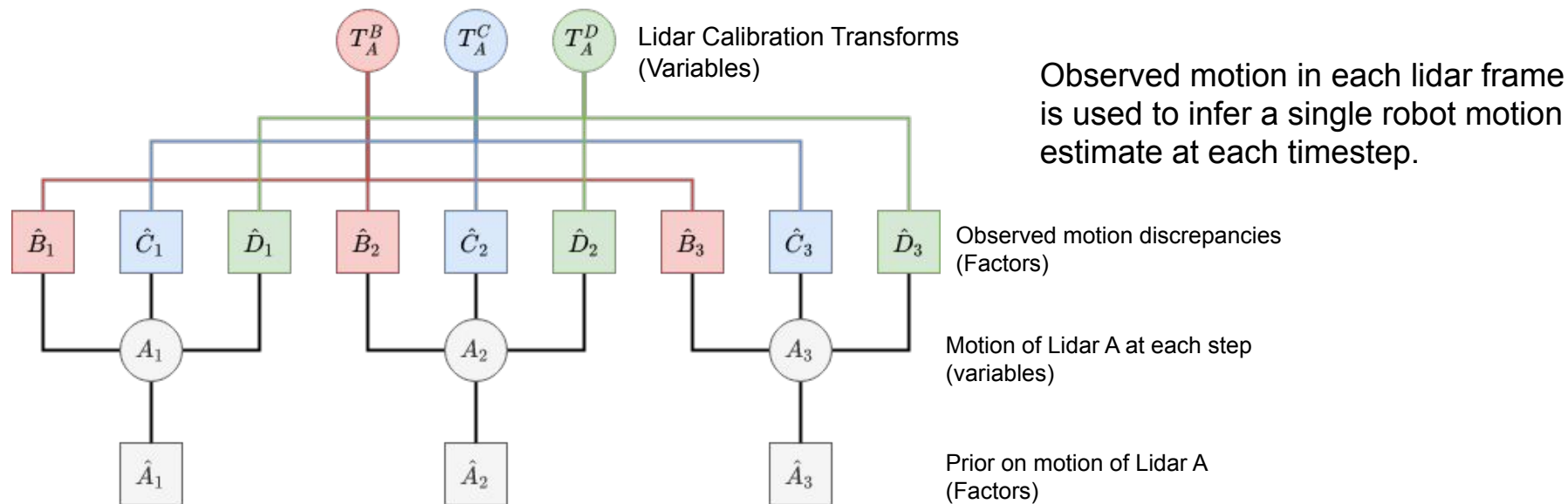


**Fig. 3** By limiting the depth of a query, multiple resolutions of the same map can be obtained at any time. Occupied voxels are displayed in resolutions 0.08 m, 0.64 , and 1.28 m.

(A. Hornung et al, "OctoMap: An Efficient Probabilistic 3D Mapping Framework Based on Octrees" [4])

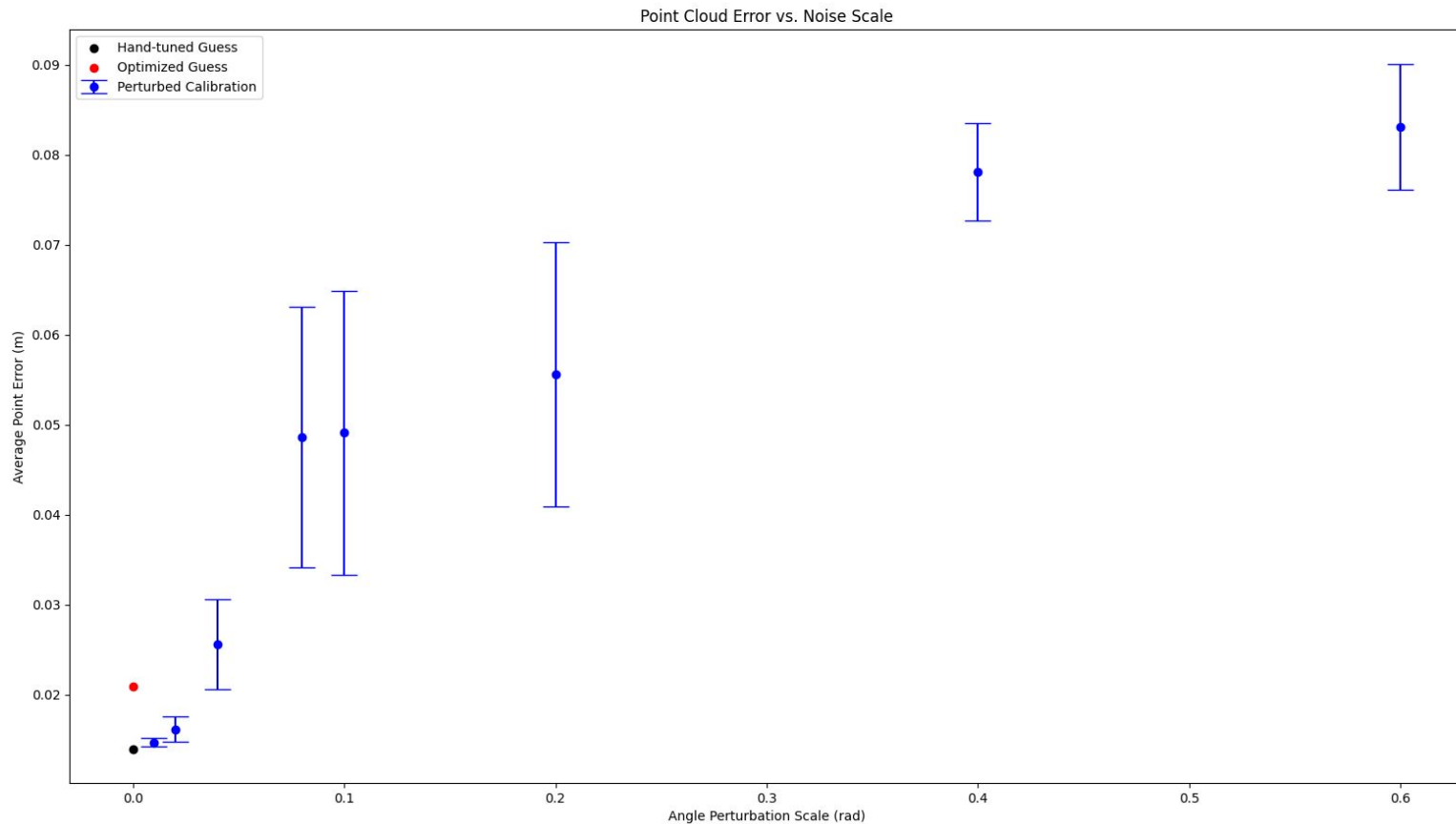
# Improving Accuracy

- The manual calibration performed better on our point cloud metric
- Improve solution quality by **jointly estimating calibration and robot motion**
  - *Evaluation of Combined Time-Offset Estimation and Hand-Eye Calibration on Robotic Datasets*, Furrer, Fehr, Novkovic, Sommer, Gilitschenski, and Siegwart. 2017 [3].



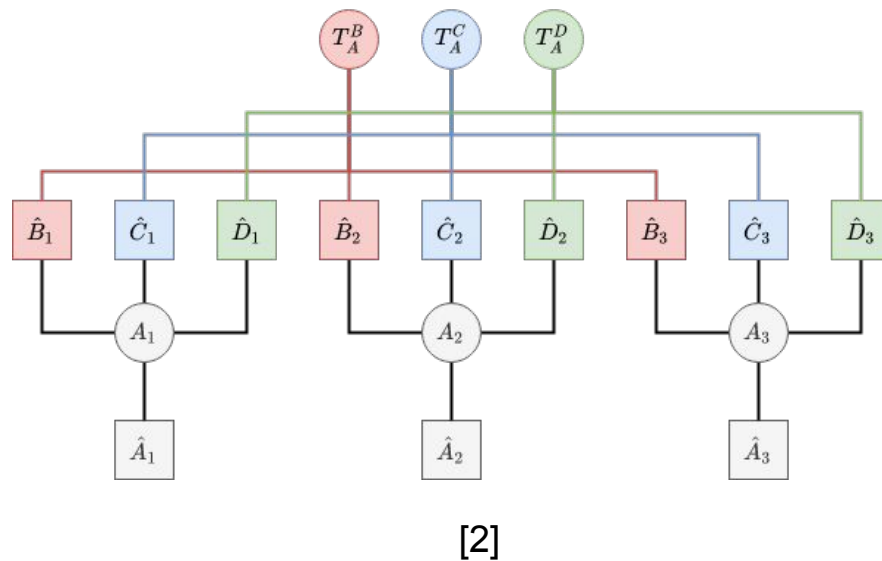
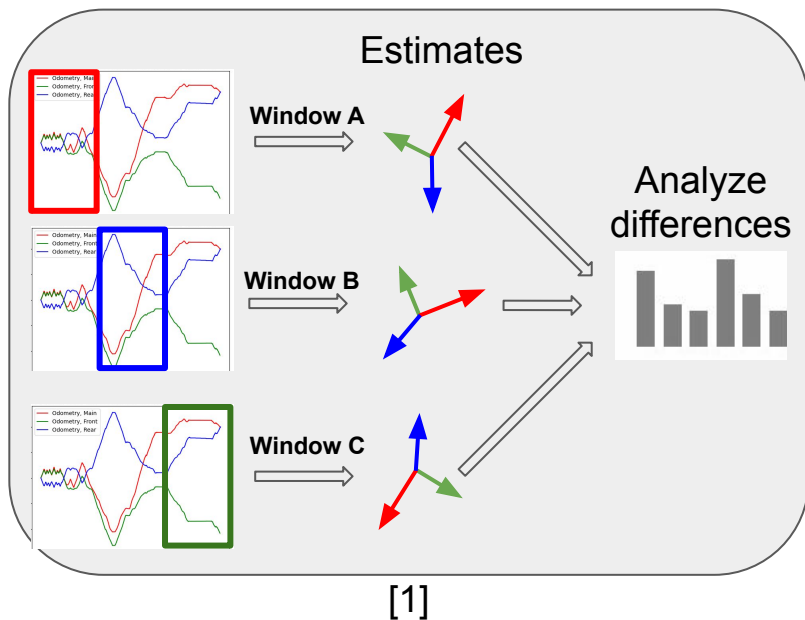


# Point Cloud Accuracy Analysis



# Future Work / Finishing Up

1. **Rolling average pose estimation** to see if transformation changes over time
2. Accuracy improvement through **joint** estimation of **rotation** and **translation**



# Acknowledgements

Thank you to:

1. **Professor Luca Carlone and the VNAV staff** for their meaningful support and mentorship on this project and throughout the semester.
2. **Ben and Fadhil** for their insightful support with understanding our data from the DARPA Sub-T challenge.
3. **Mike and Phil** for collaborating with us on this project, enabling us to build a framework that complements their findings.

# References

- [1] Y. Shiu, S. Ahmad. "Calibration of Wrist-Mounted Robotic Sensors by Solving Homogeneous Transform Equations of the Form  $AX = XB$ ." In IEEE Transactions on Robotics and Automation, 5(1):16–29, 1989.
- [2] R. Tsai, R. Lenz. "A New Technique for Fully Autonomous and Efficient 3D Robotics Hand/Eye Calibration." In IEEE Transactions on Robotics and Automation, 5(3):345-358, 1989.
- [3] Furrer, Fadri, et al. "Evaluation of combined time-offset estimation and hand-eye calibration on robotic datasets." Field and Service Robotics. Springer, Cham, 2018.
- [4] Hornung, Armin, et al. "OctoMap: An efficient probabilistic 3D mapping framework based on octrees." Autonomous robots 34.3 (2013): 189-206.

Questions?

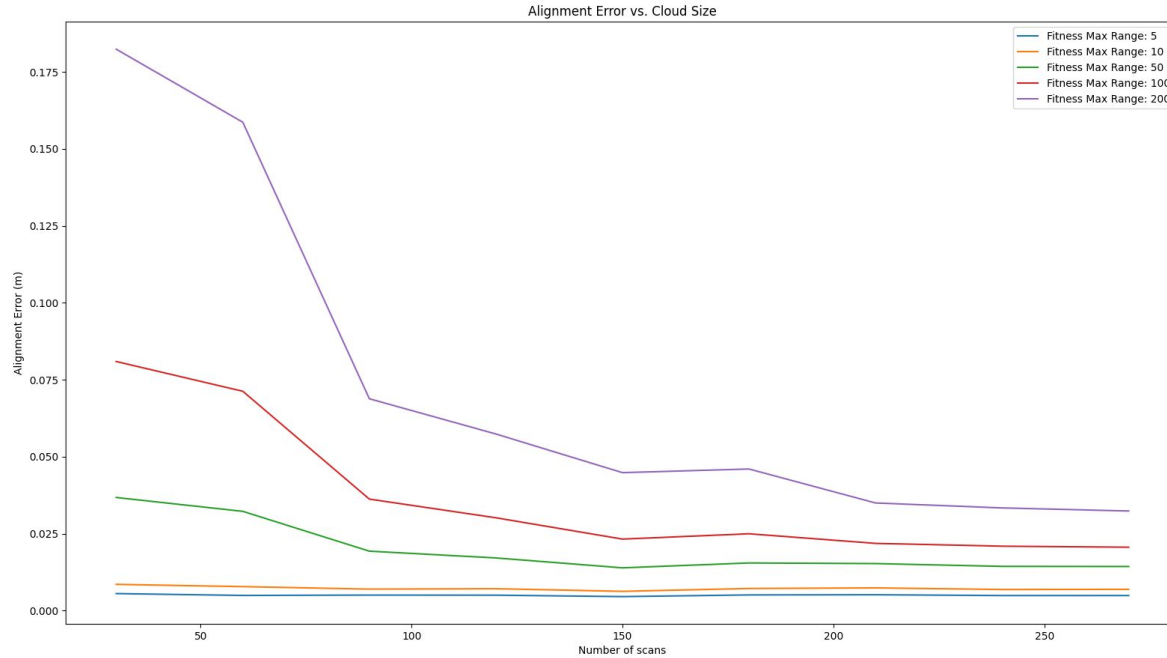
# Appendix

# Computing Weights in Weighted Estimates

- Weights for **translation**  $\rho_i$  computed using the **translation inverse variance of the odometry data**.
- Weights for **rotation**  $\omega_i$  computed using the **rotation inverse variance of the odometry data**.
- Rejection weights  $r_i \in \{0, 1\}$  indicate whether a sample should be rejected due to high uncertainty in the particularly sample, based off of whether:  
 **$\max(\text{diag}(\Sigma_{\text{ICP}})) > \text{rejection\_threshold}$** .

# Point Cloud Consistency

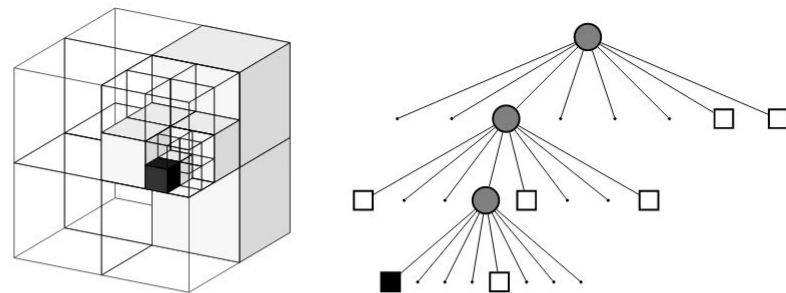
- Larger point clouds give more accurate validation
- Over long periods, drift in the map obscures alignment information



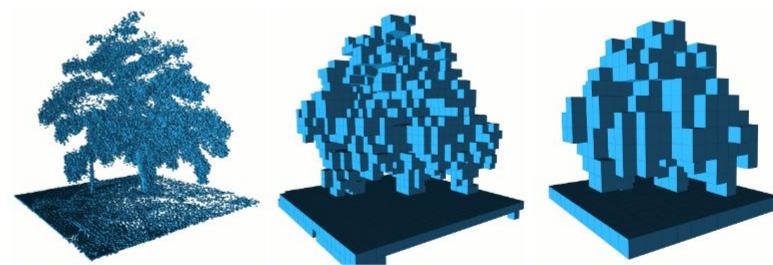


# Accumulating Points

- How do we accumulate multiple scans into a single point cloud?
- Use an Octree!
  - Occupancy grid with multiple levels of resolution
  - Very efficient data structure
  - ROS integration with Octomap



**Fig. 2** Example of an octree storing free (shaded white) and occupied (black) cells. The volumetric model is shown on the left and the corresponding tree representation on the right.



**Fig. 3** By limiting the depth of a query, multiple resolutions of the same map can be obtained at any time. Occupied voxels are displayed in resolutions 0.08 m, 0.64 , and 1.28 m.

# SD Version of Movie

