2 Project Scope

2.1 Problem description

There is a given set of n types of small, three-dimensional, rectangle-shaped items, called boxes, $B = \{b_1, b_2, ..., b_n\}$, of which each box type is characterized by its length l_i, width w_i, height h_i and quantity m_i, $i = \{1, 2, ..., m\}$. The boxes are loaded in the rectangular container with length L, width W and height H. Suppose that the right-hand face of the container is opened. Let a container be located in the first octant of a three-dimensional Cartesian coordinate system with the front-left-bottom vertex in the coordinate origin, i.e. $O(0, 0, 0)$ and the length, width and height of the container are oriented in accordance with the directions of the $x-$, $y-$ and $z-$axes, respectively. Each box must be loaded orthogonally in to the container, that is, the faces of the loaded box are parallel to the walls of the container. A box b_{ij} (i.e. the jth box of type i where $i = \{1, 2, ..., n\}$ and $j = \{1, 2, ..., m_i\}$) is represented by its front-left-bottom vertex coordinate (x_{ij}, y_{ij}, z_{ij}), length l_i, width w_i and height h_i aligned with $x-$, $y-$ and $z-$axes respectively.

2.1.1 Problem Constraints

Apart from the aforementioned rules for orthogonal loading without overlapping (product positioning constraint), the following practical constraints are included in the project scope.

1. **Weight limit (overloading) constraint**. The total weight of loaded boxes cannot exceed the maximum weight limit of the container.

2. **Connectivity (collocation) constraint**. Identical boxes are loaded together where possible to save the loading/unloading time.

3. **Weight distribution (axle-weight balance) constraint**. The centre of gravity(COG) lies close to the geometrical midpoint of the container floor.

4. **Orientation constraint**. Boxes may be loaded in one or more of the six orientations through rotating. Some boxes cannot be rotated in some orientations, and some boxes can be rotated in all six orientations.

5. **Stability constraint**. The loaded boxes cannot be suspended in the air. Their bottom faces must completely touch either the top faces of other boxes or the floor of the container underneath. At the same time at least one side of the box is also required to touch the walls of the container or the sides of other boxes.

6. **Delivery order based placement priority**. The arrangement of the boxes within the container should also reflect the sequence according to which they have to be delivered at their various destinations in order to avoid unnecessary unloading and reloading operations.

2.2 Suggested Solution Methodology

According to (Liu et al. 2012), metaheuristics can obtain a high space utilisation but be ineffective to tackle the practical constraints, whereas the heuristics with the human knowledge is superior to metaheuristics for tackling the practical constraints, particularly for the stability and load bearing strength. However, heuristics result in a lower space utilisation compared to metaheuristics. Considering the trade-off between volume utilisation and practical constraints, we suggest a novel hybrid approach to the problem which is effective for both strongly and weakly heterogeneous box types where the practical constraints are considered. The hybrid approach incorporates a loading heuristic and a handling method for remaining spaces with tabu search techniques. We note that our suggested approach is a modification of the algorithm that is presented in (Liu et al. 2012). Below we present a general view of our suggested hybrid approach.

2.2.1 Input Format

The following information are inserted in the hybrid algorithm as inputs.

- The dimensions of the container.
- The maximum weight limit.
- The axle positions of the container.
- The dimensions, quantity and weight of each box type.
- Allowed orientations to rotate each box type.
- The delivery order of the boxes.

2.2.2 Suggested Placement Heuristic

Among the described placement heuristics in Section 1, we suggest a block arrangement heuristic since packing container with homogeneous blocks results in several advantages. First of all, it is easy to arrange and requires less loading time. Re-sorting cargo is avoided after unloading as boxes of same type are stowed closely to one another. The constraint of load-bearing strength is less problematic when identical boxes are stacked. The block structure provides extra stability as identical boxes packed in a rectangular shape cannot easily slip. Moreover, generating blocks are helpful to address the connectivity constraint. Because of these advantages, the block arrangement method is applied in the most remarkable articles that are similar to SLOPP in presence of the practical constraints according to (Zhao et al. 2014). We simply describe the block arrangement heuristic as follows.

The algorithm sorts the boxes in decreasing order of their volumes. The dimension of each box type is then re-arranged in decreasing order of their lengths, followed by width and then height.

An initial remaining space is generated as the entire container space. The initial loading sequence is generated in decreasing order of box volumes. During the loading process, the boxes are loaded using the following heuristic strategies.

- Orientation strategy: Each box is loaded such that its largest face is parallel to the floor of the container in order to provide maximum support. The initial orientation of the boxes is such that the two faces of the largest dimensions (i.e.length and width) are parallel to the floor of the container if the orientation is permitted.
- Position strategy: The first box must be loaded into a remaining space with its back-left bottom vertex located at the back-left- bottom vertex of the space. Then, other boxes touch the first loaded boxes to left furthest and back without overlapping. Thus, the boxes are loaded in a sequence, that is, from back to front, from bottom to top and from left to right of the container during the loading process.
- Block strategy: Identical or different box types are combined to form a rectangular block. Loading the block not only obtains a higher volume utilisation than loading single boxes one-by-one, but also offers the advantages form any practical applications, that is (1) easily constructing a large top face of the block to provide high stability;(2) loading and unloading the block with identical boxes in a relatively short time;(3) enhancing the load-bearing strength of boxes. To obtain a block with best fit in a remaining space, an evaluation function and 6 evaluation rules are described.

2.2.3 Suggested Improvement Heuristic

To improve the initial solution, we suggest a hybrid tabu search method with our suggested placement heuristic. As described in Section 1, TS is widely used in the literature to improve the initial solution of the problem that is obtained from the placement (loading) heuristic. The method can be described in the following steps. An initial solution to the problem is generated by using the aforementioned placement heuristic. This initial solution is then transformed into a representation to be used in a tabu search. A number of feasible solutions are generated using the tabu search method. Each of the feasible solutions generated by the tabu search is transferred into a loading arrangement of the boxes using the loading heuristic. Each loading arrangement is evaluated using the defined evaluation criterion ($f(TS(\theta))$). The best arrangement is chosen as an optimal solution. Further feasible solutions are generated using a tabu search with this optimal solution as an initial solution. The procedure is repeated iteratively until a given termination criterion (i.e.number of iterations or computational time) is achieved. Finally, an optimal solution is obtained.

The applied evaluation function in the hybrid approach. In addition to volume utilisation, the constraints of positioning, orientation, stability and connectivity are considered in the placement heuristic. Because the constraints of weight limit, weight distribution and delivery based shipment priority can be described by mathematical formulas, the evaluation of these constraints and volume utilisation are defined together in an evaluation function $f(TS(\theta))$, where θ denotes an arbitrary solution. If the weight limit constraint is violated, $f(TS(\theta))$ equals zero. Otherwise, $f(TS(\theta))$ is written of the form $\alpha * f_1(TS(\theta)) + \beta * f_2(TS(\theta)) + \gamma * f_3(TS(\theta)) + \delta * f_4(TS(\theta))$, where $\alpha, \beta, \gamma, \delta$ belong to $[0, 1]$ and $\alpha + \beta + \gamma + \delta = 1$. $f_1(TS(\theta))$ is a function that addresses the volume utilisation, $f_2(TS(\theta))$ is the loaded weight ratio which covers the weight limit constraint. Finally, the functions $f_3(TS(\theta))$ and $f_4(TS(\theta))$ address the weight distribution and delivery order based loading priority constraints, respectively. Note that α, β, γ and δ are weight coefficients. To ensure the above objectives, the coefficients of evaluation function are assigned different weight values. To obtain high volume utilisation, we set the weight value of the coefficient of $f_1(TS(\theta))$ greater than the other weight coefficients. All candidate solutions generated are evaluated by $f(TS(\theta))$. If the value of $f(TS(\theta))$ is maximal, the solution is considered as an optimal solution.

The performance of the modified hybrid tabu search algorithm. Experimental results with benchmark data in (Liu et al. 2012) show that the hybrid approach provides a better space utilisation than the published approaches under the condition of all loaded boxes with one hundred percent support from below. However, the performance of the algorithm decreases when practical constraints are taken into account on the same data. Moreover, Liu et al (2012) do not consider the delivery order based placement priority in their algorithms that is considered in our suggested modified approach. According to these reasons, we request to reduce the expected performance of the algorithm in the technical document of the project.

2.3 Project Objective

The objective of the project is to achieve a pre-determined volume utilisation (occupied-space utilisation) in presence of the practical constraints. Below we describe the criteria.

- **Volume Utilisation**. The volume utilisation criteria is defined as the ratio of the total volume of the loaded products over the volume of the container.
- **Occupied-Space Utilisation**. The occupied-space utilisation criteria is defined as the ratio of the total volume of the loaded orders over the occupied volume in the container.

We consider the product positioning, weight limit, orientation and stability constraints as the hard constraints, while the weight distribution, connectivity and delivery order based placement priority constraints will be considered as the soft constraints in the project.

2.4 Benchmark Data

To evaluate the performance of the suggested hybrid approach, we suggest the benchmark dataset provided by Bischoff and Ratcliff (1995) from the OR-library. The dataset contains various classes including weakly and strongly heterogeneous box types.

3 Concluding Remarks

We presented a comprehensive review on the literature of the container loading problem. We described the relevant practical constraints to the problem. We presented an overview on the solution methodologies that are applied in the literature to solve the problem. We also presented a detailed literature review on SLOPP. We determined the scope of the project by defining the problem typology and the constraints that are considered in the project. Finally, we suggested a novel hybrid approach to deal with the problem.