

变分公式推导:

看做 Δx 的函数

$$f(x + \Delta x) = f(x + \Delta x) \Big|_{\Delta x=0} + \frac{\partial f(x + \Delta x)}{\partial (x + \Delta x)} \Big|_{\Delta x=0} \frac{\partial (x + \Delta x)}{\partial \Delta x} \cdot \Delta x + O(\Delta x^2)$$

看做 e 的函数

$$\begin{aligned} F(y(x) + e\eta(x)) &= F(y(x) + e\eta(x)) \Big|_{e=0} \\ &\quad + \int \frac{\partial F(y(x) + e\eta(x))}{\partial (y(x) + e\eta(x))} \Big|_{e=0} \eta(x) dx e \\ &\quad + O(e^2) \\ &= F(y(x)) + \int \frac{\partial F}{\partial y} \cdot \eta(x) \cdot e dx \\ &\quad + O(e^2) \end{aligned}$$

