A Queue-Based Dynamic Traffic Assignment Model with Traffic Signal Control in a Corridor Network

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ABSTRACT

In this paper, we present a queue-based dynamic traffic assignment (DTA) model in a single origin-destination (OD) corridor network with two signal-controlled intersections. In many cities across the world, operations on key, important corridors are critical to the overall performance of the urban traffic network. Coordinated signal control is an efficient way to improve the safety and mobility on a corridor. Although DTA has been an effective tool to describe traffic dynamics for traffic optimization, and many researchers have considered traffic signal control in their models, signal timings have been simplified without considering complex, but realistic, phase sequence and duration restrictions. This work formulates traffic signal timing as a component of the link performance function with three control variables: cycle length, phase split, and offset. A G/G/n/FIFO queueing network is proposed to describe experienced travel times for links and paths. In addition, numerical simulation is implemented in MATLAB to solve both user-equilibrium (UE) and system-optimal (SO) DTA problems in twelve cases with different demand inputs and traffic signal plans. Finally, this paper discusses extensions from a single OD corridor model to a larger corridor network containing more entrances and exits, formulating the optimization of signal controls at network level. The main contribution of this paper is to introduce a basic model for combination problem of traffic control and traffic assignment, which is applicable in urban area.

Keywords: Dynamic Traffic Assignment (DTA), Traffic Signal Control, Queueing Network, User-Equilibrium (UE), System-Optimal (SO)

INTRODUCTION

This paper considers the problem of traffic assignment and control in a roadway network anchored by major corridors. Corridors are the main components of cities across the world, carrying a large proportion of traffic flow. Congestion in a single corridor may cause upstream congestion across the whole network. **Figure 1** shows a core area of the City of Edmonton, the fifth largest city in Canada by population. Like many older city centers in North America, the roadway network is a grid, with streets running north-south and east-west. In this area, 114 St., 109 St., Calgary Trail, and Gateway Blvd. are major north-south corridors connecting populous suburbs in the southern reaches of the city to the core (marked light blue on the map). The high volume of traffic on these four corridors, combined with less than optimal signal control, results in significant congestion across the network, particularly at AM and PM peak travel times. If coordinated signal control for traffic on these key corridors is improved, it is anticipated that the traffic state for the entire network will also improve.

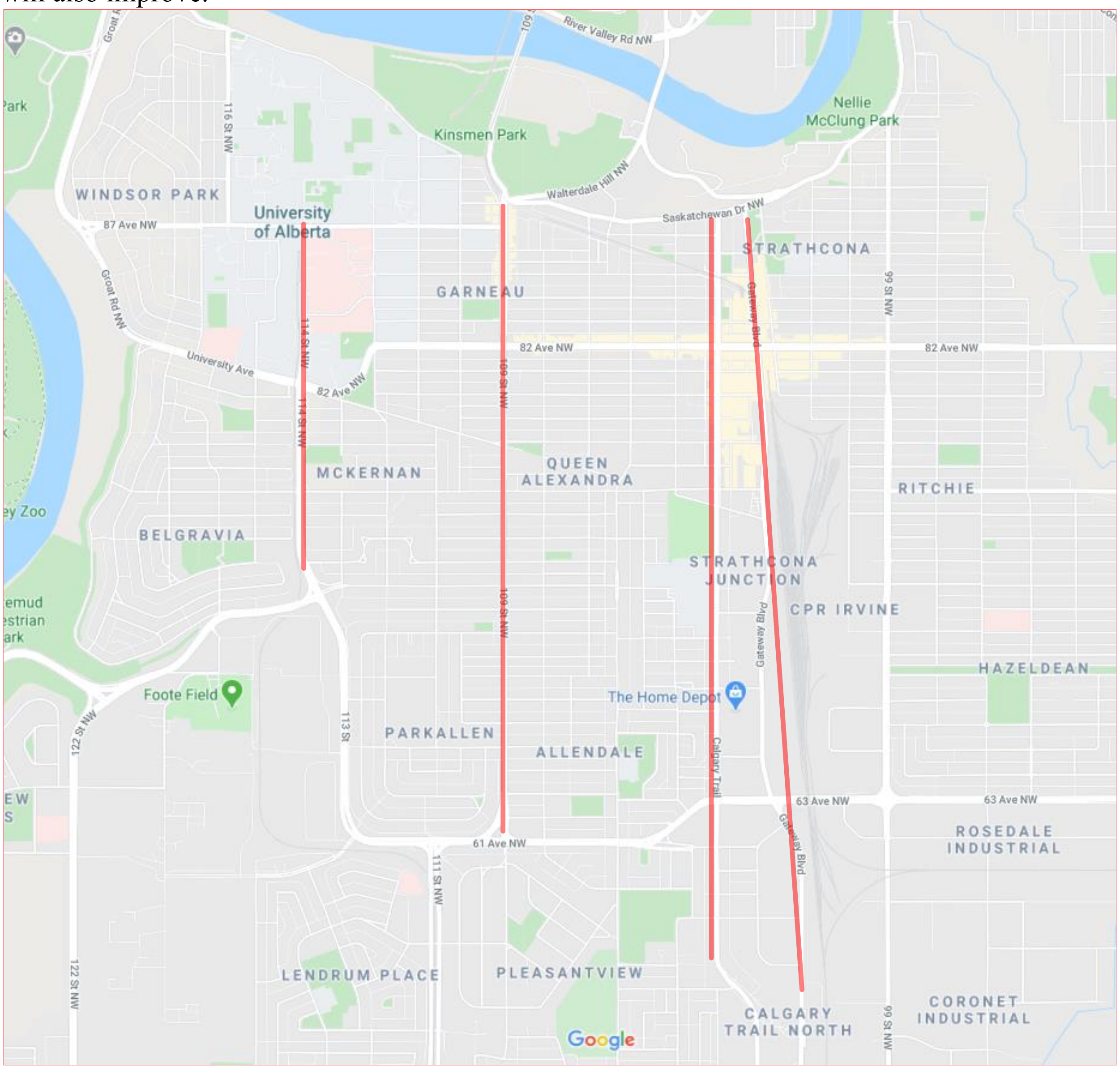


Figure 1 Four main corridors in Edmonton.

Traffic signal control is a significant factor when considering traffic congestion in urban areas, especially for corridor networks, and signal waiting time can have a high impact on travel cost. The development of Connected Vehicle to Everything (CV2X) technology can provide more detailed traffic signal timing information, more detailed traffic signal timing information can be accessed by travelers prior to their arrival at the intersection than years before, and travelers have opportunities to change routes when facing high signal wait times. With CV2X, it is assumed that all travelers have access to experienced travel times for each path immediately before they enter the corridor network.

Since the seminal work of Merchant and Nemhauser (M-N model) in 1978, Dynamic Traffic Assignment (DTA) has played a key role in traffic research (*1*, *2*). Given the time-dependent nature of demand and network characteristics, DTA models are used principally to estimate dynamic traffic flow patterns over a network.

The traffic state estimation in DTA is based primarily on two principles in the traffic assignment literature. The first is User-Equilibrium DTA (UE-DTA). Ran et al. used the optimal control approach to formulate a convex model for instantaneous UE-DTA problem-solving that considers link flows as variables *(3)*. If the actual travel times experienced by travelers departing at the same time are equal and minimal, then the dynamic flow over the network is in a travel time-based ideal dynamic user-optimal state for each OD pair at each interval of time. The second is System-Optimal DTA (SO-DTA). The objective of SO-DTA is to minimize the total travel time within the network, i.e. the travel time experienced by all users in the network. SO-DTA with full control can be formulated as a linear programing problem (*4*).

Combining DTA and traffic signal control is not an innovative pairing. As early as 1977, Allsop and Charlesworth had combined traffic signal control and traffic assignment analytically (*5*). Later, in 1995, Yang and Yagar looked at the relationship between DTA and traffic signal control in a saturated network as a bi-level problem (*6*). Sun et al. used GA by adjusting cycle, offset, and phase split in a bi-level model *(7)*. Most salient here is that, although many researchers have considered traffic signal control in their DTA models, traffic signal timing has been simplified without considering complex, but realistic, phase sequence and duration restrictions. In this work, signal wait time is included in the experienced path travel time. The signal waiting time function is dependent on the control variables of cycle length, phase split, and offset.

Queueing theory is proposed to describe traffic dynamics. The earliest well defined queueing system was developed in 1953 (*8*), with the notation written as X/Y/m/D/L, where X represents the distribution of intervals between arrivals, Y represents the distribution of service durations, m represents number of servers, D represents the queueing principle, and L represents the number of customers. Since the most popular distribution used in queueing system is exponential distribution, there are two major types of models according to the distribution type (X and/or Y), Markovian Queueing Systems and Non-Markovian Queueing Systems (*9*). There are several principles of queue discipline, such as “First In First Out” (FIFO), “Last In First Out with Preemptive” (LIFO-PR), “Processor Sharing” (PS), and “Infinite Server” (IS). Moreover, according to the number of customers, there are two major types: closed queueing systems with a constant number of customers and open queueing systems with varying numbers of customers. These definitions cover most types of queueing systems (*10*).

The literature also examines a number of queueing network model applications. Closed queuing network models are applied to the bike sharing problem(*11*). The same closed queueing network is also found in studies on the car rental system (*12*). In another application, a simulated queueing network with blocking created by traffic signal control achieves the system optimal (SO) assignment (*13*). Moreover, a multi-class queueing network is defined for air transportation in a study of transient congestion, solved by a decomposition algorithm (*14*).

With this as our background, the main objective of this research is now to solve UE-DTA and SO-DTA in a single OD corridor with traffic signal controls. A secondary objective is to formulate traffic signal optimization problem in both the UE-DTA and SO-DTA cases. To do so, this paper will introduce the model setup, assumptions, and notations used in the study. Next, the model will be developed by describing traffic dynamics as a G/G/n/FIFO open queueing network, providing details about traffic signal control, and formulating both UE-DTA and SO-DTA problem with signal controls. The third section will provide numerical examples for both UE-DTA and SO-DTA, and outline the simulation that was run for twelve cases in Matlab. Finally, we will conclude with a discussion of future research avenues.

MODEL SETUP

In our study of a single OD network with signal control, we consider a scenario where there are a maximum of three lanes, one for each of left, through, and right movements, and queues at the end of each lane for each link. Servers of the queueing network control the intersection signal. When the signal is red, the service rate is zero; when the signal is green, the service rate is constant at the maximum discharge rate. The arrival rate is equal to the inflow rate. The "First In First Out" (FIFO) principle is held for the model. In addition, vehicles will enter and exit the corridor without parking inside the network. Thus, the queueing network used is a G/G/n/FIFO open queueing network, where G represents a specific general distribution.

There are five basic assumptions for this research, as listed below.

Assumption 1 No Departure Time Choice
Since traffic signal control is real time control strategy. Travelers will not stay anywhere on their route to skip high volume traffic to achieve any other benefit. When travelers arrive at the network, they must move towards a path immediately.

Assumption 2 First In First Out (FIFO)
The first vehicle that enters the link will also be the first to leave the link.

Assumption 3 Point Queue
There is a "Point Queue" at the end of each lane of any link with a constant discharging flow rate w_0 (or saturation flow rate). "Point Queue" indicates that the vehicles in the queue occupy no space in the link, i.e. storage of links is assumed to be infinite.

Assumption 4 Experienced Travel Time
For UE-DTA cases, all travelers will choose the path with minimal experienced travel time. For SO-DTA cases, the objective is to minimize the total network travel time, i.e. the travel time experienced by all travelers in the network.

Assumption 5 No Route Changes
When travelers enter the network, they will follow the same path until they exit the network, regardless of signal control plan changes.

There are two theorems from the literature that rely on the assumptions listed above that will be used in the model development.

Theorem 1 Marginal Cost for SO-DTA (*4*)
A necessary and sufficient condition for SO-DTA in a single destination network is that all traveled paths from any cell, and departure time interval to the destination cell, have equal cost to the marginal cost of an additional unit of demand at that cell and time interval, while all untraveled paths have costs higher than or equal to the marginal cost.

Theorem 2 Dynamic Process (*15*)
The path delay operators usually do not take on any closed form. Instead they can only be evaluated numerically through the dynamic network loading procedure.

Since we are using a single OD network, there is an extended version of FIFO for the network as **Lemma 1 & 2**.

Lemma 1 FIFO for Paths
The first vehicle that enters the path will also be the first to leave the path.

Proof According to **Assumption 5**, vehicles will not change their paths. In addition, **Assumption 2** means that the first vehicle that enters the path will be the first to exit each link of the path, which proves **Lemma 1**.

Lemma 2 FIFO for single OD networks
The first vehicle that enters the network will leave the network first in UE-DTA cases for the single OD network. The latter vehicle will never overtake any previous vehicle.

Proof Suppose vehicle A enters the network early at t_a and leaves the network at t_A, and there is another vehicle B that enters the network early at t_b and leaves the network at t_B. In addition, suppose $t_a < t_b$ and $t_A > t_B$, means that vehicle A enters the network earlier than vehicle B, but exits the network later.

According to **Assumption 4**, let A choose path p_a at time t_a and let B choose path p_b at time t_b. Then we have path travel time $T_{path}^{p_a}(t_a) \leq T_{path}^{p_b}(t_a)$, since p_a is the path with minimal experienced travel cost at time t_a.

If A chooses p_b and **Assumption 2** and $t_a < t_b$, then A is in front of B on any link of p_b. Thus, the time A exits the network $t'_A < t_B$, and since $t_B < t_A$, we have $t'_A < t_A$.

Then $t'_A - t_a < t_A - t_a \implies T_{path}^{p_b}(t_a) < T_{path}^{p_a}(t_a)$.

If there is a contradiction with $T_{path}^{p_a}(t_a) \leq T_{path}^{p_b}(t_a)$, then the assumption is wrong. The lemma holds. #

Model traffic dynamics are built based on **Theorem 1**, **Lemmas 1** and **2**. Marginal cost functions with traffic signal control are formulated in the model development section, from which we can investigate SO-DTA cases according to **Theorem 2**.

Note that **Lemma 2** only holds for UE-DTA cases. For SO-DTA cases, a latter vehicle may overtake the previous vehicle. This will create conflict if we only consider each link as a queue, i.e. the vehicle entering afterwards will be in front of previous traffic in the queue due to the fact that the path with minimal marginal experienced cost is not always the same as the path with minimal experienced travel cost, something not considered when simulating previous traffic. As explained shortly, the queueing process used in our simulation can avoid these conflicts.

NOTATION

Table 1 shows the notation used in this paper.

Table 1 Notations of Variables and Parameters

Symbol	Description
i, j, o, d	Index of node.
t	Index of timestamp, a unit of t does not stand for any second or minute.
a	Index of link. Let $a = (i, j)$, then the start node for link a is i and the end node for link a is j.
A	Set of all the nodes for the network.
p	Index of path. If link a is the k^{th} link on path p, we can write $a = p(k)$.
$L(p)$	Index of length of path, i.e. number of links belonging to the path.
P	Set of all the paths for the network.
w_0	Maximum discharging flow rate for each lane of the link.
$f_{in}^{a,p}(t)$	Inflow rate for the lane on link a belongs to path p at time t.
$f_{out}^{a,p}(t)$	Outflow rate for the lane on link a belongs to path p at time t.
$f_{queue_{in}}^{a,p}(t)$	Inflow rate for the queue of lane on link a belongs to path p at time t.
$f_{queue_{out}}^{a,p}(t)$	Outflow rate for the queue of lane on link a belongs to path p at time t.
$T_{link}^{a,p}(t)$	Travel cost for the lane on link a belongs to path p at time t.
$T_{fft}^{a,p}(t)$	Free flow travel time for the lane on link a belongs to path p at time t.
$T_{queue}^{a,p}(t)$	Time spent to clear the queue of lane on link a belongs to path p at time t.
$T_{signal}^{a,p}(t)$	Signal waiting time for the lane on link a belongs to path p at time t.
$L_{queue}^{a,p}(t)$	Index of length of queue, i.e. number of vehicles in the queue of lane on link a belongs to path p at time t.
$X_{phase}^{a,p}$	Length of phase for path p at the intersection of link a.
$X_{offset}^{a,p}$	Offset for path p at the intersection of link a.
X_{cycle}^{a}	Cycle length of signal plan for the intersection of link a.
$L_{allred}^{a,p}$	Length for the red clearance phase for path p at the intersection of link a.
$T_{path}^{p}(t)$	Travel cost for path p at time t.
$\tau_k^p(t)$	Time stamp of vehicle entering the network at time t arrive at k^{th} link on path p
$\pi(t)$	Min travel cost of all the paths at time t.
$\widetilde{T_{link}}^{a,p}(t)$	Marginal travel cost for the lane on link a belongs to path p at time t.
$\widetilde{T_{path}}^{p}(t)$	Marginal travel cost for path p at time t.
$\tilde{\pi}(t)$	Min marginal travel cost of all the paths at time t.
$q^p(t)$	Demand for path p at time t.
$q(t)$	Demand for the single OD network at time t.

MODEL DEVELOPMENT

Model development will be described in three parts: link dynamics, UE-DTA, and SO-DTA.

Link Dynamics

A general link $a = (i, j)$ is constructed as per **Figure 2** in our study. The start point and end point of a link is denoted i, j. According to **Assumption 3**, there is a queue at the end of each lane of the link, which takes no space in the link. In **Figure 2**, there are three lanes and three queues for left-turn, through, and right-turn each. In addition, $(j, k_1), (j, k_2), (j, k_3)$ are links for signal phase left-turn, through, and right-turn respectively.

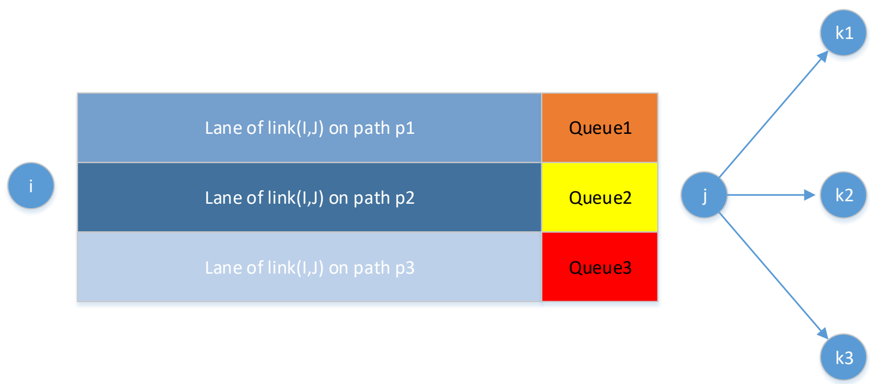


Figure 2 Sample of links.

Suppose link a is the k^{th} link on path p. We have **Equations (1)-(3)** for conservations of flow and queue.

$$f_{in}^{a,p}\left(t - T_{fft}^{a,p}(t)\right) = f_{queue_{in}}^{a,p}(t) \quad \textbf{(1)}$$

$$f_{queue_{out}}^{a,p}(t) = f_{out}^{a,p}(t) \quad \textbf{(2)}$$

$$f_{in}^{p(k),p}(t) = f_{out}^{p(k-1),p}(t) \quad \textbf{(3)}$$

Equation (1) shows that the inflow rate of the queue for the lane of link a belonging to path p at time t equals the inflow rate of the lane at time $t - T_{fft}^{a,p}(t)$. That means when a vehicle travels to the end of the lane, it will join the queue directly. **Equation (2)** shows that the outflow queue rate will equal to the outflow of the link. **Equation (3)** shows that the inflow rate of the lane on path p equals the outflow rate of the lane of the previous lane on path p.

Experienced Travel Time

It is assumed that each traveler will know the experienced travel time for the path according to **Assumption 4**. The calculation for the experienced travel time is described as below.

Link travel time

For a single link, experienced travel time has three components, as in **Equation (4)**. The first represents the time cost of travelers traveling from the start point of the link to the start point of the queue as $T_{fft}^{a,p}(t)$. Travelers arrive at the start point of the queue at time $t + T_{fft}^{a,p}(t)$. The time cost for travelers to get the front of queue is $T_{queue}^{a,p}\left(t + T_{fft}^{a,p}(t)\right)$.

$$T_{link}^{a,p}(t) = T_{fft}^{a,p}(t) + T_{queue}^{a,p}\left(t + T_{fft}^{a,p}(t)\right) + T_{signal}^{a,p}\left(t + T_{fft}^{a,p}(t) + T_{queue}^{a,p}\left(t + T_{fft}^{a,p}(t)\right)\right) \quad \textbf{(4)}$$

When travelers reach front of queue at $t + T_{fft}^{a,p}(t) + T_{queue}^{a,p}\left(t + T_{fft}^{a,p}(t)\right)$ and the signal is red, they will wait for it to turn green. The signal wait time is expressed as $T_{signal}^{a,p}\left(t + T_{fft}^{a,p}(t) + T_{queue}^{a,p}\left(t + T_{fft}^{a,p}(t)\right)\right)$.

If $T_{fft}^{a,p}(t)$ is given, the second component $T_{queue}^{a,p}\left(t + T_{fft}^{a,p}(t)\right)$ indicates time spent to clear the queue, which is formulated as **Equation (5)**. It is the ceiling for the ratio of queue length (number of vehicles in the queue) to maximum discharging flow rate w_0.

$$T_{queue}^{a,p}(t) = \left\lceil \frac{L_{queue}^{a,p}(t)}{w_0} \right\rceil \quad \textbf{(5)}$$

Figure 3 shows the relationship between queue length and time to clear queue. It is a step function that increases after the number of points that represent the maximum discharging rate w_0.

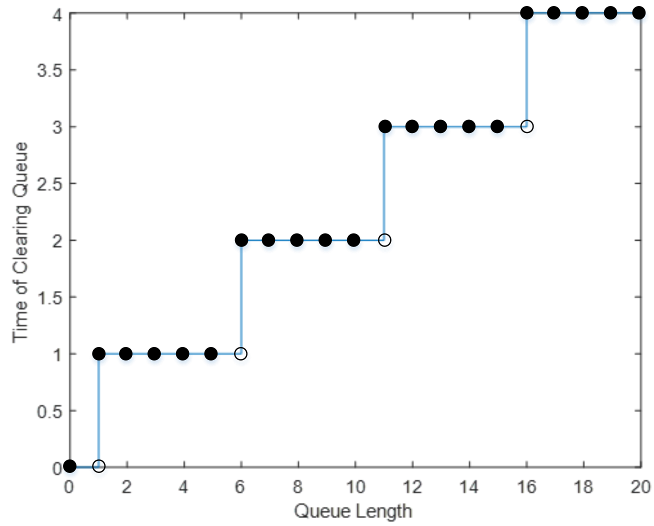


Figure 3 Relationship between queue length and time to clear queue.

Queue length function is calculated as **Equation (6)**. It is the queue inflow rate minus the queue outflow rate, summed over previous period. When time index $t < T_{fft}^{a,p}(t)$, the initial queue length is 0. According to **Equation (1)-(2)**, we get **Equation (7)**.

$$L_{queue}^{a,p}(t) = \sum_{\tau=T_{fft}^{a,p}(t)}^{t} \left(f_{queue_{in}}^{a,p}(\tau) - f_{queue_{out}}^{a,p}(\tau) \right) \quad \textbf{(6)}$$

$$L_{queue}^{a,p}(t) = \sum_{\tau=T_{fft}^{a,p}(t)}^{t} \left(f_{in}^{a,p}\left(\tau - T_{fft}^{a,p}(\tau)\right) - f_{out}^{a,p}(\tau) \right) \quad \textbf{(7)}$$

The outflow rate for the link is dependant on signal control as **Equation (8)**. When the traffic signal is red, i.e. signal wait time is positive, the outflow rate is 0. When the traffic signal is green, i.e. signal wait time is 0, the outflow rate is the minimum of queue length and maximum discharging flow rate w_0.

$$f_{out}^{a,p}(t) = \begin{cases} 0 & T_{signal}^{a,p}(t) > 0 \\ \min\left(L_{queue}^{a,p}(t), w_0\right) & T_{signal}^{a,p}(t) = 0 \end{cases} \quad \textbf{(8)}$$

The signal waiting time function is a cyclic function with a cycle length based on the cycle length of the signal control in **Figure 4**.

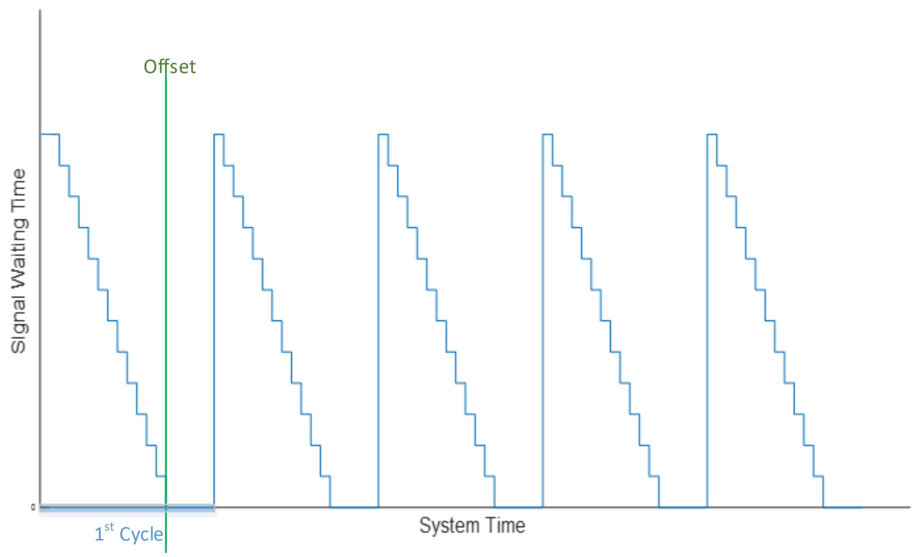


Figure 4 Signal waiting time function.

In addition, we will formulate the signal waiting time function with cycle length X^{a}_{cycle}, phase split $X^{a,p}_{phase}$ (green time for the phase), and offset $X^{a,p}_{offset}$ as **Equation (9)**. $\hat{t}$ is the reference time point in one cycle, which is equal to the remainder of $t + X^{a,p}_{offset}$ dividing X^{a}_{cycle} as per **Equation (10)**. $X^{a,p}_{offset}$ is defined as the start point of green in the first cycle for the phase on link a belonging to path p. For example, offset is the green vertical line in **Figure 4**. Each phase will start with a red clearance as $L^{a,p}_{allred}$.

$$T^{a,p}_{signal}(t) = \max\{X^{a}_{cycle} - X^{a,p}_{phase} + L^{a,p}_{allred} - \hat{t}, 0\} \quad \textbf{(9)}$$

$$\hat{t} \equiv \left(t + X^{a,p}_{offset} + X^{a,p}_{phase} - L^{a,p}_{allred}\right) \, mod \, X^{a}_{cycle}, 0 \le \hat{t} < X^{a}_{cycle} \quad \textbf{(10)}$$

Note that the arrival rate and the service rate for the queue in each lane, before the traffic signals, are the multiplicative inverses of inflow rate $f^{a,p}_{queue_{in}}(t)$ and outflow rate $f^{a,p}_{queue_{out}}(\tau)$ respectively. Thus, the time-varying demand and traffic assignment principle (UE or SO) will determine the arrival rate for each queue as a specific distribution, and signal control patterns (cycle length, phase split, offset) will determine the service rate as a periodic uniform distribution. In addition, the number of queues in the network is fixed. Travelers will follow the FIFO principle to clear the queue, and the total number of users for the network is not fixed. The network dynamics can be regarded as a G/G/n/FIFO open queueing network.

Path travel time

Path p is defined as a vector of ordered links$\{a_i\}$ that can be written as $p = \{a_i\}$.. If link a is the k^{th} link on path p, we can write $a = p(k)$. $L(p)$ is length of path p, i.e. number of links in path p.

When a traveler on path p arrives at the k^{th} link $p(k)$, we record the time as $\tau^{p}_{k}(t)$. $\{\tau^{p}_{k}(t)\}_{k}$ is a sequence calculated via **Equation (11)-(12)**.

$$\tau^{p}_{1}(t) = t \quad \textbf{(11)}$$

$$\tau^{p}_{k+1}(t) = \tau^{p}_{k}(t) + T^{p(k),p}_{link}\left(\tau^{p}_{k}(t)\right), 2 \le k < L(p) \quad \textbf{(12)}$$

Then the path travel time can be formulated as **Equation (13)**.

$$T_{path}^{p}(t) = \sum_{k=1}^{L(p)} T_{link}^{p(k),p}\left(\tau_k^p(t)\right) \quad \textbf{(13)}$$

Marginal travel time for link

The marginal cost of a link is the additional cost of an additional demand added into the link. The calculation of differentials is based on this principle to compute the limit of $\frac{\Delta T}{\Delta f}$. The marginal cost of link can be formulated as **Equation (14)**. Since $T_{link}^{a,p}(t)$ has three components as **Equation (4)** (free-flow travel time, queuing time, signal wait time), $\frac{\partial T_{link}^{a,p}(t)}{\partial f_{in}^{a,p}(t)}$ will be calculated as **Equation (15)**.

$$\widetilde{T_{link}}^{a,p}(t) = T_{link}^{a,p}(t) + f_{in}^{a,p}(t) \times \frac{\partial T_{link}^{a,p}(t)}{\partial f_{in}^{a,p}(t)} \quad \textbf{(14)}$$

$$\frac{\partial T_{link}^{a,p}(t)}{\partial f_{in}^{a,p}(t)} = \frac{\partial T_{fft}^{a,p}(t)}{\partial f_{in}^{a,p}(t)} + \frac{\partial T_{queue}^{a,p}\left(t+T_{fft}^{a,p}(t)\right)}{\partial f_{in}^{a,p}(t)} + \frac{\partial T_{signal}^{a,p}\left(t+T_{fft}^{a,p}(t)+T_{queue}^{a,p}\left(t+T_{fft}^{a,p}(t)\right)\right)}{\partial f_{in}^{a,p}(t)} \quad \textbf{(15)}$$

Since $T_{fft}^{a,p}(t)$ is a constant function during the control period, we have

$$\frac{\partial T_{fft}^{a,p}(t)}{\partial f_{in}^{a,p}(t)} = 0 \quad \textbf{(16)}$$

According to **Figure 3**, **Equation (1)**, and **Equation (5)-(7)**, we have

$$\frac{\partial T_{queue}^{a,p}\left(t+T_{fft}^{a,p}(t)\right)}{\partial f_{in}^{a,p}(t)} = \frac{\partial T_{queue}^{a,p}\left(t+T_{fft}^{a,p}(t)\right)}{\partial f_{queue_{in}}^{a,p}\left(t+T_{fft}^{a,p}(t)\right)}\begin{cases}0 & L_{queue}^{a,p}\left(t+T_{fft}^{a,p}(t)\right) \bmod w_0 \not\equiv 0 \\ 1 & L_{queue}^{a,p}\left(t+T_{fft}^{a,p}(t)\right) \bmod w_0 \equiv 0\end{cases} \quad \textbf{(17)}$$

According to **Figure 4**, and **Equation (9)-(10)**, we have

$$\frac{\partial T_{signal}^{a,p}(t^*)}{\partial f_{in}^{a,p}(t)} = \frac{\partial T_{signal}^{a,p}(t^*)}{\partial t^*}\frac{\partial t^*}{\partial f_{in}^{a,p}(t)} \quad \textbf{(18)}$$

$$t^* = t + T_{fft}^{a,p}(t) + T_{queue}^{a,p}\left(t+T_{fft}^{a,p}(t)\right) \quad \textbf{(19)}$$

$$\frac{\partial T_{signal}^{a,p}(t^*)}{\partial t^*} = \begin{cases}X_{cycle}^{a} - X_{phase}^{a,p} + L_{allred}^{a,p} & t^* \equiv X_{phase}^{a,p} + X_{offset}^{a,p} - L_{allred}^{a,p} \bmod X_{cycle}^{a} \\ 0 & otherwise\end{cases} \quad \textbf{(20)}$$

$$\frac{\partial t^*}{\partial f_{in}^{a,p}(t)} = \frac{\partial T_{queue}^{a,p}\left(t+T_{fft}^{a,p}(t)\right)}{\partial f_{in}^{a,p}(t)} \quad \textbf{(21)}$$

Marginal path cost for path

Marginal cost of a path is the additional cost of one additional demand unit added into the path. According to **Equation (13) & (15)**, we get

$$\widetilde{T_{path}^{p}}(t) = T_{path}^{p}(t) + f_{in}^{p(1),p}(t) \times \frac{\partial T_{path}^{p}(t)}{\partial f_{in}^{p(1),p}(t)} \quad \textbf{(22)}$$

$$\frac{\partial T_{path}^{p}(t)}{\partial f_{in}^{p(1),p}(t)} = \sum_{k=1}^{L(p)} \frac{\partial T_{link}^{p(k),p}\left(\tau_k^p(t)\right)}{\partial f_{in}^{p(k),p}\left(\tau_k^p(t)\right)} \quad \textbf{(23)}$$

To explain **Equation (23)**. The additional user entering the first link of path p at time t will arrive at the k^{th} link at time $\tau_k^p(t)$.

UE-DTA & SO-DTA

With the discussion above, the dynamic user equilibrium (DUE) conditions can be stated as **Equations (24)-(27)** according to **Assumption 4**. And the dynamic system optimal (DSO) conditions can be stated as **Equations (28)-(31)** according to **Theorem 1**. The UE-DTA and SO-DTA problems are to find feasible solutions for time-varying path demand $d^p(t)$ for all the paths constraint with DUE conditions and DSO conditions respectively (*16*).

DUE conditions:

$$T_{path}^{p}(t) - \pi(t) \geq 0 \quad \textbf{(24)}$$

$$d^p(t)\left(T_{path}^{p}(t) - \pi(t)\right) = 0 \quad \textbf{(25)}$$

$$d^p(t) \geq 0 \quad \textbf{(26)}$$

$$\sum_{p \in P} d^p(t) = d(t) \quad \textbf{(27)}$$

DSO conditions:

$$\widetilde{T_{path}^{p}}(t) - \tilde{\pi}(t) \geq 0 \quad \textbf{(28)}$$

$$d^p(t)\left(\widetilde{T_{path}^{p}}(t) - \tilde{\pi}(t)\right) = 0 \quad \textbf{(29)}$$

$$d^p(t) \geq 0 \quad \textbf{(30)}$$

$$\sum_{p \in P} d^p(t) = d(t) \quad \textbf{(31)}$$

The main difference between UE-DTA and SO-DTA is that users choose paths with minimal experienced travel time in UE-DTA cases, while users choose paths with minimal marginal experienced travel time in SO-DTA cases.

NUMERICAL EXAMPLE

The simulation in this section will use a single OD network with seven paths. Both UE-DTA and SO-DTA cases are tested. Simulations are based on traffic dynamics discussed in the model development part.

Network Description

Figure 5 illustrates the single OD network with seven possible paths demarcated by colored arrows. There is a major corridor in the middle, labeled as path #5. The numbers on each link represents the free flow link travel time, while the numbers on the arrows at the bottom of the figure represent the free flow path travel time. There are two intersections. We call the intersection with nodes 7,8,9,10 '**Intersection A**', and the other intersection as '**Intersection B**'. For links inside each intersection, the left-turn lane costs 3 time units, the right-turn lane 2 time units, and the through lane 1 time unit. Each link is a one-way link with three lanes at most. For middle links $(1,10), (8,14)$, they each have 3 lanes for left, right, and through movements each. For the SO-DTA case in this network, we can see that the traffic flow entering later has no impact on the previous flow. If they are on different paths sharing lanes as $(1,10), (10,8), or\ (8,14)$, they will abide by **Lemma 1**; otherwise, they will not share lanes. The maximum discharge rate is set as $w_0 = 3$. And the red clearance $L_{allred}^{a,p} = 2$ time units for each phase of the two intersections.

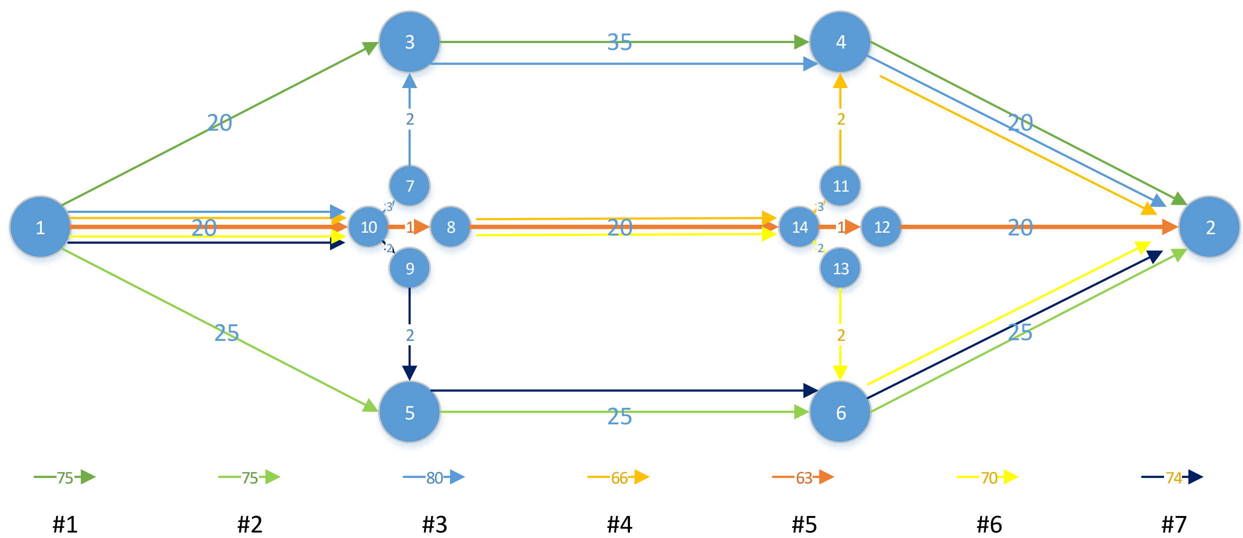


Figure 5 Single OD Network.

The 3-phase signal plan for each intersection is shown as **Figure 6**. Offset is generally referred as the time relationship between coordinated phases-defined reference point and a defined master reference (master clock or sync pulse). Here offset for each intersection is defined as the same value as offset for the left-turn phase in **Figure 6**, which was mentioned before **Equation (6).**

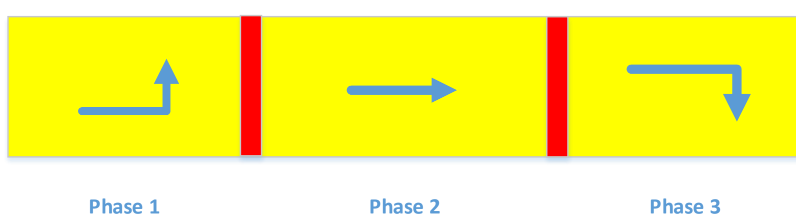


Figure 6 3-phase traffic signal plan.

Case Design

Each scenario has a demand level and signal control plan for both UE-DTA and SO-DTA cases. There are three time-varying demand levels of 1, 3, and 5 vehs per time unit. The demand profile length is 100 time units. In addition to 4 phasing plan groups as per **Table 2**, there are 12 scenarios.

Table 2 Three Groups of Phase Plan for Two Intersections

Time	**Intersection A**				**Intersection B**			
Units	Phase 1	Phase 2	Phase 3	Offset	Phase 1	Phase 2	Phase 3	Offset
Group 1	10	10	10	0	10	10	10	0
Group 2	20	20	20	0	20	20	20	0
Group 3	10	40	10	0	10	40	10	0
Group 4	10	40	10	0	10	40	10	20

The phase split and offset are the same for **Groups 1** and **2**, we will investigate the impact of changing cycle length with comparison between **Groups 1** and **2**. The cycle length and offset are the same for **Groups 2** and **3**, and we will investigate the impact of changing phase split with a comparison between these two groups. The cycle length and phase split are the same for **Groups**

3 and **4**, and we will investigate the impact of changing offset with comparison between these two groups.

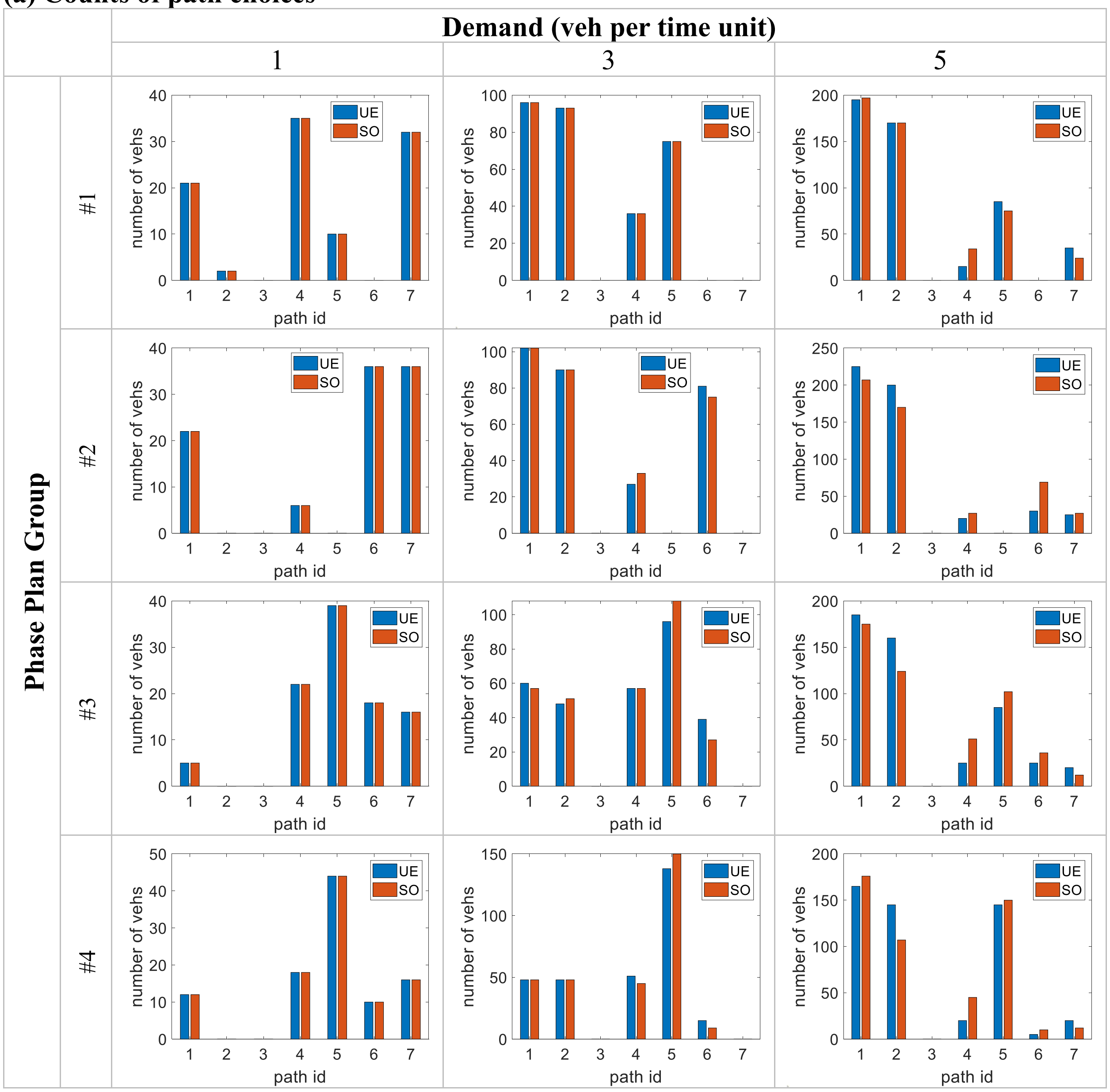
According to **Assumption 4**, travelers will choose paths with minimal experienced travel time in cases of UE-DTA. According to **Theorem 1**, travelers will choose paths with minimal marginal experienced travel time in cases of SO-DTA. Traffic assignment follows DUE and DSO conditions as **Equations (24)-(31)**.

Results

Results are show in **Table 3**.

Table 3 Results of 12 Scenarios

(a) Counts of path choices



(b) Average travel cost for each user

UE(SO)		Demand (veh per time unit)		
		1	3	5
Group of Phase Plan	**Group 1**	72.46(72.46)	74.18(74.18)	79.13(75.17)
	Group 2	72.76(72.76)	74.55(74.53)	79.70(75.16)
	Group 3	68.50(68.50)	73.02(72.84)	78.87(74.45)
	Group 4	67.50(67.50)	71.41(71.24)	78.03(74.51)

For different demand levels in both **Tables 3(a)** and **3(b)**, there are no differences between the UE-DTA cases and SO-DTA cases in either path sharing or average travel cost when demand is 1 veh per time unit. There are slight differences when demand increases to 3 vehs per time unit. However, there are significant differences when demand increases to 5 vehs per time unit. As is made clear in **Equations (17)** and **(20)**, the marginal cost increases only when 1) a vehicle joins the queue and it costs more to clear the queue than the vehicle ahead of it, and/or 2) when it becomes the head of the queue at a red signal. This situation rarely happens when demand is low but becomes more frequent as demand increases. As a result, the SO-DTA cases perform better for average travel cost than UE-DTA cases when demand is 5 vehs per time unit. When demand increases by the phase plan group, the average travel cost increases for both UE-DTA and SO-DTA cases, since there is greater likelihood that travelers will face queues, and the average queuing time will increase at higher demand levels.

The path sharing rates differ by the different signal plans (shown in columns of **Table 2 (a)**) because the signal wait times (and thus, queueing) are largely responsible for the path cost changes.

Group 2 has a phase plan with a longer cycle length than that of **Group 1**. In most cases, the average travel time of cases in **Group 2** is higher than in **Group 1**. However, when demand increases to 5, the SO-DTA case in **Group 2** is slightly better than that of **Group 1** (See in **Table 3 (b)**, 75.16 vs 75.17). We cannot draw conclusions on any trend for average travel cost when the cycle length changes, but evidence shows that the trend may be different between UE-DTA and SO-DTA, something to be investigated in our future research.

When increasing the green times for phase 2 in the signal plans of both intersections, there is a significant increase in volumes for path #5 in cases for **Group 3**, from no volumes for path # 5 in cases of **Group 2**, since the main path (#5) benefits the most among all paths. Increasing the green time for specific phases will absolutely change the path choices of travelers.

The offset of **Intersection B** is changed to 20 in **Group 4** from that in **Group 3**. As shown in **Figure 5,** the free flow travel time for link (8,14) is 20s. This increase provides greater likelihood for travelers on path #5 to meet green when arriving at **Intersection B**. This simple optimization for the signal plan of the one OD network improved most cases (except the SO-DTA cases at demand of 5) in **Group 4** that have the minimal average travel cost of the same demand level.

CONCLUSIONS AND FUTURE WORK

This work investigates the impact of traffic signal control – specifically, combinations of cycle length, phase spilt, and offset – on traffic assignment according to UE-DTA and SO-DTA.

To simulate the traffic dynamics, this paper first introduced a G/G/n/FIFO open queueing network. Equations were formulated for path travel time and marginal path travel time. In addition, DUE and DSO conditions help describe cases of UE-DTA and SO-DTA respectively.

We generated 12 cases for simulation based on three different demand inputs and four different signal phase plans. The average travel cost was used to evaluate all the cases. As expected, SO-DTA cases were shown to be superior to UE-DTA cases in terms of average travel cost, with this difference increasing with increasing demand. In addition, when we compared the impact of variables for signal controls as cycle length, phase split, and offset, the results showed that all three variables have significant effects on both path choices and average travel cost of users.

The purpose this research was to investigate the potential relationship between traffic assignment and traffic signal control. The main contribution of this paper is to introduce a basic model for combination problem of traffic control and traffic assignment, which is applicable in urban network. Thus, here are two main steps for future research: extend the current study to a larger scale network, and solve the signal optimization problem for both UE-DTA and SO-DTA cases.

(1) Corridor decomposition

Corridors are much more complex than those simulated here. However, large-scale corridor networks can involve large amounts of decision-making variables that make them a Nondeterministic Polynomial (NP) problem. The way to resolve this is to use a decentralized system that decomposes the whole network into subnetworks. Thus, traffic estimation, route guidance, and signal optimization will be simplified inside each subnetwork.

(2) Signal optimization for UE-DTA and SO-DTA

The variables for signal control in this research are cycle length X_{cycle}^{a}, phase split $X_{phase}^{a,p}$ (green time for the phase) and offset $X_{offset}^{a,p}$. There will be two cases for the optimization of signal control. For the UE-DTA case, users will choose the minimal experienced travel cost path. For the SO-DTA case, a virtual traffic management center (TMC) will assign traffic to achieve SO-DTA. In both cases, when we change the signal control, the traffic assignment will differ significantly. The optimization problem targeting at total travel time (TTT) can be formulated as below:

$$\begin{aligned} &\textbf{Min } \sum_{p\in P}\sum_{t=1}^{\Delta T} d^p(t) T_{path}^{p}(t) \\ &\textit{Subject to} \\ &\{d^p(t)\}_{0\le t\le \Delta T}^{p\in P} = \Phi_{UE/SO}\left(\{X_{cycle}^{a}\}_{a\in A^*}, \{X_{phase}^{a,p}\}_{a\in A^*}^{p\in P_a}, \{X_{offset}^{a,p}\}_{a\in A^*}^{p\in P_a}\right) \\ &X_{cycle}^{a} = \sum_{p\in P_a} X_{phase}^{a,p}, \forall a \\ &L_{allred}^{a,p} \le X_{phase}^{a,p} < X_{cycle}^{a}, \forall a, \forall p \end{aligned} \tag{32}$$

$\Phi\left(\{X_{cycle}^{a}\}_{a\in A^*}, \{X_{phase}^{a,p}\}_{a\in A^*}^{p\in P_a}, \{X_{offset}^{a,p}\}_{a\in A^*}^{p\in P_a}\right)$ is function with inputs as cycle length, phase splits and offsets of all the signalized intersections in the network, and outputs as time-varying demand of all paths. Here A^* is the set of links connecting to the signalized intersections. The principle of traffic assignment used is either the DUE or DSO condition.

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AUTHOR CONTRIBUTION STATEMENT

The authors confirm contributions to the paper as follows: study conception and design: Can Zhang, Amy Kim, and Zhijun Qiu; data collection: Can Zhang; analysis and interpretation of results: Can Zhang; draft manuscript preparation: Can Zhang, Amy Kim. All authors reviewed the results and approved the final version of the manuscript.

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