Problem Set 7

Problem 1. *Find the QR decomposition for the matrix*

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 2 & 1 \end{bmatrix}$$

Solution 1.

$$a_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, a_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$a_2' = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \frac{1}{5}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 6/5 \\ -3/5 \end{bmatrix}$$

$$q_1 = \begin{bmatrix} \frac{\sqrt{5}}{5} \\ \frac{2\sqrt{5}}{5} \end{bmatrix}, q_2 = \begin{bmatrix} \frac{2\sqrt{5}}{5} \\ -\frac{\sqrt{5}}{5} \end{bmatrix}$$

$$q_1{}^T a_1 = \sqrt{5}, q_1{}^T a_2 = -\frac{\sqrt{5}}{5}, q_2{}^T a_2 = \frac{3\sqrt{5}}{5}$$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{5}}{5} & \frac{2\sqrt{5}}{5} \\ \frac{2\sqrt{5}}{5} & -\frac{\sqrt{5}}{5} \end{bmatrix} \begin{bmatrix} \sqrt{5} & -\frac{\sqrt{5}}{5} \\ 0 & \frac{3\sqrt{5}}{5} \end{bmatrix} = QR$$

$$B = \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ \frac{2}{3} & \frac{\sqrt{2}}{2} \\ \frac{2}{3} & -\frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 3 & 3 \\ 0 & \sqrt{2} \end{bmatrix} = QR$$

Problem 2. *Calculate the area of triangle on the plane $\mathbb{R}^2$ with vertices $(2,1),(3,4),(0,5)$ using determinants. Also calculate the volume of parallelepiped on $\mathbb{R}^3$ created by vectors $(2,1,1),(3,4,1),(0,5,1)$.*

Solution 2.
Firstly, calculate the area by adding an one column:

$$A = \frac{1}{2}\begin{vmatrix} 2 & 1 & 1 \\ 3 & 4 & 1 \\ 0 & 5 & 1 \end{vmatrix} = 5$$

Then, calculate the volume of parallelepiped:

$$V = \begin{vmatrix} 2 & 1 & 1 \\ 3 & 4 & 1 \\ 0 & 5 & 1 \end{vmatrix} = 10$$

Problem 3. *Consider the following matrix A:*

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

If there exist λ that makes $\textbf{det}(A - \lambda I) = 0$? Find all of them. (Those are the eigenvalues of matrix A.)

Solution 3.

$$\begin{vmatrix} 1-\lambda & 1 & 1 & 1 \\ 1 & 1-\lambda & 1 & 1 \\ 1 & 1 & 1-\lambda & 1 \\ 1 & 1 & 1 & 1-\lambda \end{vmatrix} = \lambda^3(\lambda - 4) = 0$$

$$\lambda_1 = 0, \lambda_2 = 4$$