MA113 Linear Algebra
Peer Supporting Class
Provided by Zhang Ce

Problem Set 3

SUSTech Southern University of Science and Technology

October 25, 2022

Problem 1. *(2020 Fall Midterm - 16 points)Let*

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Please give a basis for each of the four fundamental subspaces $C(A)$, $N(A)$, $C(A^T)$, $N(A^T)$.

Solution 1. *Simplify to RREF form firstly.*

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 2 & 0 & -2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A basis for $C(A)$ (for reference):

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

A basis for $C(A^T)$ (for reference):

$$\begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

A basis for $N(A)$ (for reference):

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 1 & 0 & 0 \\ 0 & 1 & 2 & 4 & 6 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 \end{bmatrix}$$

A basis for $N(A^T)$ (for reference):

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 2 & 0 \\ 3 & 4 & 1 \\ 4 & 6 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 2 & 2 & 0 \\ 3 & 4 & 1 \\ 4 & 6 & 2 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

A basis for $C(A^T)$ (for reference):

$$\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 4 \\ 6 \end{bmatrix}$$

A basis for $N(A^T)$ (for reference):

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

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Problem 2. *(2019 Fall Midterm - 14 points)Let*

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 2 & 2 & 3 \\ -1 & -2 & 0 & 2 & 3 \end{bmatrix}$$

Determine the dimension and also give a basis for each of the four fundamental subspaces $C(A)$, $N(A)$, $C(A^T)$, $N(A^T)$.

Solution 2. *Simplify to RREF form firstly.*

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 2 & 2 & 3 \\ -1 & -2 & 0 & 2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 & -2 & -3 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank $r = 2$, $dim(C(A)) = r = 2$, $dim(C(A^T)) = r = 2$, $dim(N(A)) = n - r = 3$, $dim(N(A^T)) = m - r = 1$.
A basis for $C(A)$ (for reference):

$$\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

A basis for $C(A^T)$ (for reference):

$$\begin{bmatrix} 1 \\ 2 \\ 0 \\ -2 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

A basis for $N(A)$ (for reference):

$$\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 2 & 2 & 2 & 3 & 0 & 1 & 0 \\ -1 & -2 & 0 & 2 & 3 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 3 & 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -1 & 1 \end{bmatrix}$$

A basis for $N(A^T)$ (for reference):

$$\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 3 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 3 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

A basis for $C(A^T)$ (for reference):

$$\begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 3 \end{bmatrix}$$