Problem Set 2

Problem 1. *Let*

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 2 & 2 & 3 \\ -1 & -2 & 0 & 2 & 3 \end{bmatrix}$$

(a) *Find the complete solution to* $Ax = 0$.

(b) *Explain why* $Ax = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$ *is inconsistent.*

(c) *Find the complete solution to* $Ax = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$.

Solution 1.

(a)

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 2 & 2 & 3 \\ -1 & -2 & 0 & 2 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} x_1 + 2x_2 + x_3 = 0 \\ x_3 + 2x_4 + 3x_5 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = -2x_2 + 2x_4 + 3x_5 \\ x_3 = -2x_4 - 3x_5 \end{cases}$$

The nullspace solution $x_n = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2x_2 + 2x_4 + 3x_5 \\ x_2 \\ -2x_4 - 3x_5 \\ x_4 \\ x_5 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 3 \\ 0 \\ -3 \\ 0 \\ 1 \end{bmatrix}$

(b)

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 0 & 2 \\ 1 & 2 & 2 & 2 & 3 & 2 \\ -1 & -2 & 0 & 2 & 3 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 2 & 3 & 5 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

The third row gives $0 = 5$, *so the equation system is inconsistent.*

(c)

One column 3 gives b, *so a particular solution is* $x_p = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$.

The complete solution is

$$x = x_p + x_n = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 3 \\ 0 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

Problem 2. ***True*** *or* ***False****. Give explanations or find counter examples.*

1. *If x_p is a particular solution to $Ax = b$, then x_p is in the nullspace of A.*
2. *Linear equation systems $Ax = 0$ always have a solution.*
3. *The column space and the nullspace of the 5×3 rectangular matrix A have the same dimension.*

Solution 2.

1. ***False****.*

If the right-hand side $b = 0$, that is the case. The nullspace contains all the solutions to linear equation system $Ax = 0$. But if $b \neq 0$, we can not guarantee x_p is in the nullspace of A.

2. ***True****.*

Zero solution! Zero vector is in the nullspace of any matrix A.

3. ***False****.*

The 5×3 zero matrix have the column space of only the origin in $\mathbb{R}^5$ space (0 dimension), while the nullspace contains all the $\mathbb{R}^3$ vectors (3 dimensions).