Problem Set 12

Problem 1. *(2020 Fall Final Exam, 12 marks)Consider the quadratic form*

$$f(x_1, x_2, x_3) = (x_1 - x_2 + x_3)^2 + (x_2 + x_3)^2 + (x_1 + ax_3)^2$$

where $a \in \mathbb{R}$ is a parameter.
(a) What are the possible values of a if the quadratic form f is positive definite?
(b) What are the possible values of a if the equation $f(x_1, x_2, x_3) = 0$ has infinitely many solutions.
(c) Let y be a new system of variables and equation $f(x_1, x_2, x_3) = 0$ has infinitely many solutions. Find an invertible linear transformation $y = Px$, such that the quadratic form f has a diagonal form.

Problem 2. *(2019 Fall Final Exam, 15 marks) Consider the quadratic form*

$$f(x_1, x_2, x_3) = -2{x_1}^2 - 4{x_2}^2 - 5{x_3}^2 + 4x_1x_3$$

(a) Find the matrix A for the quadratic form $f(x_1, x_2, x_3)$.
(b) Decide for or against the positive definiteness of A.
(c) Find an orthogonal matrix Q to diagonalize A.
(d) Is there a real solution to the quadratic form $f(x_1, x_2, x_3) = 1$? Explain why.

Problem 3. *(2020 Fall Final Exam, 4 marks) Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}$. Find all the singular values of A.*