ECON 512 project: Replicating Chernozhukov and Hong 2003

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Problem with EE estimator

Extremum estimator with nonsmooth objective function has nice theoretic asymptotics, but computing extremum is problematic.

Example (Censored quantile regression)

$$\hat{\theta}_{EE} = \arg\max_{\theta \in \Theta} \frac{1}{n} \sum_{i=1}^{n} \rho_\tau(Y_i - \max(0, g(X_i, \theta))).$$

The objective function has too many angles and flat area. Smoothing does not seem to help, because computation problem comes from flatness.

Quasi-Bayesian estimator

If the extremum estimator is MLE

$$\hat{\theta}_{MLE} = \arg\max_{\theta \in \Theta} L_n(Y_i, X_i, \theta),$$

where L_n is log-likelihood. We can do MCMC use $e^{L_n(Y_i, X_i, \theta)}$ as posterior.

Quasi-posterior

$$p_n(Y_i, X_i, \theta) = \frac{e^{nQ_n(Y_i, X_i, \theta)} \pi(Y_i, X_i, \theta)}{\int_\Theta e^{nQ_n(Y_i, X_i, \theta)} \pi(Y_i, X_i, \theta) \, \mathrm{d}\theta}.$$

By this definition, $p_n(\theta)$ is a proper posterior.

Censored median regression

Data generating process:

$$\begin{aligned} Y^* =& \theta_0 + X\theta + \varepsilon, \\ X \sim& N(0, I_3), \\ \varepsilon \sim& N(0, X_2^2 I), \\ Y =& \max(0, Y^*). \end{aligned}$$

Use $\theta_0 = -6$, $\theta = (3, 3, 3)'$ to generate the data, which generates about 80% censoring.

median EE estimator

$$\hat{\theta} = \arg\max_{\theta \in \Theta} \frac{1}{n} \sum_{i=1}^{n} |Y_i - \max(0, g(X_i, \theta))|.$$

Surface of objective function

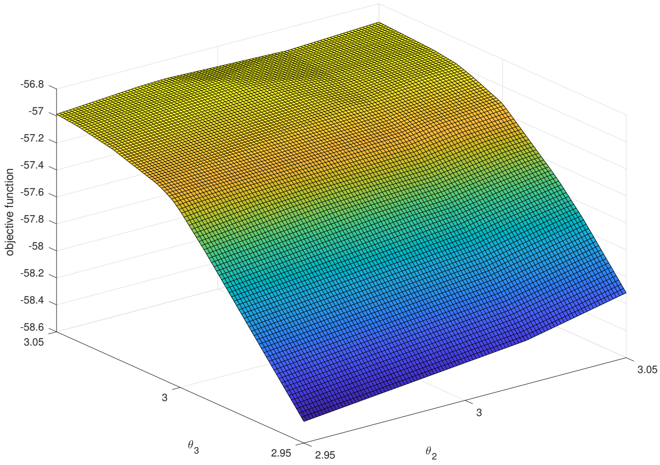

Figure 1: Surface of objective function

Iterative linear programming

Buchinsky (1994) proposed ILP to solve censored quantile regression.

1. Start with some $\hat{\theta}^{(0)}$ for $j = 0$.
2. Compute $X\hat{\theta}^{(j)}$ and collect the subsample $S_j = \{i : x_i'\hat{\theta}^{(j)} \geq 0\}$.
3. Use only subsample S_j, run the standard quantile regression with linear programming. With new $\hat{\theta}^{(j+1)}$, compute S_{j+1}.
4. If $S_{j+1} = S_j$, then stop and set estimate to $\hat{\theta}^{(j+1)}$. Otherwise, set $j = j + 1$ and repeat step.3.

Quasi-Bayesian method

Choose prior uniform over $\Theta = [\theta_0 - 10, \theta_0 + 10]$, use

$$p_n(Y_i, X_i, \theta) = \frac{e^{nQ_n(Y_i, X_i, \theta)}\pi(Y_i, X_i, \theta)}{\int_\Theta e^{nQ_n(Y_i, X_i, \theta)}\pi(Y_i, X_i, \theta)\, \mathrm{d}\theta}$$

as posterior, run MCMC.
Each parameter is updated via Gibss-Metroplis procedure, which modifies slightly the basic Metroplis-Hastings algorithm: for each update, only update one component each time. Adjust variance of innovation every 200 draws so that the rejection probability is roughly 50%.

Performance of QBE and ILP

	RMSE	MAD	mean bias
n = 400			
QBE-mean	1.9882	0.5445	0.5651
QBE-median	1.6338	0.4796	0.4120
ILP (18)	0.9424	0.2867	0.0345
	3.1080	0.9505	0.0845
n = 1600			
QBE-mean	0.3067	0.1051	0.0933
QBE-median	0.2913	0.1004	0.0717
ILP (11)	0.2236	0.0742	0.0071
	2.6528	0.7371	0.0234

Table 1: ex