ECON 512: Computation project

Chen Zhang

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1 Introduction

A lot of econometric problems work well in theory, but they may be computational challenging. For example, many models of extremum estimators are known to be difficult to compute due to highly nonconvex criterion functions with many local optima (but well pronounced global optimum), e.g. instruement quantile regression, censored and nonlinear quantile regression. In Chernozhukov and Hong (2003, JoE) they proposed a class of estimators, quasi-Bayesian estimators or Laplace type estimator (LTE), which are defined similar to Bayesian estimators and can be computed by MCMC method. This estimator is computational attractive, because it transforms the optimization problem of extremum estimators to an numerical integration problem, which does not suffer to the problem of nonconvexity of objective function.

2 Laplacian or quasi-Bayesian estimator

This paper takes advantage of LTE to investigate computation problem in censored data. Consider the model

Suppose we have a MLE estimator defined as

$$\hat{\theta}_{MLE} = \arg\sup_{\theta\in\Theta} L_n(\theta),$$

where $L_n(\theta)$ is the log-likelihood function. To implement Bayesian estimation for any prior $\pi(\theta)$, the posterior is

$$p(\theta|y,x) = e^{L_n(\theta)}\pi(\theta),$$

where $L_n(\theta)$ is the objective function of maximum likelihood estimator, the likelihood function. We can see there is a natural connection between maximum likelihood and Bayesian method.

Now, we consider a general extremum estimator problem

$$\hat{\theta}_{EE} = \arg\sup_{\theta\in\Theta} Q_n(\theta),$$

where $Q_n(\theta)$ can be an objective function of any extremum estimator. If $Q_n(\theta) = \frac{1}{n}L_n(\theta)$ is the objective function of MLE, for any prior $\pi(\theta)$ we can use $e^{nQ_n(\theta)}\pi(\theta)$ as posterior and do Bayesian with no effort. If it is not, the transformation

$$p_n(\theta) = \frac{e^{nQ_\theta}\pi(\theta)}{\int_\Theta e^{nQ_n\pi(\theta)}\ \mathrm{d}\theta},$$

is a proper distribution density and can be used as posterior, called here the *quasi-posterior*. Here $\pi(\theta)$ is a weight function or prior probability density that is strictly positive over Θ. Note that $p_n(\theta)$ is generally not a true posterior in Bayesian sense, since $nQ_n(\theta)$ may not be a likelihood.

The quasi-posterior mean is then defined as

$$\hat{\theta} = \int_\Theta \theta p_n(\theta)\ \mathrm{d}\theta = \int_\Theta \theta \frac{e^{nQ_n}\pi(\theta)}{\int_\Theta e^{nQ_n}\pi(\theta)\ \mathrm{d}\theta}.$$

Follow the spirit of Bayesian, using Markov chain Monte Carlo method, we can draw a Markov chain,

$$S = (\theta^{(1)}, \theta^{(2)}, ..., \theta^{(B)}),$$

whose marginal density is given by $p_n(\theta)$. Then the estimate $\hat{\theta}$ can be computed as

$$\hat{\theta} = \frac{1}{B}\sum_{i=1}^{B}\theta^{(i)}.$$

The confidence interval can be constructed based on the quantile of $S = (\theta^{(1)}, \theta^{(2)}, ..., \theta^{(B)})$.

3 Censored median regression

In this section, I will apply the quasi-Bayesian estimator to a simulated censored median model and compare it with the extensively used iterative linear programming algorithm.

3.1 The model

Consider the model

$$\begin{aligned} Y_1 =& X\beta_1 + u_1, \\ Y_2 =& X\beta_2 + u_2, \\ \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} &\sim N(0, X_2^2 I_2), \\ Y^1 &\text{ is observed when } Y^1 > Y^2. \end{aligned}$$

We can think of Y_2 as an alternative of Y_1, Y_1 is chosen only if $Y_1 > Y_2$. So the data of Y_1 will be censored at Y_2.

Although in this model the censored point Y_2 is not fixed, by assumption we can observe all data of Y_2. Therefore, we can get a consistent estimate β_2 in the first step with no effort, and then obtain the fitted value $\hat{Y}_2$. By the following transformation the model reduced to censored regression with fixed censoring point

$$Y_3 = Y_1 -$$