ECON 512 project: Replicating Chernozhukov and Hong (2003)

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Problem with EE estimator

Extremum estimator with nonsmooth objective function has nice theoretic asymptotics, but computing extremum is problematic.

Example (Censored quantile regression)

$$\hat{\theta}_{EE} = \arg\max_{\theta \in \Theta} \frac{1}{n} \sum_{i=1}^{n} \rho_\tau(Y_i - \max(0, g(X_i, \theta))).$$

The objective function has too many angles and flat area. Smoothing does not seem to help, because computation problem comes from flatness.

Quasi-Bayesian estimator

If the extremum estimator is MLE

$$\hat{\theta}_{MLE} = \arg\max_{\theta \in \Theta} L_n(Y, X, \theta),$$

where L_n is log-likelihood. We can do MCMC use $e^{L_n(Y,X,\theta)}$ as posterior.

Quasi-posterior

$$p_n(Y, X, \theta) = \frac{e^{nQ_n(Y,X,\theta)}\pi(Y, X, \theta)}{\int_\Theta e^{nQ_n(Y,X,\theta)}\pi(Y, X, \theta)\, \mathrm{d}\theta}.$$

By this definition, $p_n(\theta)$ is a proper posterior.

Censored median regression

Data generating process:

$$
\begin{aligned}
Y^* &= \theta_0 + X\theta + \varepsilon, \\
X &\sim N(0, I_3), \\
\varepsilon &\sim N(0, X_2^2 I), \\
Y &= \max(0, Y^*).
\end{aligned}
$$

Use $\theta_0 = -6$, $\theta = (3, 3, 3)'$ to generate the data, which generates about 80% censoring.

median EE estimator

$$
\hat{\theta} = \arg\max_{\theta \in \Theta} \frac{1}{n} \sum_{i=1}^{n} |Y_i - \max(0, g(X_i, \theta))|.
$$

Surface of objective function

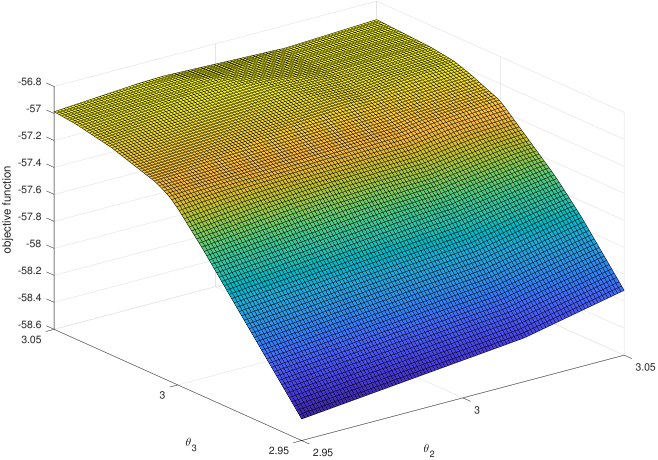

Figure 1: Surface of objective function

Iterative linear programming

Buchinsky (1994) proposed ILP to solve censored quantile regression.

1. Start with some $\hat{\theta}^{(0)}$ for $j = 0$.
2. Compute $X\hat{\theta}^{(j)}$ and collect the subsample $S_j = \{i : x_i'\hat{\theta}^{(j)} \geq 0\}$.
3. Use only subsample S_j, run the standard quantile regression with linear programming. With new $\hat{\theta}^{(j+1)}$, compute S_{j+1}.
4. If $S_{j+1} = S_j$, then stop and set estimate to $\hat{\theta}^{(j+1)}$. Otherwise, set $j = j + 1$ and repeat step.3.

Quasi-Bayesian method

Choose prior uniform over $\Theta = [\theta_0 - 10, \theta_0 + 10]$, use

$$p_n(Y, X, \theta) = \frac{e^{nQ_n(Y,X,\theta)}\pi(Y, X, \theta)}{\int_\Theta e^{nQ_n(Y,X,\theta)}\pi(Y, X, \theta)\, \mathrm{d}\theta}$$

as posterior, run MCMC.
Each parameter is updated via Gibss-Metroplis procedure, which modifies slightly the basic Metroplis-Hastings algorithm: for each update, only update one component each time. Adjust variance of innovation every 200 draws so that the rejection probability is roughly 50%.

Performance of QBE and ILP

	RMSE	MAD	Median bias
n = 1600			
QBE-mean	0.307	0.105	-0.006
QBE-median	0.291	0.100	-0.005
ILP (11)	0.227	0.074	0.006
	2.653	0.737	0.005
CH(2003)			
QBE-mean	0.155	0.121	0.009
QBE-median	0.155	0.121	0.002
ILP (7)	0.134	0.106	0.067
	3.547	0.511	-0.384

Table 1: Performance of estimators