

$$\begin{array}{ccccc}
\mathbf{mod}_{R^{\mathrm{op}}} & \xleftarrow{+\lim^{<\infty}} & \mathbf{add}(R) & & (\mathrm{cok}(f)_R, -)_R \\
& & & & \uparrow \\
- \otimes_R R^m & & {}_R R^m & & (R_R^m, -)_R \\
\downarrow f_! & & \downarrow f & & \uparrow f^* \\
- \otimes_R R^n & & {}_R R^n & & (R_R^n, -)_R \\
\downarrow & & & & \\
- \otimes_R \mathrm{cok}(f) & & & & \\
& & & & \mathbf{add}(R) \xrightarrow{+\lim^{<\infty}} (\mathbf{mod}_R)^{\mathrm{op}}
\end{array}$$