

$$\begin{array}{ccccccc}
& & & (S, ?)_\mathcal{T} & & & \\
& & \nearrow & \downarrow (\kappa_i)_* & \searrow 0 & & \\
0 & \longrightarrow & \ker[(\pi_i)_S] & \longrightarrow & (S, X_i)_\mathcal{T} & \xrightarrow{(\pi_i)_S} & FS \longrightarrow 0 \\
& & \downarrow & & \downarrow (\phi_i)_* & & \parallel \\
0 & \longrightarrow & \ker[(\pi_{i+1})_S] & \longrightarrow & (S, X_{i+1})_\mathcal{T} & \xrightarrow{(\pi_{i+1})_S} & FS \longrightarrow 0
\end{array}$$