

$$\begin{array}{ccccccc}
& & \coprod_{i \in I} (P_i \oplus Q_i) & \longrightarrow & \coprod_{i \in I} P_i & & \\
& & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \ker \pi \cap \varinjlim^{fil} (P_\bullet \oplus Q_\bullet) & \hookrightarrow & \varinjlim^{fil} (P_\bullet \oplus Q_\bullet) & \twoheadrightarrow & \varinjlim^{fil} P_\bullet \longrightarrow 0 \\
& & \downarrow & & \downarrow & & \parallel \\
0 & \longrightarrow & \ker \pi & \hookrightarrow & P & \twoheadrightarrow & F \longrightarrow 0
\end{array}$$