$$C \xrightarrow{f_0} \prod_{X' \in \mathscr{X}} \prod_{f \in (C, X')} X'$$