

$$\begin{array}{ccc}
 X & \xrightarrow{\quad} & M_k \\
 \downarrow & \circlearrowleft & \downarrow \\
 M_j & \xrightarrow{\text{split}} & M_j \sqcup_X M_k
 \end{array}$$

A commutative diagram with two rows and two columns. The top row consists of X followed by a horizontal arrow pointing to M_k . The bottom row consists of M_j followed by a horizontal arrow labeled "split" pointing to $M_j \sqcup_X M_k$. A vertical arrow points from X down to M_j . Another vertical arrow points from M_k down to $M_j \sqcup_X M_k$. A dashed diagonal arrow points from M_j up and to the right to M_k . A small circle with a clockwise arrow is located in the upper right quadrant of the diagram.

$$\begin{array}{ccc}
 X & \xrightarrow{\quad} & \varinjlim^{fil} M_k \\
 \downarrow & \circlearrowleft & \downarrow \text{split} \\
 M_j & \xrightarrow{\text{pure.}} & M_j \sqcup_X \varinjlim^{fil} M_k
 \end{array}$$

A commutative diagram similar to the one on the left, but with a more complex top-right object. The top row consists of X followed by a horizontal arrow pointing to $\varinjlim^{fil} M_k$. The bottom row consists of M_j followed by a horizontal arrow labeled "pure." pointing to $M_j \sqcup_X \varinjlim^{fil} M_k$. A vertical arrow points from X down to M_j . Another vertical arrow points from $\varinjlim^{fil} M_k$ down to $M_j \sqcup_X \varinjlim^{fil} M_k$, and this arrow is labeled "split" to its right. A dashed diagonal arrow points from M_j up and to the right to $\varinjlim^{fil} M_k$, and this arrow is labeled e_j to its right. A small circle with a clockwise arrow is located in the upper right quadrant of the diagram.