$$0 \longrightarrow M \hookrightarrow X_{\beta} \longrightarrow C_{\beta} \longrightarrow 0 \qquad \theta_{\beta}$$

$$\downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow$$

$$0 \longrightarrow M \hookrightarrow \operatorname{im}(\alpha_{\beta}) \longrightarrow \operatorname{im}(\alpha_{\beta}|_{\operatorname{cok}}) \longrightarrow 0 \qquad \varepsilon$$

$$\downarrow \qquad \qquad \downarrow$$

$$0 \longrightarrow M \hookrightarrow X_{\beta+1} \longrightarrow C_{\beta+1} \longrightarrow 0 \qquad \theta_{\beta+1}$$