$$
\begin{tikzcd}
\coprod_{g\in\ker_n} |\partial\Delta^{n+1}| \arrow[rr, "\coprod_{g\in\ker_n} |i^{n+1}|"] \arrow[d, "(a_g)_{g\in\ker_n}"'] \arrow[drr, phantom, "\mathrm{PO}"] & & \coprod_{g\in\ker_n} |\Delta^{n+1}| \arrow[d] \arrow[ddr, bend left] & \\
K_n \arrow[rr, hook, "\iota_n"'] \arrow[drrr, hook, bend right=10, "f_n"'] & & K_n' \arrow[dr, dashed, "f_{n+1}'"'] & \\
& & & X
\end{tikzcd}
$$