

$$\begin{array}{ccc}
S & \mathcal{A} & S \times S \\
& \begin{array}{c} \xleftarrow{(X,Y) \mapsto X \oplus Y} \\ \xrightarrow{X \mapsto (X,X)} \end{array} & \mathcal{A} \times \mathcal{A} \\
& \downarrow Q_S & \downarrow Q_{S \times S} \\
& & (\mathcal{A} \times \mathcal{A})[(S \times S)^{-1}] \\
& & \downarrow \sim \\
\mathcal{A}[S^{-1}] & \begin{array}{c} \xleftarrow{(X,Y) \mapsto X \oplus Y} \\ \xrightarrow{X \mapsto (X,X)} \end{array} & (\mathcal{A}[S^{-1}]) \times (\mathcal{A}[S^{-1}])
\end{array}$$