

$$\begin{array}{ccccc}
 \partial\Delta^1 \times \Lambda_k^n & \longrightarrow & \Delta^1 \times \Lambda_k^n & & \\
 \downarrow & & \downarrow & & \searrow p|_{\Delta^1 \times \Lambda_k^n} \\
 \partial\Delta^1 \times \Delta^n & \xrightarrow{\text{PO}} & \square & & \\
 \nearrow & \searrow & \downarrow & & \\
 \Lambda_k^n & & & & \Lambda_k^n \\
 \downarrow & & & & \downarrow \\
 \Delta^n & \xrightarrow{\iota_1} & \Delta^1 \times \Delta^n & \xrightarrow{p} & \Delta^n
 \end{array}$$