

A commutative diagram illustrating the relationships between various spaces and maps in a cochain complex across indices $n-1$, n , and $n+1$.

The diagram consists of the following nodes and maps:

- Top Row:**
 - X^{n-1} maps to B^n via a horizontal arrow.
 - B^n maps to X^n via a horizontal arrow.
 - X^n maps to B^{n+1} via a horizontal arrow.
 - B^{n+1} maps to X^{n+1} via a horizontal arrow.
 - A curved arrow labeled d^{n-1} maps X^{n-1} to X^n .
 - A curved arrow labeled d^n maps X^n to X^{n+1} .
- Bottom Row:**
 - H^{n-1} maps to C^{n-1} via a diagonal arrow.
 - C^{n-1} maps to B^n via a diagonal arrow.
 - B^n maps to Z^n via a diagonal arrow.
 - Z^n maps to H^n via a diagonal arrow.
 - H^n maps to C^n via a diagonal arrow.
 - C^n maps to B^{n+1} via a diagonal arrow.
 - B^{n+1} maps to Z^{n+1} via a diagonal arrow.
 - Z^{n+1} maps to H^{n+1} via a diagonal arrow.
- Intermediate Row:**
 - C^{n-1} maps to Z^n via a diagonal arrow.
 - Z^n maps to C^n via a diagonal arrow.
 - C^n maps to Z^{n+1} via a diagonal arrow.
 - Z^{n+1} maps to C^{n+1} via a diagonal arrow.

The diagram shows the following commutative properties:

- $X^{n-1} \xrightarrow{d^{n-1}} X^n \xrightarrow{d^n} X^{n+1}$ is a cochain complex.
- $B^n \subset X^n$ and $B^{n+1} \subset X^{n+1}$ are boundaries.
- $Z^n \subset B^n$ and $Z^{n+1} \subset B^{n+1}$ are cycles.
- $C^{n-1} \subset X^{n-1}$ and $C^n \subset X^n$ are coboundaries.
- $H^{n-1} \subset C^{n-1}$ and $H^n \subset C^n$ are cohomology classes.