

$$\begin{array}{ccc}
 P & \xrightarrow{a} & \mathcal{H}\mathcal{I}\mathcal{I}(Y, A) \\
 m \downarrow & \nearrow s & \downarrow \Phi(f, g) \\
 Q & \xrightarrow{b} & \mathcal{H}\mathcal{I}\mathcal{I}(X, A) \times_{\mathcal{H}\mathcal{I}\mathcal{I}(X, B)} \mathcal{H}\mathcal{I}\mathcal{I}(Y, B)
 \end{array}
 \qquad
 \begin{array}{ccc}
 (Y \times P) \sqcup_{(X \times P)} (X \times Q) & \xrightarrow{(a^*, (\text{pr}_1 \circ b)^*)} & A \\
 m \wedge f \downarrow & \nearrow t & \downarrow g \\
 Y \times Q & \xrightarrow{(\text{pr}_2 \circ b)^*} & B
 \end{array}$$