$$
\begin{array}{ccccc}
S & \mathscr{A} & \xleftarrow{\;(X,Y)\mapsto X\oplus Y\;} & \mathscr{A}\times\mathscr{A} & S\times S \\
 & & \xrightarrow{\;X\mapsto(X,X)\;} & \\
 & \Big\downarrow{\scriptstyle Q_S} & & \Big\downarrow{\scriptstyle Q_{S\times S}} & \\
 & & & (\mathscr{A}\times\mathscr{A})[(S\times S)^{-1}] & \\
 & & & \Big\downarrow{\scriptstyle\sim} & \\
 & \mathscr{A}[S^{-1}] & \xleftarrow{\;(X,Y)\mapsto X\oplus Y\;} & (\mathscr{A}[S^{-1}])\times(\mathscr{A}[S^{-1}]) & \\
 & & \xdashrightarrow{\;X\mapsto(X,X)\;} & \\
\end{array}
$$