

$$\begin{array}{ccc}
(X, -)_{\mathcal{X}} & \xrightarrow{f^*} \twoheadrightarrow & (C, i(-))_{\mathcal{C}} \\
\downarrow \theta & & \parallel \\
(X, -)_{\mathcal{X}} & \xrightarrow{f^*} \twoheadrightarrow & (C, i(-))_{\mathcal{C}}
\end{array}
\quad \Longrightarrow \quad \alpha \in \operatorname{Aut}(h^X)$$