$$\operatorname{coCone}(S^{\perp},\mathbf{Fil}(S)) \quad \ni \quad X \longleftarrow X_F \longrightarrow X_C \qquad : \qquad \theta_X$$
 
$$\tau_X \qquad : \qquad K_F \longleftarrow \mathbf{Free}(X) \sqcup_K K_F \longrightarrow X \quad \in \quad \operatorname{Cone}(S^{\perp},\mathbf{Free} * \mathbf{Fil}(S))$$