

$$\begin{array}{ccccc}
0 & \longrightarrow & ? & \longrightarrow & 0 \\
\downarrow & & \downarrow & & \downarrow \\
\prod_{i \geq 0} \ker[(\pi_i)_{S \amalg J}] & \longrightarrow & \prod_{i \geq 0} (S \amalg^J, X_i)_{\mathcal{T}} & \xrightarrow{(\pi_i)_{S \amalg^J}} & \prod_{i \geq 0} F(S \amalg^J) \\
\downarrow 1 - \sum 0 & & \downarrow 1 - \sum (\phi_i)_* & & \downarrow 1 - \sum 1 \\
\prod_{i \geq 0} \ker[(\pi_i)_{S \amalg J}] & \longrightarrow & \prod_{i \geq 0} (S \amalg^J, X_{i+1})_{\mathcal{T}} & \xrightarrow{(\pi_{i+1})_{S \amalg^J}} & \prod_{i \geq 0} F(S \amalg^J) \\
\downarrow & & \downarrow & & \downarrow \\
0 & \longrightarrow & ? & \longrightarrow & F(S \amalg^J)
\end{array}$$