

Fix size input equal probability random sampling

Each element i , has a probability of $x = \frac{K-|S|}{N-i} = \frac{K}{N}$ to put into the sample set S .

When $i = 0, x = \frac{K}{N}, as |S| = 0$.

When $i = 1, x = \frac{K}{N} \frac{K-1}{N-1} + \frac{N-K}{N} \frac{K}{N-1} = \frac{K}{N}$

There are 2 cases, first element didn't select with probability $\frac{N-K}{N}$, or first element selected into the sample set with probability $\frac{K}{N}$.

When $i = 2, x = \left(\frac{K}{N}\right)^2 \frac{K-2}{N-2} + 2 \frac{K}{N} \frac{N-K}{N} \frac{K-1}{N-2} + \left(\frac{N-K}{N}\right)^2 \frac{K}{N-2} = \frac{K}{N}$

There are 3 cases, both first and second elements are selected with probability $\left(\frac{K}{N}\right)^2$, either one of the first two elements is selected with probability $2 \frac{K}{N} \frac{N-K}{N}$, or both first and second elements aren't selected with probability $\left(\frac{N-K}{N}\right)^2$.

Up until $i = N - 1$