Homework 8

Problem 1. Two aliens have arrived on Earth, each claiming to possess a machine that can solve the SAT problem in polynomial time. However, only one of these machines is genuine, while the other is a counterfeit. Your task is to design a protocol for solving the SAT problem in polynomial time by asking questions to their machines.

Solution. As shown in class, if we have a yes/no machine D deciding the SAT problem, we can find the solution in polynomial time by checking, for i in $1, 2, 3, ..., n$, $D(\phi(a_1, a_2, ..., a_{i-1}, 0, x_{i+1}, x_{i+2}, ..., x_n))$ and let $a_i = 0$ if it accepts and $a_i = 1$ if otherwise. Finally we check if $a_1, a_2, ..., a_n$ is a solution.

Denote the two machines as T_1 and T_2. Following the above procedure, we can construct a machine T as follows:

On input ϕ:

1. For i in $1, 2, ..., n$:
2. (a) check $T_1(\phi(a_1, a_2, ..., a_{i-1}, 0, x_{i+1}, x_{i+2}, ..., x_n))$
   (b) Set $a_i = 0$ if it accepts and $a_i = 1$ if otherwise.
   (c) check $T_2(\phi(b_1, b_2, ..., b_{i-1}, 0, x_{i+1}, x_{i+2}, ..., x_n))$
   (d) Set $b_i = 0$ if it accepts and $b_i = 1$ if otherwise.
3. check $\phi(a_1, a_2, ..., a_n)$ and $\phi(b_1, b_2, ..., b_n)$ by classical method. If at least one of them is correct, accept. Else, reject.

Because one of the two machines is correct, we can suppose it's T_1. This way, $a_1, ..., a_n$ should be the correct solution if ϕ is satisfiabe, and T will accept ϕ. If ϕ isn't satisfiable, then apparently $a_1, ..., a_n$ and $b_1, ..., b_n$ both can't satisfy ϕ, so T will reject ϕ. Therefore, T is a correct machine to decide the SAT problem.

Meanwhile, since we can do the varification in polynomial time, and T_1 and T_2 can give the answer in polynomial time and we run the two machines for n times, so T can decide the SAT problem in polynomial time.