

Project 2: Creation of Spatial Structure by AICA

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1. Experiment Design

In this project, we are aiming at test various combinations of the five parameters ($J1$, $J2$, $R1$, $R2$, and h) and investigate their effect on the quantitative measures and the qualitative behavior of the system. Three experiments are designed to test each parameter respectively. The first experiment set disables inhibition by assign $J2 = 0$, and the second set disables activation by setting $J1 = 0$, while the third set enables both inhibition and activation.

In set one, there are 9 sub-experiments with different $R1$, $R2$ and h values, and set two and three have 17 sub-experiments with different $R1$, $R2$ and h values. In all of the experiments, each running for 4 times, there are quite a lot nice behavior AICA, but also has some system converged to extreme states of white or black board.

2. Convergence of AICA

To prove the synchronous update of grids will lead to failure of convergence, we exhibit a counter-example of 4x4 grids generating an AICA that oscillates between two states.

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***** Activator/Inhibitor CA *****
J1 = 1, J2 = -2, R1 = 2, R2 = 5, h = 1

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Figure 1. Counter-example of asynchronous updating

3. Correlation, Joint entropy and Mutual information

a. Disable inhibition and enable activation ($J1 = 1, J2 = 0$)

Disable the inhibition system by setting $J2 = 0$ and $J1 = 1$. This means that $R2$ has no effect on the system, so the system only depends on $R1$ and h , and we set $R2 = 15$, which will not influence the results.

Table 1. Parameters with Inhibition Disabled ($J1 = 1, J2 = 0$)

$R1$	$R2$	h	Observation
1	15	-1	Chaotic, converge very quickly; Very low rho
1	15	0	As above
1	15	1	Converge to black board
3	15	-1	Converge to regular pattern, the edges of patter are thin
3	15	0	Converge to regular pattern, faster, the edges of patter are bolder
3	15	2	Converge to black board, faster than above (rho=1)
6	15	-2	Converge to black board, faster than above (rho=1)
6	15	0	Separated black and while pattern, very bold pattern
6	15	1	Converge to black board

For the first three experiments, since system converges to very chaotic board, rho value is less than 0.1, and both entropy and mutual information are not changed too much with distance l . If system eventually converges to black board, rho values are always 1 during the converging process. This makes sense since high spatial correlation and activation only force the whole board converge to all +1 state. In the case of very low spatial correlation such as the first three experiments in Table 1, system almost halts in the initial chaotic state, since weak spatial correlation and short activation radius can hardly effect cells in the board.



Figure 2a. $J1=1, J2=0, R1=3, R2=15, h=0$

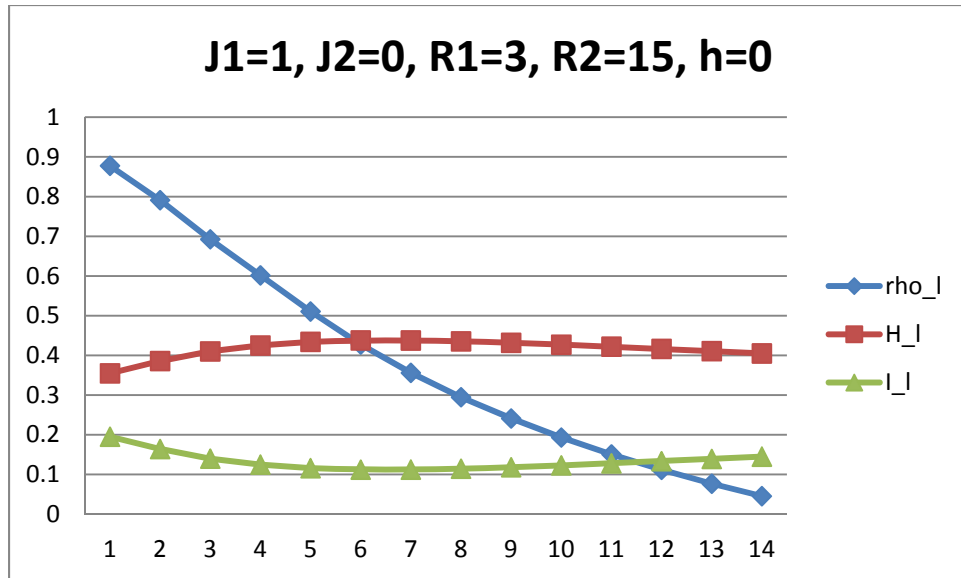


Figure 2b. Correlation, Joint entropy and Mutual information

Notice that the curves for entropy and mutual information are essentially mirror images of each other. This makes sense since entropy and mutual information are basically inverses of one another.

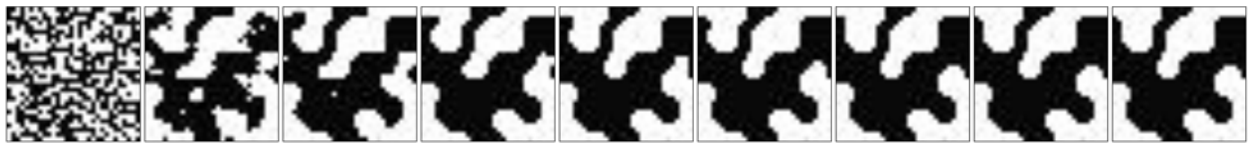


Figure 3a. J1=1, J2=0, R1=3, R2=15, h= -1

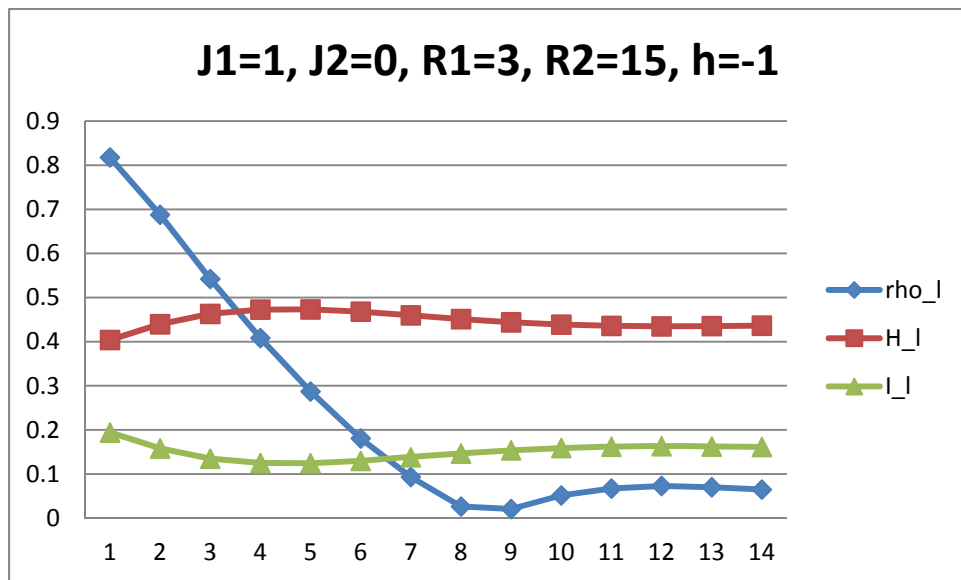


Figure 3b. Correlation, Joint entropy and Mutual information



Figure 4a. $J1=1, J2=0, R1=3, R2=15, h=-2$

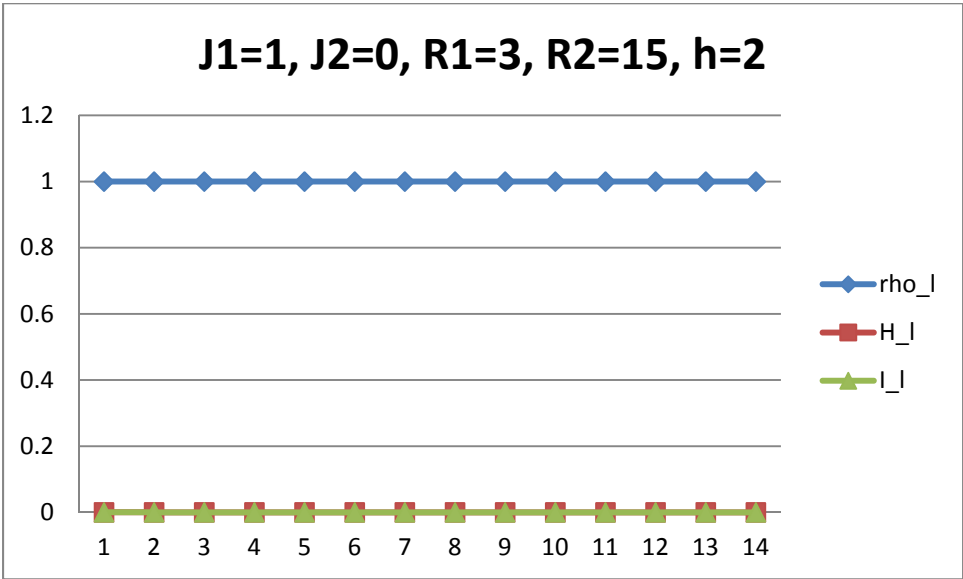


Figure 4b. Correlation, Joint entropy and Mutual information



Figure 5a. $J1=1, J2=0, R1=6, R2=15, h=0$

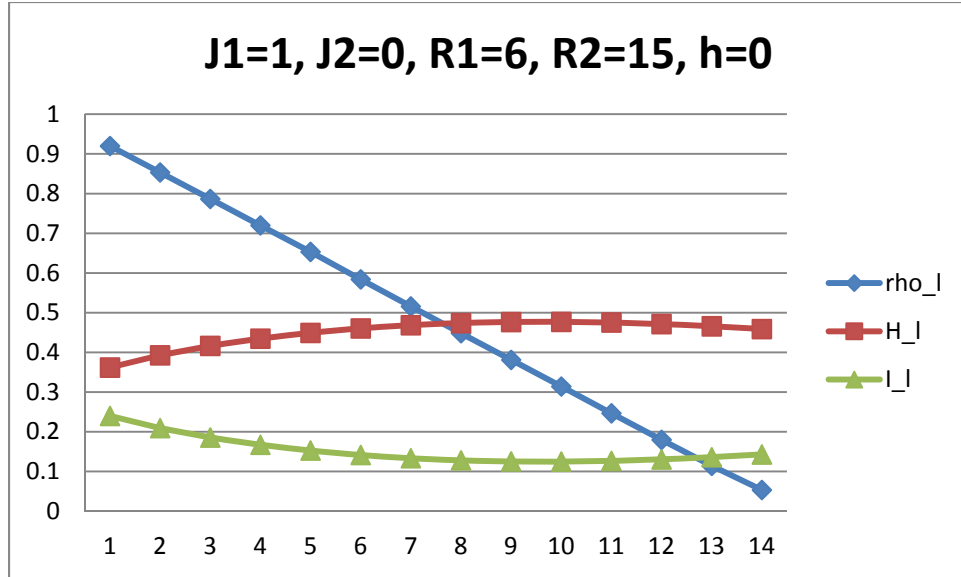


Figure 5b. Correlation, Joint entropy and Mutual information

From this experiment set, we can draw the conclusion that disabling inhibition leads to more black board(all +1 state), and increase the radius of $R1$ will increase the influence of activation, which results in bolder blocks or lines in final pattern.

b. Disable activation and enable inhibition ($J1 = 0, J2 = -0.1$)

Disable the activation system by setting $J1 = 0$ and $J2 = -0.1$. This means that $R1$ has no effect on the system, so the system only depends on $R2$ and h .

Table 2. Parameters with Inhibition Disabled ($J1 = 0, J2 = -0.1$)

$R1$	$R2$	h	Observation
1	2	0	Regular pattern, short and thin lines
1	4	-2	Regular pattern, short white lines
1	4	-1	Regular pattern, short black lines
1	4	0	Regular pattern, bolder lines
1	6	-5	Regular black lines
1	6	-3	Regular black dot lines, bolder
1	6	0	Regular black dot lines, bolder
1	9	0	Regular black lines, bolder
1	13	0	Regular black lines, bolder
4	5	0	Regular grids
4	7	-5	Black dots
4	7	-3	Small black blocks
4	7	0	Regular black lines, bolder
4	12	0	Regular black lines, bolder
9	12	-6	larger black blocks
9	12	-3	Regular black lines, bolder
9	12	0	Regular black lines, bolder

In this experiment set, almost every system exhibits very nice pattern. If we fixed two parameters and tuning the third one, many interesting observation can be made.

1) Tuning h while fix $R2$ ($R1$ has no effect)

Given experiments $R1 = 1$, $R2 = 6$, $h = -5, -3, 0$, we display the final states of three experiments. From Fig 6, it is obvious that when bias h increase, more cells become $+1$ state, so that black lines are bolder.

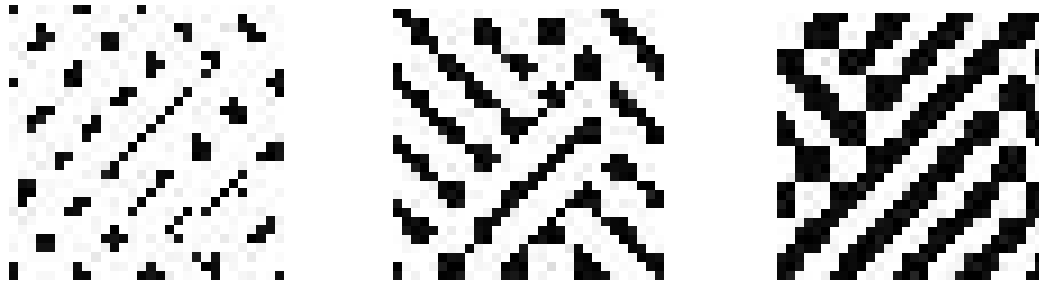


Figure 6. Compare AICA systems with different h values

2) Tuning $R2$, while fix h

Given experiments $h = 0$, $R2 = 2, 4, 5, 6, 7, 9, 13, 12$ we display the final states of three experiments. If bias is set to be zero, as inhibition radius $R2$ increases, pattern granularity increase, which means cells tend to behavior the in the same way, except those in the edge of pattern.

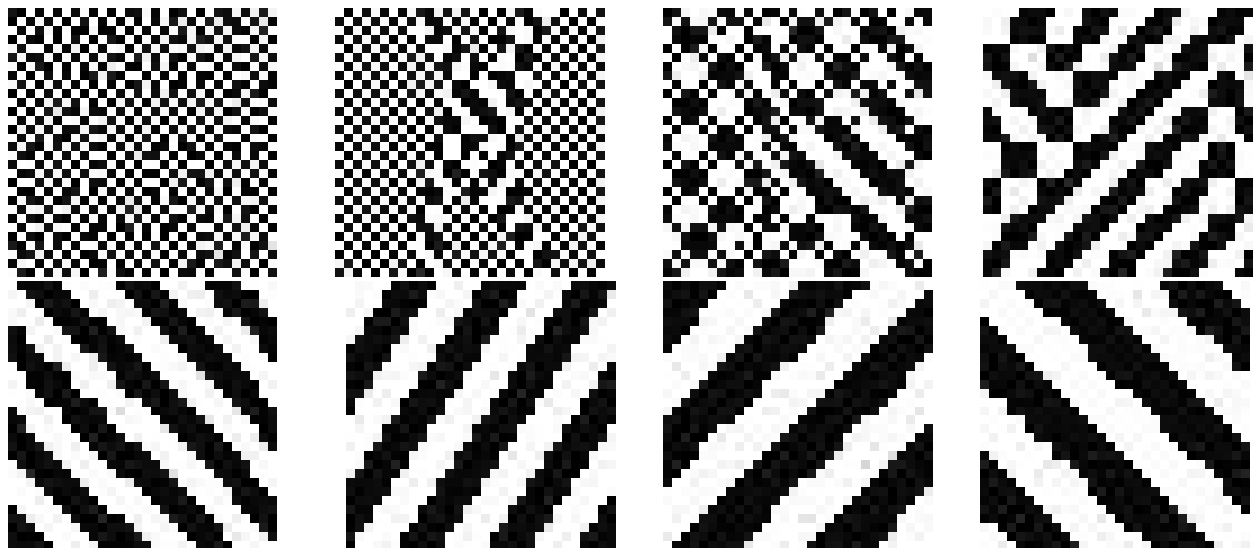


Figure 7. Compare AICA systems with different $R2$ values

c. Enable both inhibition and activation

The third experiment set has both inhibition and activation enabled, and results are shown in Table 3. From these experiments, we find lots of chaotic final state similar with systems in the first experiment set, which have every low rho values. Also, nice patterns are found in these experiments.

Table 3. Parameters with Inhibition Disabled ($J1 = 1, J2 = -0.1$)

$R1$	$R2$	h	Observation
1	2	0	Chaotic final state
1	5	-4	Converge to small black dot, not obvious pattern
1	5	-2	Converge to small black dot, pattern appears
1	5	0	more patterns appear
1	9	-6	Regular pattern
1	9	-3	Regular pattern, bolder
1	9	0	Lines, bolder
1	14	0	Lines, bolder
3	5	-1	Blocks
3	5	0	Larger blocks
3	9	-6	Converge to white board
3	9	-3	Blocks
3	9	0	Bold grid
3	14	0	Bolder grid
7	9	0	Large block
7	14	0	Large block
12	14	0	Black board

1) Tuning h while fix $R2$ and $R1$

Display final states of $R1=3, R2 = 9, h = -3, 0$ as below. According to Fig 8, increasing h means bias to $+1$ state, so that black cells become more and more from the left sub-figure to right sub-figure.



Figure 8. Compare AICA systems with different h values

2) Tuning $R1$, while fix h and $R2$

Display final states of $h=0$, $R_2 = 9$, $R_1 = 1, 3, 7$ as below. From Fig9, as R_1 increase, power of activation is enhanced that more +1 state accumulated.



Figure 9. Compare AICA systems with different R_1 values

3) Tuning R_2 , while fix h and R_1

Display final states of $h=0$, $R_1 = 1$, $R_2 = 2, 5, 9$, as below. As Fig 10 shows, inhibition shapes the edge of pattern.



Figure 10. Compare AICA systems with different R_2 values

4. Characteristic of correlation length λ

From experiments, we can observe that spatial correlation often decreases quickly with distance, so a metric of how fast spatial correlation decreases with distance can be defined, which is the characteristic correlation length.

By assuming that spatial correlation is exponentially decreasing, we can estimate $\rho[\lambda] \approx \rho[1] / e$. By searching for the λ such that $\rho[\lambda]$ most nearly approximates $\rho[1]/e$. The result will be that at a distance of $k = \lambda + 1$ from a cell, $\rho[k]$ will have decayed to approximately $\rho[1]/e$.

As claimed in experiment design section, some AICA systems with certain parameters do not have regular pattern, or converge to totally white and black board, in which cases ρ values is 0 or 1.

Therefore, in this section we only pick systems with regular pattern to observe the characteristic of correlation length λ .

For each experiment set, we select one system to evaluate λ .

1) Disable inhibition ($J1 = 1, J2 = 0$)

Take example of the condition $R1 = 3, R2 = 15, h = 0, J1 = 1, J2 = 0$. From Table 1, $\rho[1] / e = 0.32291648$, which is close to $\rho[8] = 0.294722$. So that at distance $k = \lambda + 1 = 9$, $\rho[9]$ decays to approximately $\rho[1]/e$. Other two examples are also showed below.

Table 4. rho value vs. l

l	Rho(R1=3,R2=15, $h = -1$)	Rho(R1=3,R2=15, $h = 0$)	Rho(R1=6,R2=15,h=0)
1	0.817778	0.877778	0.92
2	0.687778	0.791111	0.853333
3	0.542222	0.692593	0.786667
4	0.408333	0.601667	0.72
5	0.287111	0.510667	0.653333
6	0.180741	0.427407	0.584074
7	0.093333	0.35619	0.515873
8	0.026389	0.294722	0.448333
9	0.020741	0.241235	0.381235
10	0.051556	0.193333	0.314222
11	0.067071	0.150707	0.246667
12	0.072963	0.111852	0.179815
13	0.070085	0.076923	0.114359
14	0.064762	0.045556	0.053492

2) Disable activation ($J1 = 0, J2 = -0.1$)

Take example of the condition $R1 = 1, R2 = 6, h = -3, J1 = 0, J2 = -0.1$. From Table 1, $\rho[1] / e = 0.174947276$, which is close to $\rho[6] = 0.172963$. So that at distance $k = \lambda + 1 = 7$, $\rho[7]$ decays to approximately $\rho[1]/e$.

Table 5. rho value vs. l

l	Rho(R1 = 1, R2 = 6, $h = -3$)	Rho(R1 = 1, R2 = 4, $h = -1$)	Rho(R1 = 9, R2 = 12, $h = -6$)
1	0.475556	0.08	0.822222
2	0.206667	0.14	0.704444
3	0.026667	0.254815	0.571852
4	0.091667	0.365	0.441667
5	0.070222	0.027556	0.317778
6	0.172963	0.224444	0.197407
7	0.358095	0.108254	0.090794

8	0.384167	0.137778	0.002222
9	0.215802	0.052099	0.069136
10	0.042	0.170667	0.096222
11	0.06101	0.064646	0.088687
12	0.045185	0.054444	0.047037
13	0.084274	0.075214	0.014872
14	0.256508	0.076349	0.087143

3) $J1 = 1, J2 = -0.1$

Take example of the condition $R1 = 3, R2 = 9, h = 0, J1 = 1, J2 = -0.1$. From Table 1, $\rho[1] / e = 0.262421$, which is close to $\rho[3] = 0.292593$. So that at distance $k = \lambda + 1 = 4$, $\rho[3]$ decays to approximately $\rho[1]/e$.

Table 6. ρ value vs. l

l	$\text{Rho}((R1 = 3, R2 = 9, h = 0))$	$\text{Rho}((R1 = 7, R2 = 14, h = 0))$	$\text{Rho}((R1 = 3, R2 = 9, h = -6))$
1	0.713333	0.913333	0.824444
2	0.517778	0.846667	0.69
3	0.292593	0.777037	0.551852
4	0.084444	0.705556	0.434444
5	0.093778	0.633333	0.343556
6	0.21963	0.561481	0.291111
7	0.27619	0.490476	0.273968
8	0.248056	0.42	0.290278
9	0.17358	0.34963	0.320741
10	0.082	0.28	0.355333
11	0.015354	0.211111	0.384242
12	0.096667	0.142963	0.403889
13	0.143761	0.076239	0.411111
14	0.159841	0.012857	0.407619

From all the three experiment sets, we can observe that if $h = 0$, λ is almost $R1$ value. But if h is less than zero, λ is larger than $R1$.