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HW#: 1

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#### I. INTRODUCTION

# A. Purpose

- Question 1: Draw a expanding tree stricture for the graph using depth-first search algorithm and the breadth-first search algorithm
- Question 2: Calculate the shortest path from node A to E using UCS method. Write step by step update of the fringe list and closed list.
- Question 3: Write out the complete path finding process from the green grid to the red grid using  $A^*$  algorithm

## B. Equipment

There is a minimal amount of equipment to be used in this lab. The few requirements are listed below:

• Python 3.7.0 (Anaconda)

#### C. Procedure

#### 1. Problem 1

The expanding tree stricture for the graph is shown in FIG.1. The schematic form for depth-first search algorithm is shown in FIG.2 and the schematic form for breadth-first search algorithm is shown in FIG.3.

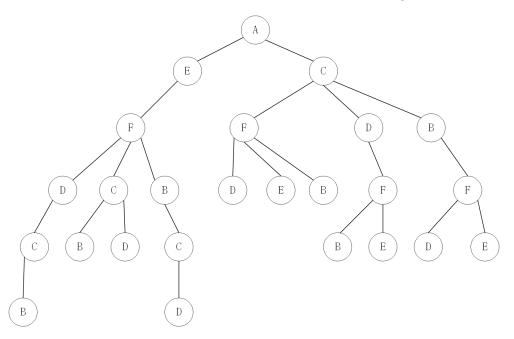


FIG. 1: expanding tree stricture

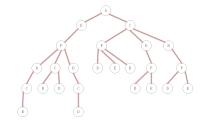


FIG. 2: depth-first search algorithm

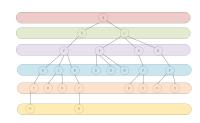


FIG. 3: breadth-first search algorithm

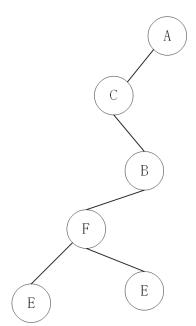


FIG. 4: depth-first search tree

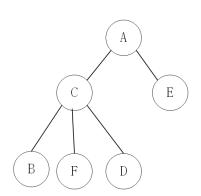


FIG. 5: breadth-first search tree

Question: If the graph has n nodes and the maximum degree for each node is d, what is the complexity of BFS and DFS?

If the graph has n nodes and the maximum degree for each node is d, the time complexity of BFS and DFS is both O(nd). And the space complexity of BFS and DFS is both O(n).

#### 2. Problem 2

- Step 1:
  - Expand: A
  - Fringe List: A-C(3), A-B(10), A-D(20)
  - Closed Set: A
- Step 2:
  - Expand: A-C
  - Fringe List: A-C-B(5), A-B(10), A-C-E(18), A-D(20)
  - Closed Set: A, A-C
- Step 3:
  - Expand: A-C-B
  - Fringe List: A-C-B-D(10), A-B(10), A-C-E(18), A-D(20)
  - Closed Set: A, A-C, A-C-B
- Step 4:
  - Expand: A-C-B-D
  - Fringe List: A-C-B-D-E(21), A-B(10), A-C-E(18), A-D(20)
  - Closed Set: A, A-C, A-C-B, A-C-B-D
- and we can obtain the shortest path A-C-B-D-E, which lenth is 21 from A to E.

#### 3. Problem 3

As the heuristic h is admissible if  $0 \le h(n) \le h^*(n)$ . The Manhattan distance is chosen as heuristic h. For the grid (x, y), h is given by:

$$h(n) = |x - 5| + |y - 2|$$

FIG. shows the value of h(n) of each grid and the value of g(n) of each grid. To determine the path finding process, I write the code Q3 which uses  $A^*$  algorithm. The code is included in its entirety in Appendix. With the help of  $A^*$  algorithm, the route is chosen by:

- step 1 [1, 2] fringe list [(4, [[1, 2], [2, 2], 1]), (6, [[1, 2], [0, 2], 1]), (6, [[1, 2], [1, 3], 1]), (6, [[1, 2], [1, 1], 1])] closed set [[1, 2], [1, 2]]
- step 2 [2, 2]
  fringe list [(6, [[1, 2], [0, 2], 1]), (6, [[1, 2], [1, 1], 1]), (6, [[1, 2], [1, 3], 1]), (6, [[2, 2], [2, 1], 2]), (6, [[2, 2], [2, 3], 2])]
  closed set [[1, 2], [1, 2], [2, 2]]
- step 3 [0, 2]
  fringe list [(6, [[1, 2], [1, 1], 1]), (6, [[2, 2], [2, 1], 2]), (6, [[1, 2], [1, 3], 1]), (6, [[2, 2], [2, 3], 2]), (8, [[0, 2], [0, 1], 2]), (8, [[0, 2], [0, 3], 2])]
  closed set [[1, 2], [1, 2], [2, 2], [0, 2]]

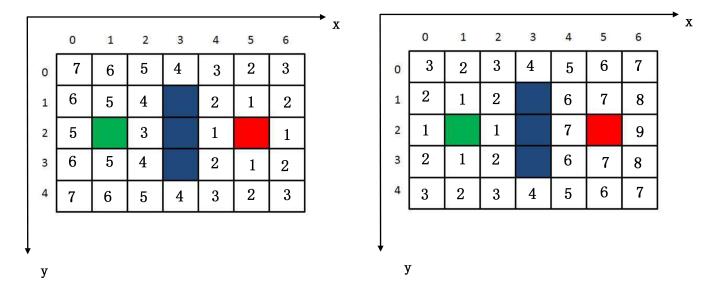


FIG. 6: the value of h(n) and g(n) of each grid

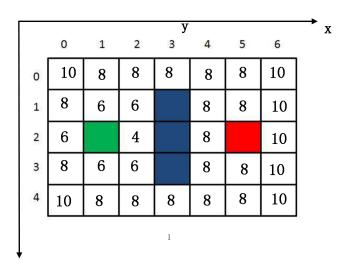


FIG. 7: the value of f(n) of each grid

- step 4 [1, 1]
  fringe list [(6, [[1, 1], [2, 1], 2]), (6, [[1, 2], [1, 3], 1]), (8, [[0, 2], [0, 3], 2]), (6, [[2, 2], [2, 1], 2]), (8, [[0, 2], [0, 1], 2]), (8, [[1, 1], [0, 1], 2]), (8, [[1, 1], [1, 0], 2]), (6, [[2, 2], [2, 3], 2])]
  closed set [[1, 2], [1, 2], [2, 2], [0, 2], [1, 1]]
- step 5 [2, 1]
  fringe list [(6, [[1, 2], [1, 3], 1]), (6, [[2, 2], [2, 1], 2]), (8, [[0, 2], [0, 3], 2]), (6, [[2, 2], [2, 3], 2]), (8, [[0, 2], [0, 1], 2]), (8, [[1, 1], [0, 1], 2]), (8, [[1, 1], [1, 0], 2]), (8, [[2, 1], [2, 0], 3])]
  closed set [[1, 2], [1, 2], [2, 2], [0, 2], [1, 1], [2, 1]]
- step 6 [1, 3]

fringe list [(6, [[1, 3], [2, 3], 2]), (6, [[2, 2], [2, 1], 2]), (8, [[0, 2], [0, 3], 2]), (8, [[1, 3], [0, 3], 2]), (6, [[2, 2], [2, 3], 2]), (8, [[1, 1], [0, 1], 2]), (8, [[1, 1], [1, 0], 2]), (8, [[2, 1], [2, 0], 3]), (8, [[1, 3], [1, 4], 2]), (8, [[0, 2], [0, 1], 2])] closed set [[1, 2], [1, 2], [2, 2], [0, 2], [1, 1], [2, 1], [1, 3]]

• step 7 [2, 3]

 $\begin{array}{l} \text{fringe list } [(6,\,[[2,\,2],\,[2,\,1],\,2]),\,(6,\,[[2,\,2],\,[2,\,3],\,2]),\,(8,\,[[0,\,2],\,[0,\,3],\,2]),\,(8,\,[[1,\,3],\,[0,\,3],\,2]),\,(8,\,[[0,\,2],\,[0,\,1],\,2]),\,(8,\,[[1,\,1],\,[0,\,1],\,2]),\,(8,\,[[1,\,1],\,[2,\,0],\,3]),\,(8,\,[[1,\,3],\,[1,\,4],\,2]),\,(8,\,[[2,\,3],\,[2,\,4],\,3])] \end{array}$ 

closed set [[1, 2], [1, 2], [2, 2], [0, 2], [1, 1], [2, 1], [1, 3], [2, 3]]

• step 8 [0, 1]

fringe list [(8, [[0, 2], [0, 3], 2]), (8, [[1, 3], [0, 3], 2]), (8, [[1, 1], [0, 1], 2]), (8, [[1, 3], [1, 4], 2]), (8, [[2, 3], [2, 4], 3]), (8, [[2, 1], [2, 0], 3]), (8, [[1, 1], [1, 0], 2]), (10, [[0, 1], [0, 0], 3])] closed set [[1, 2], [1, 2], [2, 2], [0, 2], [1, 1], [2, 1], [1, 3], [2, 3], [0, 1]]

• step 9 [0, 3]

fringe list [(8, [[1, 1], [0, 1], 2]), (8, [[1, 3], [0, 3], 2]), (8, [[1, 1], [1, 0], 2]), (8, [[1, 3], [1, 4], 2]), (8, [[2, 3], [2, 4], 3]), (8, [[2, 1], [2, 0], 3]), (10, [[0, 1], [0, 0], 3]), (10, [[0, 3], [0, 4], 3])] closed set [[1, 2], [1, 2], [2, 2], [0, 2], [1, 1], [2, 1], [1, 3], [2, 3], [0, 1], [0, 3]]

• step 10 [1, 0]

fringe list [(8, [[1, 0], [2, 0], 3]), (8, [[1, 3], [0, 3], 2]), (8, [[2, 1], [2, 0], 3]), (8, [[1, 3], [1, 4], 2]), (8, [[2, 3], [2, 4], 3]), (10, [[0, 3], [0, 4], 3]), (10, [[1, 0], [0, 0], 3]), (10, [[0, 1], [0, 0], 3])] closed set [[1, 2], [1, 2], [2, 2], [0, 2], [1, 1], [2, 1], [1, 3], [2, 3], [0, 1], [0, 3], [1, 0]]

• step 11 [2, 0]

fringe list [(8, [[1, 3], [0, 3], 2]), (8, [[1, 3], [1, 4], 2]), (8, [[2, 1], [2, 0], 3]), (8, [[2, 0], [3, 0], 4]), (8, [[2, 3], [2, 4], 3]), (10, [[0, 3], [0, 4], 3]), (10, [[1, 0], [0, 0], 3]), (10, [[0, 1], [0, 0], 3])] closed set [[1, 2], [1, 2], [2, 2], [0, 2], [1, 1], [2, 1], [1, 3], [2, 3], [0, 1], [0, 3], [1, 0], [2, 0]]

• step 12 [1, 4]

fringe list [(8, [[1, 4], [2, 4], 3]), (8, [[2, 0], [3, 0], 4]), (8, [[2, 1], [2, 0], 3]), (8, [[2, 3], [2, 4], 3]), (10, [[0, 0], 3]), (10, [[0, 3], [0, 4], 3]), (10, [[1, 4], [0, 4], 3]), (10, [[0, 1], [0, 0], 3])] closed set [[1, 2], [1, 2], [2, 2], [0, 2], [1, 1], [2, 1], [1, 3], [2, 3], [0, 1], [0, 3], [1, 0], [2, 0], [1, 4]]

• step 13 [2, 4]

fringe list [(8, [[2, 0], [3, 0], 4]), (8, [[2, 3], [2, 4], 3]), (8, [[2, 1], [2, 0], 3]), (8, [[2, 4], [3, 4], 4]), (10, [[1, 0], [0, 0], 3]), (10, [[0, 3], [0, 4], 3]), (10, [[1, 4], [0, 4], 3]), (10, [[0, 1], [0, 0], 3])] closed set [[1, 2], [1, 2], [2, 2], [0, 2], [1, 1], [2, 1], [1, 3], [2, 3], [0, 1], [0, 3], [1, 0], [2, 0], [1, 4], [2, 4]]

• step 14 [3, 0]

fringe list [(8, [[2, 1], [2, 0], 3]), (8, [[2, 3], [2, 4], 3]), (10, [[0, 1], [0, 0], 3]), (8, [[2, 4], [3, 4], 4]), (10, [[1, 0], [0, 0], 3]), (10, [[0, 3], [0, 4], 3]), (10, [[1, 4], [0, 4], 3]), (8, [[3, 0], [4, 0], 5])] closed set [[1, 2], [1, 2], [2, 2], [0, 2], [1, 1], [2, 1], [1, 3], [2, 3], [0, 1], [0, 3], [1, 0], [2, 0], [1, 4], [2, 4], [3, 0]]

• step 15 [3, 4]

fringe list [(8, [[3, 0], [4, 0], 5]), (10, [[0, 3], [0, 4], 3]), (8, [[3, 4], [4, 4], 5]), (10, [[1, 4], [0, 4], 3]), (10, [[1, 0], [0, 0], 3]), (10, [[0, 1], [0, 0], 3])] closed set [[1, 2], [1, 2], [2, 2], [0, 2], [1, 1], [2, 1], [1, 3], [2, 3], [0, 1], [0, 3], [1, 0], [2, 0], [1, 4], [2, 4], [3, 0], [3, 4]]

# • step 16 [4, 0]

fringe list [(8, [[3, 4], [4, 4], 5]), (10, [[0, 3], [0, 4], 3]), (8, [[4, 0], [4, 1], 6]), (10, [[1, 4], [0, 4], 3]), (10, [[1, 0], [0, 0], 3]), (10, [[0, 1], [0, 0], 3]), (8, [[4, 0], [5, 0], 6])] closed set [[1, 2], [1, 2], [2, 2], [0, 2], [1, 1], [2, 1], [1, 3], [2, 3], [0, 1], [0, 3], [1, 0], [2, 0], [1, 4], [2, 4], [3, 0], [3, 4], [4, 0]]

## • step 17 [4, 4]

fringe list [(8, [[4, 0], [4, 1], 6]), (8, [[4, 4], [5, 4], 6]), (8, [[4, 0], [5, 0], 6]), (10, [[0, 3], [0, 4], 3]), (10, [[1, 0], [0, 0], 3]), (10, [[0, 1], [0, 0], 3]), (8, [[4, 4], [4, 3], 6]), (10, [[1, 4], [0, 4], 3])] closed set [[1, 2], [1, 2], [2, 2], [0, 2], [1, 1], [2, 1], [1, 3], [2, 3], [0, 1], [0, 3], [1, 0], [2, 0], [1, 4], [2, 4], [3, 0], [3, 4], [4, 0], [4, 4]]

# • step 18 [4, 1]

fringe list [(8, [[4, 0], [5, 0], 6]), (8, [[4, 1], [4, 2], 7]), (8, [[4, 4], [4, 3], 6]), (8, [[4, 1], [5, 1], 7]), (10, [[1, 0], [0, 0], 3]), (10, [[0, 1], [0, 0], 3]), (10, [[1, 4], [0, 4], 3]), (10, [[0, 3], [0, 4], 3]), (8, [[4, 4], [5, 4], 6])] closed set [[1, 2], [1, 2], [2, 2], [0, 2], [1, 1], [2, 1], [1, 3], [2, 3], [0, 1], [0, 3], [1, 0], [2, 0], [1, 4], [2, 4], [3, 0], [3, 4], [4, 0], [4, 4], [4, 1]]

## • step 19 [5, 0]

fringe list [(8, [[4, 1], [4, 2], 7]), (8, [[4, 1], [5, 1], 7]), (8, [[4, 4], [4, 3], 6]), (8, [[4, 4], [5, 4], 6]), (10, [[1, 0], [0, 0], 3]), (10, [[0, 1], [0, 0], 3]), (10, [[1, 4], [0, 4], 3]), (10, [[0, 3], [0, 4], 3]), (8, [[5, 0], [5, 1], 7]), (10, [[5, 0], [6, 0], 7])]

closed set [[1, 2], [1, 2], [2, 2], [0, 2], [1, 1], [2, 1], [1, 3], [2, 3], [0, 1], [0, 3], [1, 0], [2, 0], [1, 4], [2, 4], [3, 0], [3, 4], [4, 0], [4, 4], [4, 1], [5, 0]]

# • step 20 [4, 2]

fringe list [(8, [[4, 1], [5, 1], 7]), (8, [[4, 2], [5, 2], 8]), (8, [[4, 4], [4, 3], 6]), (8, [[5, 0], [5, 1], 7]), (8, [[4, 4], [5, 4], 6]), (10, [[0, 1], [0, 0], 3]), (10, [[1, 4], [0, 4], 3]), (10, [[0, 3], [0, 4], 3]), (10, [[5, 0], [6, 0], 7]), (10, [[4, 2], [4, 3], 8]), (10, [[1, 0], [0, 0], 3])]

closed set [[1, 2], [1, 2], [2, 2], [0, 2], [1, 1], [2, 1], [1, 3], [2, 3], [0, 1], [0, 3], [1, 0], [2, 0], [1, 4], [2, 4], [3, 0], [3, 4], [4, 0], [4, 4], [4, 1], [5, 0], [4, 2]]

# • step 21 [5, 1]

fringe list [(8, [[4, 2], [5, 2], 8]), (8, [[4, 4], [5, 4], 6]), (8, [[4, 4], [4, 3], 6]), (8, [[5, 0], [5, 1], 7]), (8, [[5, 1], [5, 2], 8]), (10, [[0, 1], [0, 0], 3]), (10, [[1, 4], [0, 4], 3]), (10, [[0, 3], [0, 4], 3]), (10, [[5, 0], [6, 0], 7]), (10, [[4, 2], [4, 3], 8]), (10, [[1, 0], [0, 0], 3]), (10, [[5, 1], [6, 1], 8])]closed set [[1, 2], [1, 2], [2, 2], [0, 2], [1, 1], [2, 1], [1, 3], [2, 3], [0, 1], [0, 3], [1, 0], [2, 0], [1, 4], [2, 4], [3, 0], [3, 4], [4, 0], [4, 4], [4, 1], [5, 0], [4, 2], [5, 1]]

• step 22 [5, 2]

final path ['(1,2)', '(1,1)', '(1,0)', '(2,0)', '(3,0)', '(4,0)', '(4,1)', '(4,2)', '(5,2)']

#### 4. Problem 4

## • BFSvsDFS.py

The file provides the class Graph, which includes a dictionary edges and function neighbors. edges is treated as dictionary to look up. Function neighbors pass in an id and returns a list of neighboring node

- function reconstruct\_path(came\_from, start, goal)

Given a dictionary named *came\_from*. The key of the dictionary is the node character and its value is the parent node, the start node and the goal node, the function will compute the path from start to the end.

To accomplish this function, I start with *goal* because each node might have more than one children but one father node only. With the *came\_from* provided, I can search for the father node of every node. Start with *goal* and append its father to the *path* list. Loop until *start* appears as a node's father. At last, reverse the list and return it as result.

```
def reconstruct_path(came_from, start, goal):
    path = []

### START CODE HERE ### ( 6 line of code)
    if came_from is None:
        print("Path reconstruction failed!!")
    return

current_node = goal
    while(current_node is not start):
        path.append(current_node)
    current_node = came_from[current_node]
    path.append(current_node)

path.reverse()

### END CODE HERE ###
    return path
```

- function breadth\_first\_search(graph, start, goal)

Given a graph, a start node and a goal node, the function utilizes breadth first search algorithm to find the path from start node to the goal node. It will return a dictionary whose key is each node and corresponding value is its parent node

To accomplish this function, I use *queue* to expand the search region. The elements in the queue are stored as format [father, child]. Firstly put the [None, start] into the queue, While the queue is not empty, get the element from the head of queue as variable Expand. Add the father and child of Expand into the came\_from dictionary. To realize **early stoping** mentioned in the code, I check if goal neighbors to the child of Expand. If so, add the goal to the dictionary as child of child in Expand, then return came\_from.

It needs to be aware that for a gragh search, you should never expand a state twice. So I set the list *closed\_set* to store the node that has been put into the queue. Before putting any node into the queue, check the node if it is in the *closed\_set*.

```
def breadth_first_search(graph, start, goal):
                       came_from[start] = None
### START CODE HERE ### ( 10 line of code)
                        ## check goal and start ========
                                  ((goal in graph.edges) and (start in graph.edges) ):
tt("Error! Goal or Start is not in the graph")
  9
                            return None
11
                        closed\_set = [start]
13
                       BFS_queue = Queue(maxsize=0)
                      BrS_queue = Queue(maxsize=0)
BrS_queue.put([None, start])
while not BrS_queue.empty():
Expand = BrS_queue.get()
came_from[Expand[1]] = Expand[0]
if goal in graph.neighbors(Expand[1]):
came_from[goal] = Expand[1]
return came_from
15
17
19
                               return came_from
                           for value in graph.neighbors(Expand[1]):
if value not in closed_set:
BFS_queue.put([Expand[1], value])
closed_set.append(value)
21
23
25
                       ### END CODE HERE ###
return came_from
```

- function depth first search(graph, start, goal)

Given a graph, a start node and a goal node, the function utilizes depth first search algorithm to find the path from start node to the goal node. It will return a dictionary whose key is each node and corresponding value is its parent node

To accomplish this function, I use *stack* to expand the search region. The elements in the stack are stored as format [father, child]. Firstly put the [None, start] into the stack, While the stack is not empty, get the element from the head of stack as variable Expand. Add the father and child of Expand into the came\_from dictionary. To realize **early stoping** mentioned in the code, I check if goal neighbors to the child of Expand. If so, add the goal to the dictionary as child of child in Expand, then return came\_from.

It needs to be aware that for a gragh search, you should never expand a state twice. So I set the list *closed\_set* to store the node that has been put into the stack. Before putting any node into the stack, check the node if it is in the *closed set*.

```
def depth_first_search(graph, start, goal):
    came_from = {}
    came_from[start] = None
    ### START CODE HERE ### ( 10 line of code)
                            not ((goal in graph.edges) and (start in graph.edges) ): print("Error! Goal or Start is not in the graph")
  9
                        11
                       closed_set = [start]
DFS_queue = LifoQueue(maxsize=0)
DFS_queue.put([None, start])
while not DFS_queue.empty():
Expand = DFS_queue.egt()
came_from[Expand[1]] = Expand[0]
if goal in graph.neighbors(Expand[1]):
came_from[goal] = Expand[1]
13
15
17
                                came_from[goal] = Expand[1]
return came_from
r value in graph.neighbors(Expand[1]):
if value not in closed_set:
19
21
                                   DFS_queue.put([Expand[1], value]) closed_set.append(value)
23
25
                        ### END CODE HERE ###
27
                        return came from
```

The demonstrations for functions will be shown in part II EXPERIMENT

#### • UniformCostSearch.py

The file provides the class *Graph*, which includes the dictionary *edges*, the dictionary *edge Weights*, the function *neighbors* and the function *get\_cost*. *edges* and *edge Weights* are treated as dictionary to look up. Pass in an id to function *neighbors*, it will return a list of neighboring node, while *get\_cost* accepts two adjacent nodes and returns the cost.

## - function reconstruct\_path(came\_from, start, goal)

Given a dictionary named *came\_from*. The key of dictionary is the node character and its value is the parent node, the start node and the goal node, the function will compute the path from start to the end.

To accomplish this function, I start with *goal* because each node might have more than one children but one father node only. With the *came\_from* provided, I can search for the father node of every node. Start with *goal* and append its father to the *path* list. Loop untill *start* appears as a node's father. At last, reverse the list and return it as result.

```
def reconstruct_path(came_from, start, goal):
    path = []
    ### START CODE HERE ### ( 6 line of code)

if came_from is None:
    print("Path reconstruction failed!!")

return

current_node = goal
    while(current_node is not start):
```

#### - function uniform\_cost\_search

Given a graph, a start node and a goal node, the function utilizes uniform cost search algorithm to find the path from start node to the goal node. It will return two dictionaries, whose key is each node and corresponding value is its parent node, and whose key is each node and corresponding value is the cost from start to the node.

To accomplish this function, I use *PriorityQueue* to expand the search region. The elements in the PriorityQueue are stored as format [value,[father, child]]. Firstly put the [0,[None, start]] into the PriorityQueue. While the PriorityQueue is not empty, get the element from the head of PriorityQueue as variable Expand. Add the father and child of Expand into the came\_from dictionary, then add the child and value of Expand into the cost\_so\_far. To realize early stoping mentioned in the code, I check if goal neighbors to the child of Expand. If so, add the goal to the dictionary as child of child in Expand, then return came from and cost so far.

It needs to be aware that for a gragh search, you should never expand a state twice. So I set the list *closed\_set* to store the node that has been put into the PriorityQueue. Before putting any node into the PriorityQueue, check the node if it is in the *closed set*.

```
def uniform_cost_search(graph, start, goal):
                             came\_from = \{\}
                            cost_so_far = {}
came_from[start]
                             ### START CODE HERE ### ( 15 line of code)
                             ### q.put((num,value)), smaller the num is, higher the priority is
                             ### ## the cost_so_far should store the value that the nodes cost from start closed_set = [start]
UCS_queue = PriorityQueue(maxsize=0)
UCS_queue.put((0, [None, start]))
11
 13
                             UCS_queue.put((0, [None, start]))
while not UCS_queue.empty():
    ### Expand[1][1] is current node, Expand[1][0] is its father
    ### Expand[0] is the cost that from start to the Expand[1][1]
Expand = UCS_queue.get()
    came_from[Expand[1][1]] = Expand[0][0]
    if goal in graph.neighbors(Expand[1][1]):
        came_from[goal] = Expand[1][1]:
        cost_so_far[soal] = graph.get_cost(Expand[1][1], goal) + Expand[0]
        return came_from, cost_so_far
    for value in graph.neighbors(Expand[1][1]):
    if value not in closed_set:
15
17
19
21
23
25
                                      if value not in closed_set:

UCS_queue.put((graph.get_cost(Expand[1][1], value)+Expand[0], [Expand[1][1], value]))

closed_set.append(value)
27
                             ### END CODE HERE ###
29
```

The demonstration for functions will be shown in part II EXPERIMENT

# • AStarSearch.py

The file provides the class *Graph*, which includes the dictionary *edges*, the dictionary *edge Weights*, the function *neighbors* and the function *get\_cost*. *edges* and *edge Weights* are treated as dictionary to look up. Pass in an id to function *neighbors*, it will return a list of neighboring node, while *get\_cost* accepts two adjacent nodes and returns the cost.

- function reconstruct\_path(came\_from, start, goal)

  The function is same as the previous reconstruct\_path(came\_from, start, goal).
- function def heuristic(graph, current\_node, goal\_node)

I choose *Euclidean distance* between a node and the goal as the heuristic of the node. To check whether the graph satisfies the consistency of heuristics, in the function *heuristic*, I import the function *uniform\_cost\_search* to compare the heuristic and the least cost of the node. If the

```
def heuristic(graph, current_node, goal_node):

heuristic_value = 0
### START CODE HERE ### ( 15 line of code)

from UniformCostSearch import uniform_cost_search import math

came_from_UCS, _ = uniform_cost_search(graph, current_node, goal_node)
path = reconstruct_path(came_from_UCS, current_node, goal)

actural_distance = 0
for i in range(len(path)-1):
    actural_distance += graph.get_cost(path[i],path[i+1])

heuristic_value = math.sqrt( (graph.locations[current_node][0] - graph.locations[goal_node][0])**2 + (graph.locations[current_node][1] - graph.locations[current_node][1] + graph.locations[current_
```

If heuristic value is more than actral distance, it means that graph doesn't satisfies the consistency of heuristics

#### - function A star search

Given a graph, a start node and a goal node, the function utilizes A\* search algorithm to find the path from start node to the goal node. It will return two dictionaries, whose key is each node and corresponding value is its parent node, and whose key is each node and corresponding value is the cost from start to the node.

To accomplish this function, I use PriorityQueue to expand the search region. The elements in the PriorityQueue are stored as format [value,[father, child, CostToChild]]. The value computed by adding the cost from start to the current node and the heuristic of the current node. The CostToChild is the cost from start to the child node. Firstly put the [0+heuristic(graph,start,goal),[None, start,0]] into the PriorityQueue. While the PriorityQueue is not empty, get the element from the head of PriorityQueue as variable Expand. Add the father and child of Expand into the  $came\_from$  dictionary, then add the child and CostToChild of Expand into the  $cost\_so\_far$ . To realize early stoping mentioned in the code, I check if goal neighbors to the child of Expand. If so, add the goal to the dictionary as child of child in Expand, then return  $came\_from$  and  $cost\_so\_far$ .

```
def A_star_search(graph, start, goal):
                           came_from = {}
cost_so_far = {}
came_from[start] = None
cost_so_far[start] = 0
   6
                            ### START CODE HERE ### ( 15 line of code)
                            closed set = set(start)
                            closed_set = set(start)
Astar_queue = PriorityQueue(maxsize=0)
Astar_queue.put((0 + heuristic(graph,start,goal), [None, start, 0]))
while not Astar_queue.empty():
# Expand[1][1] is current node, Expand[1][0] is its father
# Expand[0][2] is the value of h()+g()
# Expand[1][2] is the cost that from start to the Expand[1][1]
Expand = Astar_queue.get()
closed_set.add(Expand[1][1])
came_from[Expand[1][1]] = Expand[1][0]
cost_so_far[Expand[1][1]] = Expand[1][2]
 10
 12
14
 16
18
20
                                if\ goal\ in\ graph.neighbors(Expand[1][1]):\\
                                     goal in graph.nerghrous Expand[1][1].

came_from[goal] = Expand[1][1]

cost_so_far[goal] = graph.get_cost(Expand[1][1], goal) + Expand[1][2]

return came_from, cost_so_far
22
24
                                     return came_iron, cost_so_iar
r value in graph.neighbors(Expand[1][1]):
if value not in closed_set:
Astar_queue.put((graph.get_cost(Expand[1][1], value)+Expand[1][2] + heuristic(graph,value,goal),
26
28
                                                                                     [Expand[1][1], value, graph.get_cost(Expand[1][1], value)+Expand[1][2]]
                             ### END CODE HERE ###
```

The demonstration for functions will be shown in part II EXPERIMENT

#### II. EXPERIMENT

This section consists of screenshots taken during the laboratory procedure.

# A. BFSvsDFS.py

For large graph, the search starts from S to E. For small graph, the search starts from A to E.

```
small_graph.edges = {
    'A': ['B','D'],
    'B': ['A', 'C', 'D'],
    'C': ['A'],
    'D': ['E', 'A'],
    'E': ['B']
}
```

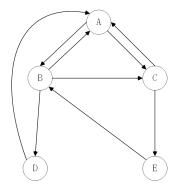


FIG. 8: small graph

```
large_graph.edges = {
    'S': ['A','C'],
    'A': ['S','B','D'],
    'B': ['S', 'A', 'D','H'],
    'C': ['S','L'],
    'D': ['A', 'B','F'],
    'E': ['G','K'],
    'F': ['H','D'],
    'G': ['H','E'],
    'H': ['B','F','G'],
    'I': ['L','J','K'],
    'J': ['L','J','K'],
    'K': ['I','J','E'],
    'L': ['C','I','J']
}
```

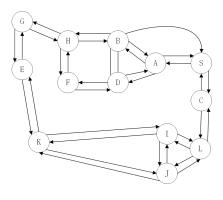


FIG. 9: large graph

```
Large graph

came from DFS {'S': None, 'C': 'S', 'L': 'C', 'J': 'L', 'K': 'J', 'E': 'K'}

path from DFS ['S', 'C', 'L', 'J', 'K', 'E']

came from BFS {'S': None, 'A': 'S', 'C': 'S', 'B': 'A', 'D': 'A', 'L': 'C', 'H': 'B', 'F': 'D', 'I': 'L', 'J': 'L', 'G': 'H', 'E': 'G'}

path from BFS ['S', 'A', 'B', 'H', 'G', 'E']

Small graph

came from DFS {'A': None, 'D': 'A', 'E': 'D'}

path from DFS ['A', 'D', 'E']

came from BFS ['A': None, 'B': 'A', 'D': 'A', 'E': 'D'}

path from BFS ['A': None, 'B': 'A', 'D': 'A', 'E': 'D'}

path from BFS ['A': None, 'B': 'A', 'D': 'A', 'E': 'D'}
```

FIG. 10: result for BFS and DFS

#### B. UniformCostSearch.py

For large graph, the search starts from S to H. For small graph, the search starts from A to E.

```
small_graph.edges = {
    'A': ['B','D'],
    'B': ['A', 'C', 'D'],
    'C': ['A'],
    'D': ['E', 'A'],
    'E': ['B']
}
small_graph.edgeWeights={
    'A': [2,4],
    'B': [2, 3, 4],
    'C': [2],
    'D': [3, 4],
    'E': [5]
}
```

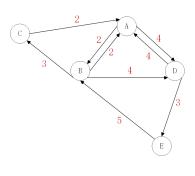


FIG. 11: small graph

```
large_graph.edges = {
    'S': ['A','B','C'],
    'A': ['S','B','D'],
    'B': ['S', 'A', 'D','H'],
    'C': ['S','L'],
    'D': ['A', 'B','F'],
    'E': ['G','K'],
    'F': ['H','D'],
    'G': ['H','E'],
    'H': ['B','F','G'],
    'I': ['L','J','K'],
    'J': ['L','I','K'],
    'K': ['I','J','E'],
    'L': ['C','I','J']
}
```

```
large_graph.edgeWeights = {
    'S': [7, 2, 3],
    'A': [7, 3, 4],
    'B': [2, 3, 4, 1],
    'C': [3, 2],
    'D': [4, 4, 5],
    'E': [2, 5],
    'F': [3, 5],
    'G': [2, 2],
    'H': [1, 3, 2],
    'I': [4, 6, 4],
    'J': [4, 6, 4],
    'K': [4, 4, 5],
    'L': [2, 4, 4]
}
```

FIG. 12: large graph

#### C. AStarSearch.py

The graphs are same as the previous graphs.

```
Small graph
came from UCS {'A': None, 'B': 'A', 'D': 'A', 'E': 'D'}
cost form UCS {'A': 0, 'B': 2, 'D': 4, 'E': 7}
path from UCS ['A', 'D', 'E']
Large graph
came from UCS {'S': None, 'B': 'S', 'H': 'B', 'C': 'S', 'L': 'C', 'G': 'H', 'E': 'G'}
cost form UCS {'S': 0, 'B': 2, 'H': 3, 'C': 3, 'L': 5, 'G': 5, 'E': 7}
path from UCS ['S', 'B', 'H', 'G', 'E']
```

FIG. 13: result for UCS

```
Small Graph
graph doesn't satisfies the consistency of heuristics
came from Astar {'A': None, 'D': 'A', 'E': 'D'}
cost form Astar {'A': 0, 'D': 4, 'E': 7}
path from Astar ['A', 'D', 'E']
Large Graph
graph doesn't satisfies the consistency of heuristics
came from Astar {'S': None, 'B': 'S', 'H': 'B', 'G': 'H', 'E': 'G'}
cost form Astar {'S': 0, 'B': 2, 'H': 3, 'G': 5, 'E': 7}
path from Astar ['S', 'B', 'H', 'G', 'E']
```

FIG. 14: result for A\*

#### III. DISCUSSION & CONCLUSION

Homework1 is more difficult than HW0. I really spent lots of time on the homework. During the process of learning, I made many mistakes, which help me further understand the algorithms involved. The most inspiring thing is that I successfully solve the A\* Search problem in Q3 by writing code. The process is pretty hard but I enjoyed it.

#### IV. APPENDIX

A. code of Q3.py

```
from queue import PriorityQueue import math

class Graph:
"""

Defines a graph with edges, each edge is treated as dictionary look up. function neighbors pass in an id and returns a list of neighboring node

"""

def __init__(self):
    self.locations = {}

def heuristic(graph, node, target = [5,2]):
    distance = abs(graph.locations['({},{})'.format(node[0],node[1])][0]-target[0]) + abs(graph.locations['({},{})'.format(node[0],node[1])][1]-target[1])

return distance
```

```
18
         def reconstruct_path(came_from, start, goal):
               20
22
24
                \label{eq:current_node} \begin{array}{l} break \\ current\_node = \c^*(\{\},\{\})\c^*.format(\came\_from[current\_node][0], came\_from[current\_node][1]) \\ path.append(current\_node) \end{array}
26
                path.reverse()
return path
30
           \label{eq:def-astarSearch(graph, start = [1,2], target = [5,2]): came\_from = {} \# \ key \ is \ child \ and \ value \ is \ father \ cost\_so\_far = {} \\ }
32
34
               \begin{array}{l} came\_from['(\{\},\{\})'.format(start[0],start[1])] = None \\ cost\_so\_far['(\{\},\{\})'.format(start[0],start[1])] = 0 \end{array}
36
38
                \begin{split} & closed\_set = [start] \\ & Astar\_queue = PriorityQueue(maxsize=0) \\ & Astar\_queue.put((0 + heuristic(graph,start), \ [None, \ start, \ 0])) \end{split}
40
42
44
                while not Astar_queue.empty():
    Expand = Astar_queue.get()
    if Expand[1][1] in closed_set and step is not 0:
46
48
                   step+=1
print('\item '+"step "+ str(step))
print(Expand[1][1])
print('\n')
50
52
54
                    \begin{split} & closed\_set.append(Expand[1][1]) \\ & came\_from['(\{\},\{\})'.format(Expand[1][1][0],Expand[1][1][1])] = Expand[1][0] \\ & cost\_so\_far['(\{\},\{\})'.format(Expand[1][1][0],Expand[1][1][1])] = Expand[1][2] \end{split}
56
58
                    \begin{array}{lll} & \text{if } (Expand[1][1][0] \text{ is } target[0]) \text{ } \mathbf{and} \text{ } (Expand[1][1][1] \text{ is } target[1]) : \\ & \text{return } came\_from,cost\_so\_far \end{array} 
60
                  62
64
66
68
70
                   print(Amgenst)
print(Astar_queue.queue)
print('\n')
print('closed set')
print(closed_set)
72
74
                    print('\n')
76
               {\bf return\ came\_from,} {\bf cost\_so\_far}
78
               mame__=="_main__":
graph = Graph()
for i in range (0,7):
    for j in range (0,5):
        graph.locations['({},{})'.format(i,j)] = [i, j]
80
82
84
               \begin{array}{l} \text{del graph.locations} ['(3,1)'] \\ \text{del graph.locations} ['(3,2)'] \\ \text{del graph.locations} ['(3,3)'] \end{array}
86
88
               came_from, cost_so_far = AstarSearch(graph)
90
92
                \mathtt{path} = \mathtt{reconstruct\_path}(\mathtt{came\_from},"(1,2)","(5,2)")
                print('final path ')
print(path)
94
```