

特殊 MPC 协议构造

隐私集合操作 (Private Set Operations) 协议

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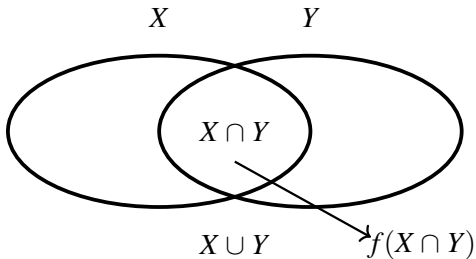


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- 3 隐私集合求并：PSU
- 4 参考文献

- #### 4 参考文献

PSO 介绍

隐私集合操作 (Private Set Operation, PSO) 协议是一类特殊的 MPC 协议。在 PSO 协议中，协议参与方的输入是一个集合，各方想要计算这个集合的函数，例如，交集、并集，交集大小，交集的秘密分享等。



PSO 介绍

最早关于 PSO 的研究集中于隐私集合求交 (Private Set Intersection, PSI)，目前 PSI 也是研究成果最多，效率最高，最具实用价值的一类 PSO 协议。近年来也有越来越多的其他 PSO 协议被提了出来，例如隐私集合求并 (Private Set Union, PSU)，隐私交集求势 (Private Set Intersection Cardinality, PSI-CA/PSI-card)，隐私交集的势与和 (PSI-card-sum)，电路隐私集合交集 (Circuit-PSI) 等。相比 PSI，这些相关的协议效率依然很低，有待进一步提升。

PSO 介绍

PSI 的相关工作:

- 两方半诚实: [Mea86; HFH99; FNP04; CT10; ACT11; CT12; DCW13; PSZ14; Pin+15; Kol+16; PSZ18; FNO19; Pin+19b; CM20; RS21; Gar+21a; RT21; RR22; Cho+22; KBM23; BC23]
- 两方恶意: [FNP04; HL08; JL09; JL10; HN10; CKT10; Dac+09; Fre+16; RR17a; RR17b; OOS17; Pin+20; RS21; RT21; RR22; Cho+22; GHL22; BC23]
- 多方半诚实: [FNP04; KS05; HV17; Kol+17; IOP18; Cha+21; Bay+22]
- 多方恶意: [KS05; HV17; GN19; AMZ21; Zha+19; Ben+22; NTY21; GHL22; Qiu+22]
- 非平衡场景: [Pin+15; CLR17; Che+18; Con+21; Kis+17; Kal+19; RA18; Li+19] [Dit+22]
- 第三方辅助计算: [Ker12; Kam+14; ATD16; Aba+19; LRG19; Aba+22]

PSO 介绍

Circuit-PSI 相关工作:

- 两方半诚实: [HEK12; Pin+15; CO18; Pin+18; Pin+19a; Gar+21b; RS21; CGS22; HMS22]
- 多方半诚实: [Cha+21]

PSU 相关工作:

- 两方半诚实: [BS05; Fri07; BA12; DC17; Kol+19; Gar+21b; Jia+22a; Jia+22b; Zha+23; Gor+22; Tu+22; Che+22b]
- 两方恶意: [HN10; Gor+22]
- 多方半诚实: [KS05; Fri07; Hon+11; SCK12; BA12]
- 多方恶意: [Fri07; Hon+11; SCK12; BA12]

PSO 介绍

PSI-card 相关工作:

- 两方半诚实: [HFH99; AES03; FNP04; CZ09; CGT12; Ege+15; DPT20; Gar+21b; Che+22b]
- 多方半诚实: [KS05; VC05; Nar+09; Cha+21; TYG22; Che+22a]
- 多方恶意: [KS05]

PSI-card-sum 相关工作:

- 两方半诚实: [lon+17; lon+20; Gar+21b; Che+22b]
- 两方恶意: [Mia+20]
- 多方半诚实: [Che+22a]

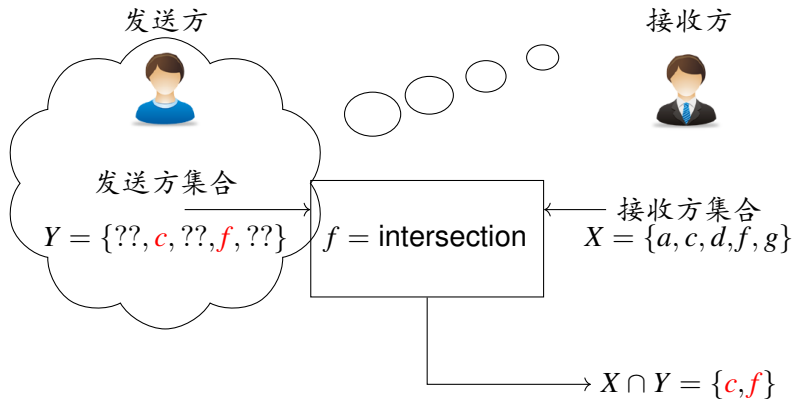
PSO 介绍

其他 PSI 相关工作:

- 标签 PSI(Labeled PSI): [CGN98; FNP04; Fre+05; Che+18; Con+21]
- 门限 PSI(Threshold PSI): [HOS17; ZC18; GS19; Bad+21; BDP21]
- 可更新 PSI(Updatable PSI): [BMX22]
- 简洁 PSI(Laconic PSI): [Ala+21; Ara+22]
- 输出交集中的一项: [BXR22]
- 模糊 PSI(Fuzzy PSI): [FNP04; CFR21; GRS22; GRS23]
- 输入是秘密分享: [MRR20]
- Private Over-Threshold Set-Union/Quorum PSI: [KS05; Cha+21]

- #### 4 参考文献

Private Set Intersection



Naive Hash

最简单的一个想法是，设 H 是一个密码学 hash 函数，两方进行如下协议：

$$S(X = \{x_1, \dots, x_n\})$$

$$R(Y = \{y_1, \dots, y_n\})$$

计算 $H(x_i), i \in [n]$

$$\Omega := \{H(x_i)\}_{i \in [n]}$$

$$X \cap Y := \{y | H(y) \in \Omega, y \in Y\}$$

$$R(Y = \{y_1, \dots, y_n\})$$

PSI 思路

目前 PSI 主要构造思路有以下几种:

- ① 基于公钥: [Mea86; HFH99; CT10; ACT11; CT12]
- ② 基于多项式: [FNP04; KS05; HN10; Dac+09; Fre+16; HV17; GN19; GHL22; Cho+22]
- ③ 基于 GBF: [DCW13; PSZ14; RR17a; IOP18; Bay+22; Zha+19; Ben+22]
- ④ 基于 OPRF: [HL08; PSZ14; Pin+15; Kol+16; RR17b; OOS17; PSZ18; FNO19; Pin+19b; Pin+20; CM20; RS21; Gar+21a; RT21; RR22; KBM23; BC23]
- ⑤ 基于电路: [HEK12; Pin+15; CO18; Pin+18; Pin+19a; Gar+21b; RS21; CGS22; HMS22]

- #### 4 参考文献

基于 DDH 的构造 [Mea86; HFH99](DH-PSI)

最早的 PSI 协议就是基于公钥的 DH 密钥交换协议，其特点是构造简单，通信低，缺点是计算开销大。

$$S(X = \{x_1, \dots, x_n\})$$

$$R(Y = \{y_1, \dots, y_n\})$$

$$a \leftarrow \mathbb{Z}_q, A := \{H(x_i)^a\}_{i \in [n]}$$

A

→

$$b \leftarrow \mathbb{Z}_q, B := \{H(y_i)^b\}_{i \in [n]}$$

$$B$$

←

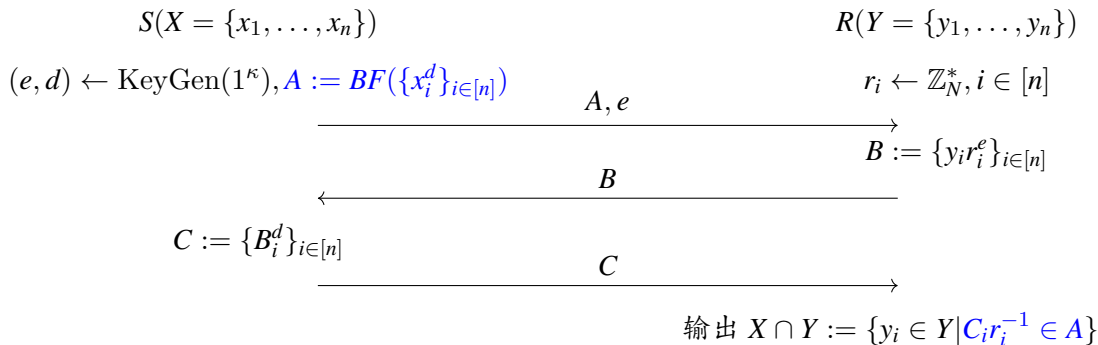
$$C := \{B_i^a\}_{i \in [n]}$$

C

输出 $X \cap Y := \{y_i \in Y | C_i \in A^b\}$

基于 RSA 盲签名的构造 [CT10]

参数: RSA 模数 N , hash 函数 H



- #### 4 参考文献

基于多项式的 PSI

[FNP04] 最早提出了 PSI 的概念, 第一个提出使用多项式表示集合, 思想是令集合元素为多项式的根, 即集合 $X = \{x_1, \dots, x_n\}$ 的多项式表示为

$f(x) := \prod_{i \in [n]} (x - x_i) = x^n + \alpha_{n-1}x^{n-1} + \dots + \alpha_0$, 然后根据要求的集合对多项式进行运算。例如, 对两个集合 X, Y , 设其多项式表示为 f, g , 则 $X \cap Y$ 的多项式表示为 $f + g$, $X \cup Y$ 的多项式表示为 fg 。使用多项式表示的好处是标准模型, 可以方便地扩展到多方情况 [KS05; HV17; GN19], 且天然地支持多重集合 (multiset) [KS05], 缺点是效率较低, 且容易出现安全问题 [AMZ21]。

[GS19] 第一次使用 $f(x) = x^{x_1} + \dots + x^{x_n}$ 表示集合 $X = \{x, \dots, x_n\}$, 用于构造门限 PSI。

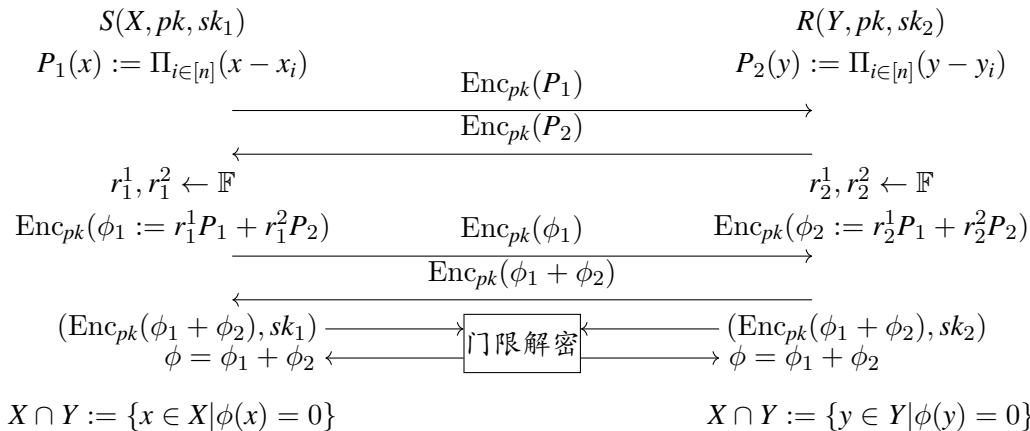
基于多项式的 PSI[FNP04]

 $S(X)$
 $R(Y)$
 $(pk, sk) \leftarrow \text{KeyGen}(1^\kappa)$
 $pk, \{\text{Enc}_{pk}(\alpha_i)\}_{i \in [0, n]} \quad P(y) := \prod_{i \in [n]} (y - y_i) = \sum_{i \in [0, n]} \alpha_i y^i$
 \longleftarrow
 $\text{Enc}_{pk}(P(x_i)) := \sum_{j \in [0, n]} x_i^j \text{Enc}_{pk}(\alpha_j), i \in [n]$
 $r_i \leftarrow \mathbb{F}, \text{Enc}_{pk}(r_i P(x_i) + x_i) := r_i \text{Enc}_{pk}(P(x_i)) + x_i$
 $\{c_i := \text{Enc}_{pk}(r_i P(x_i) + x_i)\}_{i \in [n]}$
 \longrightarrow
 $X \cap Y := \{\text{Dec}_{sk}(c_i) | \text{Dec}_{sk}(c_i) \in Y\}$

基于多项式的 PSI[FNP04]

 $S(X)$
 $R(Y)$
 $(pk, sk) \leftarrow \text{KeyGen}(1^\kappa)$
 $pk, \{\text{Enc}_{pk}(\alpha_i)\}_{i \in [0, n]} \quad P(y) := \prod_{i \in [n]} (y - y_i) = \sum_{i \in [0, n]} \alpha_i y^i$
 \longleftarrow
 $\text{Enc}_{pk}(P(x_i)) := \sum_{j \in [0, n]} x_i^j \text{Enc}_{pk}(\alpha_j), i \in [n]$
 $r_i \leftarrow \mathbb{F}, \text{Enc}_{pk}(r_i P(x_i) + x_i || p_{x_i}) := r_i \text{Enc}_{pk}(P(x_i)) + x_i || p_{x_i}$
 $\{c_i := \text{Enc}_{pk}(r_i P(x_i) + x_i || p_{x_i})\}_{i \in [n]}$
 \longrightarrow
 $X \cap Y := \{\text{Dec}_{sk}(c_i) | \text{Dec}_{sk}(c_i) \in Y\}$

基于多项式的 PSI[KS05]



- #### 4 参考文献

基于 GBF 的 PSI 协议

[DCW13] 提出了 Garbled Bloom Filter(GBF) 的概念，并用 GBF 和 OT 给出了第一个只基于 OT 和对称运算构造的 PSI 协议，使得 PSI 协议的效率有了极大的提升。目前基于 GBF 的协议已经可以扩展到多方 [IOP18] 和恶意 [RR17a; Zha+19; Ben+22] 情况，但由于后面计算和通信都更高效的 OPRF-based 的协议的提出，我个人认为 GBF-based 的 PSI 协议已经不再是 PSI 研究的主流，但 GBF 作为一个密码学组件还是值得了解的，它也是后面提出的 Oblivious Key-Value Store (OKVS) 的一个特例。

基于 GBF 的 PSI 协议

Bloom Filter (BF) [Blo70] 是一种用于集合成员测试的数据结构。表示 n 个元素的 BF 是由一个 m 长的比特串 F 和 k 个 hash 函数 $h_1, \dots, h_k : \{0, 1\}^* \rightarrow [m]$ 组成。 F 初始化为全 0 字符串, 当插入一个元素 x 时, 令 $F[h_i(x)] := 1, i \in [k]$ 。要检查一个元素是否在 F 中, 只需检查是否对所有的 $i \in [k], F[h_i(x)] = 1$ 。

BF 可以保证插入过的元素一定能检查出来，但是没有插入过的元素有可能检查出来，设这种失败的情况称为 false positive。[DCW13] 给出的参数选择为：设 false positive 概率是 ϵ ，选取 $k = \log 1/\epsilon, m \approx 1.44kn$ 。在实际应用中，一般需要 $\epsilon = 2^{-\lambda}, k = \lambda, m = 1.44\lambda n$ 。

基于 GBF 的 PSI 协议

Garbled Bloom Filter (GBF) [DCW13] 和 BF 类似，也是使用 k 个 hash 函数 $h_1, \dots, h_k: \{0, 1\}^* \rightarrow [m]$ 做映射，但和 BF 的区别是，GBF 的每个位置不再是一个比特，而是一个 l 长字符串，表示集合元素的输入分享。一个 GBF $G \in \{0, 1\}^{ml}$ 初始化为空字符串，当插入一个元素 x 时，对每个位置 $h_i(x)$ ，令其位置的 GBF 为 x 的一个加法分享，即令 $\bigoplus_{i \in [k]} G[h_i(x)] := x$ 。当插入完 n 个元素后，如果还有位置为空，则赋一个随机值。要检查一个元素是否在 G 中，只需检查是否有 $\bigoplus_{i \in [k]} G[h_i(x)] = x$ 。

GBF 的参数选择和 BF 相同, 即 $\epsilon = 2^{-\lambda}, k = \lambda, m = 1.44\lambda n$.

注：GBF 可进一步扩展为插入 key-value 对 [PSZ14; Gar+21a]，即插入 (x, y) 时，令 $\bigoplus_{i \in [k]} G[h_i(x)] := y$ 即可。

基于 GBF 的 PSI 协议 [DCW13]

$H := \{h_1, \dots, h_k\}$ 是 BF/GBF 方案用到的 hash 函数集合。

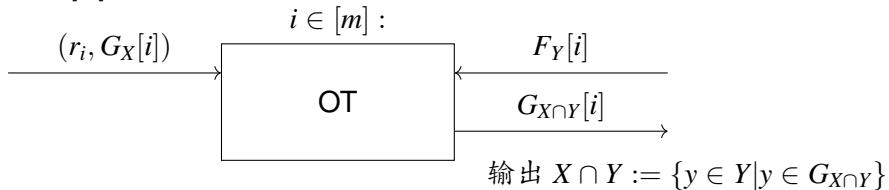
$$S(X)$$

$$R(Y)$$

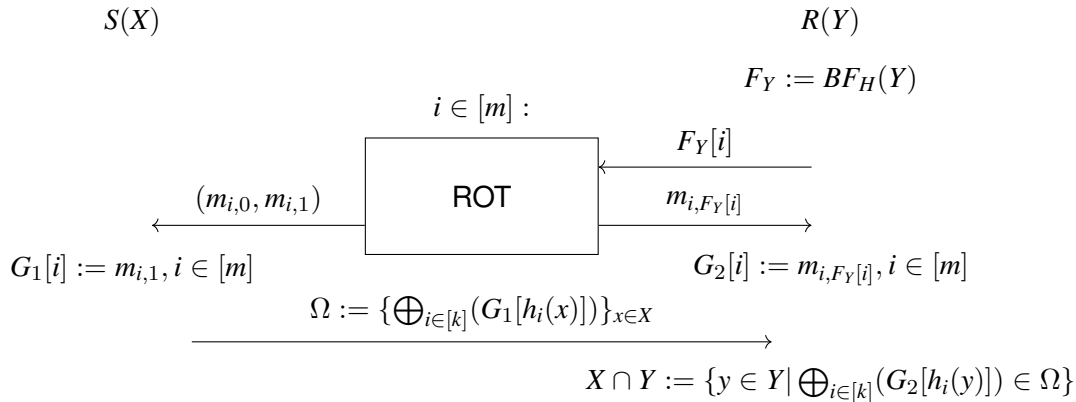
$$G_X := \text{GBF}_H(X)$$

$$F_Y := \text{BF}_H(Y)$$

$$r_i \leftarrow \{0, 1\}^l, i \in [m]$$



基于 GBF 的 PSI 协议 [PSZ14]



1 介绍

2 隐私集合求交: PSI

- 基于公钥
- 基于多项式
- 基于 GBF
- 基于 OPRF
 - 单点 OPRF
 - 多点 OPRF
- 基于电路

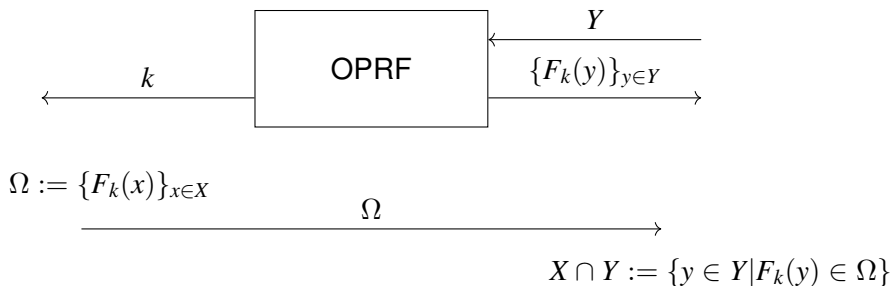
3 隐私集合求并: PSU

4 参考文献

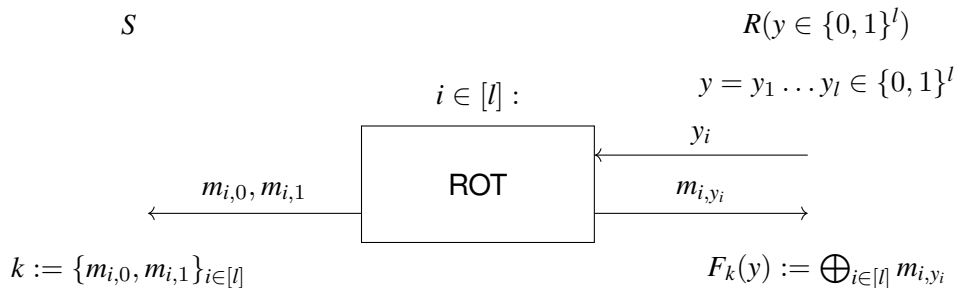
基于 OPRF 的 PSI 协议框架

$$S(X = \{x_1, \dots, x_n\})$$

$$R(Y = \{y_1, \dots, y_n\})$$



单点 OPRF [PSZ14]



单点 OPRF [PSZ14]

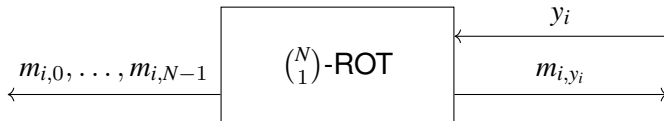
参数: $\log N = \eta, t = l/\eta$

S

$R(y \in [0, \eta - 1]^t)$

$i \in [t] :$

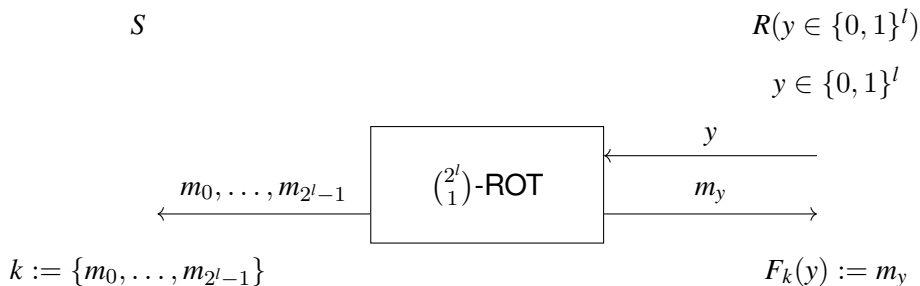
$y = y_1 \dots y_t \in [0, \eta - 1]^t$



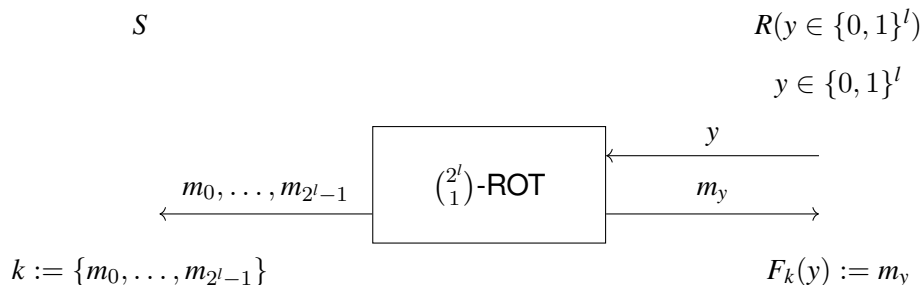
$k := \{m_{i,0}, \dots, m_{i,N-1}\}_{i \in [t]}$

$F_k(y) := \bigoplus_{i \in [t]} m_{i,y_i}$

单点 OPRF [KOl+16]



单点 OPRF [KOl+16]

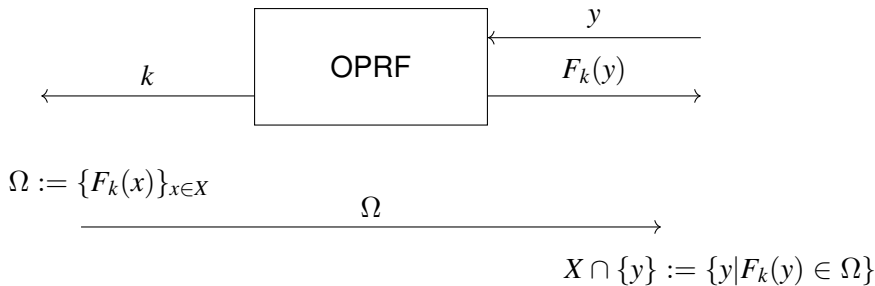


注：实际上 $k := (q, s)$, $m_x = F_k(x) = H(q \oplus (C(x) \cdot s))$, C 是伪随机编码 (PRC).

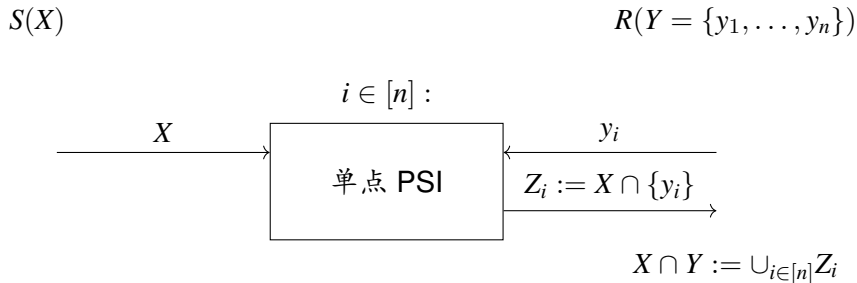
单点 OPRF \rightarrow 单点 PSI

$$S(X = \{x_1, \dots, x_n\})$$

$$R(y)$$



单点 PSI \rightarrow PSI

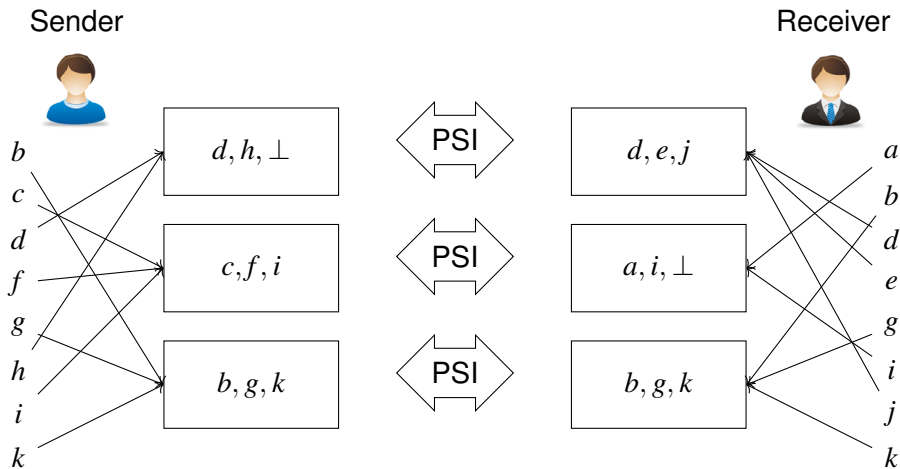


问题：对每个 y 都要重新执行一次 n 长的单点 OPRF，效率太低。

Hash to bin 技术

由于平凡地执行 n 次单点 PSI 开销太大，一般考虑使用 hash to bin 技术进行优化。hash to bin 技术的思想是双方使用相同的 hash 函数将各自的元素映射到很多 bin 中，即将 n 个元素 e_1, \dots, e_n 用 hash 函数映射到 m 个 bin 中，记为 B_1, \dots, B_m ，之后在每个 bin 中求 PSI。这样就将两个大集合元素的 PSI 问题转化到了求很多小集合元素的 PSI 问题，设 $X = \cup_{i \in [m]} X_i, Y = \cup_{i \in [m]} Y_i, X \cap Y = \cup_{i \in [m]} X_i \cap Y_i$ ，能够节省开销。

Hash to bin 技术



simple hash

设 $h: \{0, 1\}^\sigma \rightarrow [m]$ 是随机 hash 函数。对每个输入项 e , 映射到 $B_{h(e)}$ 中。

参数分析:

考虑将 n 个球放入 m 个 bin 的游戏, 则:

$$\begin{aligned} & Pr[\exists \text{ 某个 bin 中球的个数} \geq \max_m] \\ & \leq \sum_{i=1}^m Pr[\text{第 } i \text{ 个 bin 中球的个数} \geq \max_m] \\ & = m \cdot Pr[\text{第 } i \text{ 个 bin 中球的个数} \geq \max_m] \\ & = m \cdot (\sum_{i=\max_m}^n \binom{n}{i} \cdot (\frac{1}{m})^i \cdot (1 - \frac{1}{m})^{n-i}) \leq 2^{-\lambda} \end{aligned}$$

当固定 n 和 m 时, 可求得 \max_m 的大小。

[Gon81] 分析表明, 当 $m = n$ 时, $\max_m = \frac{\ln n}{\ln \ln n} (1 + o(1))$ 。

当 $m = O(n/\log n)$ 时, $\max_m = O(\log n)$ 。

balanced hash

设 $h_1, h_2 : \{0, 1\}^\sigma \rightarrow [m]$ 是 2 个随机 hash 函数。对每个输入项 e ，映射到 $B_{h_1(e)}$ 与 $B_{h_2(e)}$ 中较少的 bin 中。

[Aza+94] 表明, $m = n$ 时, $\max_m = \frac{\ln \ln n}{\ln 2} (1 + o(1))$

cuckoo hash [PR04]

设 $h_1, \dots, h_k : \{0, 1\}^\sigma \rightarrow [m]$ 是 k 个随机 hash 函数。对每个输入项 e ：首先将其放入 $B_{h_1(e)}$ 中，如果 $B_{h_1(e)}$ 已有元素 o ，则把 o 放到新的 $B_{h_i(o)}$ 中， $i \in [k], h_i(o) \neq h_1(e)$ 。重复此过程，直到没有元素被重放，或重放次数达到门限值。如果达到门限值，则将最后一个元素放入 stash 中。stash 大小记为 s 。

注: cuckoo hash 每个 bin 中至多有一个元素。

参数分析 [Pin+18]:

cuckoo hash 有 3 个参数需要分析: stash 大小 s , hash 函数数量 k , bin 的个数 m 。

一般取 $m = O(n)$ (具体来说 $k = 3, m = 1.2n, k = 2, m = 2.4n$)。

s : 当 $m = O(n)$ 时, 失败概率为 $O(n^{-s})$ 。要使 $n^{-s} < 2^{-\lambda}$, 取 $s = O(\lambda/\log n)$

k : 一般 k 取 2, 但是此时 bin 的数量大概为 $2n$, 即利用率为 50%。若 $s = 0$, $\lambda = 40$, 实验表明, 当 $k = 3$ 时, bin 的数量大概为 $1.27n$, 当 $k = 4$ 时, bin 的数量大概为 $1.09n$, 当 $k = 5$ 时, bin 的数量大概为 $1.05n$ 。

Hash to bin 技术

基于置换的 hash [ANS10]

为了降低每个 bin 中元素的存储长度, 可以考虑使用基于置换的 hash。

设每个元素 $x \in \{0, 1\}^l$, bin 的数量是 m 。考虑利用 bin 位置的信息降低存储 x 需要的空间, 设 π 是 $\{0, 1\}^l$ 上的随机置换, 令 $\pi(x) = \pi_L(x) || \pi_R(x) \in \{0, 1\}^{\log m + (l - \log m)}$ 。要存储 x , 只需将 $\pi_R(x)$ 存储在第 $\pi_L(x)$ 个 bin 中。此时每个元素的存储空间由 l 降低到 $l - \log m$, 当 l 和 $\log m$ 接近时 (e.g. $l = 32, m = 2^{20}$), 开销会有很大提升。

使用这种方法, 如果两个元素 x, x' 都存储在同一个 bin 中, 且存储值相同, 则一定有 $x = x'$ 。即 $\pi_L(x) = \pi_L(x'), \pi_R(x) = \pi_R(x') \implies \pi^{-1}(\pi(x)) = \pi^{-1}(\pi(x')), x = x'$ 。

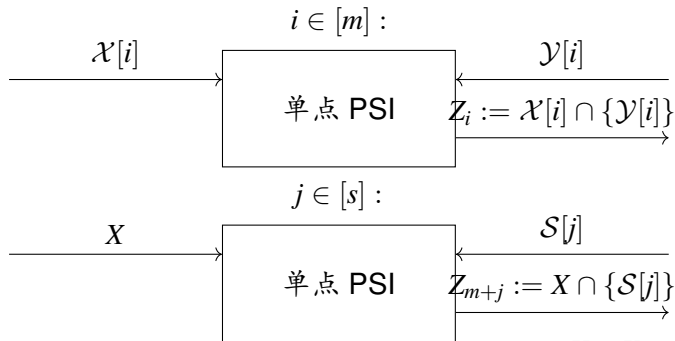
单点 PSI+Hash \rightarrow PSI

$$S(X = \{x_1, \dots, x_n\})$$

$$\mathcal{X} \leftarrow \text{SimpleH}_{h_1, h_2, h_3}^m(X)$$

$$R(Y = \{y_1, \dots, y_n\})$$

$$(\mathcal{Y}, \mathcal{S}) \leftarrow \text{CuckooH}_{h_1, h_2, h_3}^m(Y)$$

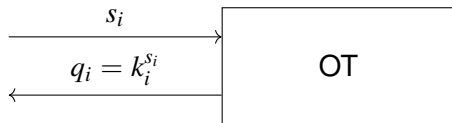


$$X \cap Y := \bigcup_{i \in [m+s]} Z_i$$

多点 OPRF [Pin+19a]

回忆 OTe 协议 [Ash+13]:

S

$$s \leftarrow \{0, 1\}^\kappa$$
$$i \in [\kappa] :$$

$$R(r \in \{0, 1\}^N)$$
$$k_i^0, k_i^1 \leftarrow \{0, 1\}^\kappa, i \in [\kappa]$$
$$(k_i^0, k_i^1)$$
$$T := [G(k_1^0) | \dots | G(k_\kappa^0)]_{N \times \kappa}$$
$$U := [G(k_1^1) | \dots | G(k_\kappa^1)]_{N \times \kappa}$$
$$C := [r \mid \dots \mid r]_{N \times \kappa}$$
$$P := T \oplus U \oplus C$$
$$Q := [G(q_1) \mid \dots \mid G(q_\kappa)]_{N \times \kappa}$$
$$Q(j) = T(j) \oplus s(P(j) \oplus r_j \cdot 1^\kappa)$$
$$m_{j,0} := H(Q(j) \oplus s \cdot P(j)), j \in [N]$$
$$m_{j,1} := H(Q(j) \oplus s \cdot P(j) \oplus s), j \in [N]$$
$$m_{j,r_j} := H(T(j)), j \in [N]$$

多点 OPRF [Pin+19a]

核心观察: OTe 的每一行表示一个 OT 实例, 如果把 OTe 的行数 i 看做 PRF 输入, 考虑 $F: [N] \rightarrow \{0, 1\}^k$, 令 $F_k(i) := m_{i,0}$, 这里 $k = (s, Q)$ 。此时, 发送方可以对任意 $i \in [N]$ 计算 $F_k(i)$, 而接收方只能得到对应 $r_i = 0$ 那些行的 $F_k(i)$ 。

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问题 1：这里 N 表示矩阵的行数，只能是多项式长度，即 $N = \text{poly}(\kappa)$ ，当双方元素很长时，例如 $l = 128$ ，需要取 $N = 2^{128}$ ，由于要发送的矩阵 P 是 $N \times \kappa$ ，通信太大无法实现。

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解决思路：只需要让接受者发送 $y \in Y$ 对应的 n 行即可。

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问题 2: 这样会暴露所有 y 的信息。

解决思路：利用多项式插值 P 隐藏 y 的信息。即，令 $P(y)$ 是需要发送的目标行，发送方只能求 $P(x)$ 那些行

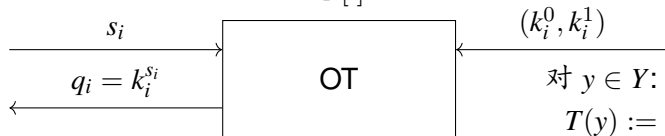
多点 OPRF [Pin+19a]

参数: $l = \log |\mathbb{F}|$, PRF $F : \{0, 1\}^\kappa \times \{0, 1\}^l \rightarrow \{0, 1\}$, CRHFH : $\{0, 1\}^l \rightarrow \{0, 1\}^{\lambda+2\log n}$

S

$s \leftarrow \{0, 1\}^\kappa$

$i \in [l] :$



$R(Y = \{y_1, \dots, y_n\})$

$k_i^0, k_i^1 \leftarrow \{0, 1\}^\kappa, i \in [\kappa]$

(k_i^0, k_i^1)

对 $y \in Y$:

$T(y) := F(k_1^0, y) || \dots || F(k_l^0, y)$

$U(y) := F(k_1^1, y) || \dots || F(k_l^1, y)$

$R(y) := T(y) \oplus U(y)$

计算 $P := \text{Interp}_{\mathbb{F}}(\{(y, R(y))\}_{y \in Y})$

P

定义 $Q(x) := F(q_1, x) || \dots || F(q_l, x)$, 令 $k := (s, Q, P)$ 输出 $\{\bar{F}_k(y) := H(T(y))\}_{y \in Y}$

$\bar{F}_k(x) := H(Q(x) \oplus s \cdot P(x))$

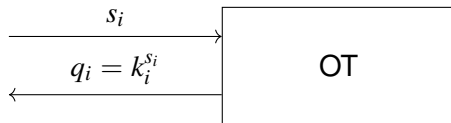
多点 OPRF [CM20]

回忆 OTe 协议 [KK13; Kol+16]:

S

$$s \leftarrow \{0, 1\}^\kappa$$

$i \in [\rho] :$



$$R(r \in [0, 2^l - 1]^N)$$

$$k_i^0, k_i^1 \leftarrow \{0, 1\}^\kappa, i \in [\kappa]$$

$$(k_i^0, k_i^1)$$

$$T := [G(k_1^0) | \dots | G(k_\rho^0)]_{N \times \rho}$$

$$U := [G(k_1^1) | \dots | G(k_\rho^1)]_{N \times \rho}$$

$$C := [\mathcal{C}(r_1) | \dots | \mathcal{C}(r_N)]_{N \times \rho}^T$$

$$P := T \oplus U \oplus C$$

$$Q := [G(q_1) | \dots | G(q_\rho)]_{N \times \rho}$$

$$Q(j) = T(j) \oplus s(P(j) \oplus \mathcal{C}(r_j))$$

$$m_{j,t} := H(Q(j) \oplus s \cdot P(j) \oplus s \cdot \mathcal{C}(t)), j \in [N], t \in [0, 2^l - 1]$$

$$m_{j,r_j} := H(T(j)), j \in [N]$$

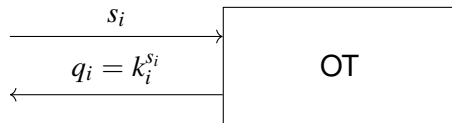
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$$C := [\mathcal{C}(r_1) | \dots | \mathcal{C}(r_N)]_{N \times \rho}^T$$

$$P := T \oplus U \oplus C$$

$$Q := [G(q_1) | \dots | G(q_\rho)]_{N \times \rho}$$

$$Q(j) = T(j) \oplus s(P(j) \oplus \mathcal{C}(r_j))$$

$$m_{j,t} := H(Q(j) \oplus s \cdot P(j) \oplus s \cdot \mathcal{C}(t)), j \in [N], t \in [0, 2^l - 1] \quad m_{j,r_j} := H(T(j)), j \in [N]$$

每一行是一个输入为 $\{0, 1\}^l$ 的单点 OPRF 实例, $k_j = (s, \mathcal{C}, Q(j))$

多点 OPRF [CM20]

主要观察：把每一行 $C(r_j)$ 的每个比特均匀随机放在不同的行中，密钥 $k = (s, Q)$ 设 PRF $F: \{0, 1\}^\kappa \times \{0, 1\}^l \rightarrow [N]^\rho$ 是将每个元素映射到所有 ρ 列对应位置的函数。考虑这样的构造：

$$\text{MatGen}(\{r_j \in \{0, 1\}^l\}_{j \in [N]}):$$

- 1 初始化 $C = [C_1 | \dots | C_\rho] := [1]_{N \times \rho}$
- 2 $k \leftarrow \{0, 1\}^\kappa$
- 3 对 $j \in [N]$:
 - 1 计算 $v_j := F_k(r_j)$
 - 2 令 $C_i[v_j[i]] := 0, i \in [\rho]$
- 4 输出 C

此算法相当于把每个 r_j 的编码随机映射到了矩阵的不同行, 而不再只是第 j 行

多点 OPRF [CM20]

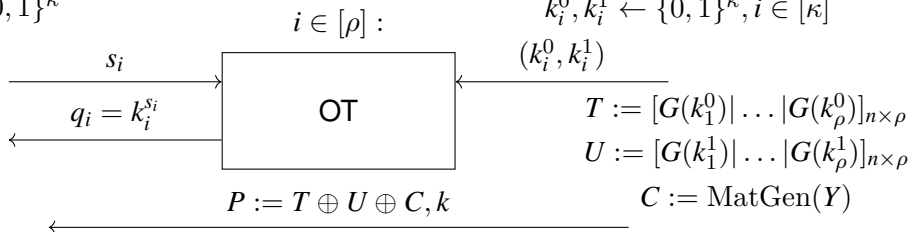
参数: PRF $F : \{0, 1\}^\kappa \times \{0, 1\}^l \rightarrow [n]^\rho$, CRHF $H : \{0, 1\}^\rho \rightarrow \{0, 1\}^{\lambda+2\log n}$

S

$s \leftarrow \{0, 1\}^\kappa$

$R(Y = \{y_1, \dots, y_n\})$

$k_i^0, k_i^1 \leftarrow \{0, 1\}^\kappa, i \in [\kappa]$



$Q := [G(q_1) \oplus s_1 P_1 | \dots | G(q_\rho) \oplus s_\rho P_\rho]_{n \times \rho}$

输出 $\bar{k} := (s, Q)$

输出 $\{\bar{F}_{\bar{k}}(y) := H(T_1[v_y[1]] || \dots || T_\rho[v_y[\rho]])\}_{y \in Y}$

$\bar{F}_{\bar{k}}(x) := H(Q_1[v_x[1]] || \dots || Q_\rho[v_x[\rho]])$

其中 $v_y = F_k(y)$

OKVS

不经意性. 对所有不同的 $\{x_1^0, \dots, x_n^0\}$ 和 $\{x_1^1, \dots, x_n^1\}$, 如果 Encode_H 对输入 $\{x_1^0, \dots, x_n^0\}$ 或 $\{x_1^1, \dots, x_n^1\}$ 不输出 \perp , 则

$$\{D|y_i \leftarrow \mathcal{V}, i \in [n], \text{Encode}_H((x_1^0, y_1), \dots, (x_n^0, y_n))\} \approx \{D|y_i \leftarrow \mathcal{V}, i \in [n], \text{Encode}_H((x_1^1, y_1), \dots, (x_n^1, y_n))\}$$

随机性. 对任意 $A = \{(x_1, y_1), \dots, (x_n, y_n)\}$ 且 $x^* \notin \{x_1, \dots, x_n\}$, $\text{Decode}_H(D, x^*)$ 的输出和 \mathcal{V} 上的均匀分布统计不可区分, 其中 $D \leftarrow \text{Encode}_H(A)$.

一个键值存储如果满足不经意性，则称为不经意键值存储 (Oblivious Key-Value Store, OKVS).

OKVS

OKVS 的效率可以用以下三个参数衡量:

- **比率 (Rate):** 令 n/m 作为 OKVS 的比率, 其中 m 是 D 的大小。最优比率为 1。
- **编码复杂度 (Encoding complexity):** Encode_H 算法的计算复杂度, 是键值对数量 n 的函数
- **解码复杂度 (Decoding complexity):** Decode_H 算法的计算复杂度。

方案	比率	编码	解码	不经意性	随机性
多项式	1	$O(n \log^2 n)$	$O(\log n)$	\checkmark	\times
GBF [DCW13]	$O(1/\lambda)$	$O(\lambda n)$	$O(\lambda)$	\checkmark	\checkmark
2H-GCT [Pin+20]	$0.42 - o(1)$	$O(\lambda n)$	$O(\lambda)$	\times	\times
XoPaXoS [RS21]	$0.42 - o(1)$	$O(\lambda n)$	$O(\lambda)$	\checkmark	\checkmark
3H-GCT [Gar+21a]	$0.81 - o(1)$	$O(\lambda n)$	$O(\lambda)$	\times	\times
3H-GCT++ [Zha+23]	$0.81 - o(1)$	$O(\lambda n)$	$O(\lambda)$	\checkmark	\checkmark

表 1: 不同 OKVS 方案比较

VOLE



- #### 4 参考文献

电路 PSI

电路 PSI (Circuit-PSI) 概念的内涵是有演变的，早期的电路 PSI 论文 [HEK12; Pin+15; Pin+18] 指的是构造 PSI 电路，使用一般的 MPC 协议 (如 GC, GMW) 计算集合交集的协议，重点在于优化电路，降低比较次数，协议的输出就是交集；后面的电路 PSI 协议 [CO18; Pin+19a; Gar+21b; RS21; CGS22; HMS22] 则主要指协议输出是交集的秘密分享，以便使用通用协议对交集进行后续计算 (即 $f(X \cap Y)$)，其构造思路不局限于只使用电路，也会使用其他的组件，如 OT, OKVS, OPPRF, HE 等。前者的电路体现在计算交集用电路，后者的电路体现在执行后续计算用电路。注：两种概念也不是互斥的，使用电路计算交集的协议自然可以轻松地改造成计算任意交集函数的协议，后者相当于使用更灵活的思路得到交集的秘密分享。

个人分析概念变化的原因：早期的电路 PSI [HEK12; Pin+15] 和基于公钥的 PSI 协议效率还是可以扳一扳手腕的，因此指的就是用电路计算 PSI，随着基于 OT 的 PSI 的提出，基于电路的 PSI 效率就完全跟不上了，因此后续的研究会强调电路 PSI 的优势是可以计算交集的函数。

电路 PSI

[HEK12] 首先构造了 SCS(sort-compare-shuffle) 电路, 然后用 Garbled circuit 的方法构造了 PSI 协议。这里 SCS 电路的设计思路是, 首先对两方集合的所有元素同时进行排序, 然后比较两两相邻的元素, 如果相等则在交集中, 最后为了隐藏顺序可能会泄露的集合信息, 把集合重新打乱。[HEK12] 设计协议的通信复杂度为 $O(l \log n)$, 这里 n 是集合元素个数, l 是集合元素的长度。

随后 Pinkas 等人 [Pin+15] 利用 Circuit-Phasing 技术, 即将元素用 hash 函数映射到不同的 bin 中, 并将 bin 的位置信息作为额外的元素信息, 可以降低每个元素的存储空间, 然后使用 pairwise-comparison (PWC) 电路对每个 bin 中的元素进行比较, 构造的协议通信复杂度为 $O(l \log n / \log \log n)$ 。

[Pin+18] 提出了新的哈希表构造方法, 2D 哈希, 通信复杂度为 $\omega(l \log n)$, 可以任意接近线性复杂度。[Pin+19a] 则构造了第一个线性通信复杂度 $O(l \log n)$ 的电路 PSI 协议。后续的工作均基于 PSTY 框架, 改进子协议提高效率 [Gar+21b; Cha+21; RS21; HMS22]。

PPRF

可编程 PRF(Programmable PRF,PPRF) [Kol+17] 指的是一类特殊的 PRF, 它可以将“编程”的输入映射到“编程”的输出。一个 PPRF 由下述算法组成:

- $\text{KeyGen}(1^\kappa, \mathcal{P}) \rightarrow (k, \text{hint})$: 输入安全参数和一个有不同 x_i 构成的点集 $\mathcal{P} = \{(x_1, y_1), \dots, (x_n, y_n)\}$, 生成一个 PRF 密钥 k 和一个公开的辅助信息 hint 。
- $F(k, \text{hint}, x) \rightarrow y$: 输入 x , 输出 y 。

正确性. 一个 PPRF 满足正确性如果 $(x, y) \in \mathcal{P}$ 且 $(k, \text{hint}) \leftarrow \text{KeyGen}(1^\kappa, \mathcal{P})$, 则 $F(k, \text{hint}, x) = y$.

PPRF

安全性. 对于安全性, 考虑下述实验:

$$\text{Exp}^A(X, Q, \kappa):$$

对每个 $x_i \in X$, 选随机的 $y_i \leftarrow \mathcal{V}$

$$(k, \text{hint}) \leftarrow \text{KeyGen}(1^\kappa, \{(x_i, y_i) | x_i \in X\})$$

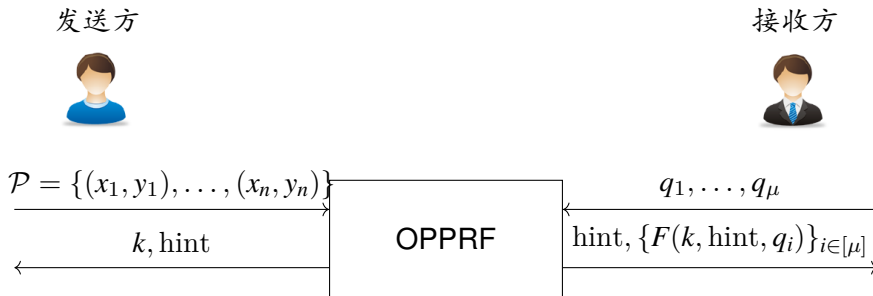
返回 $\mathcal{A}(\text{hint}, \{F(k, \text{hint}, q) | q \in Q\})$

我们称一个 PPRF 是 (n, μ) -安全, 如果对所有 $|X_0| = |X_1| = n$, 所有 $|Q| = \mu$, 和所有 PPT 的 \mathcal{A} :

$$|Pr[\text{Exp}^A(X_0, Q, \kappa) = 1] - Pr[\text{Exp}^A(X_1, Q, \kappa) = 1]| \leq \text{negl}(\kappa)$$

OPPRF

Oblivious PPRF (OPPRF) [Kol+17] 定义如下:



OPPRF

$$S(\mathcal{P} = \{(x_1, y_1), \dots, (x_n, y_n)\})$$

$$R(Q = \{q_1, \dots, q_\mu\})$$



$$P \leftarrow \text{Encode}(\{(x_i, \bar{F}_k(x_i) \oplus y_i)\}_{i \in [n]})$$

P

$$\text{hint} := P$$

输出 (k, hint)

$$\text{hint} := P$$

输出 $\{F(k, \text{hint}, q) := \text{Decode}(P, q) \oplus \bar{F}_k(q)\}_{v \in Y}$

ESG

相等分享生成 (Equality Share Generation) 功能定义如下:

发送方

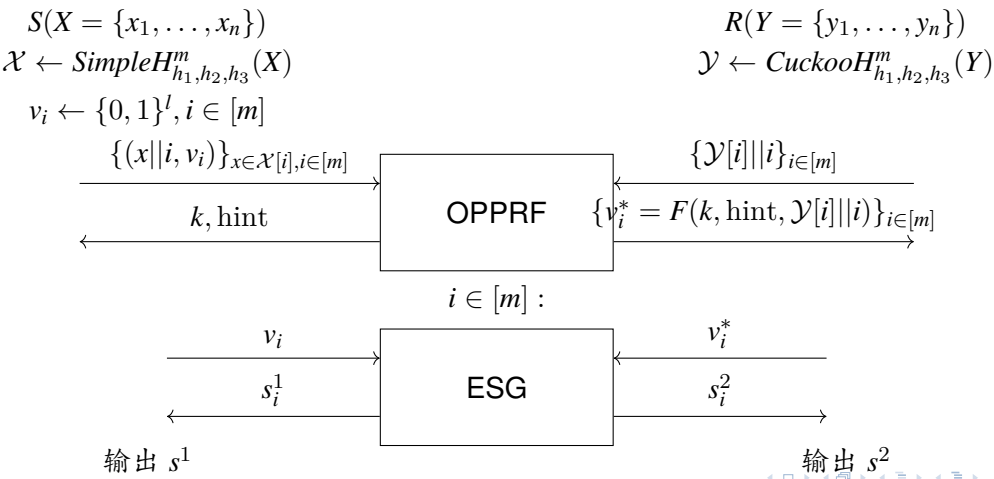


接收方



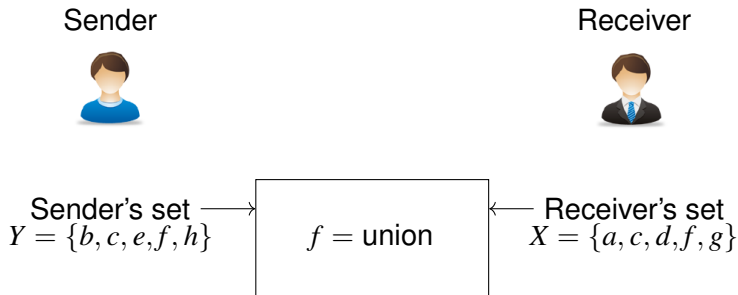
$$s_1 \oplus s_2 = \begin{cases} 1 & a = b; \\ 0 & a \neq b \end{cases}$$

PSTY 框架 [Pin+19a]



- #### 4 参考文献

Private Set Union

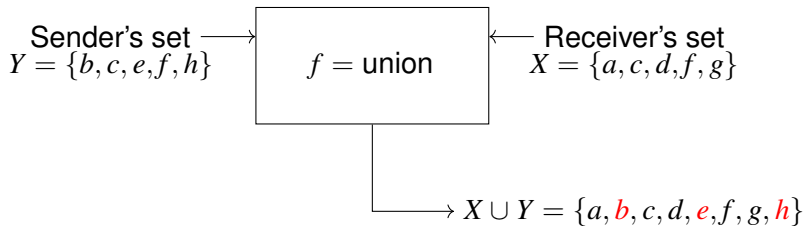


Private Set Union

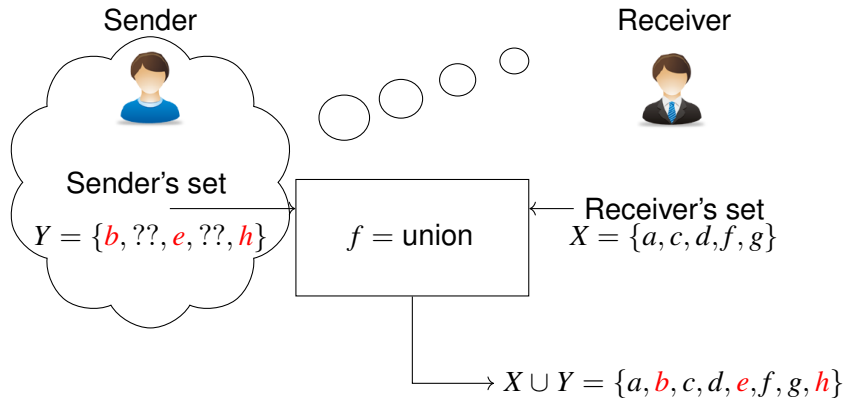
Sender



Receiver



Private Set Union



- #### 4 参考文献

基于 AHE 的 PSU [Fri07]

$$\begin{array}{ccc}
S(X) & & R(Y) \\
& & (pk, sk) \leftarrow \text{KeyGen}(1^\kappa) \\
& & P(y) := \prod_{i \in [n]} (y - y_i) = \sum_{i \in [0, n]} \alpha_i y^i \\
& \longleftarrow & \\
c_i^1 = \text{Enc}_{pk}(P(x_i)) := \sum_{j \in [0, n]} x_i^j \text{Enc}_{pk}(\alpha_j), i \in [n] & & \\
c_i^2 = \text{Enc}_{pk}(x_i \cdot P(x_i)) := x_i \cdot \text{Enc}_{pk}(P(x_i)), i \in [n] & & \\
& \xrightarrow{\{(c_i^1, c_i^2)\}_{i \in [n]}} & \\
& & X \cup Y := \{\text{Dec}_{sk}(c_i^1)^{-1} \cdot \text{Dec}_{sk}(c_i^2) \mid \text{Dec}_{sk}(c_i^1) \neq 0, i \in [n]\}
\end{array}$$

基于 AHE 的 PSU [DC17]

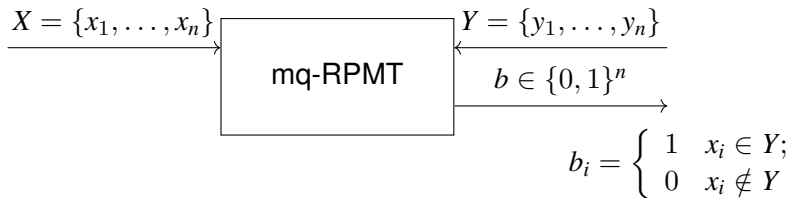
 $S(X)$
 $R(Y)$
 $(pk, sk) \leftarrow \text{KeyGen}(1^\kappa)$
 $F := \text{IBF}(Y) = f_1 f_2 \dots f_m \in \{0, 1\}^m$
 $pk, \{\text{Enc}_{pk}(f_i)\}_{i \in [m]}$
 \longleftarrow
 $c_i^1 = \sum_{j \in [\lambda]} \text{Enc}_{pk}(f_{h_j(x_i)}), i \in [n]$
 $c_i^2 := x_i \cdot c_i^1, i \in [n]$
 $\{(c_i^1, c_i^2)\}_{i \in [n]}$
 \longrightarrow
 $X \cup Y := \{\text{Dec}_{sk}(c_i^1)^{-1} \cdot \text{Dec}_{sk}(c_i^2) \mid \text{Dec}_{sk}(c_i^1) \neq 0, i \in [n]\}$

- #### 4 参考文献

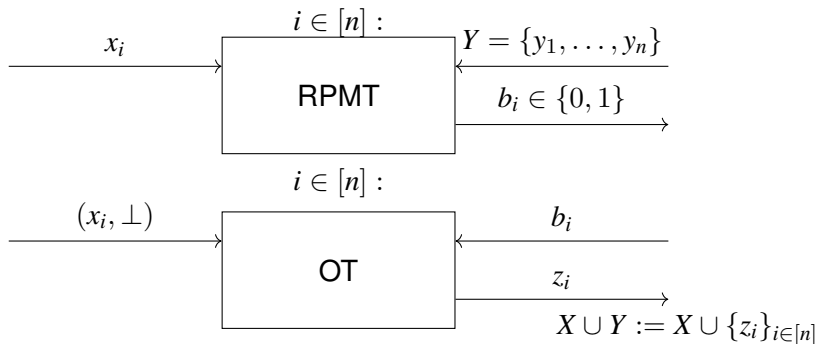
Multi-Query Reverse Private Membership Test (mq-RPMT) [Zha+23]

$$S(X = \{x_1, \dots, x_n\})$$

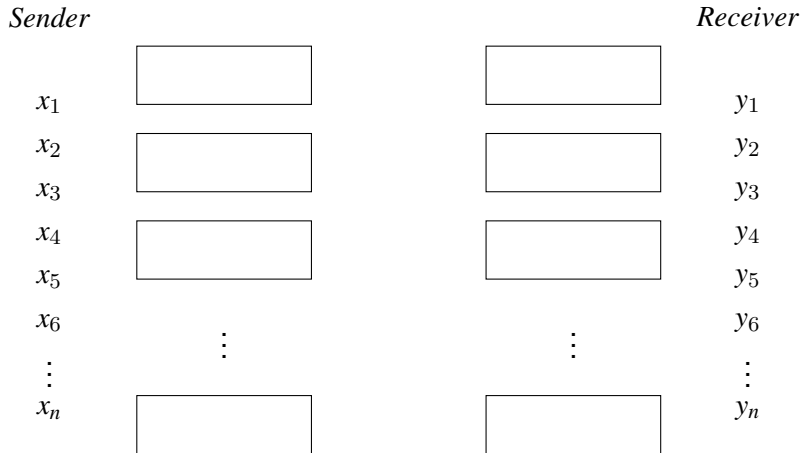
$$R(Y = \{y_1, \dots, y_n\})$$



$$R(Y = \{y_1, \dots, y_n\})$$

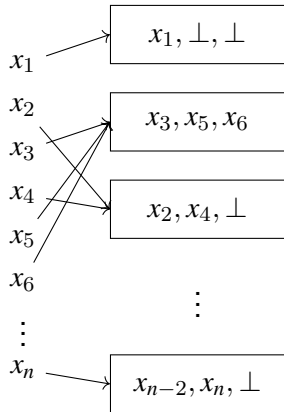


Single PSU + Simple Hash \Rightarrow PSU [Kol+19]

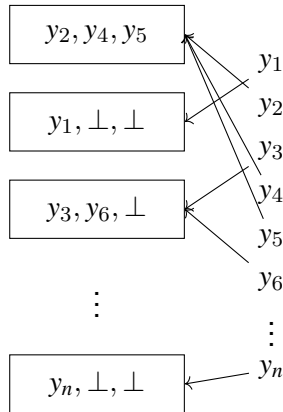


Single PSU + Simple Hash \Rightarrow PSU [Kol+19]

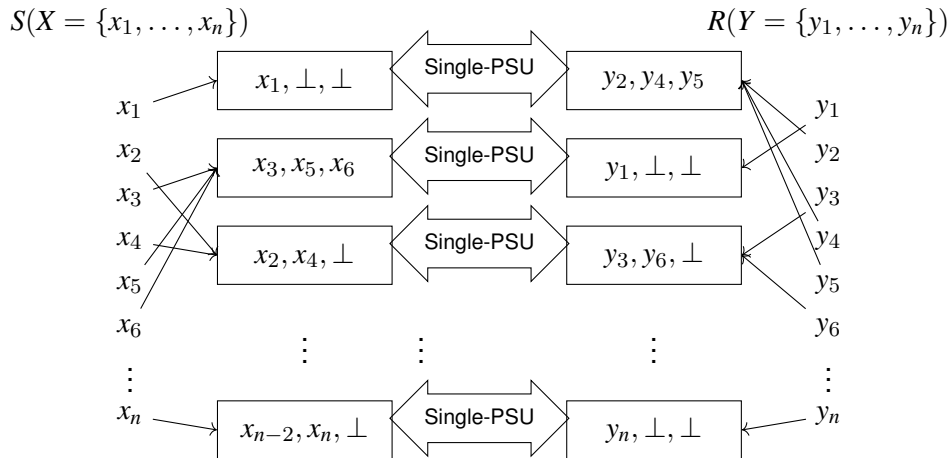
$$S(X = \{x_1, \dots, x_n\})$$



$$R(Y = \{y_1, \dots, y_n\})$$



Single PSU + Simple Hash \Rightarrow PSU [Kol+19]

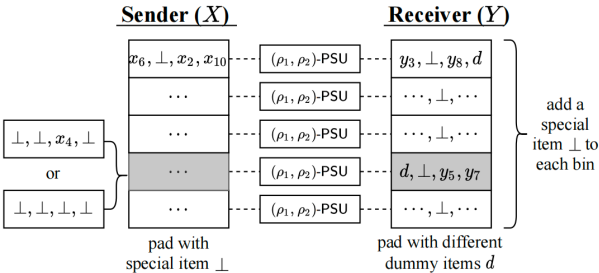


Hash to Bin 的问题

[Kol+19] 指出需要注意的地方:

- 每个 bin 需要填充 \perp 至最大值：防止 receiver 缩小交集范围
 - 只能使用 simple hash：如果 sender 使用 cuckoo hash，每个元素的位置和整个集合相关，当这个元素被 receiver 得到后会泄露整个集合的信息，无法模拟
- 即使如此，[Jia+22a] 指出上述协议依然是不安全的：
- 当 receiver 在某个 bin 发现得到的 b_i 全为 0 时，sender 在这个 bin 里全是 \perp 和至少有一个真实元素的概率是不同的，而且这个概率相差很大，因此 receiver 有很大概率会得到部分交集信息

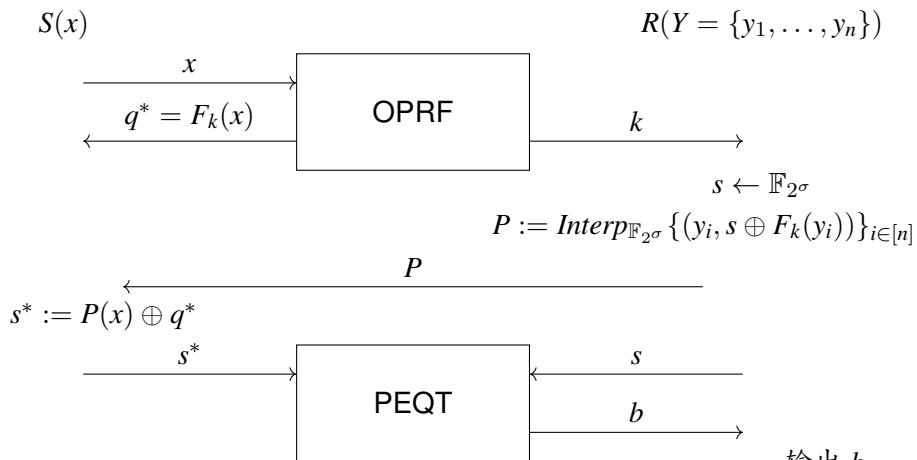
Hash to Bin 的问题

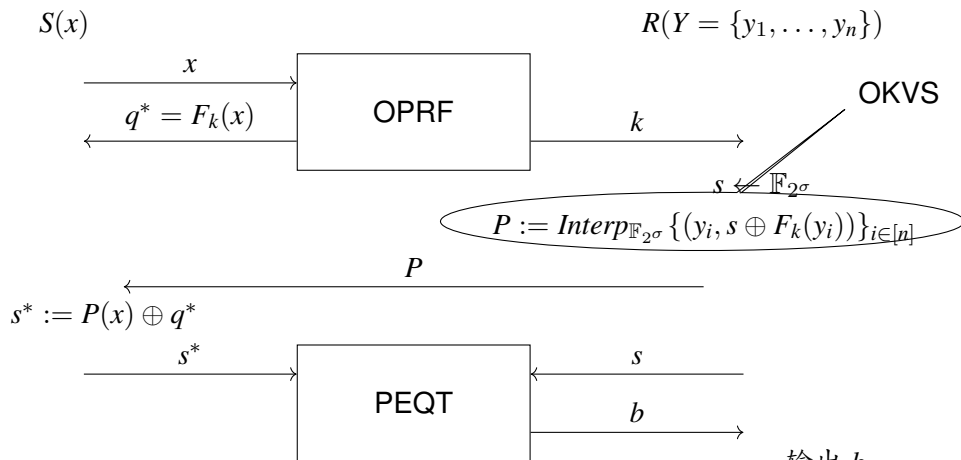


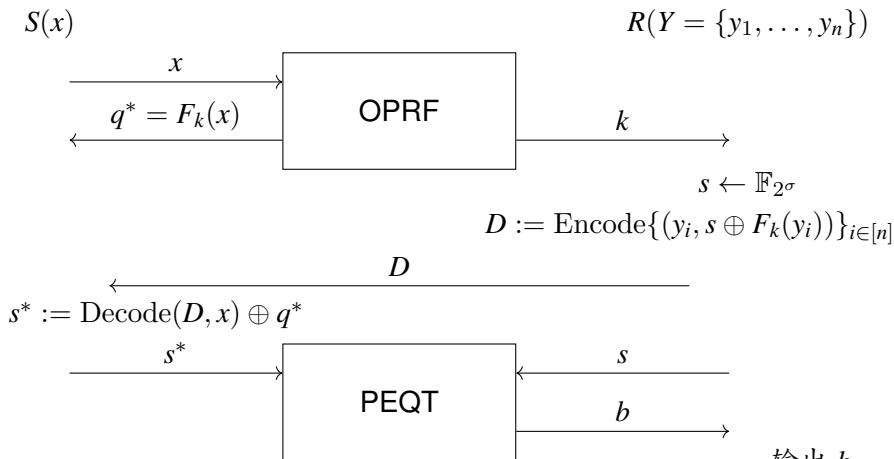
设集合大小为 n , bin 数量为 αn , 则 $Pr[case_2] = (1 - 1/\alpha n)^n \approx e^{-1/\alpha}$

parameters	set size n							
	2^8	2^{10}	2^{12}	2^{14}	2^{16}	2^{18}	2^{20}	2^{22}
α	0.043	0.055	0.05	0.053	0.058	0.052	0.06	0.051
$Pr(\times 10^{-11})$	7.946	1270	206.1	639.4	3252	444.8	5778	305.1

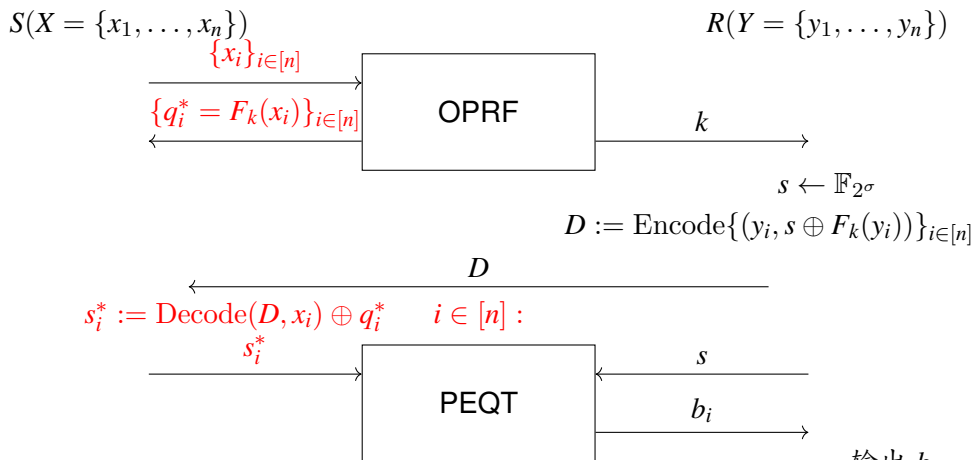
RPMT [Kol+19]

输出 b .

RPMT \rightarrow mq-RPMT [Zha+23]输出 b .

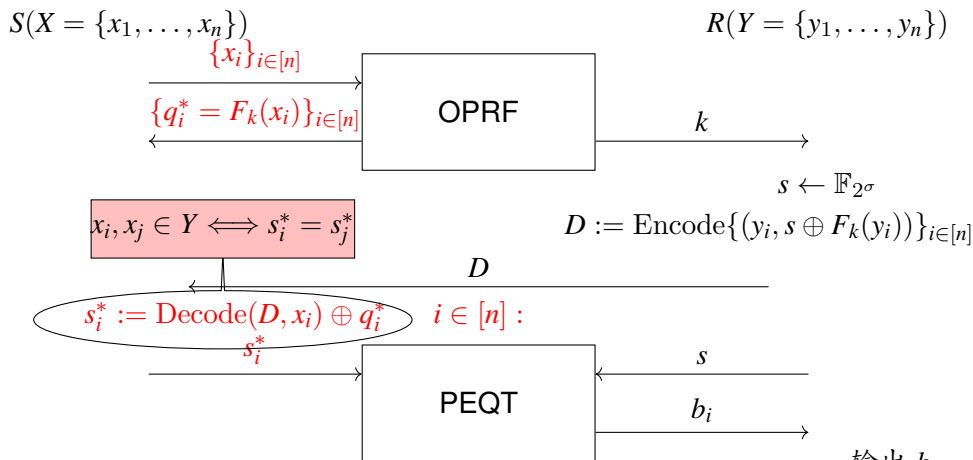
RPMT \rightarrow mq-RPMT [Zha+23]输出 b .

RPMT \rightarrow mq-RPMT [Zha+23]

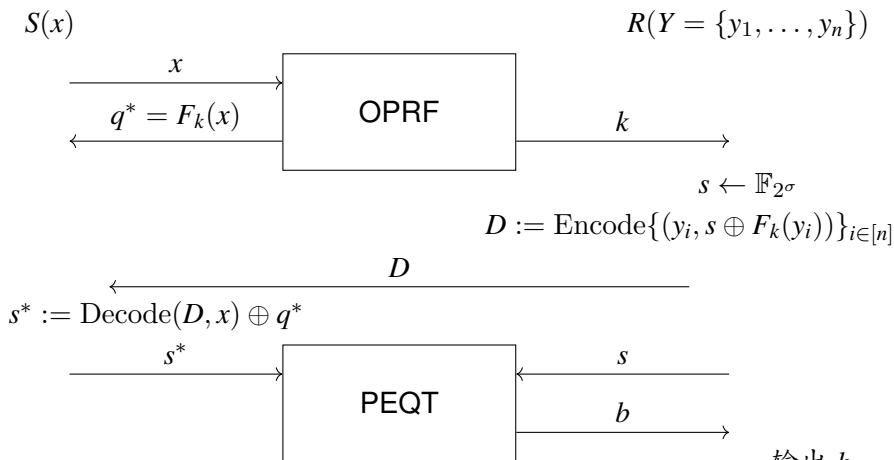


输出 b .

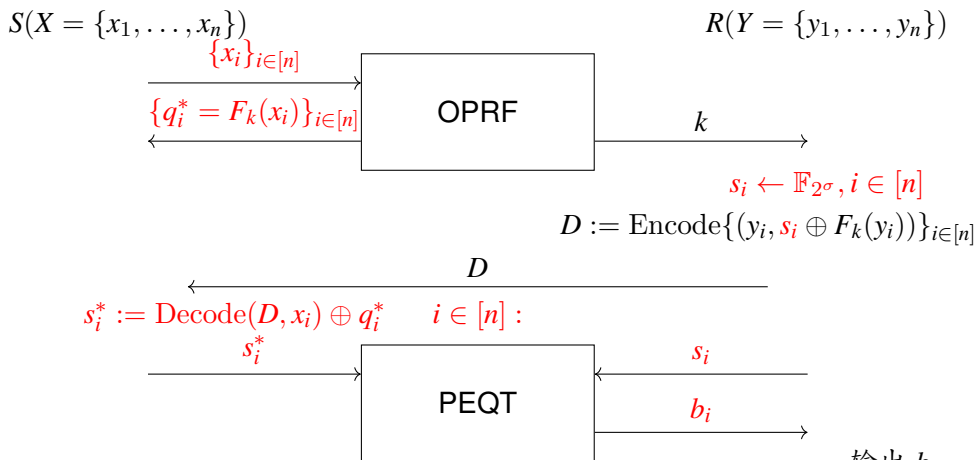
RPMT \rightarrow mq-RPMT [Zha+23]



输出 b .

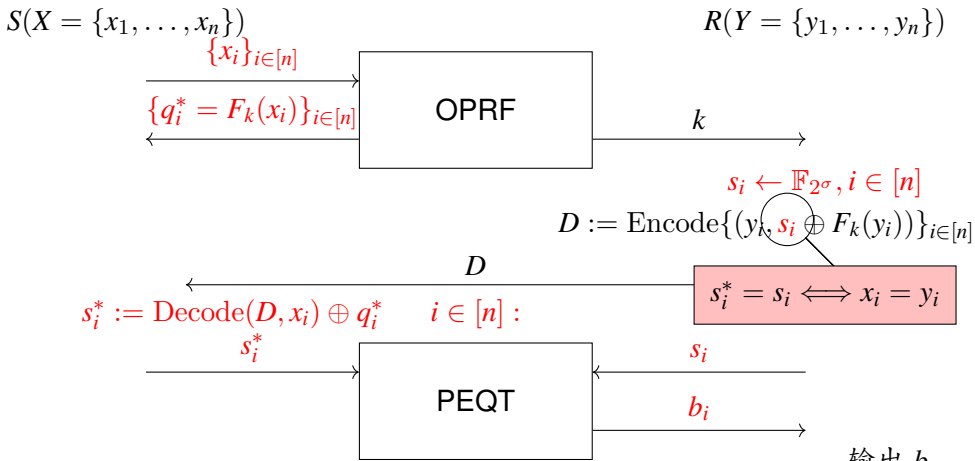
RPMT \rightarrow mq-RPMT [Zha+23]输出 b .

RPMT \longrightarrow mq-RPMT [Zha+23]



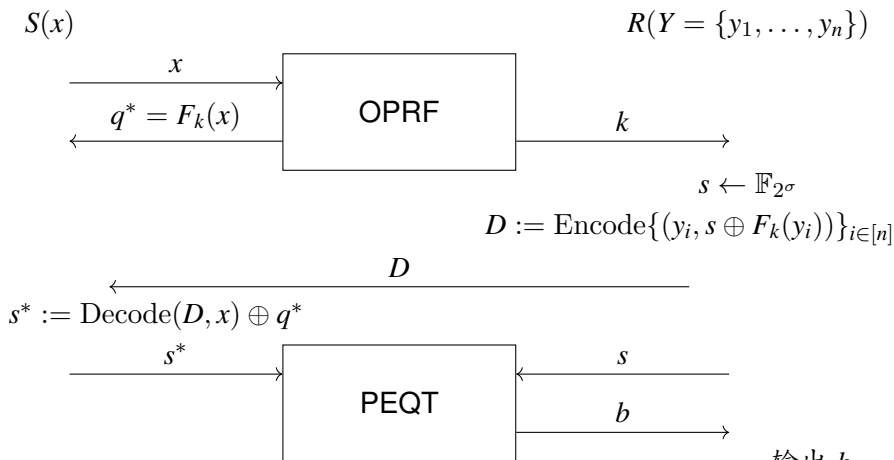
输出 b .

RPMT \rightarrow mq-RPMT [Zha+23]

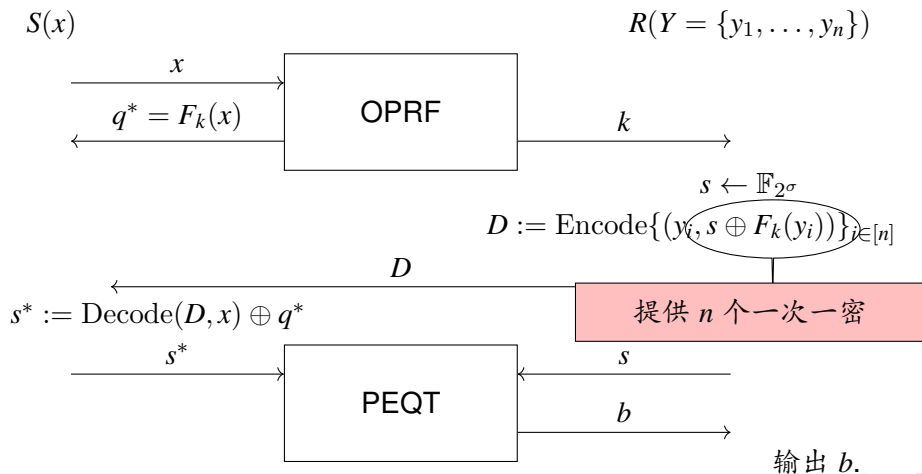


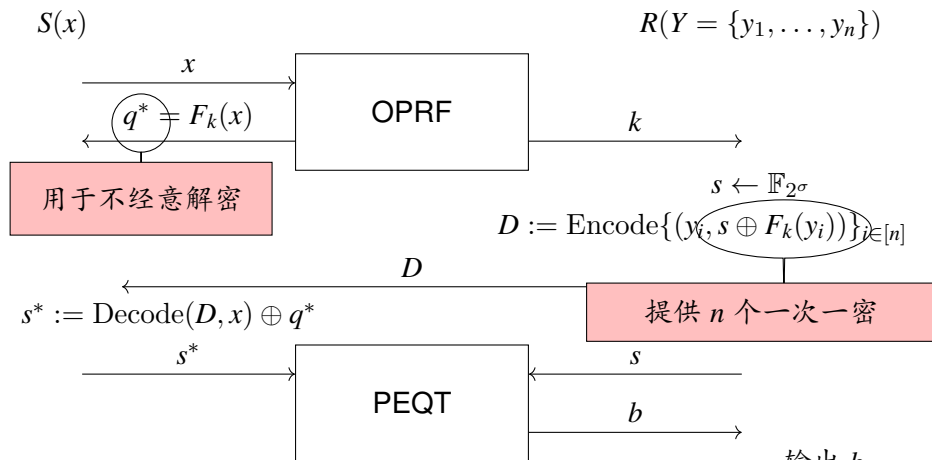
输出 b .

RPMT \rightarrow mq-RPMT [Zha+23]

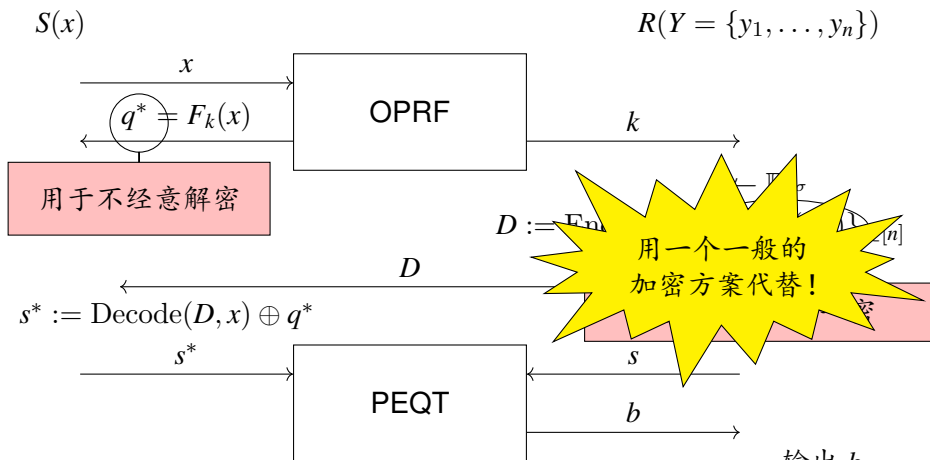


输出 b .

RPMT \rightarrow mq-RPMT [Zha+23]

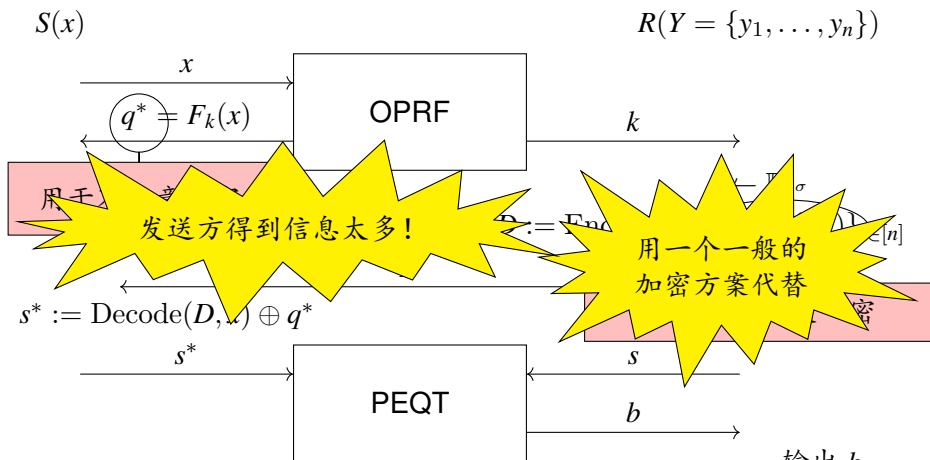
RPMT \rightarrow mq-RPMT [Zha+23]

RPMT \rightarrow mq-RPMT [Zha+23]

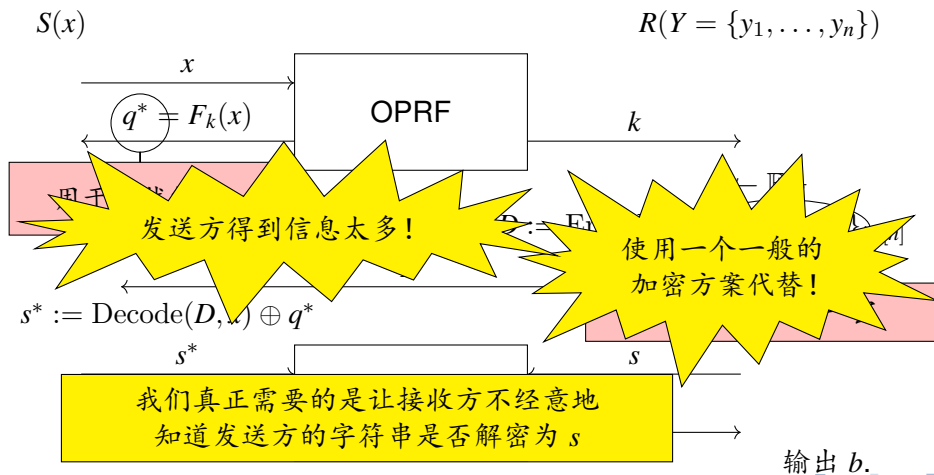


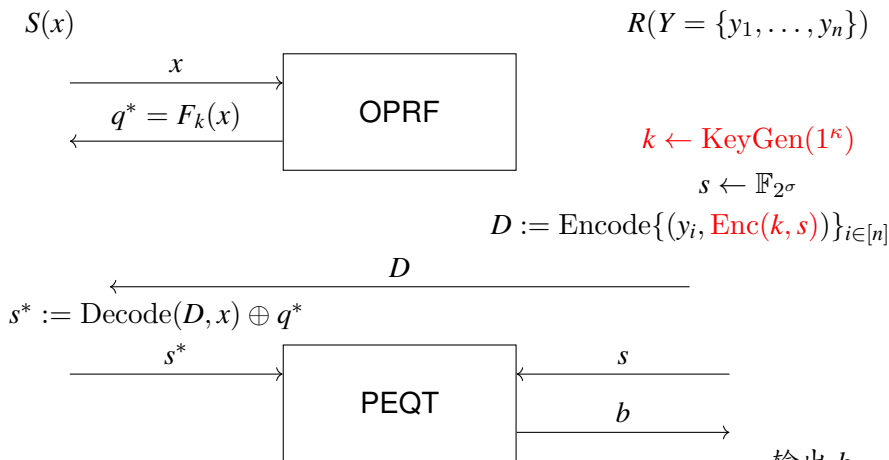
输出 b .

RPMT \rightarrow mq-RPMT [Zha+23]



输出 b .

RPMT \rightarrow mq-RPMT [Zha+23]输出 b .

RPMT \rightarrow mq-RPMT [Zha+23]

RPMT \longrightarrow mq-RPMT [Zha+23]

$$S(x)$$

$$R(Y = \{y_1, \dots, y_n\})$$

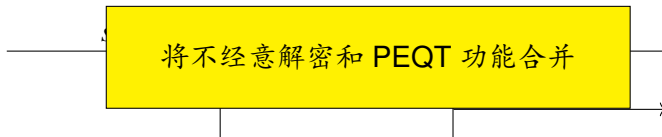
$$k \leftarrow \text{KeyGen}(1^\kappa)$$

$$s \leftarrow \mathbb{F}_{2^\sigma}$$

$$D := \text{Encode}\{(y_i, \text{Enc}(k, s))\}_{i \in [n]}$$

$$D$$

$$s_i^* := \text{Decode}(D, x_i), i \in [n]$$



输出 b .

mq-RPMT [Zha+23]

$S(X = \{x_1, \dots, x_n\})$

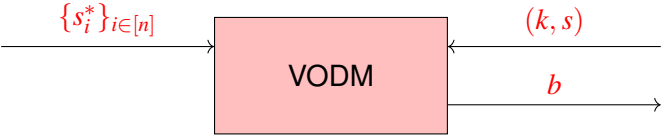
$R(Y = \{y_1, \dots, y_n\})$

$k \leftarrow \text{KeyGen}(1^\kappa)$

$s \leftarrow \mathbb{F}_{2^\sigma}$

$D := \text{Encode}\{(x_i, \text{Enc}(k, s))\}_{i \in [n]}$

$s_i^* := \text{Decode}(D, y_i), i \in [n]$



输出 b .

mq-RPMT from ReRand PKE [Zha+23]

$$S(X = \{x_1, \dots, x_n\})$$

$$R(Y = \{y_1, \dots, y_n\})$$

$$k \leftarrow \text{KeyGen}(1^\kappa)$$

$$s \leftarrow \mathbb{F}_{2^\sigma}$$

$$D := \text{Encode}\{(y_i, \text{Enc}(k, s))\}_{i \in [n]}$$

D

←

$$s_i^* := \text{Decode}(D, x_i), i \in [n]$$

$$\bar{s}_i^* := \text{ReRand}(s_i^*; r_i), i \in [n]$$

$$\{\bar{s}_i^*\}_{i \in [n]}$$

→

$$b_i = \begin{cases} 1 & \text{Dec}(sk, \bar{s}_i^*) = s; \\ 0 & \text{Dec}(sk, \bar{s}_i^*) \neq s \end{cases}$$

输出 b .

(permuted) mq-RPMT [Gar+21b]

$$S(X = \{x_1, \dots, x_n\})$$

$$\mathcal{X} \leftarrow \text{SimpleH}_{h_1, h_2, h_3}^m(X)$$

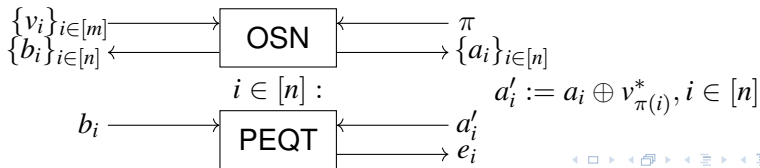
$$v_i \leftarrow \{0, 1\}^l, i \in [m]$$

$$R(Y = \{y_1, \dots, y_n\})$$

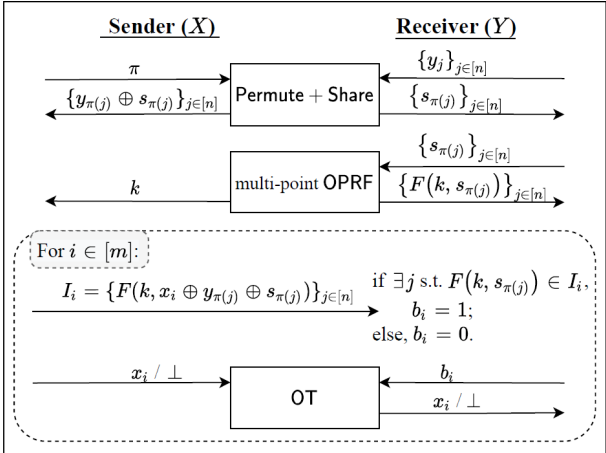
$$\mathcal{Y} \leftarrow \text{CuckooH}_{h_1, h_2, h_3}^m(Y)$$



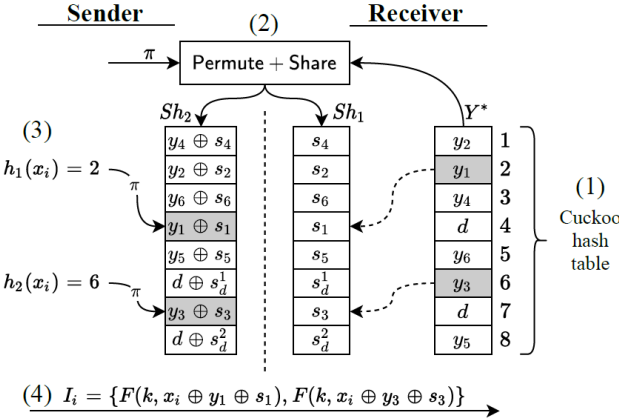
定义 $\pi : [m] \rightarrow [n]$ 是 \mathcal{Y} 中非空 bin



mq-RPMT [Jia+22a]



mq-RPMT [Jia+22a]



mq-RPMT [Che+22b]

$$S(X = \{x_1, \dots, x_n\})$$

$$R(Y = \{y_1, \dots, y_n\})$$

$$a \leftarrow \mathbb{Z}_q, A := \{H(x_i)^a\}_{i \in [n]}$$

 A

$$b \leftarrow \mathbb{Z}_q, B := \{H(y_i)^b\}_{i \in [n]}$$

 B

$$C := \{\pi(B_i^a)\}_{i \in [n]}$$

 C

$$e_i = \begin{cases} 1 & A_i^b \in C; \\ 0 & A_i^b \notin C \end{cases}$$

mq-RPMT [Che+22b]

$$S(X = \{x_1, \dots, x_n\})$$

$$R(Y = \{y_1, \dots, y_n\})$$

$$a \leftarrow \mathbb{Z}_q, A := \{H(x_i)^a\}_{i \in [n]}$$

$$b \leftarrow \mathbb{Z}_q, B := \{H(y_i)^b\}_{i \in [n]}$$

$$C := BF(\{B_i^a\}_{i \in [n]})$$

$$e_i = \begin{cases} 1 & A_i^b \in C; \\ 0 & A_i^b \notin C \end{cases}$$

- 1 介绍
- 2 隐私集合求交：PSI
- 3 隐私集合求并：PSU
- 4 参考文献

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Thanks!