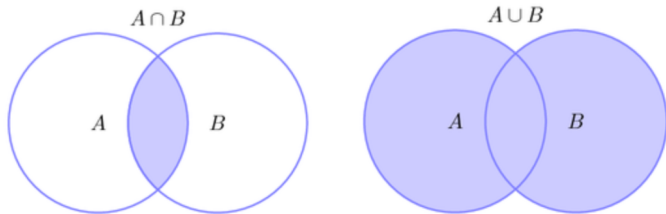


A Framework of Private Set Operations



Yu Chen
Shandong University

joint work with Min Zhang, Cong Zhang, Minglang Dong and Weiran Liu

Outline

- 1 Background
- 2 PSO Framework from mqRPMT
- 3 Construction of mqRPMT
 - 1st Construction from Commutative Weak PRF
 - 2nd Construction from Permuted Oblivious PRF
 - Connection Between mqPMT and mqRPMT
- 4 Comparison and Experimentation
- 5 Summary

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Privacy Preserving Computation

国家重大战略

国务院《关于构建要素市场化的意见》
《十四五规划和 2035 年远景目标纲要》

数据是新型生产要素 \leadsto 激活数据要素潜能

数据保护需求

数据泄露事件频发, 损失难以估量 三法五典出台

严格保护数据安全 \leadsto 数据流动性降低

Gartner 2021: 变革型前沿技术 \Rightarrow 破局的关键、数字经济的安全底座

高级密码方案

零知识证明

安全多方计算

隐私计算



打破数据孤岛

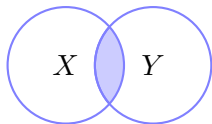
释放数据价值

市场规模

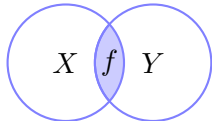
>1000亿



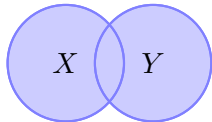
Private Set Operations (high frequency and high valuable)



$$\text{PSI} = X \cap Y$$



$$\text{PCSI} = \begin{cases} |X \cap Y| & \text{cardinality} \\ |X \cap Y|, \sum_{x_i \in X \cap Y} v_i & \text{cardinality-sum} \\ f(X \cap Y) & \text{general computation} \end{cases}$$



$$\text{PSU} = X \cup Y$$

Wide Applications of PSO

PSI

- privacy-preserving location sharing
- private contact discovery
- DNA testing and pattern matching

PCSI

- measuring the effectiveness of online advertising

PSU

- IP blacklist and vulnerability data aggregation
- private DB supporting full join
- private-ID

SOTA of PSO

PSI has been extensively studied in the last two decades

- balanced setting: [KKRT16, CM20, RR22] achieves linear complexity, and almost as efficient as insecure hash protocol
- unbalanced setting: [CLR17, CHLR18, CMdG⁺21] achieves sub-linear complexity of large set

In sharp contrast, the study of PCSI and PSU are not satisfying.

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concretely 20× slower in timing and 30× more communication than PSI

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PSU

- [KS05, Fri07, HN10, KRTW19, JSZ⁺22] have superlinear complexity
- [DC17, ZCL⁺23] achieve linear complexity, but not strict (comm. complexity additionally depends on statistical parameter $\lambda \approx 40$)

concretely 20× slower in timing and 25× more communication than PSI

Motivation

Different approaches are used for different private set operations \leadsto require much more engineering effort and maintaining cost

- **Goal:** a unified framework of PSO

¹[GMR⁺21] presented a PSO framework from permuted characteristic. However, its oblivious shuffle functionality is not necessary for PSO, and incurs superlinear complexity.

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There exists huge efficiency gap between PSI and other PSO protocols

- **Goal:** efficient instantiations to close the gap¹

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After ≈ 40 years, DH-PSI [Mea86] is still the most easily understood and implemented one among numerous PSI protocols. Surprisingly, no counterpart is known in the PSU setting yet. Existing protocols are very complicated.

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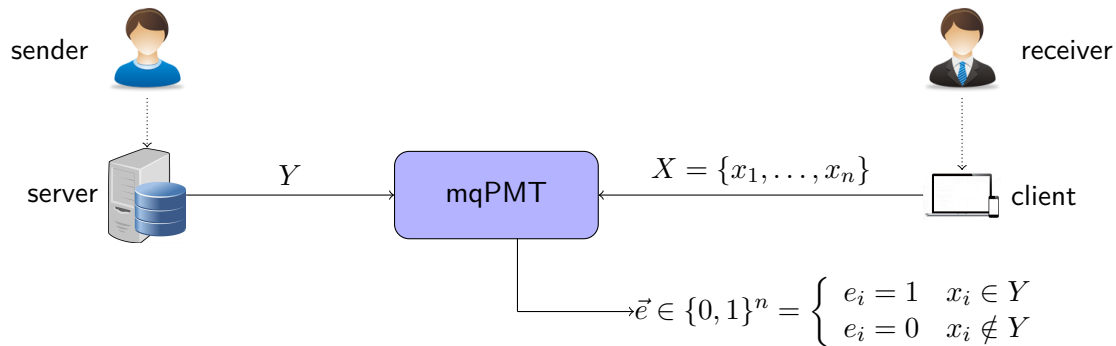
*Is there a central building block that enables a unified framework for PSO?
How to give instantiations with optimal asymptotic complexity and good concrete efficiency?
Can the DDH assumption strike back with efficient PSU protocol?*

¹[GMR⁺21] presented a PSO framework from permuted characteristic. However, its oblivious shuffle functionality is not necessary for PSO, and incurs superlinear complexity.

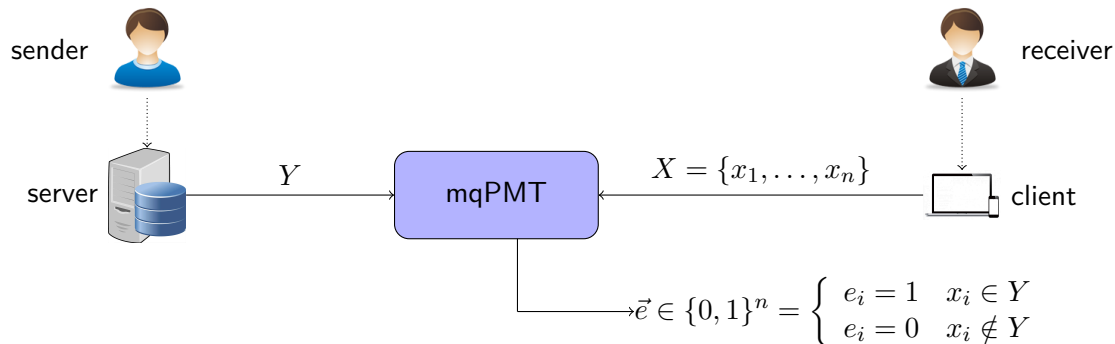
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Start Point: multi-query Private Membership Test (mqPMT) underlying PSI

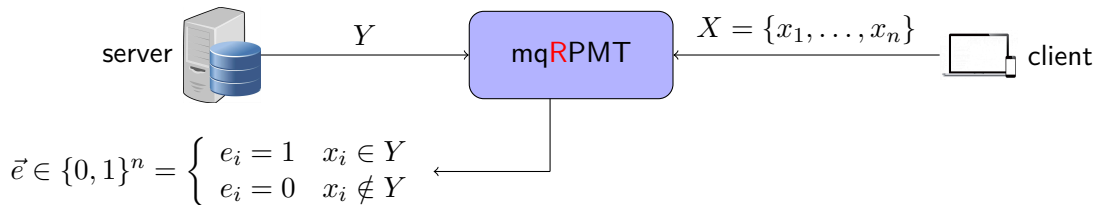


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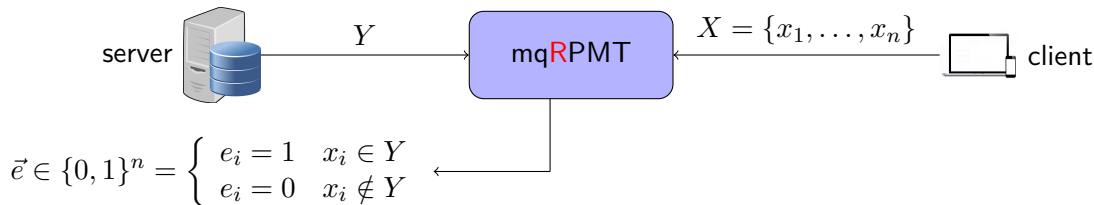


- **Problem:** the client learns both x_i and e_i , a.k.a. the intersection \leadsto not suitable for protocols that should hide intersection, such as PCSI and PSU.

The core protocol: multi-query Reverse Private Membership Test (mqRPMT)



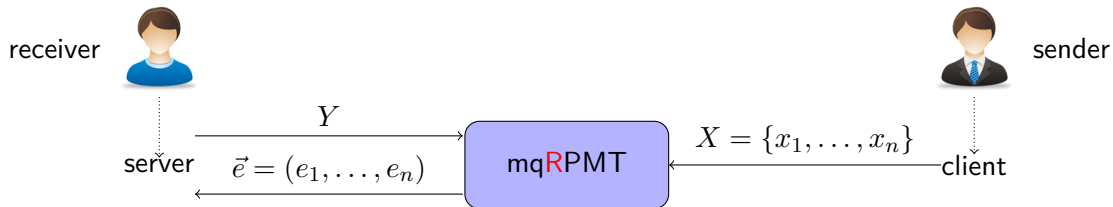
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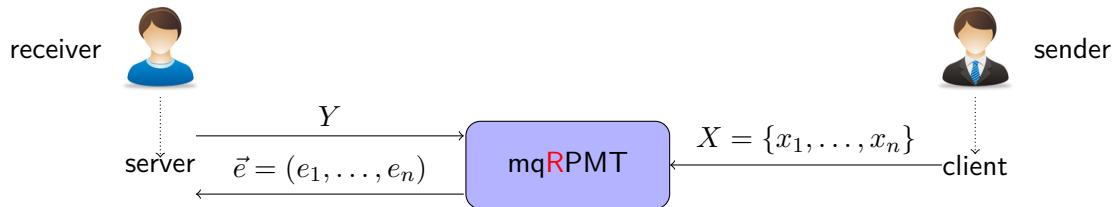
- The server learns e_i , while the client learns x_i , a.k.a. the information of intersection is shared between the two parties \leadsto suitable for all PSO protocols



PSO from mqRPMT

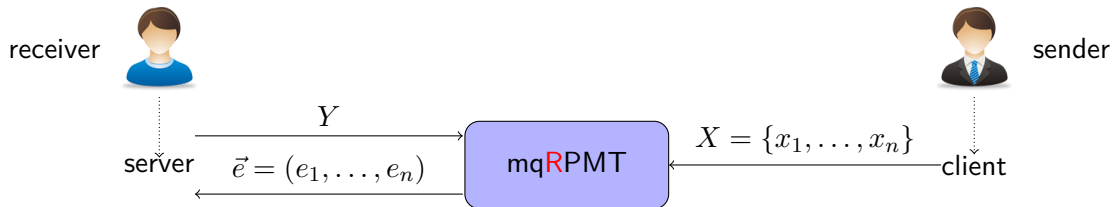


PSO from mqRPMT

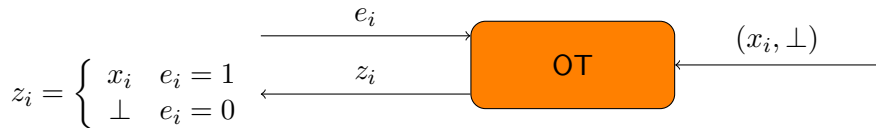


directly yields **PSI-card**: $|X \cap Y|$ is the Hamming weight of \vec{e}

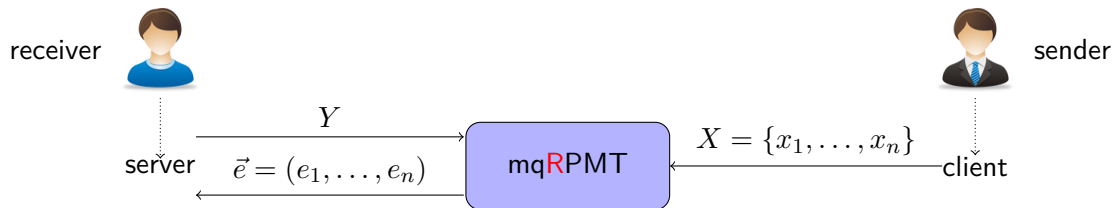
PSO from mqRPMT



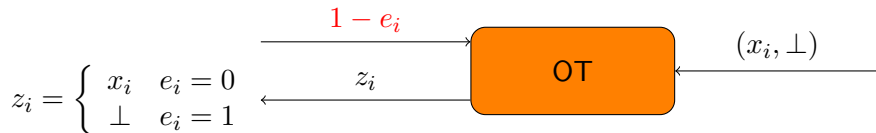
yields **PSI** coupled with OT: receiver obtains $X \cap Y$



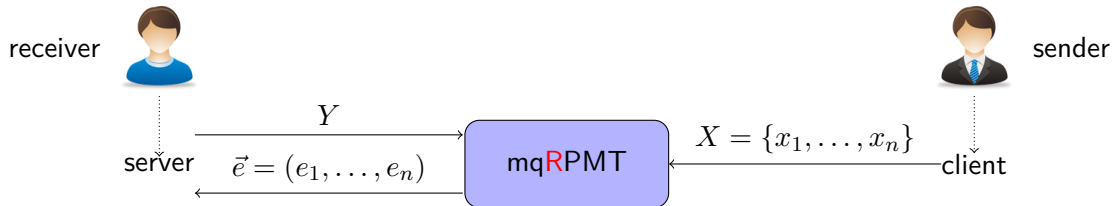
PSO from mqRPMT



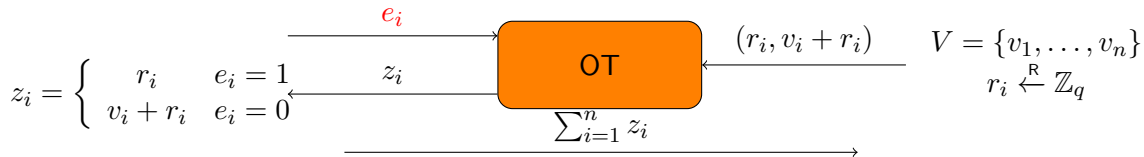
yields **PSU** coupled with OT (flipping \vec{e}): receiver obtains $X - Y$



PSO from mqRPMT



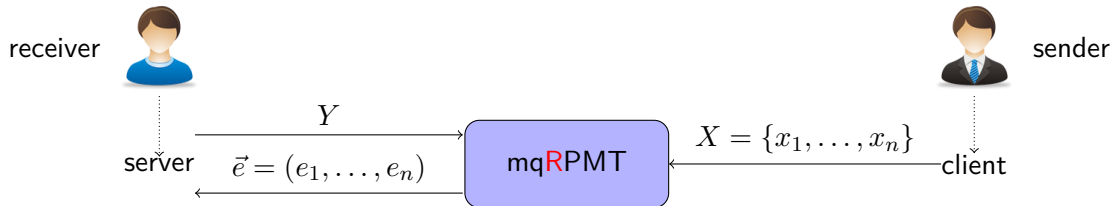
yields **PSI-card-sum** coupled with OT and masking trick



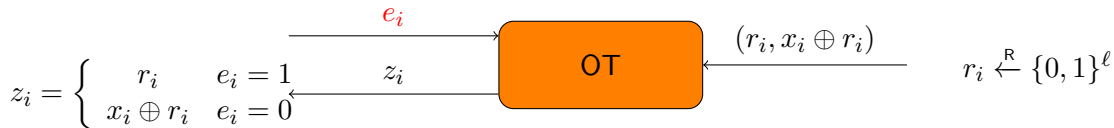
receiver obtains $|X \cap Y|$

sender obtains $\sum_{x_i \in Y} v_i = \sum_{i=1}^n z_i - \sum_{i=1}^n r_i$

PSO from mqRPMT



yields **PSI-card-secret-share** coupled with OT and masking trick



receiver obtains $|X \cap Y|$ and z_i

sender has $x_i \oplus r_i$

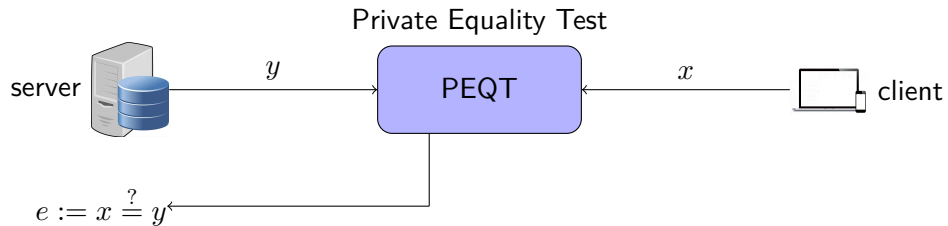
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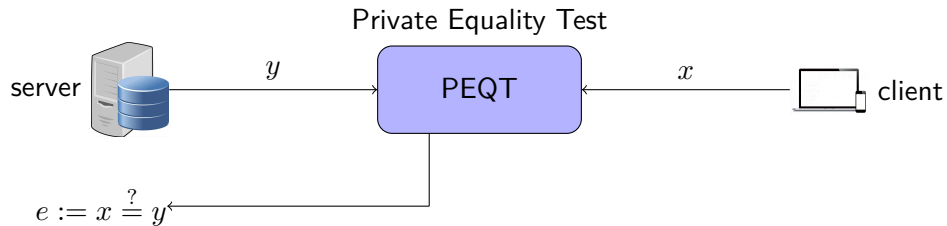
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Starting Point: PEQT



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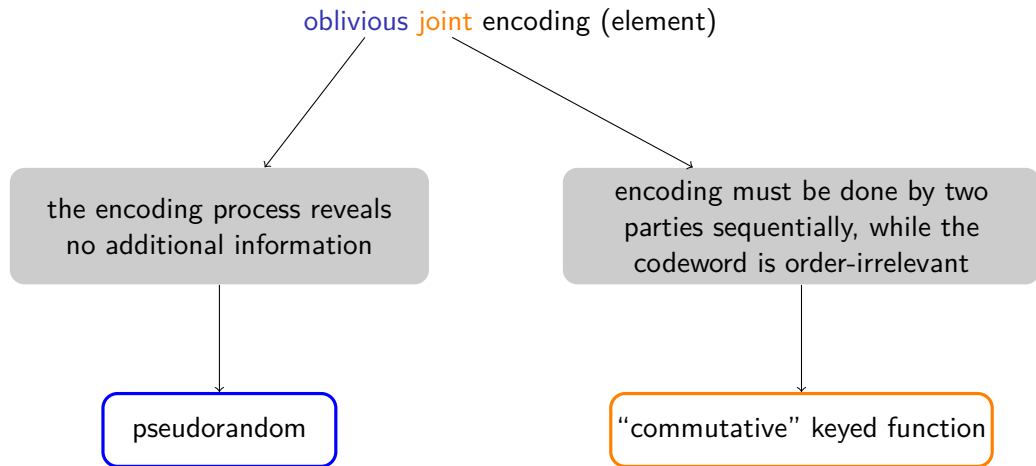


Observation: PEQT is not only an extreme case of mqPMT, but also an extreme case of mqRPMT

Goal: build PEQT amenable to extension:

$$y \rightsquigarrow Y = \{y_1, \dots, y_m\}, x \rightsquigarrow X = \{x_1, \dots, x_n\}, e \rightsquigarrow \vec{e} = (e_1, \dots, e_n)$$

High-level Idea



Commutative Weak PRF

We first formally define two standard properties for keyed functions.

Composable. For a family of keyed functions $F : K \times D \rightarrow R$, F is 2-composable if $R \subseteq D$ (special case $R = D$) $\leadsto F_{k_1}(F_{k_2}(\cdot))$ is well-defined.

Commutative. A family of composable keyed functions is commutative if:

$$\forall k_1, k_2 \in K, \forall x \in D : F_{k_1}(F_{k_2}(x)) = F_{k_2}(F_{k_1}(x))$$

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Definition 1 (Commutative Weak PRF)

$F : K \times D \rightarrow D$ is cwPRF if it satisfies **weak pseudorandomness** ($k \xleftarrow{R} K, x \xleftarrow{R} X$) and **commutative** property simultaneously. When F is a permutation, we say F is cwPRP.

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Why merely weak pseudorandomness?

Standard pseudorandomness denies commutativity. Consider the following attack:

- \mathcal{A} picks $k' \xleftarrow{R} K, x \xleftarrow{R} D$, queries the real-or-random oracle at point $F_{k'}(x)$ and x , receiving y' and y . \mathcal{A} then outputs '1' iff $F_{k'}(y) = y'$.

Construction of cwPRF

Construction (DDH-based cwPRF)

- $\text{Setup}(1^\kappa)$: runs $\text{GroupGen}(1^\kappa) \rightarrow (\mathbb{G}, g, p)$, output $pp = (\mathbb{G}, g, p)$ which defines
$$F : \mathbb{Z}_p \times D \rightarrow \mathbb{G} \text{ as } F_k(x) := x^k$$
- $\text{KeyGen}(pp)$: outputs $k \xleftarrow{R} \mathbb{Z}_p$.
- $\text{Eval}(k, x)$: on input $k \in \mathbb{Z}_p$ and $x \in D$, outputs x^k .

DDH assumption \Rightarrow weak pseudorandomness

Commutativity: $\forall k_1, k_2 \in K$ and $\forall x \in D$: $F_{k_1}(F_{k_2}(x)) = x^{k_1 k_2} = F_{k_2}(F_{k_1}(x))$

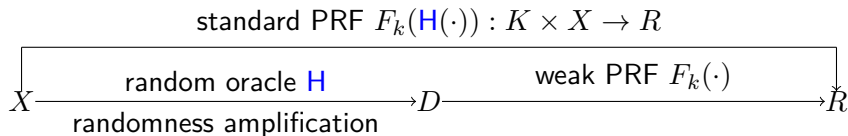
cwPRF is the “right” cryptographic abstraction of the classic DH function.

Randomness Amplification

But what we need for mqRPMT is standard pseudorandomness.

Solution: hash-then-evaluate

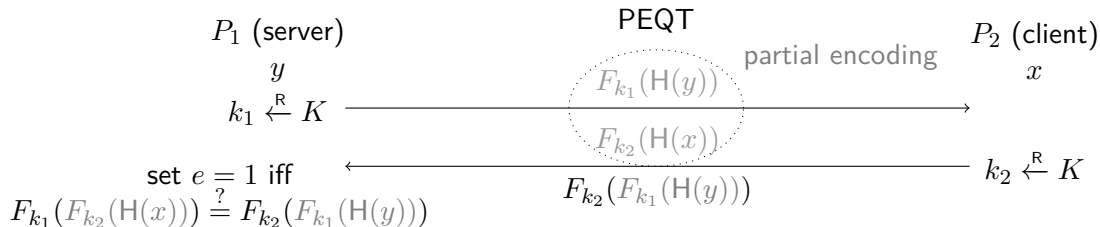
- Domain extension: handle arbitrary domain $X = \{0, 1\}^*$
- Randomness amplification: weak \rightsquigarrow standard



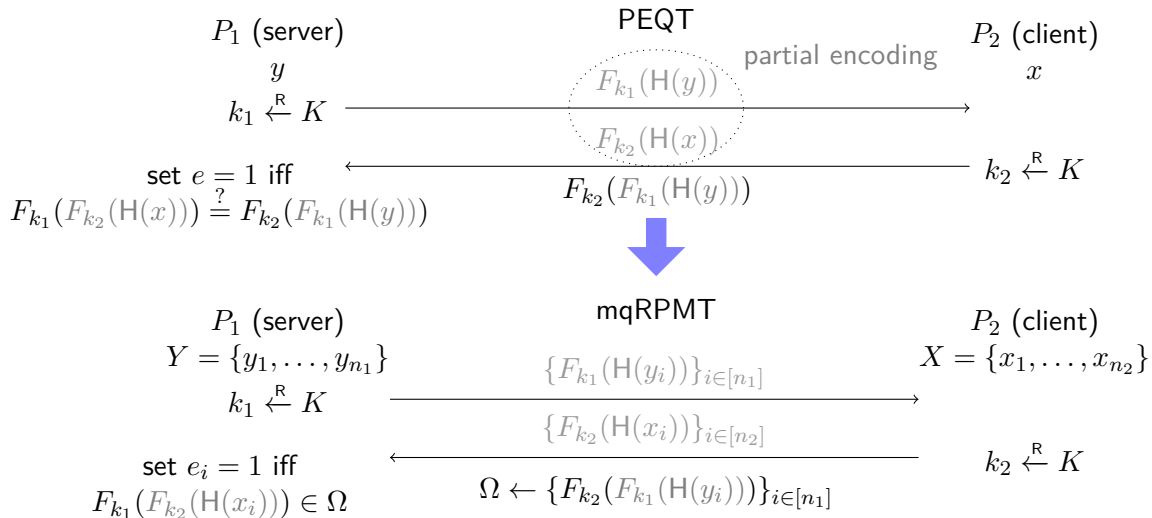
Commutativity still holds w.r.t. \mathbf{H}

$$F_{k_1}(F_{k_2}(\mathbf{H}(x))) = F_{k_2}(F_{k_1}(\mathbf{H}(x)))$$

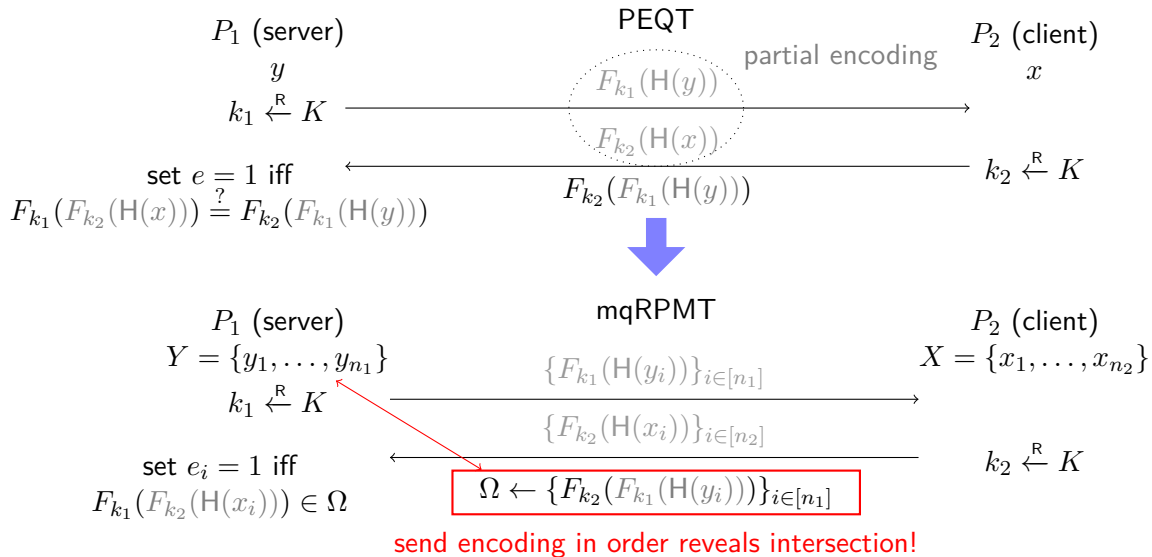
mqRPMT from cwPRF



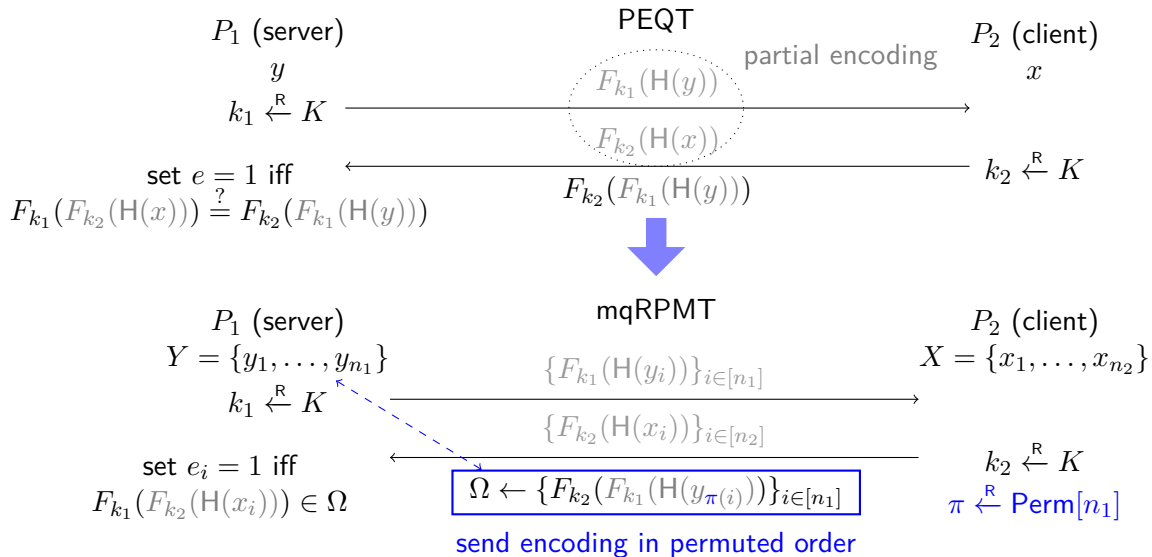
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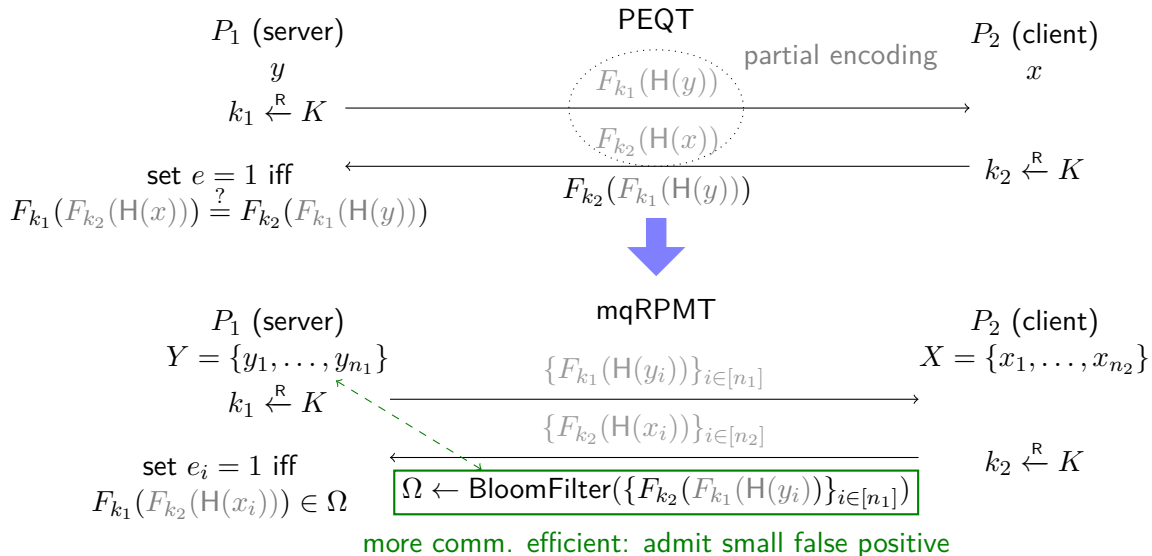
mqRPMT from cwPRF



mqRPMT from cwPRF



mqRPMT from cwPRF



Complexity Analysis

Consider the balanced setting: $n_1 = n_2 = n$

Table: Complexity of cwPRF-based mqRPMT.

Computation	$4n \times F_k(\cdot) + 2n \times H(\cdot)$ hash-to-domain
Communication	$3n \times D $ or $2n \times D + c \cdot \lambda \ll D $

cwPRF-based mqRPMT is **optimal** in the sense that both computation and communication complexities are **strictly linear** in n

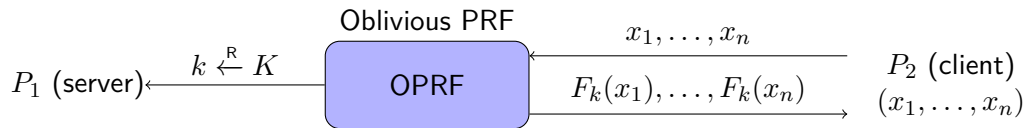
Instantiating the PSO framework with cwPRF-based mqRPMT, DDH assumption strikes back with the first **strictly linear** PSU protocol

incredibly simple and efficient

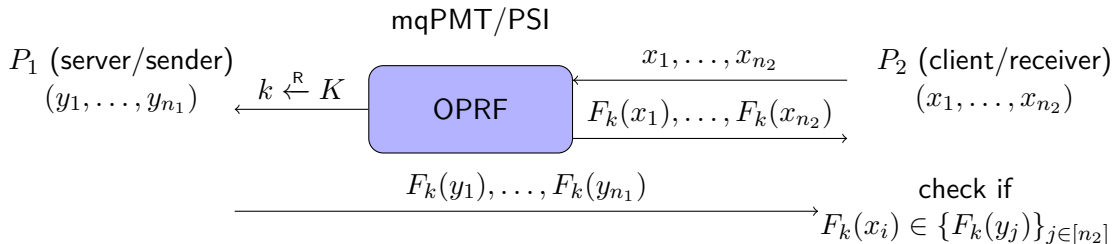
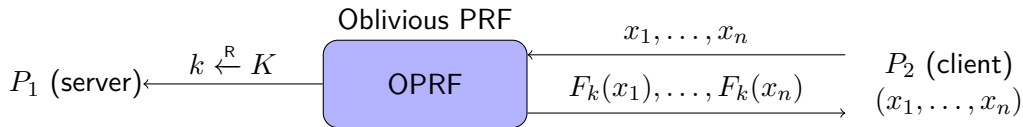
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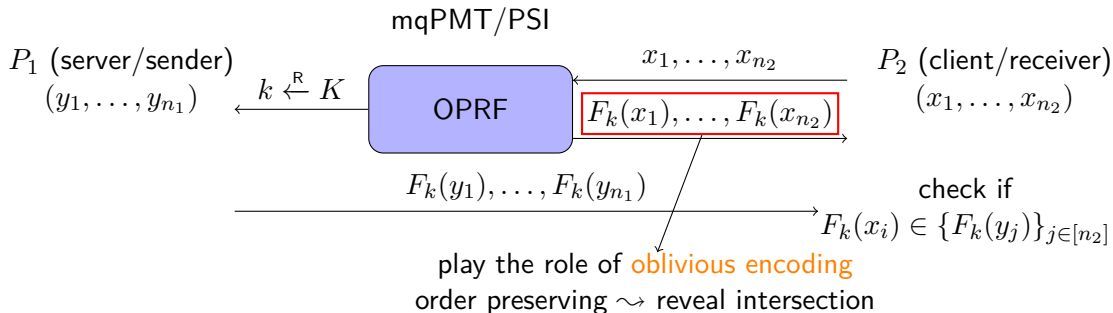
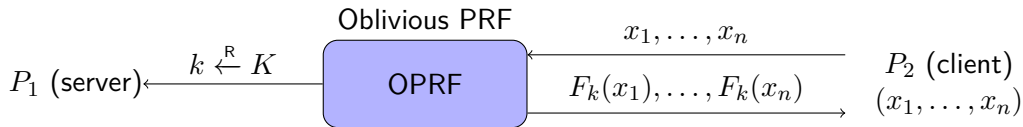
Starting Point: mqPMT/PSI from OPRF



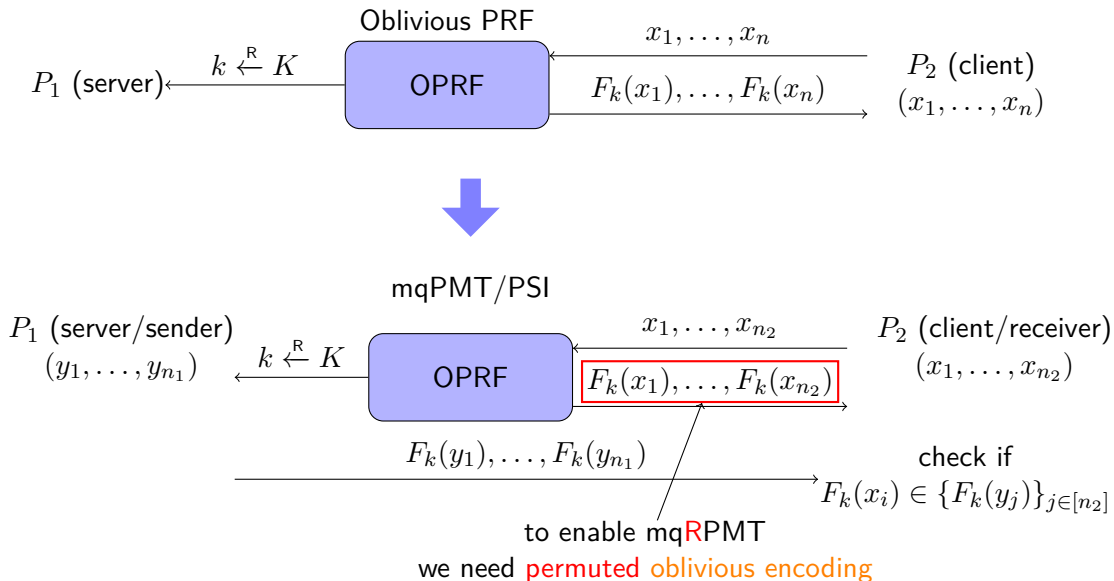
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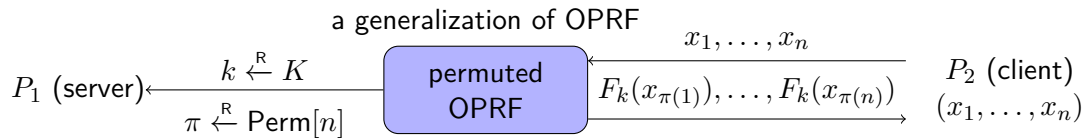
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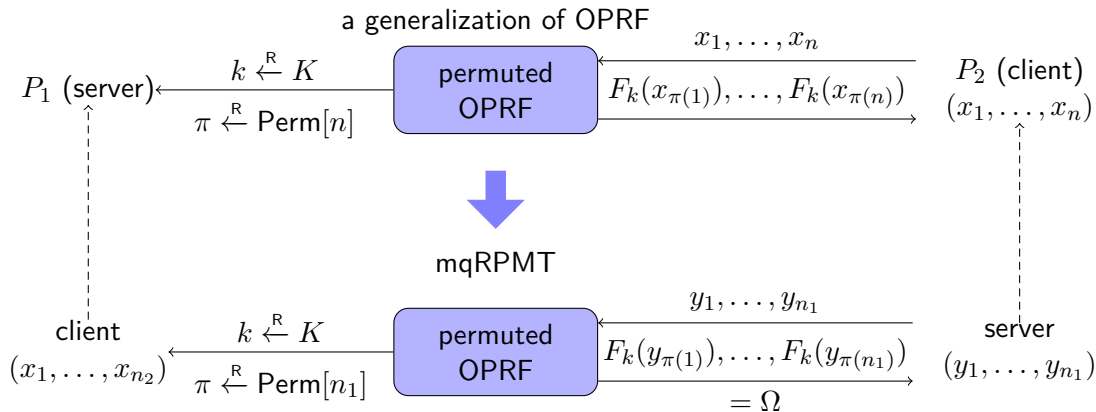
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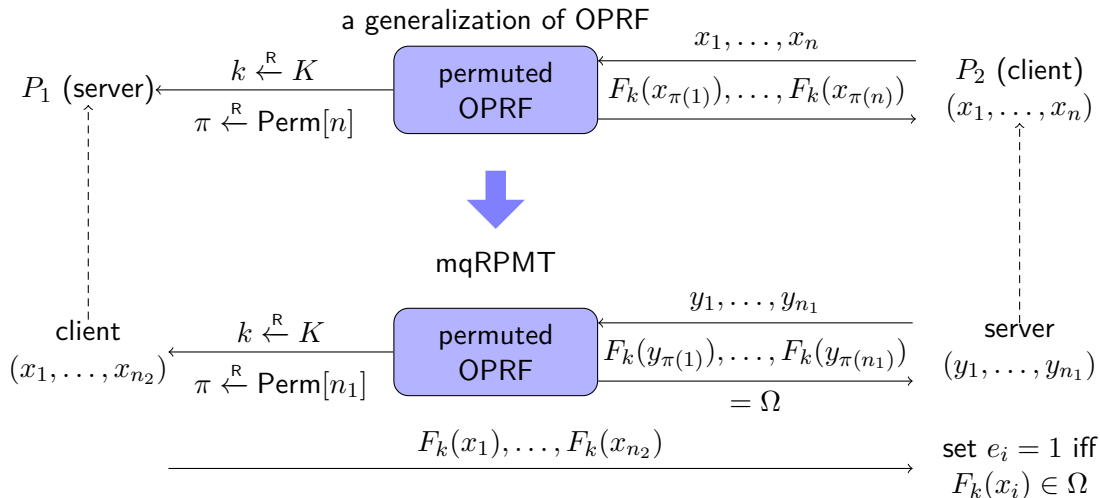
mqRPMT from Permuted OPRF



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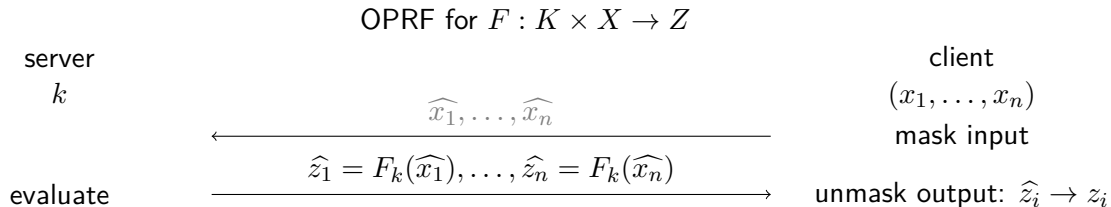


mqRPMT from Permuted OPRF



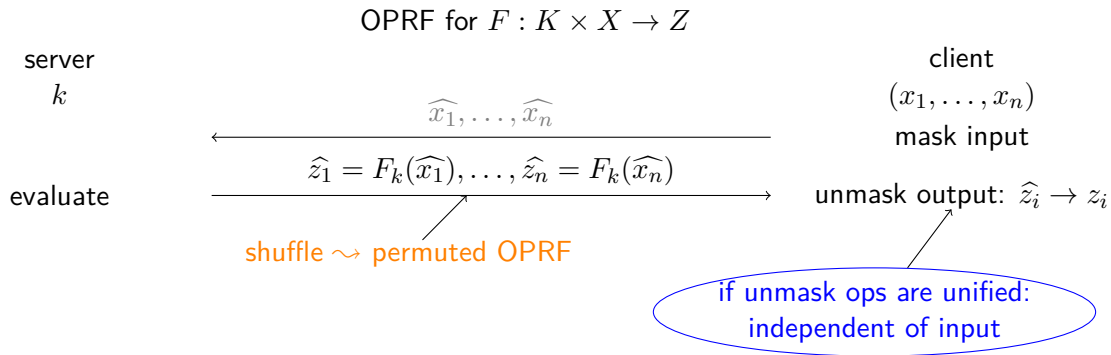
Build Permuted OPRF from cwPRP

A common approach to build OPRF is “mask-then-unmask” via **homomorphism**



Build Permuted OPRF from cwPRP

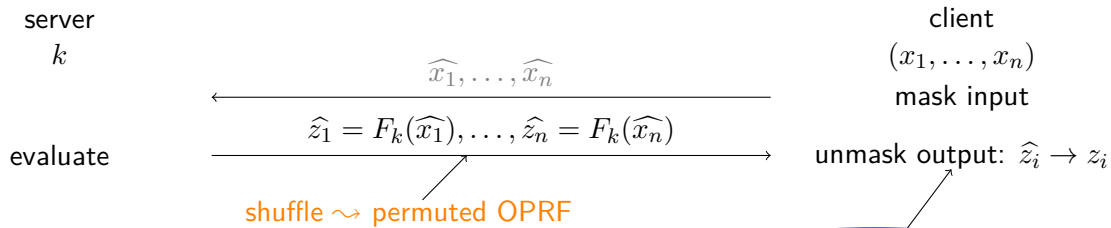
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Build Permuted OPRF from cwPRP

A common approach to build OPRF is “mask-then-unmask” via **homomorphism**

OPRF for $F : K \times X \rightarrow Z$



cwPRP enables simplest unified mask-then-unmask

mask: $\hat{x} \leftarrow F_s(x)$

evaluate: $\hat{z} \leftarrow F_k(\hat{x})$

unmask: $z \leftarrow F_s^{-1}(F_k(\hat{x}) = F_k(F_s^{-1}(\hat{x})) = F_k(x)$

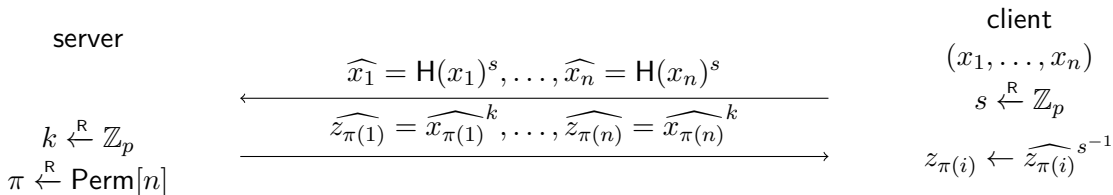
if unmask ops are unified:
independent of input

Permuted OPRF from DDH-based cwPRP

Observe that the DDH-based cwPRF is actually a cwPRP $F : \mathbb{Z}_p \times \mathbb{G} \rightarrow \mathbb{G}$.

- combine $H : \{0, 1\}^* \rightarrow \mathbb{G} \Rightarrow$ permuted OPRF protocol for $G : \mathbb{Z}_p \times \{0, 1\}^* \rightarrow \mathbb{G}$ defined as $G_k(x) = F_k(H(x))$.
-

pOPRF for $G_k(x) = F_k(H(x))$



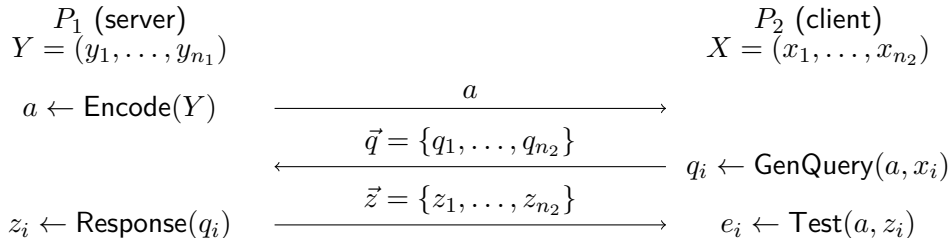
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Sigma-mqPMT

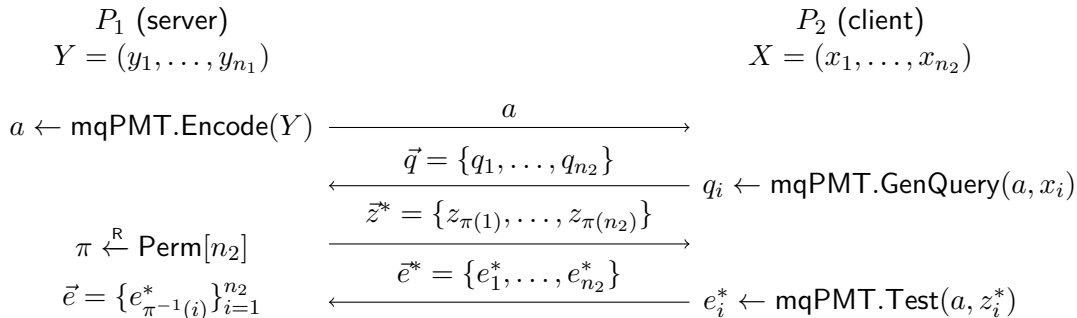
Given the efficiency gap between PSI and other PSO protocols, it is intriguing to study the connection between mqPMT and mqRPMT.

- Towards this goal, we first abstract a category of mqPMT called Sigma-mqPMT.



- **Reusable:** a (best interpreted as encoding of Y) can be safely reused.
- **Context-independent:** q_i is only related to a, x_i under test and P_2 's randomness.
- **Stateless test:** Test algorithm can work without knowing (x_i, q_i) .

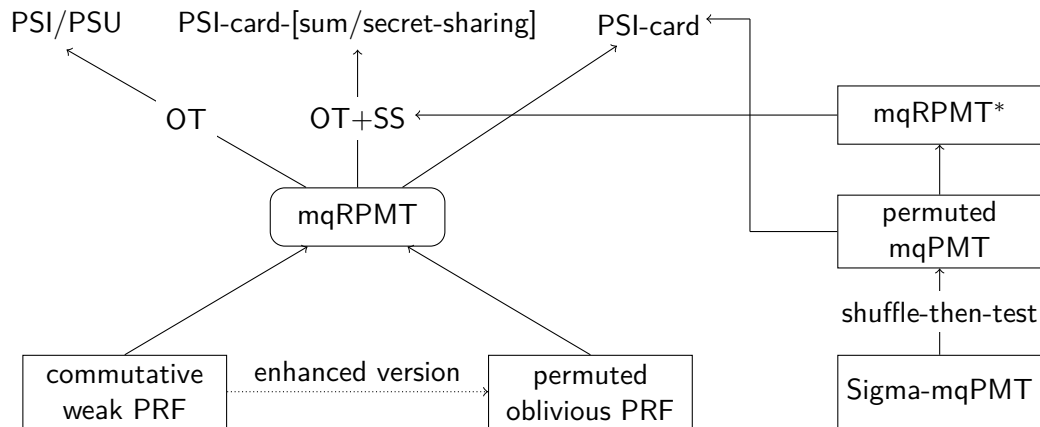
mqRPMT* from Sigma-mqPMT



Via the “permute-then-test” approach, we can tweak Sigma-mqPMT to mqRPMT* (additionally reveal intersection size to client).

- translate a category of PSI protocols (such as [\[Mea86, FIPR05, CLR17\]](#)) to other PSO protocols (allowing both parties learn the intersection size).
- make the initial step towards establishing the connection between mqRPMT and mqPMT.

Summary of Main Results



Outline

- 1 Background
- 2 PSO Framework from mqRPMT
- 3 Construction of mqRPMT
 - 1st Construction from Commutative Weak PRF
 - 2nd Construction from Permuted Oblivious PRF
 - Connection Between mqPMT and mqRPMT
- 4 Comparison and Experimentation
- 5 Summary

Cryptographic Engineering Matters

We implement our PSO framework via the following vein

EC groups DDH-based cwPRF \rightsquigarrow mqRPMT \rightsquigarrow PSO framework

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EC groups DDH-based cwPRF \leadsto mqRPMT \leadsto PSO framework

- ① NIST P-256 $\blacklozenge \blacktriangledown$ (also known as secp256r1 and prime256v1)
 - hash-to-point operation is expensive \approx non-fixed Exp
 - point compression halving communication at cost
 - \leadsto point decompression is expensive \approx non-fixed Exp

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- ② Curve25519 \star (*de facto* alternative of NIST P-256)
 - numerous merits: no backdoor, fast Exp, immunity against side-channel attacks
 - allow fast Exp in compressed form \leadsto halving comm. without **decompression**
 - any 32-byte bit string (interpreting as X -coordinate) can be ambiguously identified as a valid point \leadsto **hash-to-point operation** is almost free

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For the first time, Curve25519 fully unleashes its power in PSO area.

Correct the prejudice that “public-key operations are expensive”:

- By leveraging optimized implementation, their performances are comparable with symmetric-key operations

Implementation Features



Modular design: admit flexible combination to support various scenarios



Minimum dependency: only require OpenSSL and OpenMP



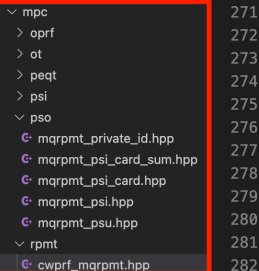
Multi-platforms: run smoothly on Linux and MacOS



Rich functionality: support all PSO operations



Highly parallelizable: scalable \leadsto support large-scale applications



```

271
272     uint8_t k1[32];
273     PRG::Seed seed = PRG::SetSeed(fixed_seed, 0); // initialize PRG
274     GenRandomBytes(seed, k1, 32); // pick a key k1
275
276     std::vector<EC25519Point> vec_Hash_Y(pp.SERVER_LEN);
277     std::vector<EC25519Point> vec_Fk1_Y(pp.SERVER_LEN);
278
279     #pragma omp parallel for num_threads(thread_count)
280     for(auto i = 0; i < pp.SERVER_LEN; i++){
281         Hash::BlockToBytes(vec_Y[i], vec_Hash_Y[i].px, 32);
282         x25519 scalar mulx(vec_Fk1_Y[i].px, k1, vec_Hash_Y[i].px);

```

Implementation Details

Dev/Test environment	Other Parameters
CPU = Intel i7 2.50 GHZ	$\kappa = 128, \lambda = 40$
Physical core = 8	item length = 128 bits
RAM = 8GB	set sizes = $\{2^{12}, 2^{16}, 2^{20}\}$
OS = Ubuntu 20.04	LAN = 10Gbps, WAN = 50Mbps, RTT = 80ms

Protocols:

- mqRPMT, PSI, PSI-card, PSI-card-sum, PSU, Private-ID

Test items:

- Functionality
- Computation cost: total running time
- Communication cost: sum of two parties

Core protocol: mqRPMT

Protocol	T	Running time (s)						Commu. (MB)		
		LAN			WAN			total		
		2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}
mqRPMT \blacklozenge	1	0.50	7.20	114.16	1.39	9.68	136.27	0.52	8.35	133.6
	2	0.31	3.89	62.09	1.14	6.54	86.60			
	4	0.22	2.37	40.41	1.11	5.08	62.77			
Speedup		1.6-2.3 \times	1.9-3.0 \times	1.8-2.8 \times	1.2-1.3 \times	1.5-1.9 \times	1.6-2.2 \times	–	–	–
mqRPMT \blacktriangledown	1	0.50	8.00	128.00	1.35	10.15	141.52	0.27	4.35	69.6
	2	0.32	5.05	80.69	1.18	7.11	94.19			
	4	0.23	3.54	58.40	1.08	5.54	71.26			
Speedup		1.6-2.2 \times	1.6-2.3 \times	1.6-2.2 \times	1.1-1.3 \times	1.4-1.8 \times	1.5-2 \times	–	–	–
mqRPMT \star	1	0.26	3.51	54.85	0.81	5.41	68.68	0.26	4.23	67.66
	2	0.15	1.79	28.24	0.75	3.83	41.38			
	4	0.10	1.07	15.32	0.72	3.09	28.31			
Speedup		1.7-2.6 \times	2.0-3.3 \times	1.9-3.6 \times	1.1-1.1 \times	1.4-1.8 \times	1.7-2.4 \times	–	–	–

strict linear complexity & high parallelism

2^{20} scale: #time < 15s using 4 threads on laptop, #communication < 70M

PSI: Performance and Comparison

PSI	Running time (s)						Comm. (MB)		
	LAN			WAN			total		
	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}
[PTY19] [★]	5.51	88.64	1418.20	5.82	90.79	1498.67	0.30	4.74	76.60
Our PSI [♦]	0.50	7.24	114.66	1.71	10.50	142.45	0.67	10.38	165.77
Our PSI [▼]	0.55	8.04	128.18	1.73	11.02	148.18	0.41	6.38	101.63
Our PSI [★]	0.29	3.56	55.11	1.19	6.38	75.56	0.40	6.25	99.71
DH-PSI [★]	0.22	3.39	54.79	0.92	5.57	69.31	0.28	4.57	74.1

compared to existing DH-PSI implementation: # time speeds up **4.9-25.7×**

PSI	Running time (ms)						Comm. (KB)		
	LAN			WAN			total		
	2^8	2^9	2^{10}	2^8	2^9	2^{10}	2^8	2^9	2^{10}
[RT21] [★]	50.0	71.0	147.3	224.1	260.2	457.9	17.9	34.1	66.3
Our PSI [★]	41.9	69.5	99.3	577.0	582.9	646.1	38.6	63.5	113.3
DH-PSI [★]	16.49	31.80	56.91	210.42	227.33	252.32	18.48	36.68	72.8

achieve the fastest speed in small set setting ($< 2^{10}$)

PSI-card: Performance and Comparison

PSI-card	Running time (s)						Comm. (MB)		
	LAN			WAN			total		
	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}
[GMR ⁺ 21]	1.00	8.41	126.01	8.60	27.46	323.52	2.93	55.49	1030
Our PSI-card [◆]	0.49	7.20	114.31	1.30	9.68	136.06	0.52	8.36	133.71
Our PSI-card [▼]	0.53	8.00	128.00	1.35	10.16	141.31	0.27	4.35	69.6
Our PSI-card [★]	0.27	3.51	54.89	0.82	5.42	68.31	0.26	4.23	67.70

compared to the SOTA

time speeds up **2.3-10.5×**, # communication reduces **11.3-15.2×**

PSI-card-sum: Performance and Comparison

PSI-card-sum	Running time (s)						Comm. (MB)		
	LAN			WAN			total		
	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}
[IKN ⁺ 20] [▼] (deployed)	23.64	176.34	—	30.10	186.29	—	2.72	43.24	—
Our PSI-card-sum [◆]	0.51	7.22	113.66	1.46	9.68	136.27	0.64	9.89	157.80
Our PSI-card-sum [▼]	0.57	8.12	129.66	1.94	11.83	157.66	0.38	5.87	93.74
Our PSI-card-sum [★]	0.31	3.73	57.44	1.36	6.53	76.16	0.37	5.75	91.70



 [google / private-join-and-compute](#) Public



compared to the SOTA

time speeds up **22.1-76.3×**, # communication reduces **7.4-7.5×**

PSU: Performance and Comparison

PSU	Running time (s)						Comm. (MB)		
	LAN			WAN			total		
	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}
[GMR ⁺ 21]	1.16	10.06	151.34	10.34	38.52	349.43	3.85	67.38	1155
[ZCL ⁺ 23] [♦]	4.87	12.19	141.38	5.78	15.75	182.88	1.35	21.41	342.38
[ZCL ⁺ 23] [▼]	5.10	15.13	187.29	5.82	17.37	210.06	0.77	12.20	195.17
[JSZ ⁺ 22]	2.29	8.50	516.04	5.33	27.00	736.30	3.59	70.37	1341.55
Our PSU [♦]	0.52	7.27	114.44	1.70	10.56	143.29	0.68	10.38	165.77
Our PSU [▼]	0.57	8.04	128.20	1.76	10.92	148.15	0.41	6.38	101.63
Our PSU [★]	0.30	3.55	55.48	1.19	6.38	74.96	0.40	6.25	99.71

compared to the SOTA: first achieves strict linear complexity
 # time speeds up **2.4-17×**, # communication reduces **2×**

Private-ID: Performance and Comparison

Private-ID	Running time (s)						Comm. (MB)		
	LAN			WAN			total		
	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}
[GMR ⁺ 21]	1.65	11.023	158.76	13.82	43.00	385.12	4.43	76.57	1293
[BKM ⁺ 20] [★]	2.21	37.56	671.75	7.98	46.97	710.94	1.00	15.97	226.70
Our Private-ID [◆]	0.77	8.40	114.45	2.91	13.62	148.48	1.46	20.52	330.40
Our Private-ID [▼]	0.89	9.57	146.73	3.07	14.53	186.98	1.13	16.43	266.31
Our Private-ID [★]	0.61	5.11	71.57	2.83	10.06	114.66	1.13	16.31	265.15

compared to the SOTA

time speeds up **2.7-4.9×**, # communication is slightly larger

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Summary of This Work

Unified PSO framework from mqRPMT

- show mqRPMT is **complete** for all PSO protocols
- greatly reduce the deployment and maintaining costs of PSO

Generic construction of mqRPMT

- cwPRF: DDH assumption is truly a golden goose
- permuted OPRF: make the concept of OPRF more useful
- mqRPMT* from Sigma-mqPMT: a initial step towards the connection to mqPMT

Efficient implementation

- identify **expensive ECC operations** in cheap disguise
- find the perfect match: Curve25519

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*From [Grothendieck], I have learned not to take glory in the **difficulty of a proof**.*

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Likewise, we do not take shame in the **simplicity of our construction** :-)

Thanks for Your Attention!

Any Questions?

<http://eprint.iacr.org/2022/652>

<https://yuchen1024.github.io>



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