Linear Private Set Union from Multi-Query Reverse Private Membership Test

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Outline

Outline

Private Set Union

Sender

Receiver

Sender's set $\longrightarrow$ $\boxed{f = \text{union}}$ $\longleftarrow$ Receiver's set

$Y = \{b, c, e, f, h\}$

$X = \{a, c, d, f, g\}$

Private Set Union

Sender

Receiver

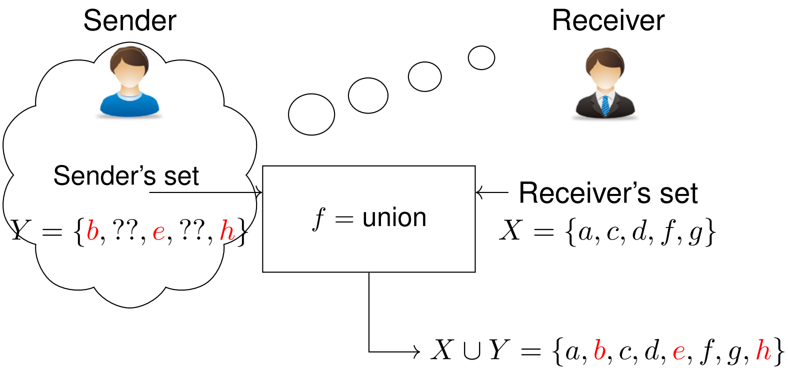
Sender's set → $f =$ union ← Receiver's set

$Y = \{b, c, e, f, h\}$

$X = \{a, c, d, f, g\}$

$\rightarrow X \cup Y = \{a, b, c, d, e, f, g, h\}$

Private Set Union



Applications

- information security risk assessment [LV04]
- IP blacklist and vulnerability data aggregation [HLS+16]
- joint graph computation [BS05]
- distributed network monitoring [KS05]
- building block for private DB supporting full join [KRTW19]
- private ID [GMR+21]

Previous Work

There are two known approaches for constructing PSU:

1. Public-key techniques, e.g. additively homomorphic encryption (AHE) : [KS05, Fri07, DC17]
   - Pros
     - Can achieve linear communication complexity.
     - Can achieve “almost” linear computation complexity.
   - Cons
     - Computation is expensive. Have to perform a non-constant number of AHE operations on each set element.
     - Inefficient.
2. Symmetric-key techniques in combination with OT : [KRTW19, GMR$^+$21, JSZ$^+$22]
   - Pros
     - Computation is cheap.
     - Running time is several orders of magnitude faster than AHE-based constructions.
   - Cons
     - Communication complexity is nonlinear.
     - Computation complexity is nonlinear.

Motivation

Can we construct efficient PSU protocols with linear complexity?

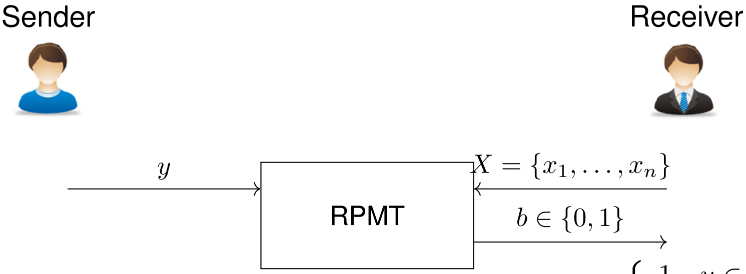
Our Result

We focus on semi-honest setting. We propose a new framework for constructing PSU protocols and instantiate it based on different encryption schemes, they are:

1. A symmetric-key-based PSU protocol
   - Linear computation and communication complexity.
   - Only symmetric operations are used (except base OT).
2. A public-key-based PSU protocol
   - Linear computation and communication complexity.
   - The lowest concrete communication.

Outline

Reverse Private Membership Test (RPMT)



$$b = \begin{cases} 1 & y \in X; \\ 0 & y \notin X \end{cases}$$

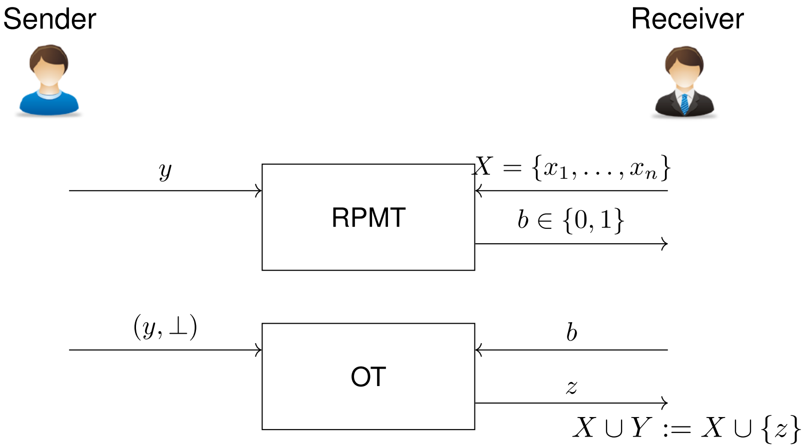
Learns nothing about X.

Learns nothing about which is the sender's item y.

Computation complexity of RPMT in [KRTW19]: $O(n \log^2 n)$.
Communication complexity of RPMT in [KRTW19]: $O(n)$.

KRTW19 Revisit

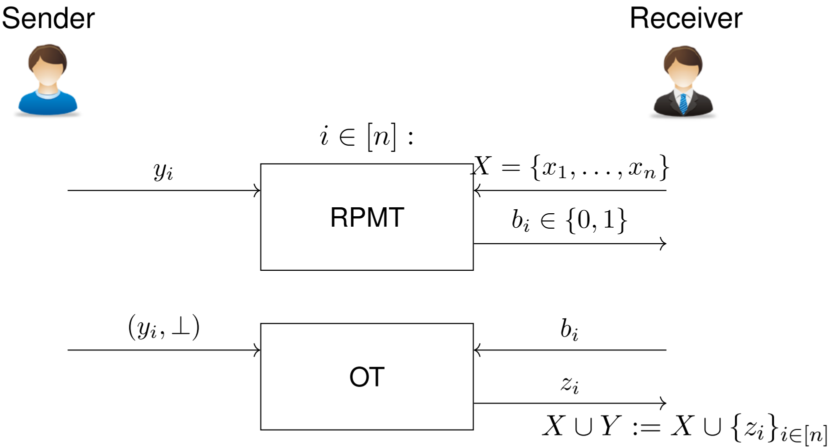
For a special case, the sender has only one item y in its set Y,



Computation complexity: $O(n \log^2 n)$.
Communication complexity: $O(n)$.

KRTW19 Revisit

Independent n times:

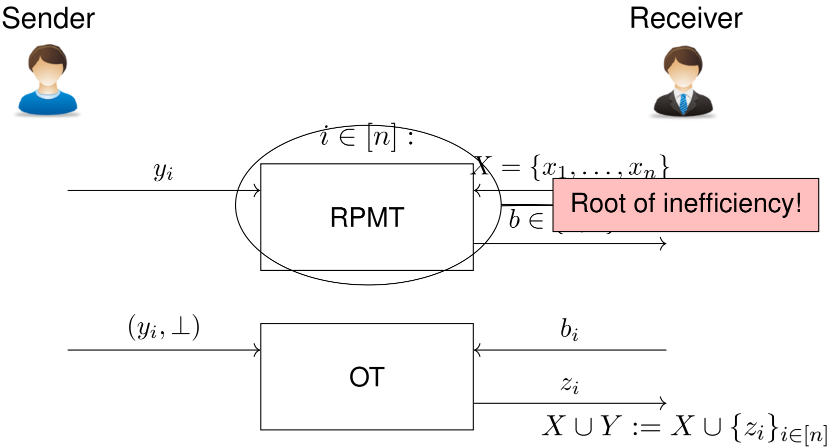


Computation complexity: $O(n^2 \log^2 n)$ → Hash to bin → $O(n \log n \log \log n)$

Communication complexity: $O(n^2)$ → $O(n \log n)$

KRTW19 Revisit

Independent n times:



Computation complexity: $O(n^2 \log^2 n)$ → Hash to bin → $O(n \log n \log \log n)$

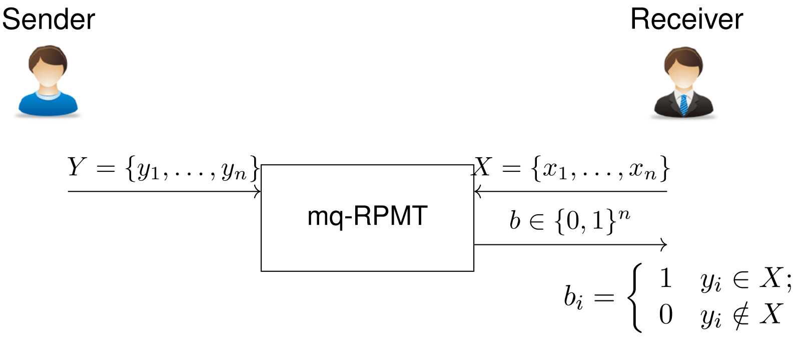
Communication complexity: $O(n^2)$ → Hash to bin → $O(n \log n)$

KRTW19 Revisit

Can we query multiple times in an RPMT instance?

Outline

Definition of mq-RPMT

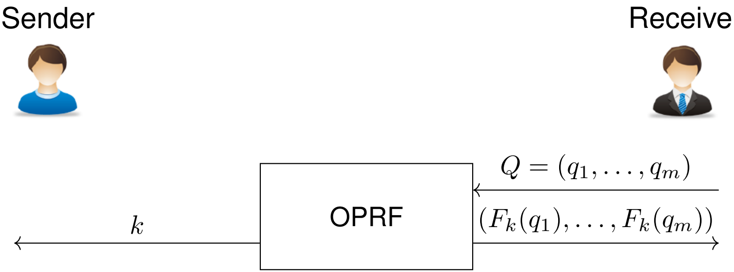


Learns nothing about X.

Learns nothing about which is the sender's item y_i.

Our expectations:
Computation complexity: $O(n)$.
Communication complexity: $O(n)$.

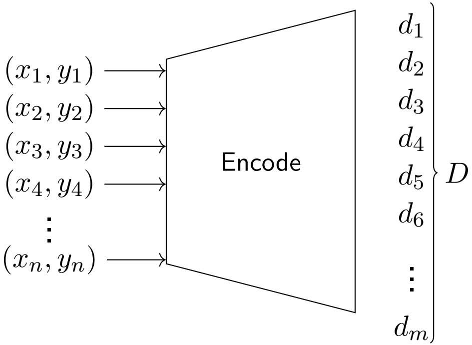
Oblivious PRF (OPRF)



Learns nothing about Q.

Learns nothing about k

Oblivious Key-Value Store



Rate:n/m
optimal is 1.

Encode Complexity:
complexity of Encode algorithm.

Decode Complexity:
complexity of Decode algorithm.

- Encode$((x_1, y_1), \ldots, (x_n, y_n)) \to D$
- Decode$(D, x) \to y$

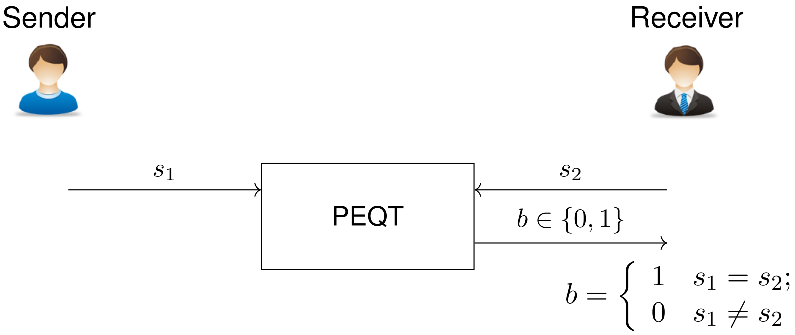
Oblivious Key-Value Store

Table: A comparison between the different OKVS schemes.

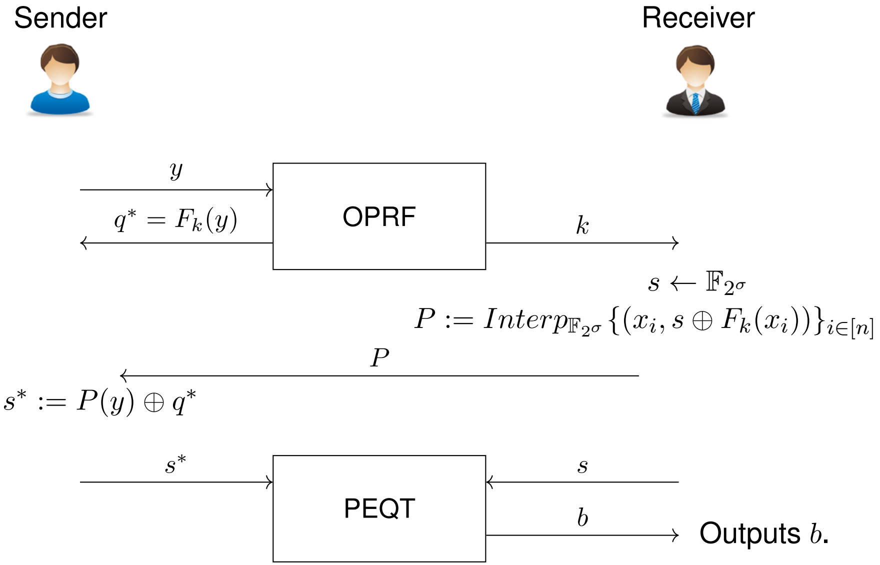
scheme	rate	encoding	decoding
Interpolation polynomial	1	$O(n \log^2 n)$	$O(\log n)$
Garbled Bloom Filter[DCW13]	$O(1/\lambda)$	$O(\lambda n)$	$O(\lambda)$
Garbled Cuckoo Table [PRTY20]	0.4	$O(\lambda n)$	$O(\lambda)$
3H-GCT [GPR$^+$21]	0.81	$O(\lambda n)$	$O(\lambda)$
RR22 [RR22]	0.81	$O(\lambda n)$	$O(\lambda)$
RB-OKVS New! [BPSY23]	0.97	$O(\lambda n)$	$O(\lambda)$

n is the number of key-value pairs, λ is a statistical security parameter (e.g.,λ = 40).

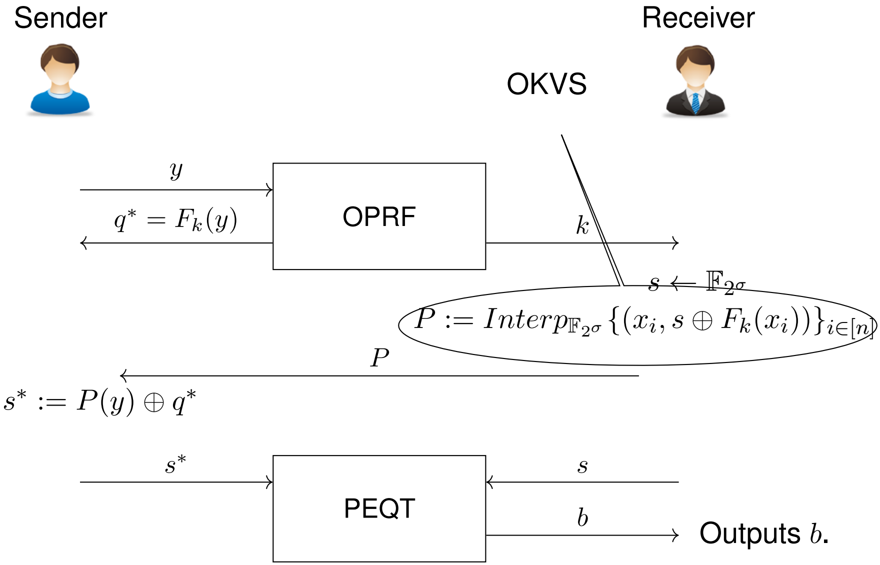
Private Equality Test (PEQT)



Zoom in on the original RPMT



Zoom in on the original RPMT



More efficient OKVS

Sender

Receiver

y → OPRF

$q^* = F_k(y)$ ← OPRF

OPRF → k

$s \leftarrow \mathbb{F}_{2^\sigma}$

$D :=$ Encode$\{(x_i, s \oplus F_k(x_i))\}_{i \in [n]}$

$\xleftarrow{\quad D \quad}$

$s^* :=$ Decode$(D, y) \oplus q^*$

s^* → PEQT ← s

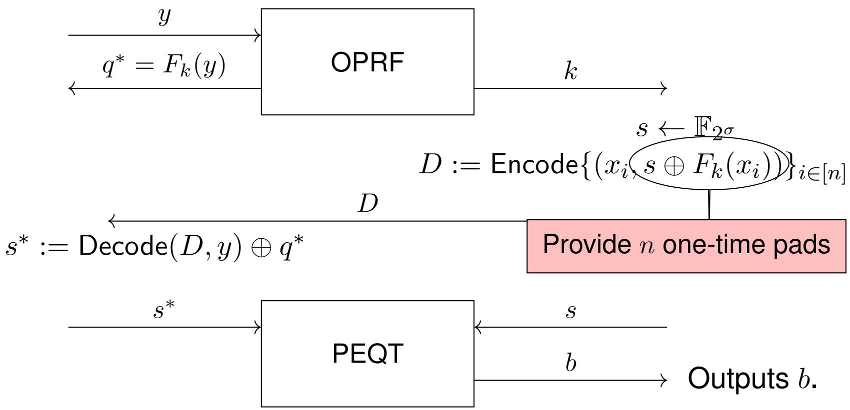
PEQT → b

Outputs b.

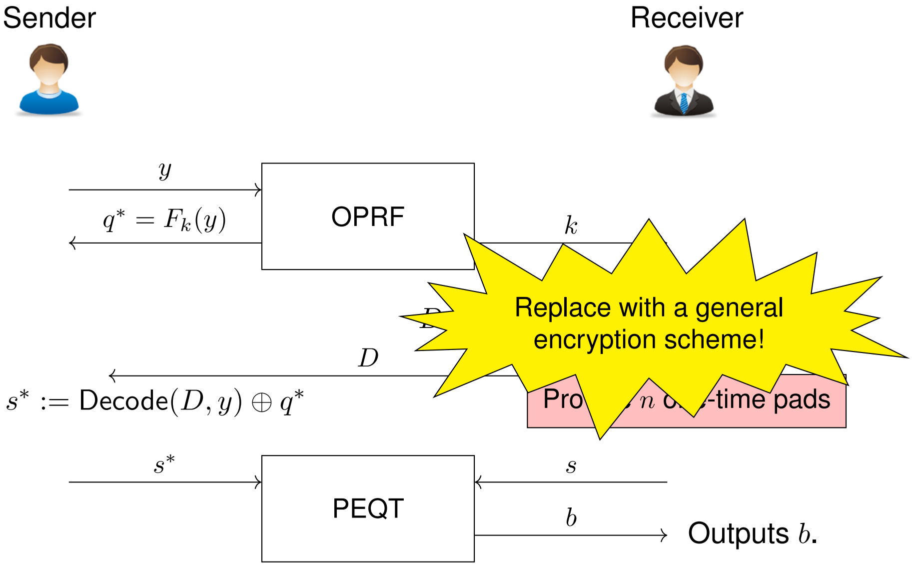
Usage of OPRF

Sender

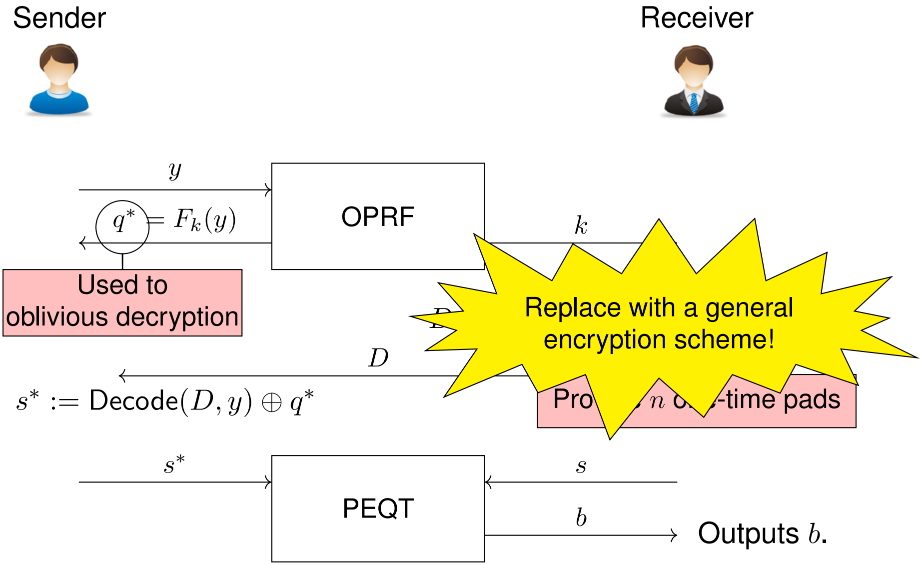
Receiver



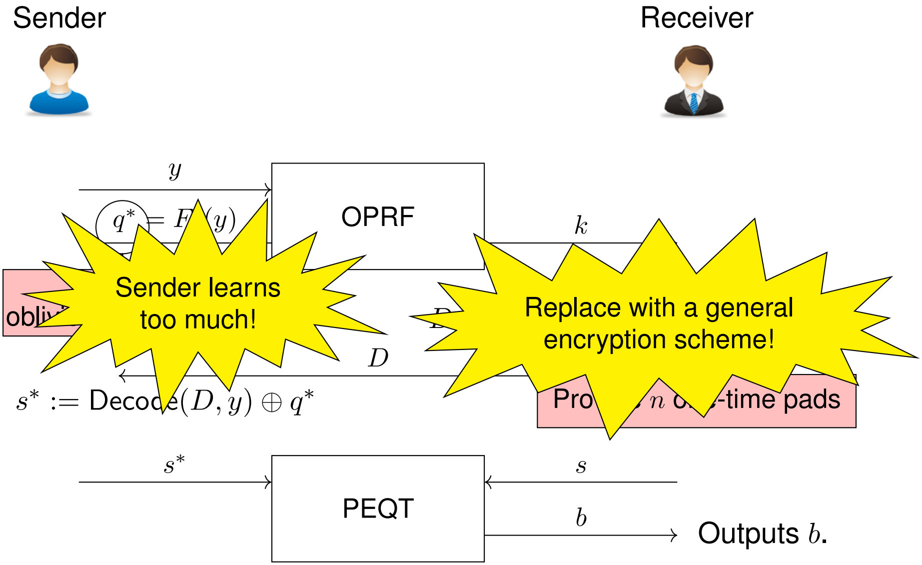
Usage of OPRF



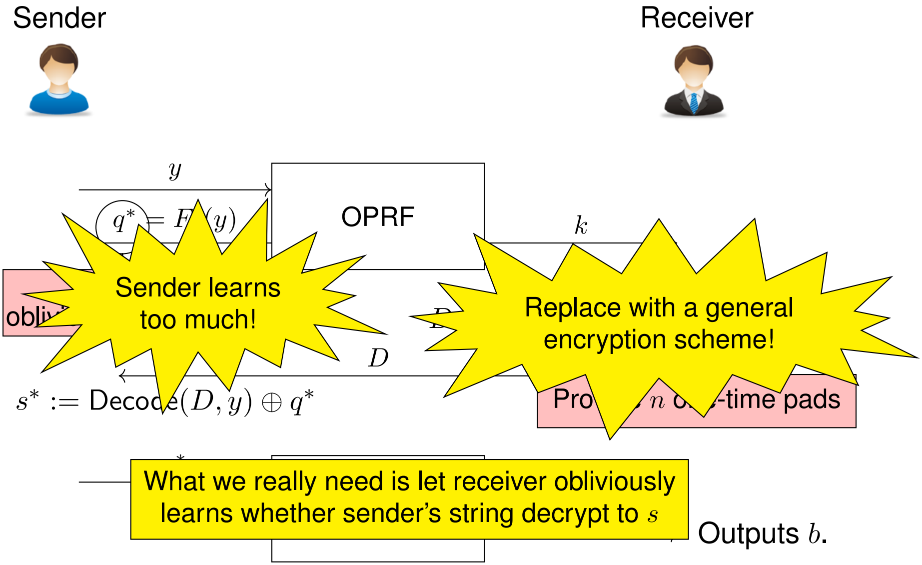
Usage of OPRF



Usage of OPRF



Usage of OPRF



Multi-Query RPMT

Let (Setup, KeyGen, Enc, Dec) be an encryption scheme.

Sender

Receiver

$k \leftarrow \mathsf{KeyGen}(1^\kappa)$

$s \leftarrow \mathbb{F}_{2^\sigma}$

$D := \mathsf{Encode}\{(x_i, \mathsf{Enc}(k, s))\}_{i\in[n]}$

$\xleftarrow{\quad D \quad}$

$s_i^* := \mathsf{Decode}(D, y_i), i \in [n]$

Merge the oblivious decryption functionality and the PEQT functionality

Outputs b.

Vector Oblivious Decryption-then-Matching (vector ODM)

Let $(\mathsf{Setup}, \mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$ be an encryption scheme.

Sender

Receiver

$\{c_i\}_{i\in[n]} \subseteq \mathcal{C}^n$

vector ODM

(k, m)

$b \in \{0,1\}^n$

$$b_i = \begin{cases} 1 & \mathsf{Dec}(k, c_i) = m; \\ 0 & \mathsf{Dec}(k, c_i) \neq m \end{cases}$$

Multi-Query RPMT

Let (Setup, KeyGen, Enc, Dec) be an encryption scheme.

Sender

Receiver

$k \leftarrow \mathsf{KeyGen}(1^\kappa)$

$s \leftarrow \mathbb{F}_{2^\sigma}$

$D := \mathsf{Encode}\{(x_i, \mathsf{Enc}(k, s))\}_{i\in[n]}$

$\xleftarrow{\quad D \quad}$

$s_i^* := \mathsf{Decode}(D, y_i), i \in [n]$

$\{s_i^*\}_{i\in[n]} \longrightarrow$ vector ODM $\longleftarrow (k, s)$

vector ODM $\xrightarrow{\quad b \quad}$ Outputs b.

Outline

SKE-based Instantiation

- $\mathsf{Setup}(1^\kappa) \to pp$.
- $\mathsf{KeyGen}(pp) \to k$.
- $\mathsf{Enc}(k, m) \to c$.
- $\mathsf{Dec}(k, c) \to m/\perp$.

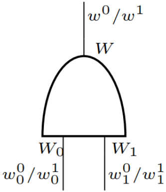
Security. For our purpose, we require a case-tailored security notion called *single-message multi-ciphertext pseudorandomness*. Formally, a SKE scheme is single-message multi-ciphertext pseudorandom if for any PPT $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$:

$$\mathsf{Adv}_{\mathcal{A}}(1^\kappa) = \Pr\left[\beta = \beta' : \begin{array}{l} pp \leftarrow \mathsf{Setup}(1^\lambda); \\ k \leftarrow \mathsf{KeyGen}(pp); \\ (m, state) \leftarrow \mathcal{A}_1(pp, k); \\ \beta \leftarrow \{0,1\}; \\ c^*_{i,0} \leftarrow \mathsf{Enc}(k, m), c^*_{i,1} \leftarrow C, \text{ for } i \in [n]; \\ \beta' \leftarrow \mathcal{A}_2(pp, state, \{c^*_{i,\beta}\}_{i \in [n]}) \end{array}\right] - \frac{1}{2}$$

is negligible in κ.

SKE-based Instantiation

Vector ODM: 2PC, e.g. Garbled Circuit [Yao86], GMW [GMW87].



W_0	W_1	W
w_0^0	w_1^0	$\mathbb{E}_{w_0^0,w_1^0}(w^0)$
w_0^0	w_1^1	$\mathbb{E}_{w_0^0,w_1^1}(w^0)$
w_0^1	w_1^0	$\mathbb{E}_{w_0^1,w_1^0}(w^0)$
w_0^1	w_1^1	$\mathbb{E}_{w_0^1,w_1^1}(w^1)$

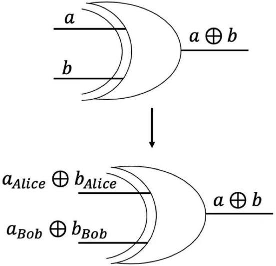


Figure: Garbled Circuit (left) and GMW (right)

Multi-Query RPMT

Let (Setup, KeyGen, Enc, Dec) be a SKE.

Sender

Receiver

$k \leftarrow \mathsf{KeyGen}(1^\kappa)$

$s \leftarrow \mathbb{F}_{2^\sigma}$

$D := \mathsf{Encode}\{(x_i, \mathsf{Enc}(k, s))\}_{i \in [n]}$

$\xleftarrow{\quad D \quad}$

$s_i^* := \mathsf{Decode}(D, y_i), i \in [n]$

$\{s_i^*\}_{i\in[n]} \longrightarrow$ vector ODM $\longleftarrow (k, s)$

vector ODM $\xrightarrow{\quad b \quad}$ Outputs b.

Multi-Query RPMT

Let (Setup, KeyGen, Enc, Dec) be a SKE.

Sender

Receiver

$k \leftarrow \mathsf{KeyGen}(1^\kappa)$

$s \leftarrow \mathbb{F}_{2^\sigma}$

$D := \text{Encode}\{(x_i, \mathsf{Enc}(k, s))\}_{i \in [n]}$

Comp: $O(n)$.

D

$s_i^* := \text{Decode}(D, y_i), i \in [n]$

$\{s_i^*\}_{i \in [n]}$

(k, s)

vector ODM

b

Outputs b.

Multi-Query RPMT

Let (Setup, KeyGen, Enc, Dec) be a SKE.

Sender

Receiver

$k \leftarrow \mathsf{KeyGen}(1^\kappa)$

$s \leftarrow \mathbb{F}_{2^\sigma}$

Comp: $O(n)$.

$D := \mathsf{Encode}\{(x_i, \mathsf{Enc}(k, s))\}_{i\in[n]}$

D

$s_i^* := \mathsf{Decode}(D, y_i), i \in [n]$

$\{s_i^*\}_{i\in[n]}$

vector ODM

(k, s)

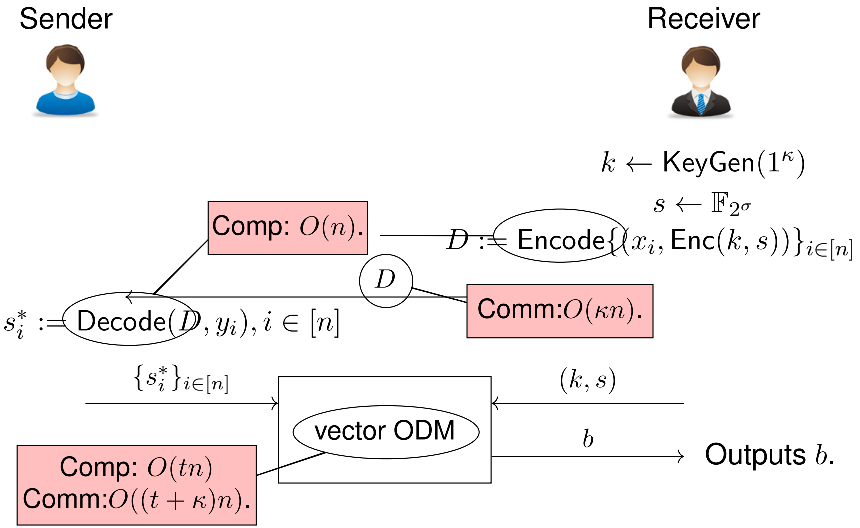
b

Outputs b.

Comp: $O(tn)$
Comm:$O((t+\kappa)n)$.

Multi-Query RPMT

Let $(\mathsf{Setup}, \mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$ be a SKE.



PKE-based Instantiation

A re-randomizable PKE (ReRand-PKE) scheme is a tuple of five algorithms:

- $\mathsf{Setup}(1^\kappa) \to pp$.
- $\mathsf{KeyGen}(pp) \to (pk, sk)$.
- $\mathsf{Enc}(pk, m) \to c$.
- $\mathsf{Dec}(sk, c) \to m/\perp$.
- $\mathsf{ReRand}(pk, c) \to c'$.

Indistinguishability. For any $pp \leftarrow \mathsf{Setup}(1^\kappa)$, any $(pk, sk) \leftarrow \mathsf{KeyGen}(pp)$, and any $m \in M$, the distribution $c_0 \leftarrow \mathsf{Enc}(pk, m)$ and the distribution $c_1 \leftarrow \mathsf{ReRand}(pk, c_0)$ are identical.

PKE-based Instantiation

Security. For our purpose, we require a case-tailored security notion called *single-message multi-ciphertext pseudorandomness*. Formally, a PKE scheme is single-message multi-ciphertext pseudorandom if for any PPT $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$:

$$\mathsf{Adv}_{\mathcal{A}}(1^\kappa) = \Pr\left[\beta = \beta' : \begin{array}{l} pp \leftarrow \mathsf{Setup}(1^\lambda); \\ (pk, sk) \leftarrow \mathsf{KeyGen}(pp); \\ (m, state) \leftarrow \mathcal{A}_1(pp, pk); \\ \beta \leftarrow \{0,1\}; \\ c^*_{i,0} \leftarrow \mathsf{Enc}(pk, m), c^*_{i,1} \leftarrow C, \text{ for } i \in [n]; \\ \beta' \leftarrow \mathcal{A}_2(pp, state, \{c^*_{i,\beta}\}_{i \in [n]}) \end{array}\right] - \frac{1}{2}$$

is negligible in κ.

PKE-based mq-RPMT

Let (Setup, KeyGen, Enc, Dec) be a ReRand PKE scheme.

Sender

Receiver

$$k \leftarrow \mathsf{KeyGen}(1^\kappa)$$

$$s \leftarrow \mathbb{F}_{2^\sigma}$$

$$D := \mathsf{Encode}\{(x_i, \mathsf{Enc}(k, s))\}_{i \in [n]}$$

$$\xleftarrow{\quad D \quad}$$

$$s_i^* := \mathsf{Decode}(D, y_i), i \in [n]$$

$$\bar{s}_i^* := \mathsf{ReRand}(s_i^*; r_i), i \in [n]$$

$$\xrightarrow{\quad \{\bar{s}_i^*\}_{i \in [n]} \quad}$$

$$b_i = \begin{cases} 1 & \mathsf{Dec}(sk, \bar{s}_i^*) = s; \\ 0 & \mathsf{Dec}(sk, \bar{s}_i^*) \neq s \end{cases}$$

Outputs b.

PKE-based mq-RPMT

Let $(\mathsf{Setup}, \mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$ be a ReRand PKE scheme.

Sender

Receiver

$k \leftarrow \mathsf{KeyGen}(1^\kappa)$

$s \leftarrow \mathbb{F}_{2^\sigma}$

$D := \mathsf{Encode}\{(x_i, \mathsf{Enc}(k, s))\}_{i\in[n]}$

Comp: $O(n)$.

D

Comp: $O(n)$.

$s_i^* := \mathsf{Decode}(D, y_i), i \in [n]$

$\bar{s}_i^* := \mathsf{ReRand}(s_i^*; r_i), i \in [n]$

$\{\bar{s}_i^*\}_{i\in[n]}$

$$b_i = \begin{cases} 1 & \mathsf{Dec}(sk, \bar{s}_i^*) = s; \\ 0 & \mathsf{Dec}(sk, \bar{s}_i^*) \neq s \end{cases}$$

Outputs b.

PKE-based mq-RPMT

Let $(\mathsf{Setup}, \mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$ be a ReRand PKE scheme.

Sender

Receiver

$k \leftarrow \mathsf{KeyGen}(1^\kappa)$

$s \leftarrow \mathbb{F}_{2^\sigma}$

$D := \mathsf{Encode}\{(x_i, \mathsf{Enc}(k, s))\}_{i \in [n]}$

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D

Comp: $O(n)$.

$s_i^* := \mathsf{Decode}(D, y_i), i \in [n]$

$\bar{s}_i^* := \mathsf{ReRand}(s_i^*; r_i), i \in [n]$

Comm: $O(\kappa n)$.

$\{\bar{s}_i^*\}_{i \in [n]}$

$$b_i = \begin{cases} 1 & \mathsf{Dec}(sk, \bar{s}_i^*) = s; \\ 0 & \mathsf{Dec}(sk, \bar{s}_i^*) \neq s \end{cases}$$

Outputs b.

Unification with Membership Encryption

Definition (Membership Encryption)

Membership encryption for set X consists of four polynomial time algorithms satisfying the following properties.

- $\mathsf{Setup}(1^\kappa)$: on input a security parameter κ, outputs public parameters pp, which include the ciphertext space C.
- $\mathsf{KeyGen}(pp, X)$: on input public parameters pp and $X \subseteq \{0,1\}^*$, outputs a key k.
- $\mathsf{Enc}(k, x)$: on input a key k and an element $x \in X$, outputs a ciphertext $c \in C$. For uttermost generality, the behavior of Enc on $x \notin X$ is unspecified. Looking ahead, such treatment suffices for the construction of mq-RPMT protocol.
- $\mathsf{Dec}(k, c)$: on input a key k and a ciphertext $c \in C$, outputs "1" indicating c is an encryption of an element x in X and "0" if not.

Final PSU

Sender

Receiver

$Y = \{y_1, \ldots, y_n\}$

$X = \{x_1, \ldots, x_n\}$

mq-RPMT

$b \in \{0,1\}^n$

$i \in [n]$:

$(y_i, \perp)$

b_i

OT

z_i

$X \cup Y := X \cup \{z_i\}_{i \in [n]}$

Computation complexity: $O(n)$.
Communication complexity: $O(\kappa n)$.

Outline

Implement

n	Protocol	Comm. (MB)					Running time (s)															
		$\mathcal{R}$		$\mathcal{S}$		total	LAN				1Gbps				100Mbps				10Mbps			
		setup	online	setup	online		$T=1$		$T=8$		$T=1$		$T=8$		$T=1$		$T=8$		$T=1$		$T=8$	
							setup	online	setup	online	setup	online	setup	online	setup	online	setup	online	setup	online	setup	online
2^{14}	KRTW	0.02	4.17	0.01	29.63	33.8	0.07	3.5	0.03	1.07	0.49	16.13	0.37	14.06	0.83	27.36	0.72	24.66	0.81	55.9	0.73	55.32
	GMRSS	0.02	5.89	0.02	7.96	13.85	0.1	1.01	0.04	0.42	0.66	1.96	0.46	1.28	1	3.53	0.91	2.97	1.06	14.44	0.93	13.97
	JSZDG-R	0.01	4.65	0.01	5.63	10.28	0.07	1.81	0.02	0.52	0.27	2.65	0.23	1.34	0.49	4.19	0.41	2.66	0.45	12.08	0.37	10.63
	SKE-PSU	0.01	3.16	0	3.36	6.52	0.03	0.65	0.02	0.29	0.12	6.76	0.11	6.48	0.21	12.66	0.19	12.09	0.2	15.62	0.19	15.59
	PKE-PSU	0.01	1.16	0	1.59	2.75	4.6	2.37	4.58	1.07	4.78	2.63	4.75	1.34	4.92	3.02	4.9	1.77	4.99	4.43	4.91	3.79
	PKE-PSU*	0.01	2.16	0	2.9	5.05	4.6	1.96	4.6	0.59	4.75	2.36	4.76	1	4.95	2.76	4.91	1.54	4.92	5.72	4.93	5.31
2^{16}	KRTW	0.02	17.64	0.01	122.05	139.69	0.07	12.57	0.03	3.76	0.46	26.27	0.39	20.96	0.82	40.09	0.73	36.3	0.81	163.48	0.75	161.63
	GMRSS	0.02	25.95	0.02	34.11	60.06	0.11	4.79	0.04	1.95	0.64	6.61	0.48	4.25	1.11	12.67	0.92	9.78	1.04	60.75	0.94	57.5
	JSZDG-R	0.01	20.75	0.01	24.74	45.49	0.07	7.5	0.02	2.25	0.3	9.29	0.2	4.45	0.44	13.78	0.4	8.58	0.47	49.41	0.42	44.58
	SKE-PSU	0.01	12.61	0	13.41	26.03	0.04	2.66	0.02	1.15	0.13	8.66	0.11	7.32	0.2	15.84	0.19	14.39	0.2	31.79	0.19	30.98
	PKE-PSU	0.01	4.62	0	6.37	10.99	4.62	9.75	4.59	4.39	4.82	10.21	4.76	5.22	4.9	10.94	4.91	5.83	5.01	16.38	4.92	13.61
	PKE-PSU*	0.01	8.63	0	11.57	20.19	4.57	7.96	4.6	2.58	4.76	8.68	4.77	3.37	4.93	9.94	4.91	4.65	4.94	21.46	4.93	19.67
2^{18}	KRTW	0.02	69.29	0.01	562.76	632.05	0.08	63.02	0.03	17.67	0.52	85.56	0.39	45.31	0.76	111.14	0.71	113.83	0.84	660.33	0.74	664.93
	GMRSS	0.02	113.7	0.02	145.11	258.81	0.13	20.74	0.03	9.8	0.58	28.62	0.55	16.63	1.09	49.68	0.93	38.82	1.03	251.84	0.97	243.63
	JSZDG-R	0.01	92.67	0.01	107.89	200.56	0.07	41.15	0.03	10.71	0.25	43.17	0.21	16.84	0.42	64.06	0.4	33.8	0.53	221.27	0.39	191.2
	SKE-PSU	0.01	50.34	0	53.51	103.85	0.04	10.78	0.02	4.88	0.12	17.83	0.1	12.32	0.2	28.38	0.18	22.54	0.21	98.96	0.19	95.72
	PKE-PSU	0.01	18.5	0	25.45	43.95	4.6	41.5	4.59	19.82	4.79	42.37	4.75	20.97	4.92	44.8	4.91	23.38	4.92	66.68	4.9	54.39
	PKE-PSU*	0.01	34.5	0	46.26	80.76	4.61	34.63	4.58	12.26	4.78	37.1	4.75	13.99	4.92	40.62	4.92	18.45	4.91	85.31	4.92	79.22
2^{20}	KRTW	0.02	300.14	0.01	2305.8	2605.95	0.11	245.37	0.04	67.97	0.52	281.96	0.38	120.35	0.82	363.95	0.74	361.12	0.84	2643.84	0.75	2638.05
	GMRSS	0.02	493.2	0.02	615.9	1109.1	0.11	100.48	0.04	48.53	0.62	119.98	0.51	75.76	1.11	207.83	0.95	164.25	1.09	1074.33	0.95	1030.3
	JSZDG-R	0.01	405.53	0.01	467.26	872.79	0.08	173.07	0.04	54.41	0.48	184.63	0.2	73.28	0.47	266.51	0.73	146.13	0.47	941.5	0.72	825.16
	SKE-PSU	0.01	200.88	0	213.55	414.43	0.05	44.73	0.03	22.78	0.13	59.65	0.11	35.71	0.2	86.11	0.2	65.18	0.21	378.57	0.4	369.24
	PKE-PSU	0.01	74	0	101.8	175.8	4.65	168.79	4.6	79.95	4.78	169.18	4.79	86.49	4.97	179.58	4.94	96.32	4.97	269.32	4.87	216.19
	PKE-PSU*	0.01	138	0	185	323	4.64	144.24	4.58	50.56	4.75	146.41	4.74	60.5	4.9	161.26	5	76.33	4.99	345	4.9	313.37

Table: Communication cost (in MB) and running time (in seconds) comparing our protocols to KRTW GMRSS, and JSZDG-R. The LAN network has 10 Gbps bandwidth and 0.2 ms RTT latency. Communication cost of $\mathcal{S}/\mathcal{R}$ indicates the outgoing communication from $\mathcal{S}/\mathcal{R}$ to the other party. The best protocol within a setting is marked in blue.

Implement

code: `http://github.com/alibaba-edu/mpc4j`

ARTIFACT EVALUATED usenix ASSOCIATION AVAILABLE

ARTIFACT EVALUATED usenix ASSOCIATION FUNCTIONAL

ARTIFACT EVALUATED usenix ASSOCIATION REPRODUCED

eprint: `https://eprint.iacr.org/2022/358`

THANK YOU!

Q & A

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