

Unbalanced Private Set Union with Reduced Computation and Communication

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Outline

- 1 Background
- 2 Preliminaries
- 3 Our construction
- 4 Implementation

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Unbalanced Private Set Union

Sender



$$X = \{x_1, \dots, x_m\}$$

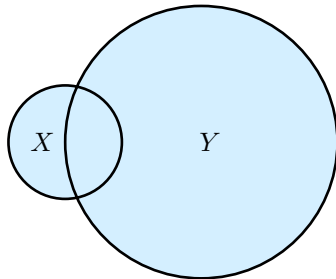
Learns nothing.

Receiver



$$Y = \{y_1, \dots, y_n\}$$

Learns $X \cup Y$.



$$|X| \ll |Y|$$

Unbalanced Private Set Union

Sender



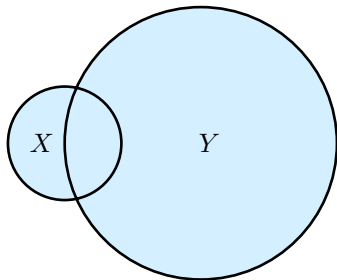
$$X = \{x_1, \dots, x_m\}$$

Receiver



$$Y = \{y_1, \dots, y_n\}$$

Learns nothing,



$$|X| \ll |Y|$$

Learns $X \cup Y$,
but knows nothing about $X \cap Y$.

Applications

PSU has found numerous applications, which include but not limit to:

- information security risk assessment [LV04]
- IP blacklist and vulnerability data aggregation [HLS⁺16]
- joint graph computation [BS05]
- distributed network monitoring [KS05]
- building block for private DB supporting full join [KRTW19]
- private ID [GMR⁺21]

Previous Work

Over the last few years, a line of works have greatly improved the efficiency of PSU in the semi-honest model.

① PSU for balanced setting [KRTW19, GMR⁺21, JSZ⁺22, ZCL⁺23, CZZ⁺24].

- Use cheap symmetric-key techniques coupled with OT.
- Do not target unbalanced case specifically.
- Communication cost is at least $O(|Y|)$ ($|Y| \gg |X|$).

② PSU for unbalanced setting [TCLZ23].

- Used lattice-based homomorphic encryption.
- Communication cost is sublinear in $|Y|$.
- Do not consider preprocessing optimization.
- Has large constant due to partition technique.

Our Result

We propose two new constructions for unbalanced PSU protocols based on different oblivious key-value store (OKVS) encoding strategies.

- ① Based on one-time pads generated from oblivious PRF.
 - Sublinear communication complexity in $|Y|$.
 - New cryptographic preliminaries: permuted private equality test (p-PEQT).
 - DDH-based construction.
 - Permute+Share based construction.
- ② Based on re-randomized public-key encryption.
 - Sublinear communication complexity in $|Y|$.
 - New "pull-down-then-lift" methodology in OKVS encoding.

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Oblivious PRF (OPRF)

Sender



Receiver



Batch Private Information Retrieval (BatchPIR)

A BatchPIR scheme consists of three algorithms (Query, Answer, Recover):

- $\text{Query}(I = \{i_1, \dots, i_b\} \in ([n]^b) \rightarrow (\text{qu}, \text{st})$
- $\text{Answer}(D, \text{qu}) \rightarrow \text{ans}$
- $\text{Recover}(\text{st}, \text{ans}) \rightarrow \{D_1, \dots, D_b\}$

Correctness. For any dataset D , all distinct inputs $I = \{i_1, \dots, i_b\}$, $(\text{st}, \text{qu}) \leftarrow \text{Query}(I)$:

$$\text{Recover}(\text{st}, \text{Answer}(D, \text{qu})) = \{D[i_1], \dots, D[i_b]\}.$$

Query privacy. For all PPT adversaries \mathcal{A} and all distinct batch query sets I_1, I_2 with $|I_1| = |I_2|$,

$$\Pr[\mathcal{A}(\text{qu}) = 1 \mid (\text{st}, \text{qu}) \leftarrow \text{Query}(I_1)] - \Pr[\mathcal{A}(\text{qu}) = 1 \mid (\text{st}, \text{qu}) \leftarrow \text{Query}(I_2)] \leq \text{negl}(\kappa).$$

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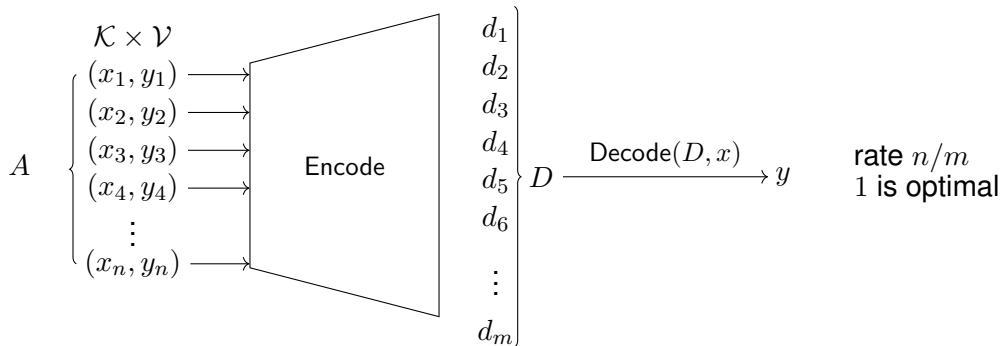
$$\text{Recover}(\text{st}, \text{Answer}(D, \text{qu})) = \{D[i_1], \dots, D[i_b]\}.$$

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Remark. Since (Batch)PIR does not protect database privacy, the server can directly send the entire database to the client, resulting in communication $O(n)$. We do not consider this trivial (Batch)PIR construction and always assume that the communication of (Batch)PIR is sublinear in n , i.e., $|\text{qu}| + |\text{ans}| = o(n)$.

Oblivious Key-Value Store



Correctness. For all $A \subseteq \mathcal{K} \times \mathcal{V}$ with distinct keys, $(x, y) \in A$, $\text{Decode}(\text{Encode}(A), x) = y$.

Obliviousness. For any $(x_1^0, \dots, x_n^0) \neq (x_1^1, \dots, x_n^1)$:

$\text{Encode}((x_1^0, y_1), \dots, (x_n^0, y_n)) \approx \text{Encode}((x_1^1, y_1), \dots, (x_n^1, y_n))$, where $y_i \xleftarrow{R} \mathcal{V}$.

Randomness. For any $A = \{(x_1, y_1), \dots, (x_n, y_n)\}, x \notin \{x_1, \dots, x_n\}$:

$\text{Decode}(D, x) \approx U_{\mathcal{V}}$, where $D \leftarrow \text{Encode}(A)$.

Oblivious Key-Value Store

All state-of-the-art OKVS schemes are binary linear OKVS. That is, the Encode algorithm is to solve the following linear equation:

$$\begin{bmatrix} -\text{row}(x_1)- \\ -\text{row}(x_2)- \\ \vdots \\ -\text{row}(x_n)- \end{bmatrix}_{n \times m} \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_m \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad (1)$$

where $\text{row} : \mathcal{K} \rightarrow \{0, 1\}^m$ is defined by the random tapes of Encode algorithm. The Decode algorithm is defined as:

$$\text{Decode}(D, x) = \langle \text{row}(x), D \rangle := \sum_{j=1}^m \text{row}(x)_j d_j = \sum_{\text{row}(x)_j=1} d_j$$

Sparse OKVS

Our key observation: the binary vector $\text{row}(x)$ has a long sparse part!

$$\text{row}(x) = \underbrace{\text{sparse}(x)}_{\text{constant weight } w} \parallel \underbrace{\text{dense}(x)}_{\text{random}} \in \{0, 1\}^{s+d}$$

where $s = O(n)$, $d = o(n)$.

As a result,

$$\begin{aligned} \text{Decode}(D, x) &= \langle \text{row}(x), D \rangle = \langle \text{sparse}(x), D_0 \rangle + \langle \text{dense}(x), D_1 \rangle \\ &= \sum_{\text{sparse}(x)_i=1} d_i + \langle \text{dense}(x), D_1 \rangle \end{aligned}$$

where $|D_0| = s = O(n)$, $|D_1| = d = o(n)$.

Typical Use Cases of OKVS

OKVS (with different instantiations) is widely used in private set operation protocols, e.g., PSI [PRTY19, GPR⁺21, RS21, RR22], PSU [KRTW19, GMR⁺21, ZCL⁺23, LG23].

A typical use case is as follows:

Sender($A = \{(x_1, y_1), \dots, (x_n, y_n)\}$)

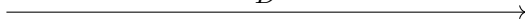


Receiver(q)



$D := \text{Encode}(A)$

D



$y^* := \text{Decode}(D, q)$

Typical Use Cases of OKVS

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Sender($A = \{(x_1, y_1), \dots, (x_n, y_n)\}$)



$D := \text{Encode}(A)$

D

Receiver(q)



$y^* := \text{Decode}(D, q)$

The obliviousness ensures sending D directly will not leak information about $\{x_1, \dots, x_n\}$.

Typical Use Cases of OKVS

OKVS (with different instantiations) is widely used in private set operation protocols, e.g., PSI [PRTY19, GPR⁺21, RS21, RR22], PSU [KRTW19, GMR⁺21, ZCL⁺23, LG23].

A typical use case is as follows:

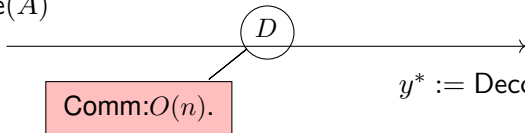
Sender($A = \{(x_1, y_1), \dots, (x_n, y_n)\}$)



Receiver(q)



$D := \text{Encode}(A)$



Communication-Efficient Decoding (CED)

Sender($A = \{(x_1, y_1), \dots, (x_n, y_n)\}$)



Receiver(q)



$$D = D_0 || D_1 := \text{Encode}(A)$$

D_1

$$I := \{i | \text{sparse}(q)_i = 1\}$$

D_0

BatchPIR

I

$\{d_i\}_{i \in I}$

$$y^* := \text{Decode}(D, q)$$

$$= \langle \text{sparse}(q), D_0 \rangle + \langle \text{dense}(q), D_1 \rangle$$

$$= \sum_{\text{sparse}(q)_i = 1} d_i + \langle \text{dense}(q), D_1 \rangle$$

Communication-Efficient Decoding (CED)

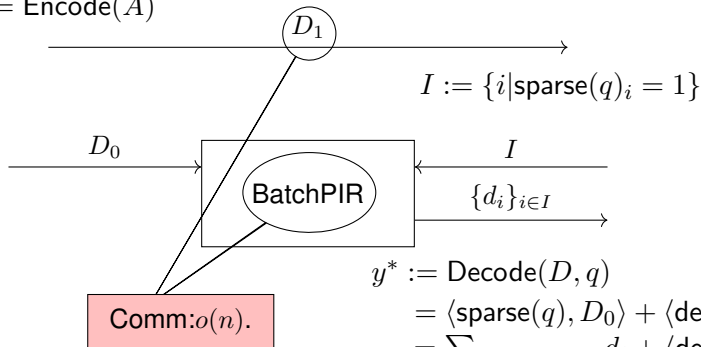
Sender($A = \{(x_1, y_1), \dots, (x_n, y_n)\}$)



Receiver(q)



$$D = D_0 || D_1 := \text{Encode}(A)$$



Outline

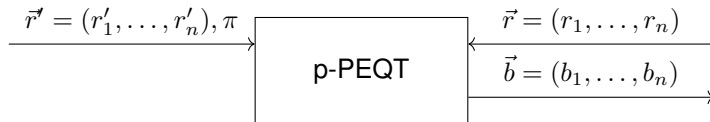
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Permuted Private Equality Test (p-PEQT)

Sender



Receiver



$$b_i = \begin{cases} 1 & r_{\pi(i)} = r'_{\pi(i)}; \\ 0 & r_{\pi(i)} \neq r'_{\pi(i)} \end{cases}$$

p-PEQT from DDH

Sender(\vec{r}', π)



Receiver(\vec{r})



$\{v_i\}_{i \in [n]}$

$a \leftarrow \mathbb{Z}_q, v_i := H(r_i)^a, i \in [n]$

$b \leftarrow \mathbb{Z}_q$

$v'_i := H(r'_{\pi(i)})^b, \bar{v}_i = (v_{\pi(i)})^b, i \in [n]$

$\{v'_i, \bar{v}_i\}_{i \in [n]}$

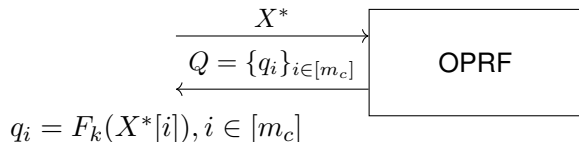
$$b_i = \begin{cases} 1 & v_i'^a = \bar{v}_i; \\ 0 & v_i'^a \neq \bar{v}_i \end{cases}$$

Our First Construction

Sender($X = \{x_1, \dots, x_m\}$)



$$X^* := \text{CuckooHash}(X)$$



Receiver($Y = \{y_1, \dots, y_n\}$)



$$Y^* := \text{SimpleHash}(Y)$$

$$\vec{r} \leftarrow \mathbb{F}^{m_c}$$

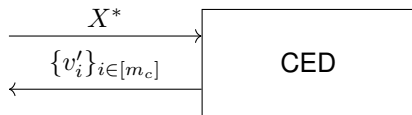
$$k$$

$$i \in [m_c], \alpha \in [|Y^*[i]|] :$$

$$k_{i,\alpha} := Y^*[i][\alpha]$$

$$v_{i,\alpha} := r_i + F_k(Y^*[i][\alpha])$$

$$\{(k_{i,\alpha}, v_{i,\alpha})\}$$



$$r'_i = v'_i - q_i, i \in [m_c]$$

Our First Construction

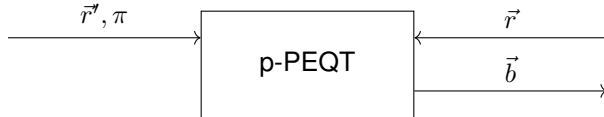
Sender($X = \{x_1, \dots, x_m\}$)



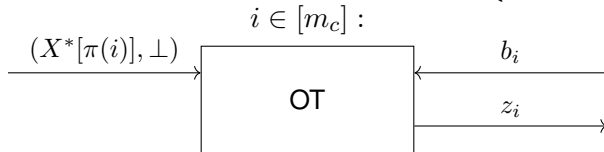
Receiver($Y = \{y_1, \dots, y_n\}$)



$\pi \leftarrow \text{Perm}([m_c])$



$$b_i = \begin{cases} 1 & X^*[\pi(i)] \in Y^*[\pi(i)]; \\ 0 & X^*[\pi(i)] \notin Y^*[\pi(i)] \end{cases}$$



$$X \cup Y := Y \cup \{z_i\}$$

Our Second Construction

Sender($X = \{x_1, \dots, x_m\}$)

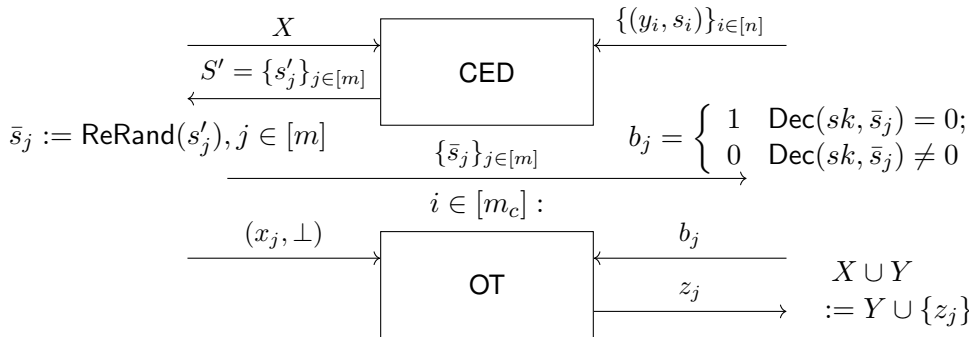


Receiver($Y = \{y_1, \dots, y_n\}$)



$(pk, sk) \leftarrow \text{KeyGen}(1^\kappa)$

$s_i := \text{Enc}(pk, 0), i \in [n]$



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Implementation

we implement our protocols and fully re-implement prior unbalanced PSU [TCLZ23] based on the same open-source library mpc4j¹.

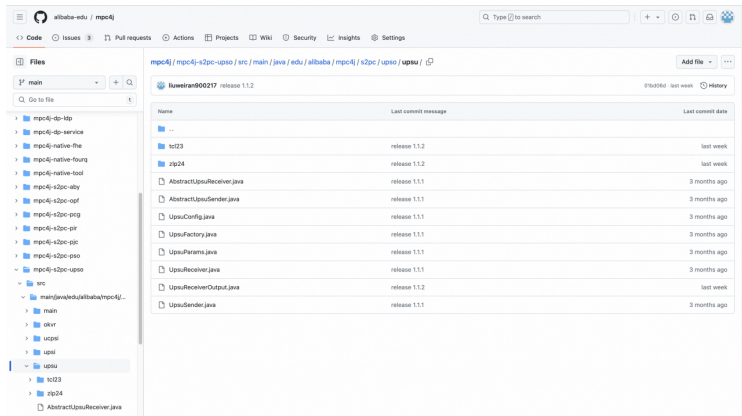


Figure: Our implementation

¹<https://github.com/alibaba-edu/mpc4j>

Performance

Param.		Protocol	Comm. (MB)		Running time (s)							
n	m		Setup	Online	LAN		100Mbps		10Mbps		1Mbps	
					Setup	Online	Setup	Online	Setup	Online	Setup	Online
2 ¹⁸	2 ⁴	TCLZ-pub [TCLZ23]	0.76	1.52	2.98	1.15	3.67	1.99	3.93	3.11	9.71	14.46
		TCLZ-sym [TCLZ23]	0.77	2.27	2.98	1.18	4.02	2.48	4.50	4.04	10.25	20.90
		Our PSU _{op} ^{ddh}	19.08	0.71	11.72	1.13	15.55	2.44	29.60	2.89	174.53	8.30
		Our PSU _{op} ^{ps}	19.09	0.73	12.05	1.14	15.02	2.71	28.79	3.27	173.45	9.29
		Our PSU _{pk}	19.08	0.74	38.01	7.52	40.16	8.11	52.31	8.32	197.53	13.92
	2 ⁶	TCLZ-pub [TCLZ23]	0.76	1.52	2.84	1.12	3.38	2.01	3.90	3.03	9.59	14.50
		TCLZ-sym [TCLZ23]	0.77	2.27	2.83	1.11	3.72	2.88	4.15	4.44	10.13	21.93
		Our PSU _{op} ^{ddh}	19.08	0.72	14.99	0.86	17.19	1.72	31.25	2.18	175.06	8.04
		Our PSU _{op} ^{ps}	19.09	0.75	14.00	0.88	17.72	2.00	30.94	2.36	175.34	7.98
		Our PSU _{pk}	19.08	0.74	34.88	2.32	37.57	2.92	52.14	3.62	196.42	9.13
	2 ⁸	TCLZ-pub [TCLZ23]	0.76	1.53	2.85	1.15	3.48	1.91	3.92	3.08	9.55	14.66
		TCLZ-sym [TCLZ23]	0.77	2.28	2.87	1.13	3.89	2.25	4.11	3.77	9.95	20.98
		Our PSU _{op} ^{ddh}	19.08	1.44	21.78	0.94	24.58	1.88	38.30	3.42	183.08	14.15
		Our PSU _{op} ^{ps}	19.09	1.52	21.43	0.95	24.41	2.13	38.66	3.13	182.88	14.53
		Our PSU _{pk}	19.08	1.70	59.15	2.24	61.30	3.07	76.01	4.35	219.35	17.46
	2 ¹⁰	TCLZ-pub [TCLZ23]	0.76	1.58	2.80	1.17	3.43	1.94	3.69	3.17	9.70	14.97
		TCLZ-sym [TCLZ23]	0.77	2.33	2.95	1.11	3.70	2.31	4.33	3.88	9.99	21.36
		Our PSU _{op} ^{ddh}	19.08	5.71	22.67	1.28	25.34	3.37	39.50	6.99	184.55	49.83
		Our PSU _{op} ^{ps}	19.09	6.07	22.17	1.16	25.39	3.17	39.45	7.43	184.40	54.09
		Our PSU _{pk}	19.08	3.90	124.79	2.96	128.02	4.21	141.98	7.05	286.85	36.41

Table: Performance comparison. The best result is marked in green. The second best result is marked in blue.

Performance

Param.		Protocol	Comm. (MB)		Running time (s)							
n	m		Setup	Online	LAN		100Mbps		10Mbps		1Mbps	
					Setup	Online	Setup	Online	Setup	Online	Setup	Online
2 ²⁰	2 ⁴	TCLZ-pub [TCLZ23]	0.76	1.86	14.47	1.68	15.27	2.55	15.62	3.90	20.76	18.51
		TCLZ-sym [TCLZ23]	0.77	3.52	14.61	1.63	15.26	3.04	15.81	5.40	21.92	31.90
		Our PSU _{op} ^{ddh}	19.08	0.76	42.34	3.07	45.15	4.17	58.63	4.76	205.86	10.83
		Our PSU _{op} ^{ps}	19.09	0.77	42.09	3.10	45.77	4.59	59.81	5.26	203.93	11.92
		Our PSU _{pk}	19.08	0.88	89.39	8.84	94.72	9.40	108.37	10.71	252.10	16.82
	2 ⁶	TCLZ-pub [TCLZ23]	0.76	1.87	14.20	1.61	14.48	2.44	15.16	3.78	21.33	17.95
		TCLZ-sym [TCLZ23]	0.77	3.52	15.23	1.55	15.42	3.63	15.64	6.19	21.37	32.77
		Our PSU _{op} ^{ddh}	19.08	1.45	42.53	1.39	46.21	2.24	58.51	3.34	203.39	15.14
		Our PSU _{op} ^{ps}	19.09	1.48	42.63	1.34	58.96	2.74	73.43	4.77	216.41	27.73
		Our PSU _{pk}	19.08	0.89	135.13	8.08	135.17	8.27	146.61	8.94	293.32	15.65
	2 ⁸	TCLZ-pub [TCLZ23]	0.76	1.88	14.66	1.59	15.14	2.41	15.05	3.81	20.70	18.00
		TCLZ-sym [TCLZ23]	0.77	3.53	14.13	1.54	15.13	2.90	15.38	5.41	21.61	32.00
		Our PSU _{op} ^{ddh}	19.08	2.86	54.80	1.29	58.14	2.59	73.32	4.56	216.79	26.45
		Our PSU _{op} ^{ps}	19.09	2.94	55.18	1.35	58.96	2.74	73.43	4.77	216.41	27.73
		Our PSU _{pk}	19.08	2.97	138.53	3.19	136.75	3.93	151.06	6.16	299.52	29.09
	2 ¹⁰	TCLZ-pub [TCLZ23]	0.76	1.92	14.42	1.60	15.22	2.45	15.15	3.86	21.41	18.43
		TCLZ-sym [TCLZ23]	0.77	3.58	14.26	1.55	15.21	2.97	16.02	5.48	21.48	32.55
		Our PSU _{op} ^{ddh}	19.08	5.76	85.49	1.93	89.58	3.71	102.54	7.90	245.92	50.95
		Our PSU _{op} ^{ps}	19.09	6.15	85.76	1.92	91.20	3.82	101.64	7.96	247.21	54.96
		Our PSU _{pk}	19.08	6.80	232.43	3.65	234.26	5.37	247.47	10.14	431.10	61.93

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Performance

Param.		Protocol	Comm. (MB)		Running time (s)							
n	m		Setup	Online	LAN		100Mbps		10Mbps		1Mbps	
					Setup	Online	Setup	Online	Setup	Online	Setup	Online
2^{22}	2^4	TCLZ-pub [TCLZ23]	0.76	3.59	60.60	3.21	60.69	4.51	60.64	6.89	67.03	34.86
		TCLZ-sym [TCLZ23]	0.77	11.13	61.39	3.02	61.74	5.28	63.20	13.31	67.48	97.47
		Our PSU _{op} ^{ddh}	19.08	0.94	182.08	7.40	187.18	9.94	202.34	10.08	345.77	17.38
		Our PSU _{op} ^{ps}	19.09	0.96	185.43	7.55	190.21	9.34	203.74	10.60	349.34	18.19
		Our PSU _{pk}	19.08	1.46	302.23	13.27	305.25	14.03	319.06	15.46	465.35	26.58
	2^6	TCLZ-pub [TCLZ23]	0.76	3.59	62.04	3.05	62.10	4.24	61.96	6.75	67.95	34.47
		TCLZ-sym [TCLZ23]	0.77	11.13	61.90	2.98	61.98	5.96	60.80	14.35	67.73	98.07
		Our PSU _{op} ^{ddh}	19.08	1.64	221.34	6.03	223.13	7.01	229.80	9.05	373.27	20.91
		Our PSU _{op} ^{ps}	19.09	1.66	216.53	6.61	217.72	7.32	235.72	8.51	380.17	21.50
		Our PSU _{pk}	19.08	2.16	424.83	11.25	417.94	11.98	442.52	13.69	584.50	29.81
	2^8	TCLZ-pub [TCLZ23]	0.76	3.60	60.62	3.26	61.03	4.39	61.71	6.89	66.99	34.31
		TCLZ-sym [TCLZ23]	0.77	11.15	62.10	3.11	61.25	5.24	62.72	13.42	68.35	98.23
		Our PSU _{op} ^{ddh}	19.08	5.80	181.34	4.07	181.20	5.79	193.74	9.17	342.64	52.94
		Our PSU _{op} ^{ps}	19.09	5.88	184.88	3.93	188.85	6.20	196.29	9.55	340.90	54.68
		Our PSU _{pk}	19.08	3.55	539.48	10.01	548.18	11.53	556.73	13.76	716.74	40.50
	2^{10}	TCLZ-pub [TCLZ23]	0.76	3.65	60.36	3.07	61.46	4.82	63.69	7.13	67.58	34.65
		TCLZ-sym [TCLZ23]	0.77	11.19	59.93	3.08	63.29	5.36	63.01	13.36	66.54	97.89
		Our PSU _{op} ^{ddh}	19.08	10.76	234.33	4.55	238.26	6.51	261.40	14.22	405.45	96.39
		Our PSU _{op} ^{ps}	19.09	11.15	236.02	4.56	239.26	6.89	254.70	14.93	409.62	99.66
		Our PSU _{pk}	19.08	11.19	551.38	9.28	554.46	13.23	567.37	18.95	706.52	103.62

Table: Performance comparison. The best result is marked in green. The second best result is marked in blue.

Thanks for Your Attention!
Any Questions?

code: `http://github.com/alibaba-edu/mpc4j`

eprint: `https://eprint.iacr.org/2024/1340`

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