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A		2412107

Ecological Perspective on Lamprey Sex Variation

Summary

Developing individuals engage in regular, active bidirectional interactions with their immediate environment. As this interactive process becomes increasingly complex, development occurs.

——Urie Bronfenbrenner
American psychologist, ecologist

Urie Bronfenbrenner's insightful words highlight the essential interactions between individuals and their surroundings for development, offering a theoretical foundation for our research on sea lamprey sex ratio variations and their ecological repercussions.

This study investigates the alterations in the sex ratio of sea lampreys and their ecological impacts. Utilizing mathematical models, we meticulously examine the potential effects of sex ratio changes on sea lamprey populations and their habitats. Initially, through comprehensive research on sea lampreys' life cycles, physiological aspects, ecological niches, and growth conditions, we constructed models based on **Lotka-Volterra equations** and **fuzzy mathematics** to simulate the population dynamics and sex ratio alterations of sea lampreys. Furthermore, the study accounts for environmental elements like temperature and food availability influencing sea lampreys' sex ratios.

Our findings suggest that in environments abundant in resources, the sex ratio of sea lampreys leans towards balance, conducive to population expansion and reproduction. Conversely, in settings with scarce resources, a skewed sex ratio (such as a surplus of males) might diminish population growth and reproductive capabilities, thereby impacting other species within the ecosystem. Through sensitivity assessments and developing stability measures, the models' robustness and dependability were validated.

Additionally, the research delves into the broader ecological consequences of sea lamprey sex ratio shifts, including their role in biodiversity and changes in species interactions. A balanced sex ratio emerged as vital for sustaining **ecosystem equilibrium and biodiversity**. In the concluding sections of model appraisal and discussions, the study's strengths, limitations, and the prospective applications and expansions of the model are outlined.

Through **Agent-Based Modeling (ABM)**, In response to the intricate dynamics of natural ecosystems, we developed a model that simulates the population dynamics of lampreys, integrating **differential and probabilistic** methods to reflect the impact of food resources on the growth rate of lamprey larvae. The model demonstrates that lamprey populations can adapt to changes in resources by adjusting their sex ratio, showing a degree of resilience and adaptability. However, in the event of a disaster, adjustments in the sex ratio may lead to a decrease in the population's disaster resistance, affecting long-term stability. This finding provides a new perspective for understanding the variations in the sex ratio of lampreys and their ecological impact, offering insights into ecosystem management.

In conclusion, by formulating mathematical models and conducting an integrated analysis, this research sheds new light on the implications of sea lamprey sex ratio variations on ecosystems and furnishes scientific underpinnings for **ecological management and safeguarding**.

Keywords: Lotka-Volterra; Fuzzy Mathematics;ABM;Differential and Probabilization;Ecosystem stability;Biodiversity;Ecological impacts;

Contents

1 Introduction

1.1 Problem Background

In the study of species population dynamics and the complex interactions within ecosystem balance, changes in the sex ratio of individual organisms present a high-profile phenomenon.Sea lampreys, act as highly detrimental parasites in certain lake habitats, while serving as vital food resources in marine regions. Their sex ratio is influenced by the external environment, especially the availability of food resources.[1]

Figure 1: Lampreys are parasitizing.[1]

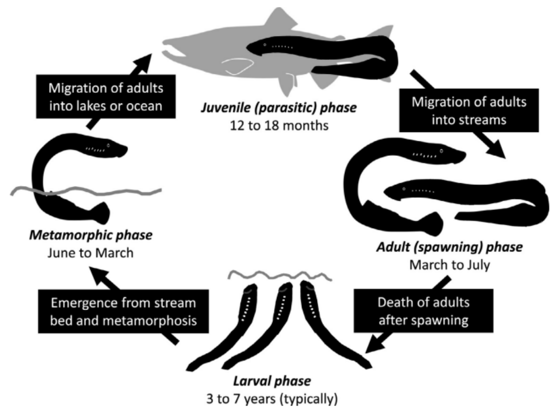

Figure 2: Life cycle of the lamprey[2]

The adaptive mechanism of gender ratio in sea lampreys not only reveals how organisms respond to environmental pressures but also offers new perspectives on the health and stability of ecosystems. In-depth research into this phenomenon contributes to a better understanding of population dynamics within ecological systems, providing a scientific basis for ecological conservation and resource management. In the following paper, we will further explore the ecological principles underlying the variation in gender ratios of marine lampreys and their potential impacts on ecosystems.

1.2 Restatement of Problem

Considering the background information, constraints outlined in the problem statement, and additional guidance provided, the issues we aim to investigate include:

- **Ecosystem Impact**: What are the implications for the broader ecosystem when the population of marine lampreys can alter their gender ratio based on resource availability? It is necessary to consider how such changes affect other species within the ecosystem, including the lamprey's prey and predators.

[1]Source: https://windsorstar.com/news/local-news/

- **Population Advantages and Disadvantages**: For the sea lamprey population itself, what are the advantages and disadvantages of the ability to regulate gender ratios? This should take into account factors such as reproductive strategies, survival rates, and population growth rates.
- **Ecosystem Stability**: How do changing sex ratios affect ecosystem stability? Examine whether fluctuations in sex ratios lead to changes in energy flow and material cycling in ecosystems, thereby affecting the balance of the ecosystem as a whole.
- **Interactions within the Ecosystem**: Can a sea lamprey population with variable gender ratios provide advantages to other organisms in the ecosystem, such as parasites? Consider how changes in gender ratios affect the social structure and interactions within the population and how these changes may influence other species in the ecosystem.

Finally, create a model that explains how variations in the gender ratio of lampreys are affected by the availability of resources and how those variations affect the dynamics of the lamprey population and the overall ecosystem.

1.3 Our Work

To avoid complex textual descriptions, we summarize the details of the model not mentioned above into a flowchart, as shown in Figure3.

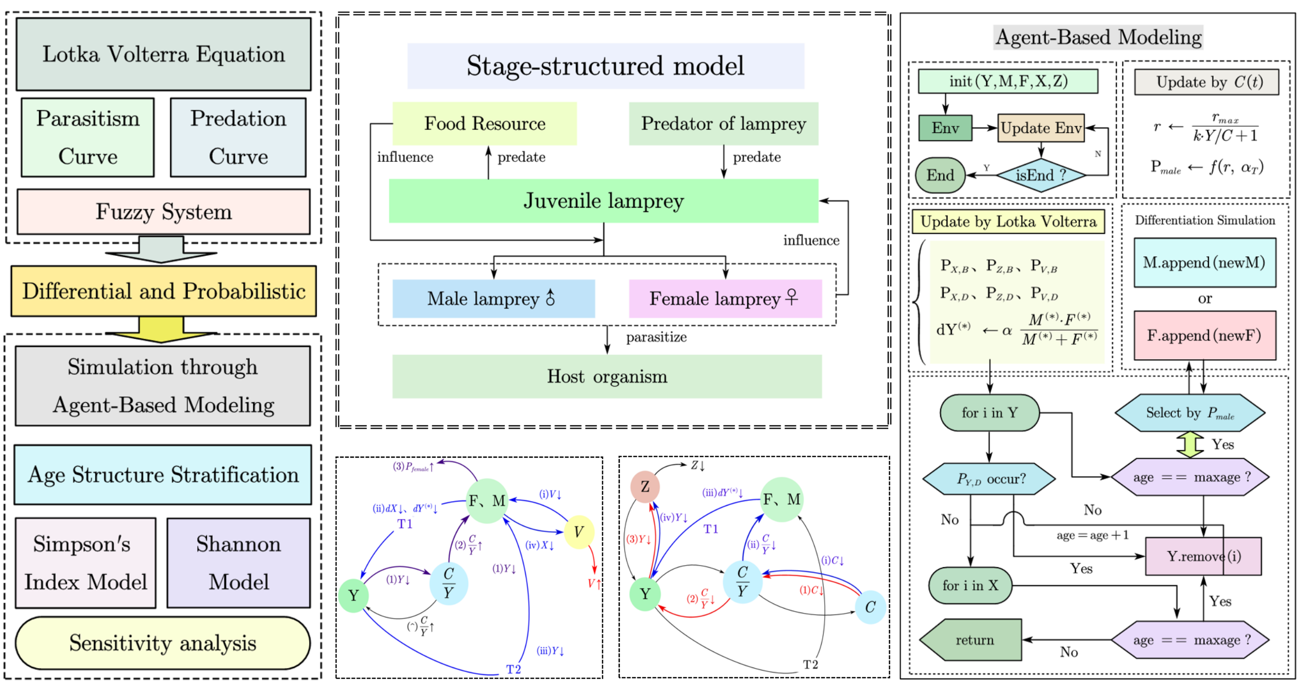

Figure 3: Our Work

2 Model Preparation

2.1 Assumptions and Justifactions

To simplify the problems, we make the following assumptions, each of which is justified.

- **Assumption 1: We assume that adult lampreys are rarely preyed upon.**
  - *Justification: Predation on lampreys occurs mainly at a young age, but adults have a unique parasitic pattern and defense mechanism and often hide in crevices in the sand and gravel, where only a few top predators are likely to prey on them.*
- **Assumption 2: Growth rates of lampreys larvae are positively correlated with resource availability and can be modeled.**
  - *Justification: Lamprey larvae growth is influenced by various factors such as food search ability and nutrient absorption. This study focuses on the link between resource availability, sex ratio, and its impact on the ecosystem.*
- **Assumption 3: The population grows in a limited environment that remains relatively stable, and the environmental carrying capacity of species is also relatively stable.**
  - *Justification: Ecosystems are capable of self-regulation and generally stabilize themselves without strong external disturbances, such as earthquakes. Populations can interact with each other to achieve ecological dynamic equilibrium.*

2.2 Notations

The key notations are shown in Table 1[2]:

2.3 Data Collection and Preparation

To better understand the lampreys so that we can solve the modeling problem efficiently, we have searched for a lot of information and literature, which is compiled as follows:

- **Life cycle of the lampreys**: Sea lampreys return to rivers to spawn, after the eggs hatch, the larvae need to develop for **3-9 years** before they can undergo metamorphosis, metamorphosis usually lasts **12-18 months**, after metamorphosis is complete the main lampreys live by parasitic bloodsucking, the adults usually survive for **2-3 years** and die soon after spawning.[3]

[2]To solve problems flexibly, this paper uses key symbols with different units. Therefore, these units are not listed individually.

Table 1: Notations

Symbols	Description
N	Total biomass of the lamprey population
F, M, X	represent the biomass of adult males and adult females, and the sum of the two, respectively
Y, V	Biomass of larvae in the population, Biomass of adult individual hosts, respectively
Z	Total biomass of predators feeding on juvenile individuals
r	Intrinsic percentage growth of young individuals
r_{max}	Maximum intrinsic percentage growth
$P_{Y,D}$	Biomass of juvenile individuals killed by a unit predator in unit time t
$P_{Z,D}$	Biomass of natural mortality per unit predator per unit time t
C	Biomass corresponding to food resources
c	Conversion rates of lampreys after acquisition of food
k	half-saturation constant
A	Amount of edible resources per unit of biomass

- **Physical condition of lampreys**: Adult lampreys (in this case especially sea lampreys) can weigh up to **2-3 kg** and reach a body length of about **120 cm**[4], with males generally weighing less, which is a strategy of sex ratio adaptation to the environment, as we will focus on below. In addition, lampreys larvae are very young, typically only a few grams to tens of grams.

- **Ecological niche of lampreys**: Larval lampreys are at the lower end of the food chain, living in freshwater environments, usually digging holes in the sediment at the bottom of lakes, rivers, or estuarine areas to feed on **plankton or organic matter**[5]; adult lampreys are at the higher end of the food chain, where they live in a semi-parasitic manner, feeding on a wide variety of fish, which is the main reason why their arrival can cause so much damage to the ecology of the Great Lakes in the United States.

Table 2: Relationship between growth rate and temperature in the lamprey

T(℃) \ W(g)	10	30	50	70	90
5	0.5	0.45	0.44	0.43	0.42
10	1.7	1.4	1.2	1.1	1.05
15	3.2	2.2	1.6	1.4	1.2
20	4.6	2.4	1.5	1.2	1

- **Growing conditions of lampreys**: Lampres can lay nearly **100,000 eggs** at a time, and the successful hatching of the eggs requires a more stringent temperature, generally needing **15 to 25 ℃**; through a search of the literature, we found that temperature also affects the growth rate of larvae, and at the right temperature, the growth rate of lampreys is also faster, and this suitable temperature range is **10 to 20 ℃**[6].

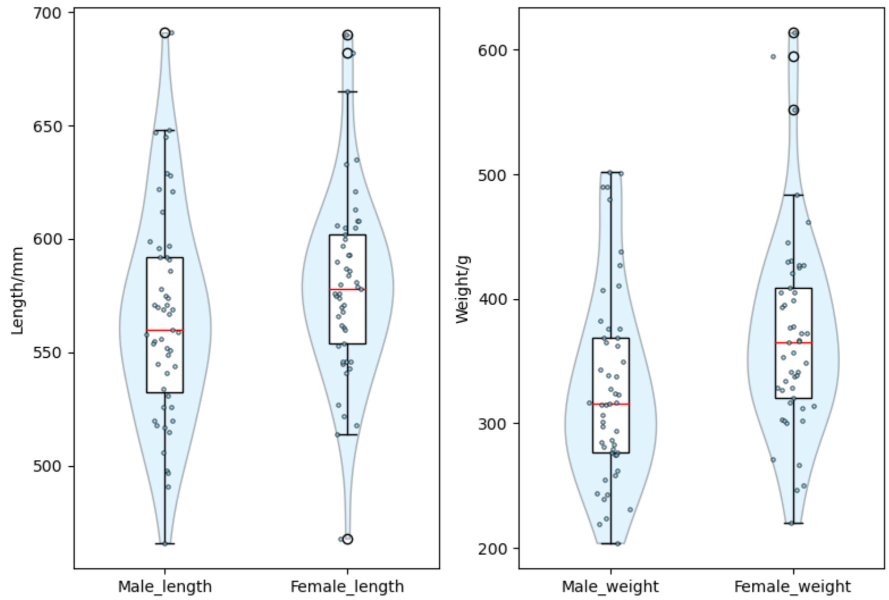

Figure 4: Box plots of length and weight for male and female lampreys

- **The special sex ratio of lampreys:** Through reviewing the data, we found that the sex ratio of lampreys is almost male than female, which is related to the resources, we collected some real data[7] and found that the body length and weight of males are slightly smaller than that of females, so this may be related to the fact that females are larger and demand more resources, which will be analyzed in the following.

3 Exhibit 1: Model based on Lotka-Volterra Equations and Fuzzy Systems

3.1 Establishment of the Lotka-Volterra Predator-Prey Equation

Considering that lamprey populations are often difficult to be directly counted, it is preferable to choose the biomass of lampreys to develop the growth equations for them.

Firstly, we consider the ideal situation, where the lampreys can grow and reproduce endlessly in the case of unlimited food and space resources. Let N denote the total biomass of the lamprey population and r is the intrinsic growth rate of young individuals. Then we can establish the equation (1) to describe the ideal growth process of lamprey:

$$\frac{dY(t)}{dt} = rY(t) \tag{1}$$

In reality, the lamprey's growth environment has limited resources. Based on the previous assumption, as the population size increases, the population growth rate gradually slows down until it reaches the environmental carrying capacity K, at which point the population size stabilizes. Therefore, we introduce the Logistic growth model then the growth equation for the lamprey population should be rewritten as:

$$\frac{dY(t)}{dt} = rY(t)(1 - \frac{Y(t)}{K}) \tag{2}$$

Populations in an ecosystem are interdependent and constrained by each other, and when examining the effects of changes in the dynamics of the lamprey population on an ecosystem, we can consider the basic relationships between populations, such as predatory and parasitic relationships.

According to assumption 1 As lampreys are very weak and at the bottom of the food chain as juveniles, they are easily preyed upon by several species, resulting in a decline in the number of juvenile individuals; after metamorphosis, adult females and males are more adaptable to survival, and the environment, so they are less susceptible to being preyed upon by other species, and usually parasitize on the bodies of other large fishes, feeding on the blood and body fluids of their hosts, resulting in a decline in the number of other populations.

Based on this, we can establish the Lotka-Volterra model to describe the predation and parasitic relationships between lampreys and other populations, and on these two relationships, two sets of equations are established separately. Where Z represents the biomass of the population that preys on juvenile lampreys, and X represents the host population biomass of adult lamprey hosts.

$$\begin{cases} \frac{dY(t)}{dt} = rY(t)(1 - Y(t)/K) - D_Y Y(t)Z(t) \\ \frac{dZ(t)}{dt} = cY(t)Z(t) - D_Z Y(t) \end{cases} \tag{3}$$

$$\begin{cases} \frac{dV(t)}{dt} = rV(t) - D_V X(t)V(t) \\ \frac{dX(t)}{dt} = c_X X(t)V(t) - D_X X(t) \end{cases} \tag{4}$$

In order to more efficiently model the growth rates of lamprey larvae under different food resource conditions, we introduced two core variables: the amount of food resource (C) and the biomass of the lamprey (Y), so that we can elicit the larval growth rate: A = C / Y.

3.2 Modeling and Discussion of Fuzzy Mathematical Systems

To better measure the impact of changes in sex ratios, we begin by establishing two basic relationships, the relationship between sex probability and growth rate, and the relationship between birth success and sex probability.

Gender probabilities are satisfied :

$$P_{male} = f(r, \alpha_T) \tag{5}$$

Given that birth rates during the breeding season are related to the ratio of males and the number of female individuals, by combining Equation (5), where P_male symbolizes the fertilization probability, and $F^*(t)$ $M^*(t)$ represent the biomass of female and male lampreys, respectively, during the breeding season, the growth rate formula for the breeding season can be derived.

We ultimately chose the negative exponential function model, i.e., $f(r) = ae^{-\mu r} + b$. The selection of this function was based on considerations of the system's overall robustness during actual operation. The specific suitability of the f function in the ABM model will be discussed in the sensitivity analysis in Section 5.

Gender probabilities are satisfied :

$$\frac{dY^*}{dt} = \alpha \frac{F^*(t)M^*(t)}{F^*(t) + M^*(t)} \tag{6}$$

In this formula, α is an adjustment factor. As shown in the figure below, the function image is expressed in terms of α = 1. As can be seen from the figure, the reproduction rate is maximum when the sex ratio of 1:1 is satisfied.

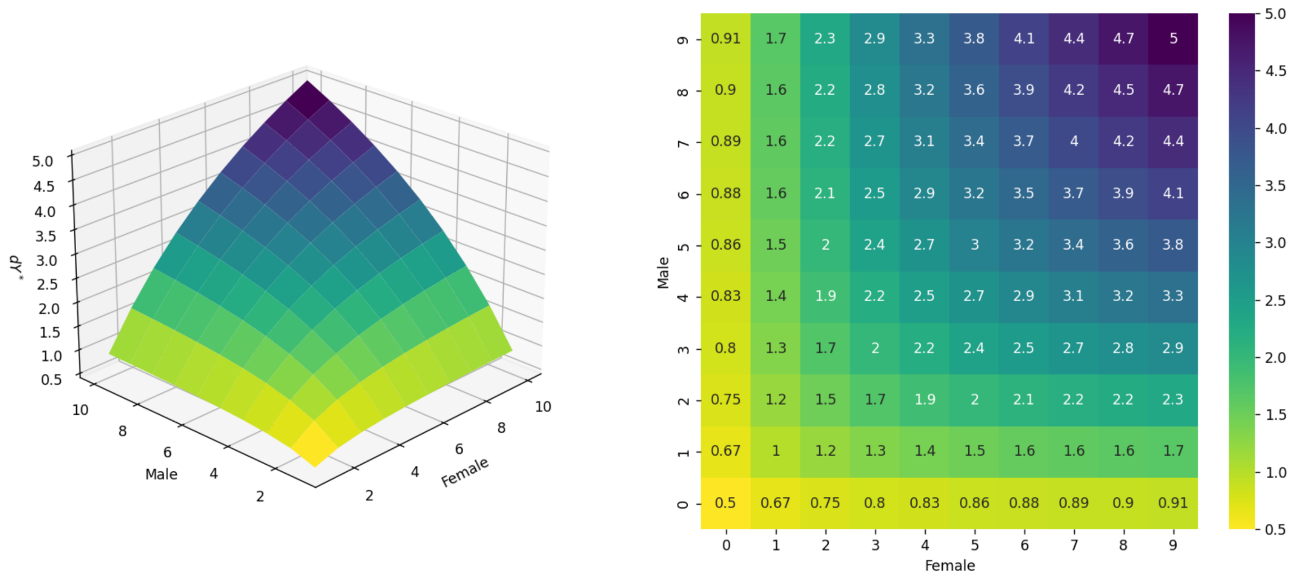

Figure 5: The graph of the function $f(\alpha, T)$ when α = 1

3.2.1 Establishment of a Fuzzy Mathematical System

Step1. Define fuzzy variables:

A fuzzy set is a set of possible values, and each value has a degree of affiliation that indicates the degree to which the value belongs to the fuzzy set. In the discussion for Problems 1 and 2, based on the establishment of Logistic and Lokata-Volterra models, we can mainly consider the definition of key variables as fuzzy variables: r,r_{max}.

Step2. Construct the affiliation function:

Define an affiliation function for each fuzzy set that describes the degree to which a specific value belongs to that fuzzy set. Obviously, the value of the affiliation function lies between 0 and 1.

Step3. Formulate fuzzy rules:
Fuzzy rules are usually formulated based on expert knowledge or experimental data. For this case, fuzzy rules can be based on basic principles of ecology and knowledge from previous studies.

Step4. Fuzzy Inference:
Fuzzy reasoning involves converting fuzzy inputs into fuzzy outputs through fuzzy rules. In this case, fuzzy reasoning can be used to predict dynamic changes in the population of lampreys, such as increase or decrease in biomass.

Step5. Defuzzification:
Defuzzification is the process of converting the results of fuzzy reasoning into a specific output value. Commonly used defuzzification methods include the center of mass method (also known as the center of gravity method) and the maximum affiliation method.

3.2.2 Comparison of Relevant Parameters

To better analyze the lamprey in the Great Lakes of Europe and the United States, we performed a comparison of the relevant parameters, and we rationalized the comparison, where E stands for the European region and L stands for the Great Lakes region of the United States.

Table 3: Comparison of relevant parameters

Class	**Comparison**	**Interpretations**
Rate	${r_{max}}^{(E)} \approx {r_{max}}^{(L)}$	The environment can affect the maximum intrinsic growth rate. When disturbances are absent, there is little difference in resources and temperatures.
	$c^{(E)} \approx c^{(L)}$	In the absence of special conditions, it can be tacitly assumed that the nutrient conversion rate of each individual of the lampreys differs very little.
Biomass	$Z^{(E)} < Z^{(L)}$	The Great Lakes are the largest freshwater lakes in the world, providing stable environments for planktonic organisms that provide larval development with ample food.
	$V^{(E)} < V^{(L)}$	The Great Lakes fisheries are more developed and can provide more food for adult lampreys than European rivers.

3.3 Model Analysis

First, we analyzed the larvae of the lamprey. To express the relationships more clearly, we created the following diagram:

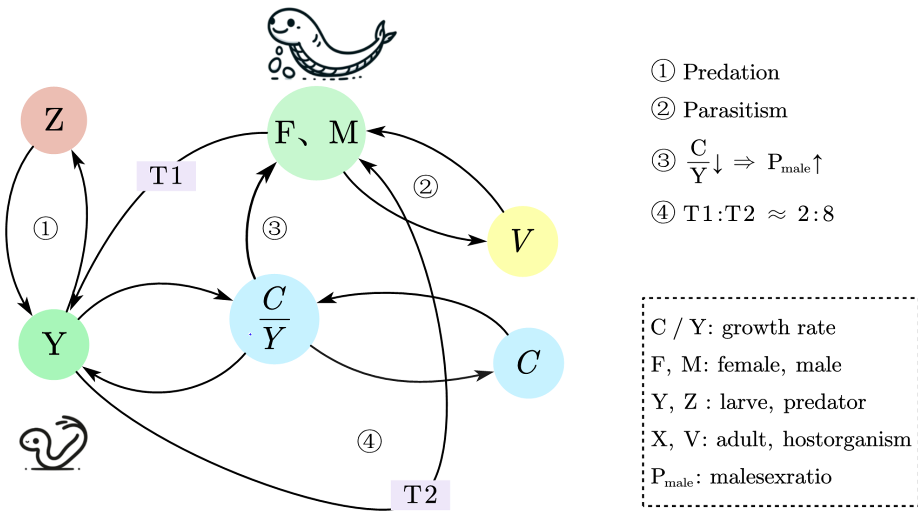

Figure 6: Diagram of the Influence of Sex Ratio on Larval Growth

For the lamprey population, two regions were categorized and discussed for more thoroughness and clarity: the European region and the Great Lakes region of the United States.We used the Lotka-Volterra model for modeling to simulate real-life scenarios by adjusting relevant parameters to obtain inter- and intra-species relationships.

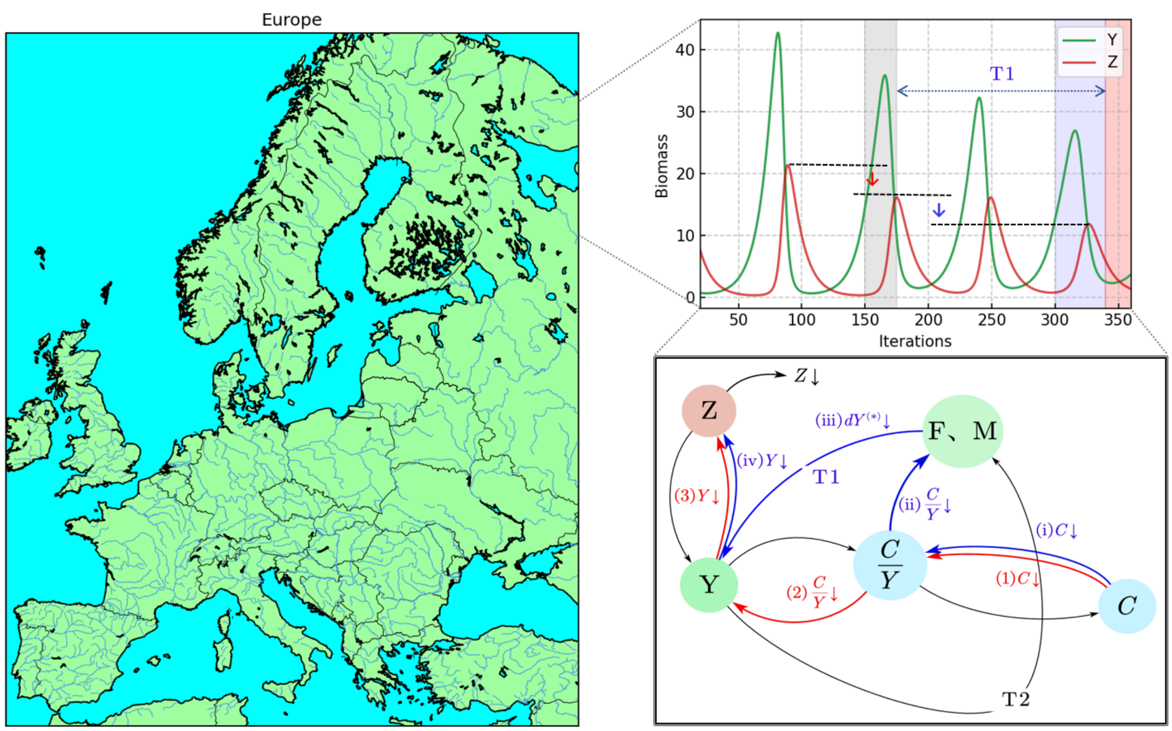

Figure 7: Relationships between European lampreys larvae and their predators

In the figure, we can clearly see two successive declines in the larvae of the lamprey and his predators, and we have described the reasons for this using clear and concise diagrams, where the red color indicates the reason for the first plunge, and the blue color indicates the reason for the second plunge.

We have appropriately reduced the food for the larvae (e.g., plankton, etc.) in the gray-colored portion of the figure, and the reduction in food for the larvae leads to a reduction in the growth rate of the larvae, as well as the death of the larvae due to the lack of food, which directly causes a decrease in the number of larvae, and with the decrease in larval numbers, their predators also decrease in numbers due to the lack of a food source, and this is the first population plunge.

The reason for the second plunge is also straightforward and reflects the fact that resource availability has had an impact on the sex ratio of lampreys. As larval food decreases, larval growth rate decreases, and larval growth rate is an important determinant of sex differentiation[8]: when larval growth rate is high, the proportion of sexes differentiating into females hi increases; conversely, the proportion of females decreases. Changes in the sex ratio result in lower larval birth rates, leading to fewer larvae and subsequently fewer larval predators due to lack of food.

It should be noted that the first and second plunges have a certain time gap, which is due to the fact that there is a period of development and spawning after sex differentiation, which requires a certain amount of time. This also verifies that our Lotka-Volterra model has a strong predictive ability.

The next place we analyzed was the Great Lakes in the U.S. We made good use of the different characteristics of the two places to better analyze, (in the Great Lakes, the adult lamprey is well-fed and is a headache species), we simulated the image using the Lotka-Volterra model.

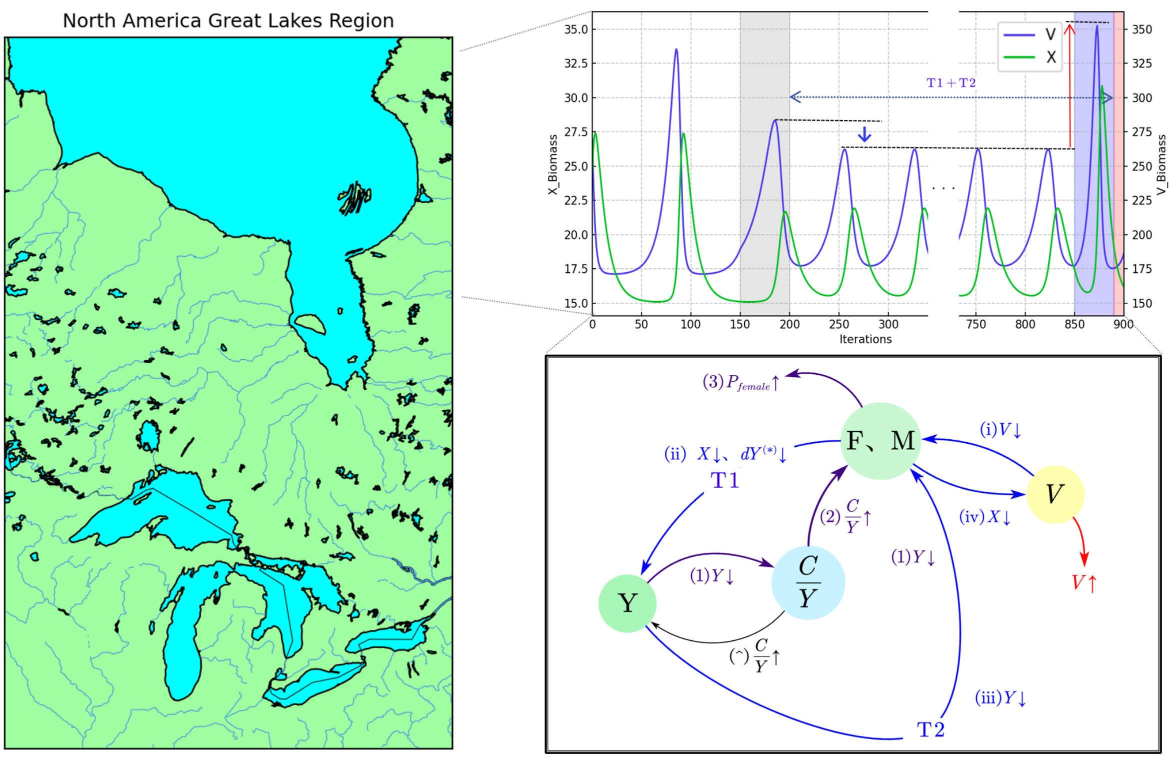

Figure 8: Relationships between the Great Lakes lampreys larvae and their prey

In the gray part of the figure above, we have simulated a reduction in the amount of food available to adult lampreys using the Lotka-Volterra model, and it is easy to see that the reduction in the amount of food has caused a rapid decline in the number of adult lampreys, which in turn has caused a decline in Y (indicated by the blue line in the lower right figure). Thus, C/Y increases, which causes the female ratio to increase somewhat, which would cause the larval birth rate to increase, which would increase the number of larvae.

In the figure above, we see that there is a section of iterations omitted from the function image. The omitted portion of the iteration is the same, and after 10 years, as the larvae increase in the first place and grow into adults, it leads to a sudden spike in the wave.

3.4 The Impacts of Lamprey Sex Ratio on the Larger Ecosystem

In larger ecosystems, species interactions like competition, predation, and parasitism are common. Variations in lamprey sex ratios can significantly impact food webs. Ecosystems, comprising both biotic and abiotic elements, are influenced by resource availability changes, as indicated by the Lotka-Volterra model. Such changes can alter lamprey populations and, consequently, affect the number of fish they parasitize. However, this effect is transient; as lampreys deplete their food sources, intra-species competition intensifies, leading to a decline in their numbers.

Our model highlights the interplay between different populations within an ecosystem, including interactions between living beings and their environment. The analysis suggests that shifts in the sex ratio impact both juvenile and adult lampreys, with potential broader ecological consequences like increased plankton levels that could trigger phenomena such as water blooms or red tides. **This underscores the dynamic equilibrium within ecosystems and the limits of their self-regulation capabilities.**

3.5 The Impacts of Lamprey Sex Ratio on Lamprey Population

Through the above analysis, we can get some information about the advantages and disadvantages of the lamprey population:

3.5.1 Advantages

- **Great adaptability**: Lampreys adapt to changes in their environment by adjusting sex ratios in response to resource availability. When resources are abundant, the production of more females increases reproductive rates to accommodate higher resource utilization.
- **Reproductive efficiency**: When sufficient resources are available in the environment, lampreys tend to produce more females, which increases the efficiency and number of reproductions.

3.5.2 Disadvantages

- **Reduced genetic diversity**: When sex ratios change, genetic diversity in populations may be affected. If a change in sex ratio results in a decrease in gene flow, it may reduce the genetic diversity of a population.
- **Environmental vulnerability**: Lamprey populations rely on environmental conditions to determine sex ratios. Extreme environmental events can cause significant imbalances, which may impact population stability.[3]

4 Exhibit 2: Simulation through Agent-Based Modeling

In this section, we employ an Agent-Based Modeling (ABM) process, adopting differential and probabilization methods of differential equations to construct an accurate model of the population dynamics of agents (lamprey larve, adult lampreys, hosts, predators), the flow of which is shown below:

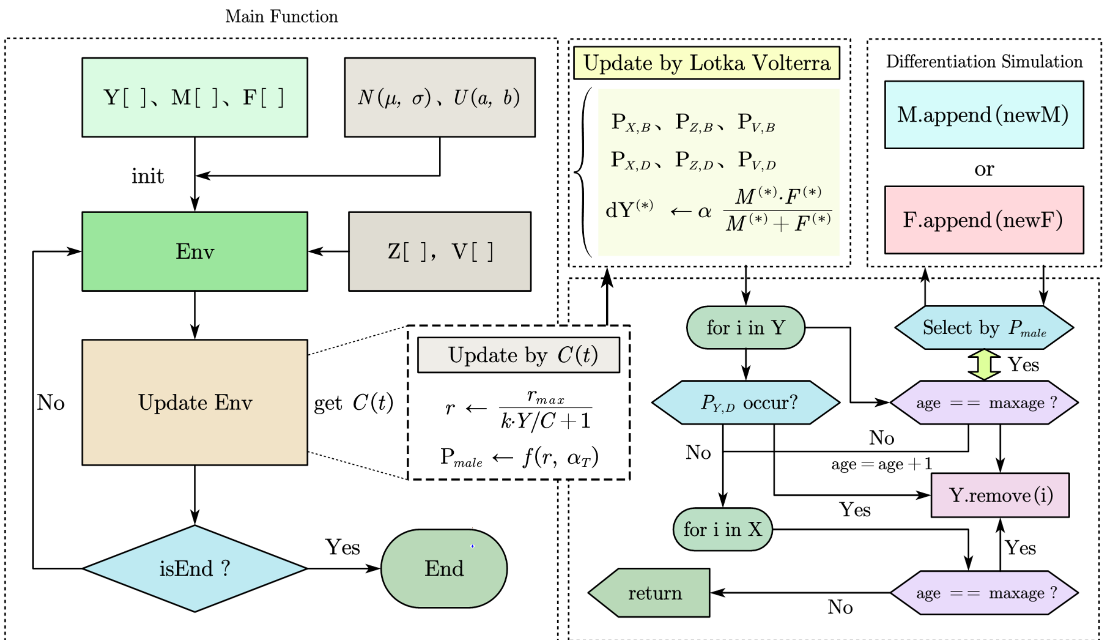

Figure 9: ABM Flowchart

Incorporating these aspects into our Agent-Based Modeling (ABM) framework, as illustrated in Figure 9, allows us to construct a nuanced model that closely mirrors the complexities of ecological systems:

- **Differentiation**: Through discretization of differential equations, our model accounts for the discrete actions and interactions of individual agents. This step is crucial for capturing the dynamic and individualistic nature of population changes, including events such as births, deaths, and other life history transitions.

- **Probabilization**: By integrating probabilistic elements, our model embraces the inherent randomness of ecological processes. This is achieved by applying probabilistic rules to key biological processes, such as reproduction, mortality, and sex determination. The inclusion of these stochastic elements lends our model the ability to more accurately reflect the unpredictable nature of ecological dynamics and population fluctuations.

- **Sampling from Historical Data**: We employ historical temperature data and lamprey population data to generate samples. Normal distributions are used to simulate the variability in individual lifespans within the agent population.

[3] There will be a more detailed explanation about the advantages and disadvantages later in the article.

- **Gaussian Noise**: To further enhance the realism of our environmental simulation, we add Gaussian noise to certain random variables. This method introduces the necessary variability and uncertainty that are characteristic of natural environments, providing a more authentic representation of the agents' habitat and life conditions.

4.1 ABM: Based on Age stratification structure

Considering the fact that the system of differential equations we listed in 3.1 could not be solved analytically, we next consider the use of differential methods to determine the numerical results, which will facilitate us to later simulate the process of population dynamics of the lamprey and the impact on the ecosystem. In the first step, we set up the differential equation for the biomass of juvenile individuals:

$$Y_{t+1} = Y_t + r_t Y_t (1 - \frac{Y_t}{K}) \tag{7}$$

By the same token, we can establish the difference equations corresponding to the two sets of relationships in which juvenile individuals are preyed upon and adults are parasitized, as follows:

$$\begin{cases} Z_{t+1} = Z_t + cY_t Z_t - P_{Z,D} Y_t \\ V_{t+1} = V_t + rV_t - P_{V,D} X_t V_t \\ X_{t+1} = X_t + c_X X_t V_t - P_{X,D} X_t \end{cases} \tag{8}$$

Now, we can calculate the birth rates for the simulated Z, V, and X populations. Since the population growth rate equals the difference between birth and death rates, and combining (7) and (8) after simplification, the birth rate for Y has been obtained in (6). We can now proceed to derive the expressions for the birth rates of Z, V, and X.

$$\begin{cases} P_{Z,B} = CY_t + P_{Z,D} - P_{Z,D} Y_t / Z_t \\ P_{V,B} = r + P_{V,D} - P_{V,D} X_t \\ P_{X,B} = P_{X,D} V_t \end{cases} \tag{9}$$

After considering the layered age structure of lampreys, it is important to note that lamprey larvae usually occupy the lower levels of the food chain in ecosystems. This phenomenon is observed in most areas, mainly due to the small size of the lamprey larvae and their diet consisting primarily of plankton. This dietary preference leads to a relative insensitivity of lamprey larvae to the predator-prey dynamics within the food chain, as described by models such as the Lotka-Volterra.

Therefore, the survival and growth of lamprey larvae depend more on the availability of lower-level food resources like plankton, whose abundance is influenced by various environmental factors, including but not limited to water temperature, nutrient concentration, and light intensity. In the simulation process, we used the monthly temperature change data collected in section 2.3 to model a more realistic food resource availability C. Additionally, to more effectively simulate the growth rate of lamprey larvae under different food resource conditions, we referred to Holling's Type II functional response principle (Holling Type II), which can be formalized through the Michaelis-Menten equation to describe the relationship between food availability and the growth rate of lamprey larvae, as shown by Equation 10:

$$r = r_{max}\frac{A}{k + A} = r_{max}\frac{C/Y(t)}{k + C/Y(t)} \tag{10}$$

Where k is the half-saturation constant, representing the food resource amount needed to reach half of the maximum growth rate r_{max}. When the food resource availability C/Y is low, the growth rate r increases significantly with A; however, as A reaches a certain level, the increase in growth rate will tend to plateau, reflecting the saturation effect of resource utilization. This principle indicates that the predation rate will increase with the density of prey, but as the prey density reaches a certain level, the increase in predation rate will slow down until it reaches the maximum feeding rate.

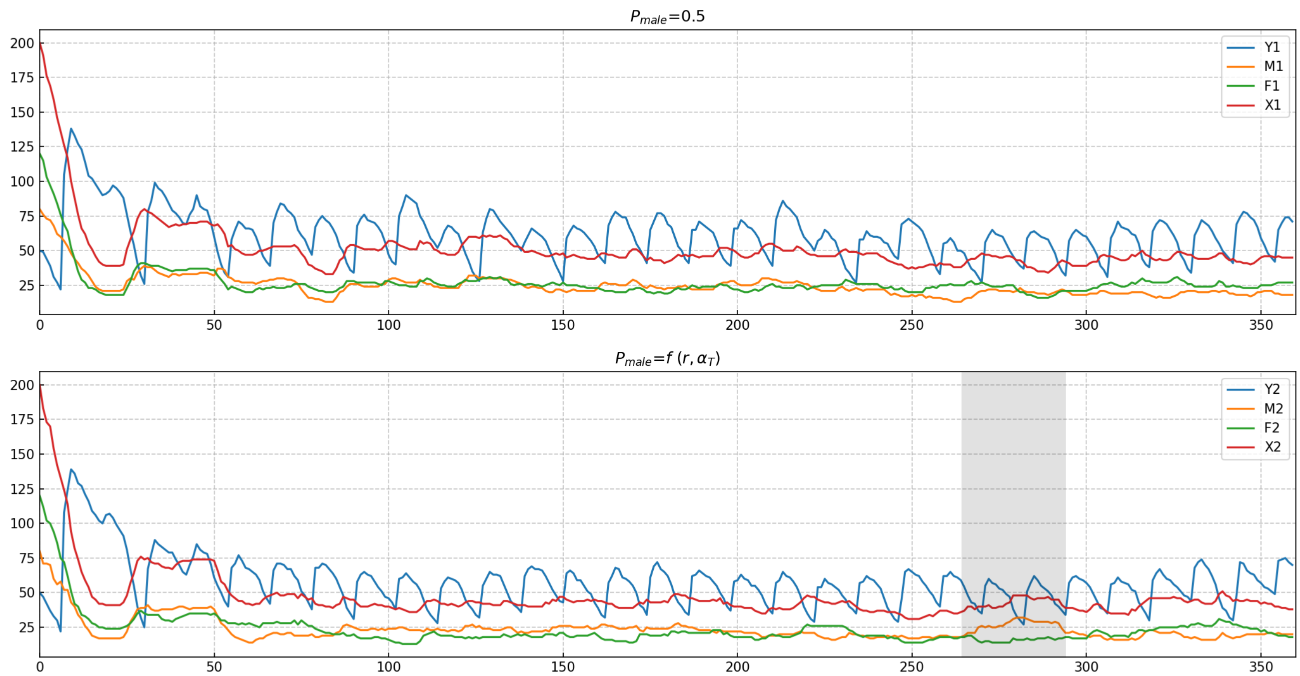

Figure 10: Comparison of population sizes for different breeding strategies

As can be seen from the above figure, comparing the ideal population (1:1 probability of male/female differentiation) with the real population there is a gray zone where the total number of adult males and adult females is greater than the number of hatchlings, but the real population adjusts to this change and it can be seen that the image can go back to the original trend. This illustrates one of the strengths of the characterization of population sex ratios as influenced by resource availability and suggests that the population is somewhat resilient and adaptable.

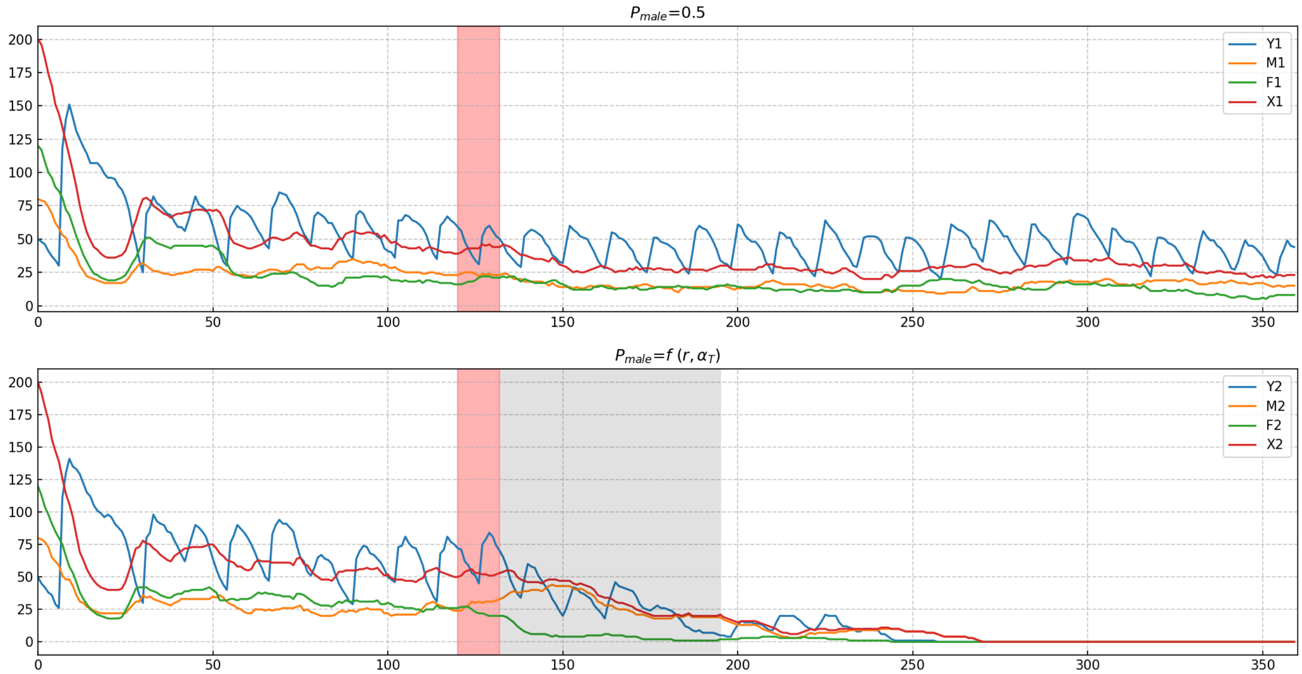

Figure 11: Comparison of population sizes of different breeding strategies after joining disasters

For the above figure, we added a certain resource pressure (i.e., disaster) at the corresponding time period, and we can see that: compared with the ideal model, the real population again has a gray zone after encountering a disaster, but at this time, the population cannot adjust back to it, and the internal equilibrium of the population is disrupted, and the loss of equilibrium leads to a gradual decrease in the number of the population and to extinction. This is one of the drawbacks of the property that the sex ratio is affected by resource availability, which results in a less resilient population of the lamprey.

4.2 Building Stability Indicators: Simpson's Index Model

Ecosystem stability includes both anti-disturbance stability and recovery stability. In general (not considering extreme environments), the higher the species richness, the higher the anti-disturbance stability of the ecosystem, and the lower the recovery stability. The lower the species richness, the lower the anti-disturbance stability and the higher the recovery stability of the ecosystem.

4.2.1 Introduction of the Model

We obtained the Simpson's Diversity Index formula by reviewing the literature[9]:

$$D = 1 - \sum_{i=1}^{S} P_i^2 \tag{11}$$

In this equation, S is the number of species, and the ratio of the number of individuals of i to the total number of individuals in the community is P_i.

4.2.2 Establishment of the Model

We can use differential equations to describe changes in the sex ratio of lampreys. We assume that the growth rates of male and female lampreys are proportional to food availability and that females grow faster when there is more food, resulting in a relative increase in the proportion of females. At this point, we can obtain a sex ratio model:

$$\begin{cases} \frac{dM}{dt} & = k_1 RM, \\ \frac{dF}{dt} & = k_2 RF, \end{cases} \tag{12}$$

In this set of equations, k_1 and k_2 are constants that need to be determined from experimental data. In this model, we can consider M and F as two different species, so we have:

$$D = 1 - \left(\frac{M}{M+F}\right)^2 - \left(\frac{F}{M+F}\right)^2 \tag{13}$$

4.2.3 Solution of the Model

Based on the modeling above, we can conclude that this is a quadratic function and that the results of the model can be obtained using Python. When the sex ratio is balanced, the value of D is the largest, indicating the highest species diversity. The value of D increases with the increase of male proportion in the range from 0 to 0.5. Whereas, in the range from 0.5 to 1, the value of D decreases as the proportion of males increases. Thus, maintaining a relatively balanced sex ratio is essential for maximizing the Simpson's diversity index.

It is worth noting that the model is based on an ideal environment and can be considered a complementary exploratory model.

4.3 Advantages of Sex Ratio Variation in Lamprey Populations

Explore the potential impact of gender ratio fluctuations in the marine Lamprey population on ecosystem equilibrium and its members, especially for parasitic organisms within the ecosystem. We need to gain a deeper understanding of how changes in gender ratios affect the interrelationships among species and ecological balance.

The following two dimensions are the focal points for analysis:

4.3.1 Contribution to Species Diversity

When discussing the impact of gender ratios in the Lamprey population on the diversity of species in the ecosystem, we can analyze it from two main aspects: an increase in inter-species interactions and an enhancement of ecosystem complexity. Gender balance in a population in resource-rich environments can promote more inter-species interactions. This is because gender balance helps maintain the population's reproduction and growth, increasing opportunities for inter-species interactions, including competition, predation, symbiosis, and other relationships. In contrast, in resource-limited environments, gender imbalance (such as an excess of males) may lead to a decrease in the population's growth and reproductive capacity, resulting in reduced inter-species interactions and affecting the diversity of the ecosystem.

To quantify the impact of gender ratio changes on ecosystem diversity, we can not only use the method described in section 4.3 to analyze it quantitatively using the Simpson Diversity Index but also use the Shannon Diversity Index as a measure of ecosystem diversity. The Shannon index considers both species richness (the number of species) and evenness (the distribution of individuals among species). Its formula is as follows:

$$H' = -\sum_{i=1}^{S} p_i \ln(p_i) \tag{14}$$

Where:

- H represents the Shannon Diversity Index.
- S is the number of species in the ecosystem.
- p_i is the proportion of individuals belonging to species i in the ecosystem.

Using the Shannon Diversity Index, we can further assess the impact of gender ratio changes on the diversity of species within the ecosystem.

Assuming a simplified ecosystem consisting of Lampreys and two other species, we can calculate the Shannon Diversity Index under both balanced and imbalanced gender ratios. For example:

- Under a balanced gender ratio scenario, let's assume the Lamprey population consists of 50% males and 50% females, and the relative abundances of species A and species B are in a 1:1 ratio, meaning each species has a relative abundance of 1/3.
- Under a scenario where males dominate due to limited reproductive capacity in Lampreys, leading to an overall decrease in the Lamprey population, the relative abundances of species A and species B increase, resulting in an abundance ratio of 0.5:1.5:1.5. In other words, the relative abundance of Lampreys becomes 0.5/3.5, while species A and B each have 1.5/3.5.
- By calculating the Shannon Diversity Index in these two scenarios, we can quantify the impact of gender ratio changes on ecosystem diversity. Typically, under a balanced gender ratio scenario, the ecosystem's diversity index is expected to be higher, reflecting a healthier and more stable ecosystem state."

By calculating the Shannon Diversity Index in these two scenarios, we can quantify the impact of gender ratio changes on ecosystem diversity. In cases where the gender ratio is more balanced, the ecosystem's diversity index is typically higher, reflecting a healthier and more stable ecosystem state.

It can be observed that gender ratio changes in the Lamprey population clearly have the potential advantage of enhancing species diversity within the ecosystem. By strengthening species diversity, there is a subsequent increase in the overall stability of the ecosystem, which, in turn, provides significant advantages for the survival of other organisms within the ecosystem. The entire logical chain is depicted in the following diagram:

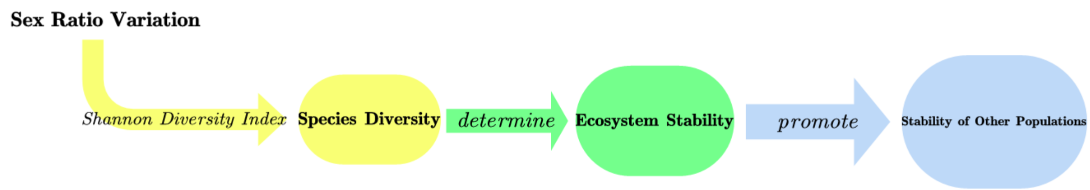

Figure 12: The logical chain for the advantages of sex ratio variation in lamprey populations.

4.3.2 Interactions among Species

The second focal dimension for analysis is the interaction among species, with a specific emphasis on species that rely on Lamprey as a food source. When gender ratios within the Lamprey population change, these species may be affected. We can elaborate on this dimension as follows and provide relevant mathematical models and data examples:

Model of Species Dependent on Lamprey as a Food Source: Let's consider a species, X, which depends on Lamprey as its primary food source. We can use the following model to represent the population size N_x of species X, which is influenced by the female Lamprey populationN_f:

$$N_x = \alpha N_f \tag{15}$$

Here, represents the food dependency coefficient of species X. When the female Lamprey population decreases, N_x will also decrease correspondingly, potentially leading to a decline in the population of species X.

Using data examples, we can gain a clearer understanding of the impact of Lamprey gender ratio fluctuations on the food chain within the ecosystem and how species dependent on Lamprey are affected. This contributes to a deeper investigation of inter-species interactions and changes in ecological balance.

5 Sensitivity Analysis

5.1 Sensitivity Analysis of Initial Total Population

We varied the proportion of juvenile and adult females and adult males in the initial population and plotted the biomass of each species in the community over time as follows:

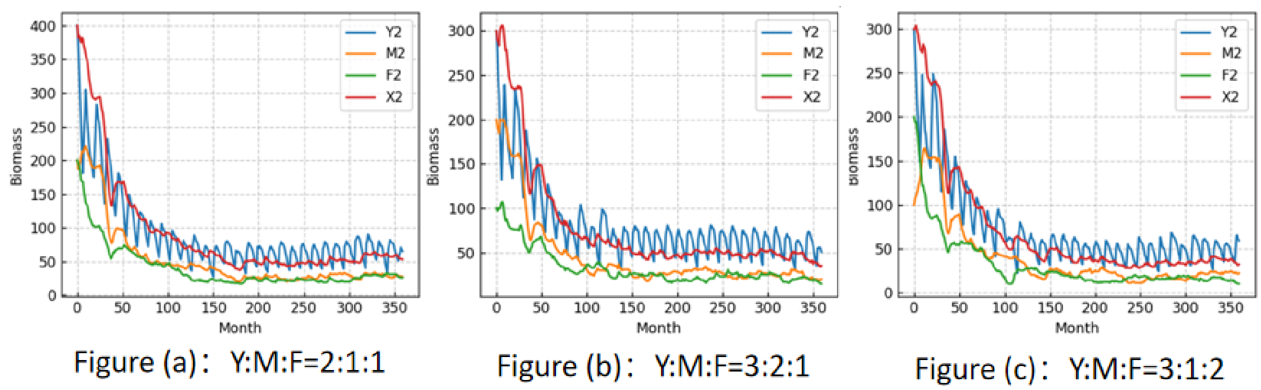

Figure 13: Changes in population biomass over 10 years

Through the literature review, we learned that under normal circumstances, the initial parameters of the lamprey population, namely the ratios of juveniles, adult females, and adult males, can be assumed to be $2:1:1$. Now, we adjust the initial population ratios of juveniles, adult females, and adult males to the following sequences respectively: $2:1:1$, $3:2:1$, and $3:1:2$.

It can be observed that by adjusting the ratios of juveniles, adult females, and adult males (i.e., the initial population parameters), the biomass trends over time are similar and tend towards stability. Therefore, we can conclude that the sensitivity of the initial ratios of juveniles, adult females, and adult males in the population is not significant.

5.2 Sensitivity Analysis of Initial Mortality Rate

The initial mortality rate refers to the proportion of natural deaths due to aging, illness, and other natural causes within the population. Through multiple calculations, we have determined that the most suitable initial mortality rate is around 0.03. Below is our sensitivity analysis of this rate. To make the results more evident, we introduced a period of resource stress (i.e., a disaster) during the iterations. In the following figure (under the condition of an initial mortality rate of 0.03), the red section indicates the time when the disaster occurred: From the above figure, under the premise of a natural mortality rate of 0.03, the introduction of a catastrophe did not lead to the extinction of the species, demonstrating that the species possesses considerable adaptability and resistance to disasters. Below, we will show how this changes with the mortality rate.

The first graph represents constant probability births, where the probability of being born male or female is 0.5. The second graph simulates the actual situation. From the graphs, it is evident that under a natural mortality rate of 0.03, the introduction of a catastrophic event does not lead to the extinction of the species, demonstrating that at this point, the species possesses strong adaptability and disaster resistance capabilities. Below, we present the trend of species extinction timing as the mortality rate changes.

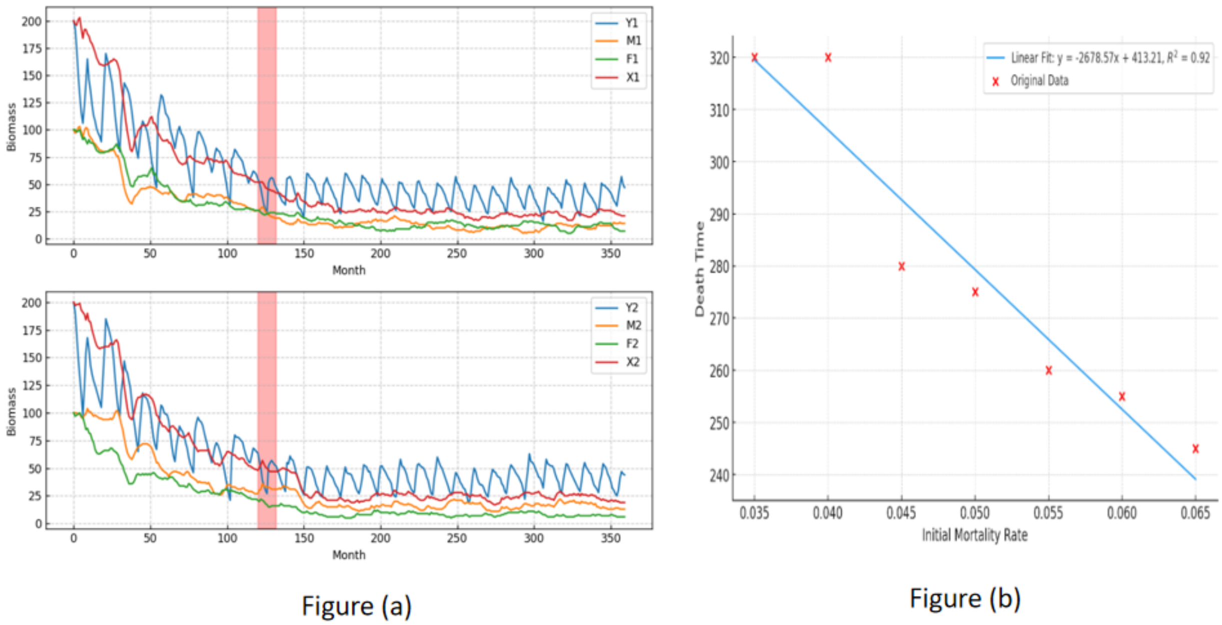

Figure 14: Sensitivity test for initial mortality

The simulation was conducted using Python, and the trend was plotted using a linear fit. The coefficient of determination for the linear equation was calculated to be 0.918, indicating an excellent fit. It is important to note that the initial mortality rate is highly sensitive and greatly affects the population.

In the test, we also fine-tuned the initial mortality rate and found that species do not become extinct when the initial mortality rate is in the range of 0.3 ± 0.03, and smaller changes in the important parameters still make the model output stable, which shows the reasonableness of our model.

5.3 Sensitivity Analysis of Sex Ratio

For the equation related to the sex ratio, in addition to α_T, when considering the relationship between P_{male} and r, we can derive the equation according to f, in addition to α_T:

$$f(r) = \begin{cases} ar + b \\ a(r-c)^{-1} + b \\ ae^{-\mu r} + b \end{cases} , P_{min} < f(n) < P_{max}. \tag{16}$$

By modeling tests, we can get $P_{min} < f(r) < P_{max}$.Exponential and inverse proportion equations have better robustness, consideration of the actual situation, we finally chose that $f(r) = ae^{-\mu r} + b$ exhibits better robustness to parameter variations.

This exponential decay model is simple and easy to understand and is very effective in describing certain natural phenomena. Its parameters are easy to estimate by mathematical methods such as least squares and its robustness means that the model is insensitive to small fluctuations in the data.

6 Model Evaluation and Further Disscussion

6.1 Strengths

- **Our model is well thought out.** We studied the impact of various factors on lamprey populations, including sex ratio, resources, predator effects on juveniles, adult host resources, and environmental conditions such as temperature.
- **Our model is innovative.** We used a logistic model to predict population growth and integrated various effects into the Lotka-Volterra equation.
- **Our model is robust.** Through sensitivity analysis, the model maintains relatively stable outputs for small changes in input parameters, so the model works reliably in the face of uncertainty in real-world data.
- **Our model visualization is well done.** We effectively visualize our models by using drawing software to create multiple clear and concise images.

6.2 Weaknesses

- The data obtained from the literature is not a definitive figure, for example, adult lampreys weigh up to 2-3 kg, but to better address the problem we quantified them into a representative figure, which may produce some errors.
- Computer simulations can be challenging with continuous data, so we performed a differential decomposition of the data. Please note that this process may result in some loss of information and potential errors.

6.3 Model Promotion

The model developed in this paper addresses the relationship between resources and the sex ratio of lampreys, causing a series of domino effects by affecting the sex ratio, which in turn affects the larger ecosystem. This modeling facilitates our ecological regulation.

Lampreys are known to have complex and varied roles. They live by parasitically feeding on the flesh and blood of fish, which at one time had a significant impact on the Great Lakes fishery; however, in some areas, they have become a delicacy because of their exceptional taste and nutritional benefits, and in some places, they are also bred.

Through the construction of our model, we can adjust some factors to change the sex ratio of the lamprey and affect the population, which can provide site-specific planning for the ecological development of the lampreys.

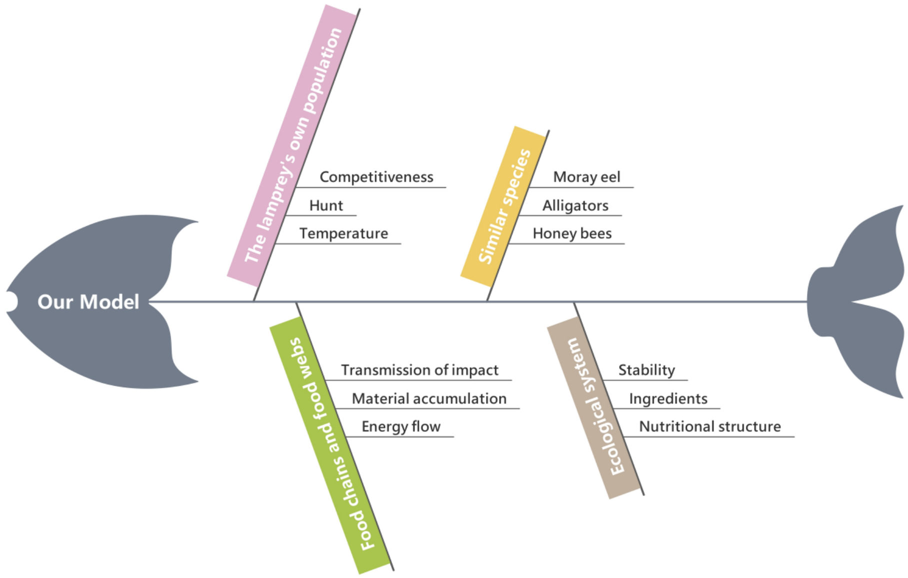

Figure 15: Expansion of the model

In addition to this, our model can be generalized to other populations with similar characteristics to the lamprey, e.g., moray eels[10], where sex ratios are influenced by resource conditions. Through our model, we can adjust the correlation factors to interfere with population growth and thus influence the larger ecosystem through population interactions.

Our model can provide a reference for the development of ecosystems and the prevention and control of ecological disasters, such as how to use endocrine disruptors correctly. Through data analysis, we can guide the relevant departments to rationally plan resource use, optimize environmental protection strategies, governance programs and long-term ecological effects to be evaluated, to achieve a friendly relationship between resources and populations.

7 Conclusion

In this paper, we focus on the adaptive adjustment of the sex ratio of marine lampreys under changing resource conditions and its impact on the balance of the entire ecosystem. Through the establishment and analysis of models, we have reached the following conclusions:

- **Interaction Between Resources and Sex Ratio:**I The abundance of resources directly influences the sex ratio of lampreys, with an increase in females when resources are plentiful and the dominance of males under scarce conditions.
- **Ecological Effects of Sex Ratio Variation:** Adjustments in the sex ratio of lampreys not only impact the structure of their population but also influence the overall stability of ecosystems, enhancing adaptability to environmental changes.

- **Impact on Other Biological Populations:** Changes in the sex ratio, acting through the food chain, affect inter-species relationships and balance within ecosystems.
- **Prospects for Sex Ratio Management:** Despite initial findings, the underlying mechanisms of sex ratio changes and their long-term ecological effects require further investigation.
- **Suggestions for Further Research:** This study provides new insights into effectively managing lamprey populations by artificially adjusting the sex ratio, aiming to mitigate its negative impacts on ecology and economic activities

In conclusion, grasping the dynamics of lamprey sex ratios and their ecological impacts is crucial for ecological preservation and resource management. Future studies need to further explore this intricate subject to better foresee the effects of environmental shifts on ecosystems.

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