EE126 Project Report: Traffic Modelling (TASEP)

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1 Objective

1. Show the relevant derivation of the model.
2. Show the method we use for state updating.
3. Interpret the simulation results.
4. Extension: Effects of cars with different rates and crossed roads.

2 Traffic Setting & Assumptions

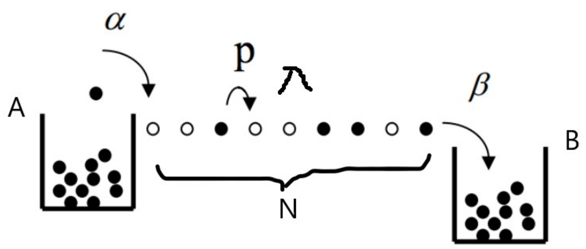


Fig. 2.0.1: Taken from `https://arxiv.org/pdf/0803.2625.pdf`

1. As shown in Fig.2.0.1, let's suppose there is one N-cell road, and cars are trying to get from point A to point B.
2. Each car has an independent exponential clock with rate λ and every time their clock goes off, they are trying to move forward with probability p. However, if there is a car in front of them, they will not move.
3. At the first and last cells, there are exponential clocks with rates α and β that introduce and remove cars from the system, respectively.

2.1 State Space

Let's first consider the case that the time is discrete. Consider a N-cell road with cells $i = 1, 2, \ldots, N$. If a car occupies cell i at time n, it can be represented by $\tau_{i,n}$

$$\tau_{i,n} = \mathbb{1}\{A\ car\ occupies\ cell\ i\ at\ time\ n\}$$

where i is the cell concerned and n is the discrete time node of the current state. Therefore, A state from state space could be symbolically represented as $\{\tau_{i,n}\} = \{\tau_{1,n}, \tau_{2,n}, \tau_{3,n}, \ldots \tau_{N,n}\}$.

The next step is to use Bernoulli approximation to fit the model into the continuous time situation. Specifically, we use small time intervals δt small enough that in each time intervals the Poisson distribution can be see as a Bernoulli distribution. Given a Poisson Process rate $X \sim Poisson(\lambda)$, then the arrival times in a small time interval δt is $X_{\delta t} \sim Poisson(\lambda \delta t)$. Let $\delta t \to 0$, we have

$$X_{\delta t} \sim Bernoulli(\lambda \delta t)$$

In each time interval, there are three kinds of independent and parallel coin tosses are ongoing.

1. (Denote as C_i, $\forall 1 \leq i \leq N$:) One of the car on the road (suppose in cell i) are attempting to move forward one cell w.p. $\lambda\delta t$ (also called 'hopping'). It succeeds if no cars in cell i+1, it fails if there is a car in cell i+1 and nothing happens.
2. (Denote as A:) The road is attempting to introduce a new car at the left side of cell 1 w.p. $\alpha\delta t$. It succeed if the cell 1 has no car and it fails if the cell 1 has a car, in which case, nothing happens.
3. (Denote as B:) The road is attempting to remove a car at the right side of cell N w.p. $\beta\delta t$. It succeed if the cell N has a car and it fails if the cell N has no car, in which case, nothing happens.

since all events above happens independently, we can set a loop for all of the events above, for each we set a coin toss test with the corresponding probability.

3 Simulation Result

3.0.1 Phase Change

To observe the phase change, we are interested in the current of each road and the car density of each road, which is defined as below:

$$J = \#of\ passing\ cars\ per\ unit\ time\ in\ steady\ state$$

$$D = percentage\ of\ occupied\ time\ of\ a\ certain\ cell.$$

According to [2], the phase diagram of 1d-TASEP is shown in Fig.3.0.2.

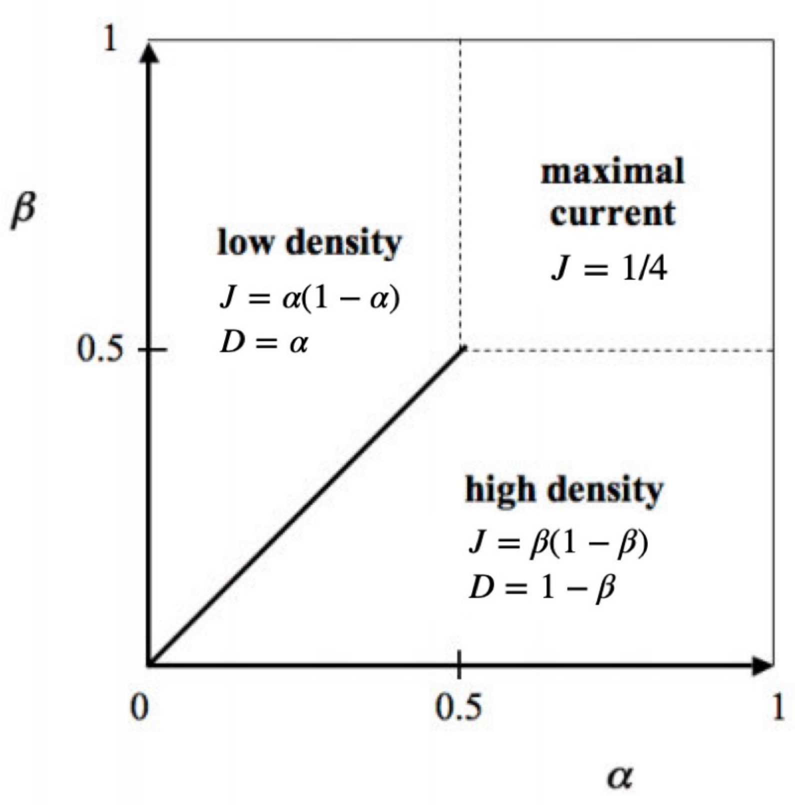


Fig. 3.0.2: The phase diagram of the 1d-TASEP.

The phase diagram shows three different phases in the 1d-TASEP scenario: high density phase (HD), low density phase (LD), and max current phase (MC).

By simulation, several facts related to the phase diagram is pointed out here:

1. In the classical 1d-TASEP setting[1][2], the introduction rate α and removal rate β shown in Fig.3.0.2 are between 0 and 1, because the car rate in the classic model is just 1 each time step. In the traffic setting, the car rate is the property of each car compared to introduction rate α and removal rate β, which is the

property of the road. It is a difference between the classical setting and ours. However, we can still test the validity of our data by scaling the classical α and β with the car rate λ.

2. In LD phase, we can see that J and D are all only dependent on the normalized α, because in this region, the removing of the cars are always quicker than introducing of the cars, so the density is low and both the current and the density are controlled by the introduction rate; In HD phase, similarly, J and D are all only dependent on the normalized β, so the density is high and both the current and the density are controlled by the introduction rate; In the maximal current phase, the current reaches the upper bound that only dependent on the car rate and the density is half-full.

We can see simulation result below matches our expectation. we can see the different phases in density and current diagram. Also, the maximal current is approximately 1/4 of the car rate when hopping rate is set to 1. Also, we can see that the HD and LD phase part of the current diagram has obvious plateau and parabolic shape.

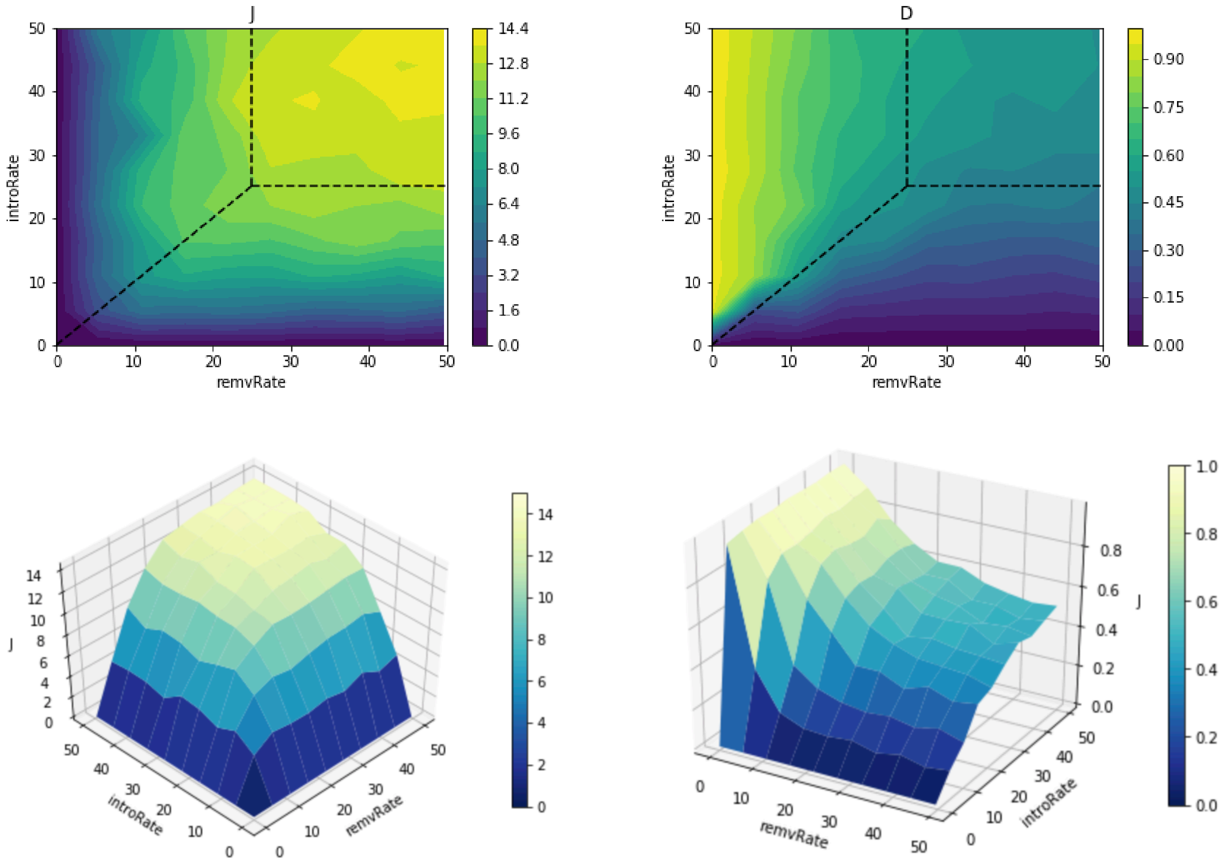


Fig. 3.0.3: The current and density diagram vs. removal rate and introduction (car rate = 50, number of cell=10, hopping prob = 1).The maximal current $J_{max} = 14.0 \pm 0.4$.

3.1 Pairwise Relation

We are also interested in what effect car rate brings to the density and current. The following Fig.?? shows the result when car rate is set on [0,25] with another rate variable, in which we can see similar J plot and D plot.

1. J-plots shows that when the car rate is less than either introduction rate or removal rate, it will be the major influence on the current. Take (a) for instance, when introduction rate is high, the current increase linearly with car rate.

2. (b) shows that when car rate is lower than both the α and β, the density is less when the car rate increase, since the introduction of other cars will be blocked by the slowly moved driving cars.

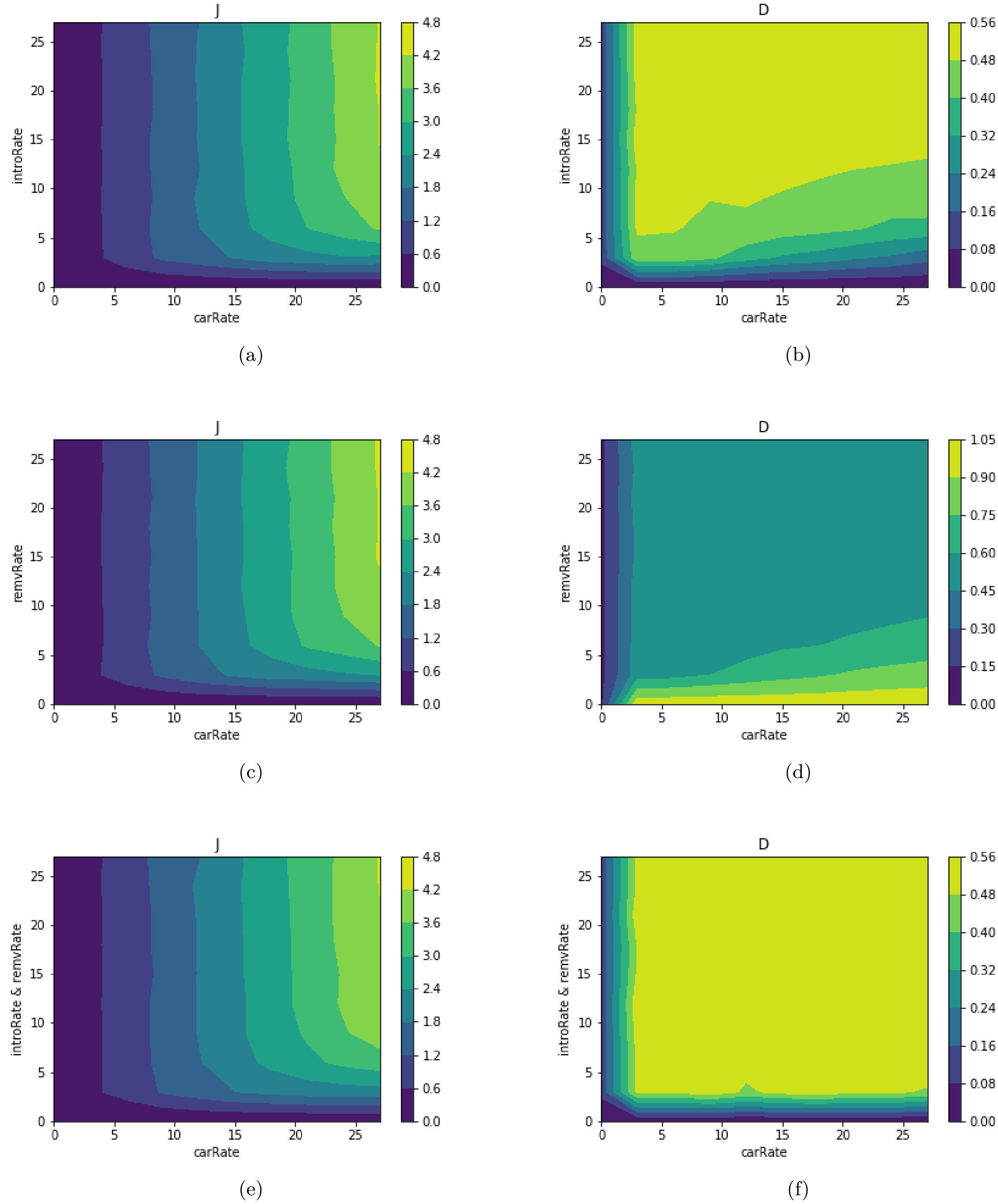


Fig. 3.1.4: Plot of J,D vs. car rate, α, β, pairwise.

4 Extension

4.1 Effects of Different Car Rates

We assume that the car rate distribution is normal.

$$R_i \sim \mathcal{N}(\mu, \sigma^2)$$

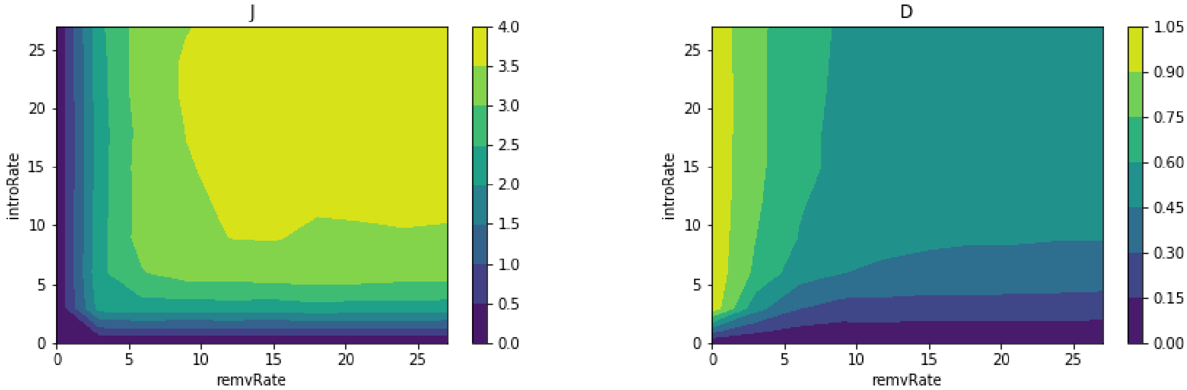


Fig. 4.1.5: The current and density diagram vs. removal rate and introduction ($R_i \sim \mathcal{N}(25, 5)$, number of cell=10, hopping prob = 0.5).

4.2 Effects of Crossed Road

what if there are two or more roads on site? In our setting, we can track the average density and total current of the roads.

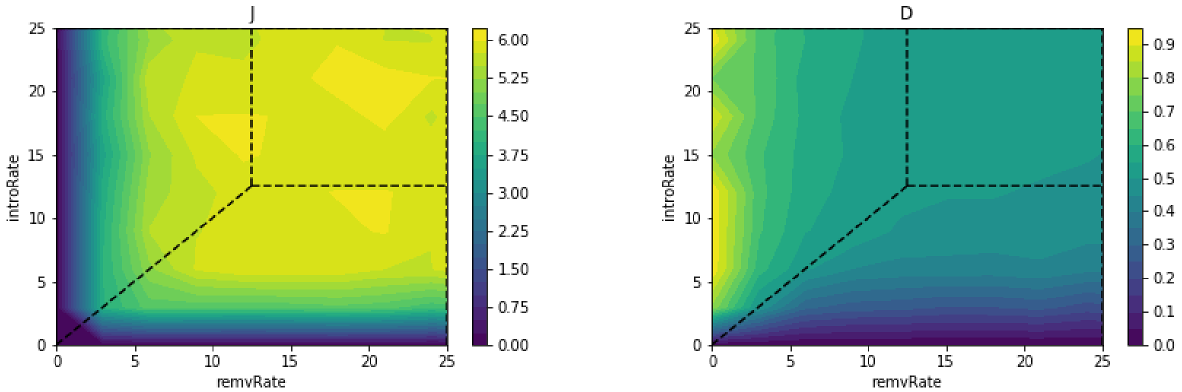


Fig. 4.2.6: The average current and density diagram vs. removal rate and introduction rate when there is a crossroad.(car rate = 25, number of cell of each road =10, hopping prob = 0.5).

Fig.?? shows that when 2 roads are crossed, the current and the density are more easy to reach the max current phase. The maximal current is of each road (6.00/2 = 3.00 in Fig.??) decreases compared to one-road scenario. which can be interpreted as when two roads are crossed, the crossing point will cause additional pause at all time.

4.3 Demonstration Metro-City Traffic!

Here are some demonstration effects our Applet can do:

1. We can change different rates by hand!

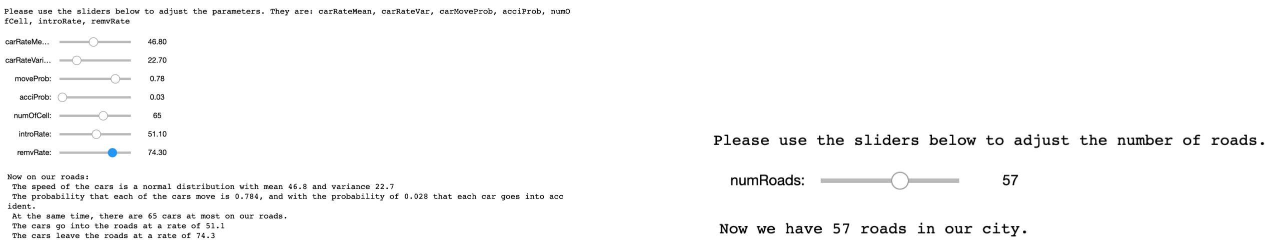


Fig. 4.3.7: The demo of our sliders that can change the parameters. See our Applet for details.

2. We can see cross-road traffic!

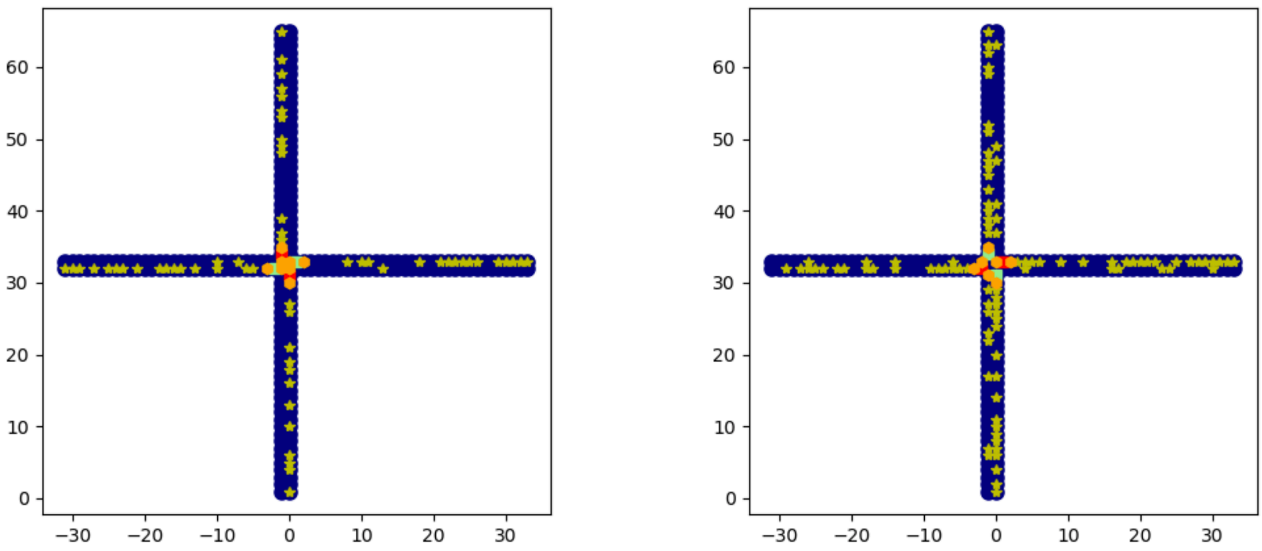


Fig. 4.3.8: The cross roads demo with traffic light (marked as red and green points) and cars (in yellow or orange). See our Applet for details.

3. And more fun! We can set random road forest!

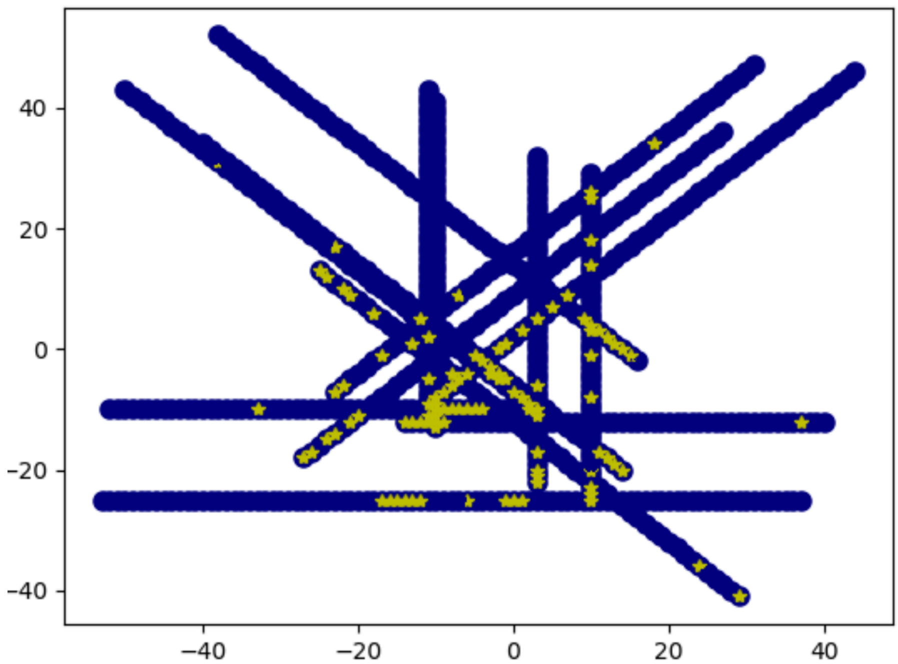
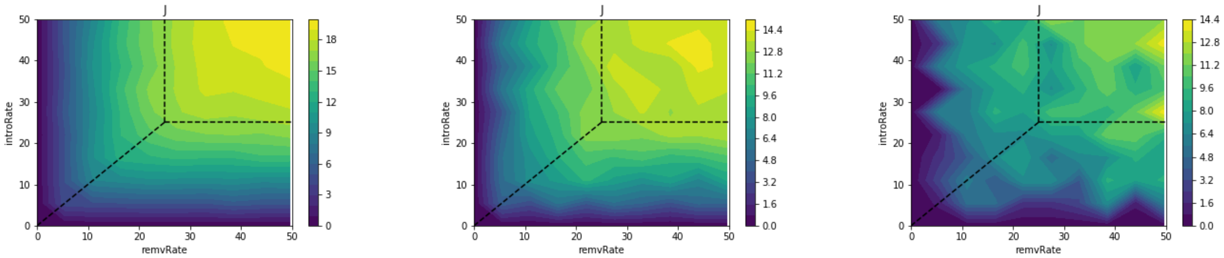


Fig. 4.3.9: You can set random roads. See our Applet for details.

5 Error Analysis



(a) interval time = 0.1 sec (b) interval time = 0.01 sec (c) interval time = 0.001 sec

Fig. 5.0.10

The plots above is the current diagram are under same running time, using different time intervals. It shows that as the interval becomes smaller, the convergence of steady state is more time-consuming. However, the smaller interval yields more accurate maximal current.

6 Bibliography

References

[1] https://www.bristol.ac.uk/physics/media/theory-theses/dwandaru-brams-thesis.pdf.

[2] Exact solution of a 1D asymmetric exclusion model using a matrix formulation. Derrida et al., J. Phys. A: Math Gen. 26 1493 (1993), G. Schutz et al., J. Stat. Phys. (1992).