CS 325 - Homework 7

1. *(7 pts)* Let X and Y be two decision problems. Suppose we know that X reduces to Y in polynomial time. Which of the following statements are true? Explain

   a. If Y is NP-complete then so is X.
   b. If X is NP-complete then so is Y.
   c. If Y is NP-complete and X is in NP then X is NP-complete.
   d. If X is NP-complete and Y is in NP then Y is NP-complete.
   e. If X is in P, then Y is in P.
   f. If Y is in P, then X is in P.
   g. X and Y can't both be in NP.

2. *(3 pts)* Two well-known NP-complete problems are 3-SAT and TSP, the Traveling Salesman Problem. The 2-SAT problem is a SAT variant in which each clause contains at most two literals. 2-SAT is known to have a polynomial-time algorithm. Are the following statements true or false? Justify your answer.

   a. 3-SAT $\leq_p$ TSP.

   b. If $P \neq NP$, then 3-SAT $\leq_p$ 2-SAT.

   c. If TSP $\leq_p$ 2-SAT, then $P = NP$.

3. *(10 pts)* A Hamiltonian path in a graph is a simple path that visits every vertex exactly once. Show that HAM-PATH = { (G, u, v): there is a Hamiltonian path from u to v in G } is NP-complete. You may use the fact that HAM-CYCLE is NP-complete.

3. *(10 pts)* K-COLOR. Given a graph G = (V,E), a k-coloring is a function c: V -> {1, 2, … , k} such that $c(u) \neq c(v)$ for every edge $(u,v) \in E$. In other words the number 1, 2, .., k represent the k colors and adjacent vertices must have different colors. The decision problem K-COLOR asks if a graph can be colored with at most K colors.

The 3-COLOR decision problem is NP-complete by using a reduction from SAT. Use the fact that 3-COLOR is NP-complete to prove that 4-COLOR is NP-complete.