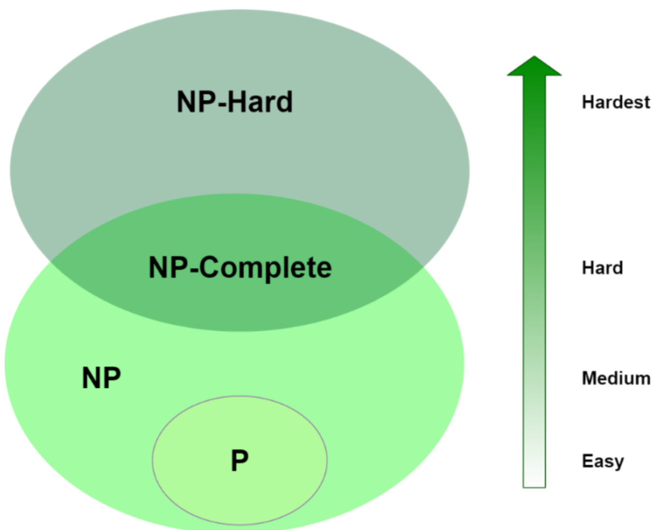
CS325

Guangyu Zhang

1. *(7 pts)* Let X and Y be two decision problems. Suppose we know that X reduces to Y in polynomial time. Which of the following statements are true? Explain

   a. If Y is NP-complete then so is X.
   b. If X is NP-complete then so is Y.
   c. If Y is NP-complete and X is in NP then X is NP-complete.
   d. If X is NP-complete and Y is in NP then Y is NP-complete.
   e. If X is in P, then Y is in P.
   f. If Y is in P, then X is in P.
   g. X and Y can't both be in NP.

Since X reduces to Y in polynomial time, X is subset of Y.



Ref: https://www.baeldung.com/cs/p-np-np-complete-np-hard

a).

False. If Y is NP-complete, X can be NP-complete or NP.

b).

False. If X is NP-complete, Y can be NP-complete or HP-hard.

c).

False. If Y is NP-complete, X in NP, X can be NP-complete or NP.

d).

True. If X is NP-complete, Y in NP, Y can only be NP-complete.

e).

False. If X in P, Y can be P or NP.

f).

True. If Y in P, X can only be P.

g).

False. X is subset of Y, so X, Y can both in NP.

2. *(3 pts)* Two well-known NP-complete problems are 3-SAT and TSP, the Traveling Salesman Problem. The 2-SAT problem is a SAT variant in which each clause contains at most two literals. 2-SAT is known to have a polynomial-time algorithm. Are the following statements true or false? Justify your answer.

a. 3-SAT $\leq_P$ TSP.

b. If P $\neq$ NP, then 3-SAT $\leq_P$ 2-SAT.

c. If TSP $\leq_P$ 2-SAT, then P = NP.

a).

True. Since 3-SAT and TSP are NP-complete problem. If 3-SAT can be solved in polynomial time, TSP can also be solved in polynomial time.

b).

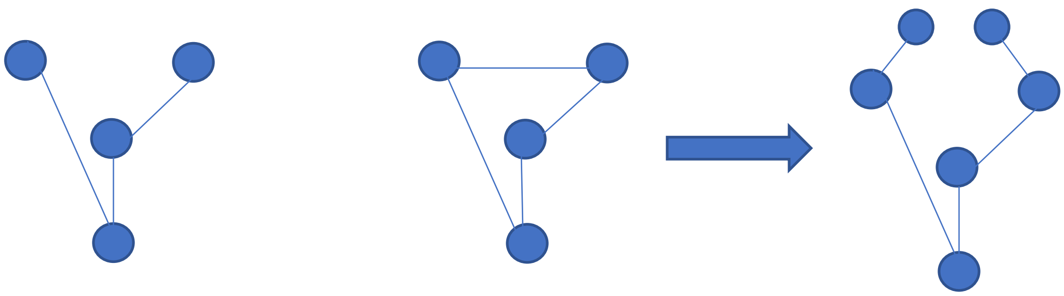
False. Since 3-SAT is NP-complete, and 2-SAT is in P, and P is not equal to NP, 3-SAT does not have a polynomial time.

c).

True. Since 2-SAT can be solved in polynomial time, and if TSP which is NP-complete can be solved in polynomial time, P=NP. Thus, proved.

3. *(10 pts)* A Hamiltonian path in a graph is a simple path that visits every vertex exactly once. Show that HAM-PATH = { (G, u, v): there is a Hamiltonian path from u to v in G } is NP-complete. You may use the fact that HAM-CYCLE is NP-complete.

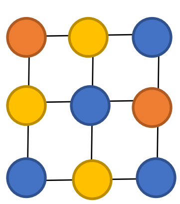
The graphs shows the HAM-PATH and HAM-CYCLE. Since HAM-PATH go through all vertices only once, it can be solved in polynomial time. So, HAM-PATH is in NP. Then, assume there is a point a that has an edge to the start point, and a point b that has an edge to the end point in HAM-CYCLE, the HAM-CYCLE can be reduced to HAM-PATH in polynomial time because it only adds some points and edges. Therefore, HAM-CYCLE is subset of HAM-PATH. And since HAM-CYCLE is NP-complete, and HAM-PATH is in NP, HAM-PATH is NP-complete.



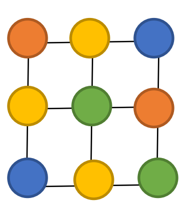
3. *(10 pts)* K-COLOR. Given a graph G = (V,E), a k-coloring is a function c: V -> {1, 2, ... , k} such that c(u) ≠ c(v) for every edge (u,v) ∈ E. In other words the number 1, 2, .., k represent the k colors and adjacent vertices must have different colors. The decision problem K-COLOR asks if a graph can be colored with at most K colors.

The 3-COLOR decision problem is NP-complete by using a reduction from SAT. Use the fact that 3-COLOR is NP-complete to prove that 4-COLOR is NP-complete.

Since 4-COLOR colors every vertex only once, it can be solved in polynomial time by changing some colors to satisfy 4-COLOR. So, 4-COLOR is in NP. Then, 3-COLOR can be reduced to 4-COLOR in polynomial time by changing some colors, here, assume we change half of blue point to green point. Therefore, since 4-COLOR reduces to 3-COLOR, 4-COLOR is subset of 3-COLOR, and 4-COLOR is in NP, 4-COLOR is NP-complete.



3-color



4-color