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Formation Control based on Flocking Algorithm in Multi-agent System*

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Abstract **- This paper mainly addresses flocking control algorithm to implement the "*boids*" model of Reynolds among the formation control by nonholonomic multi-robots. Firstly, distributed flocking of multiple autonomous agents with double integrator dynamics is studied. The proposed flocking algorithm is a gradient-based protocol combined with a velocity consensus protocol. For the gradient-based term, smooth artificial potential functions which could cope with the problem of shape generation are devised for all the agents. Moreover, the proposed coordinated protocols are applied to the formation control of a team of nonholonomic mobile robots. Finally, simulations and experiments show that the proposed controllers ensure the group formation is stabilized to a desired shape, while all the robots' velocities and directions converge to the same.**

Index Terms - Multi-agent system; Flocking; Artificial potential functions; Formation control; Nonholonomic mobile robots.

I. Introduction

Over decades the natural phenomena of flocking have invoked intensive research interests in diverse scientific and engineering fields, which can be observed in many different creatures such as flocks of migrate birds, schools of reef fish, etc [1], [2]. The collective group behaviours of these species are believed to have certain advantages over individual ones, for example, increasing the survival chances for the whole group under the danger from predators. Many research efforts in different areas, from biology to system and control, and from physics to computer science, have been tried to explain how these creatures form a collective group behavior without a global coordinator. Understanding those relative mechanisms and operational principles may provide useful ideas for developing formation control, distributed cooperative control and coordination of multiple mobile autonomous agents/robots.

Reynolds [1] introduced a computer model called *"boids"* that simulates the motion of bird flock or fish school in 1987. Each bird has a local control strategy, but the desirable group behavior is achieved. A similar model of Reynolds' was proposed in 1995 by Vicsek *et al* [2]. In this model, headings of each agent are updated as the average of the headings of agent itself with its nearest neighbours plus some additive noise. A rigorous proof of convergence for Vicsek's model was given by Jadbabaie *et al* [3] in 2003. They proved that the alignment strategy leads to the result that all the agents' headings converge to a common heading. Following the works above, Tanner *et al.* [4], [5], [6] considered a group of mobile agents moving in the plane with double integrator dynamics. In [4], [5], it is demonstrated that flocks are networks of dynamic systems with a dynamic topology. This topology is a proximity graph depends on the state of all agents and is determined locally for each agent.

Formation control of multi-agent systems has been studied extensively, because the efficient coordination can achieve better performance than a single agent. Many control approaches have been put forward to solve the problems in formation control, for example, leader-follower strategy [7], virtual structure approach [8] and behavior-based method [9]. There are many issues need to be considered when designing a distributed controller for mobile robot formation control, such as the stability of the formation, controllability of different formation patterns, safety and uncertainties in formations. In this paper, the stability is the major factor considered.

Formation control of multiple mobile vehicles is a typical application of multi-vehicle cooperation problems. To adapt the environment constraints, all vehicles maintain the pre-determined relationship to establish certain formation.

Applications of vehicle formation control are widely existed in the area of autonomous ground vehicles [10], [11] unmanned air vehicles [12], [13], autonomous underwater vehicles [14], [15], satellites [16], [17] and so on, and the formation control can carry out certain operations that the single one can't do.

The main contribution of this paper is to introduce the flocking algorithm into the formation control strategy, which is a gradient-based protocol combined with a velocity consensus protocol, and apply the algorithm to formation control problems by appropriately choosing the controllers which is based on smooth artificial potential functions. Finally, the simulations and experiments with the nonholonomic mobile robots show that the proposed algorithm could ensure the group formation be stabilized to a desired shape, while all the robots' velocities and directions converge to the same.

II. Problem Formulation

Consider a group of N agents moving in an n-dimensional Euclidean space; each has point mass dynamics described by

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$$\dot{x}_i = v_i$$
$$m_i \dot{v}_i = u_i \qquad i = 1, 2, ..., N \tag{1}$$

where $x_i \in R^n$ is the position vector of agent i, $v_i \in R^n$ is the velocity vector, $m_i > 0$ is the mass, and $u_i \in R^n$ is the control input acting on agent i. Our objective is to make the entire group move at a desired velocity and maintain constant distances between the agents. We consider the ideal case which ignores the velocity damping. In this case, we try to regulate agent velocities to the desired velocity, reduce the velocity differences between agents, and at the same time, regulate their distances such that their potentials become minima. Hence, we choose the control law u_i for agent i to be:

$$u_i = \alpha_i + \beta_i + \gamma_i \tag{2}$$

where α_i is used to regulate the potentials among agents, β_i is used to regulate the velocity of agent i to the weighted average of its flockmates and γ_i is used to regulate the momentum of agent i to the desired final momentum [18], [19].

α_i is derived from artificial potential field which is described by an artificial social potential function V_i, which is a function of the relative distances between agent i and its flockmates. Collision avoidance and cohesion in the group can be guaranteed by this term. β_i reflects the alignment or velocity matching with neighbors among agents. γ_i is designed to regulate the momentum of agent i based on the external signal (the desired velocity).

III. Flocking of Multiple Agents with a Virtual Leader

A. Preliminary

In this section, we investigate the stability properties of multiple mobile agents with point mass dynamics described in (1). Explicit control input in (2) will be presented for the terms α_i, β_i and γ_i. The control law acting on each agent is based on the neighbouring information topology. We will employ matrix analysis, algebraic graph theory and Lyapunov theory as basic tools for the discussion.

Assume that all the agents have the same communication radius R. In order to describe the information flow of the sensing network, the following definition is given below

Definition 3.1 (Neighbouring Information Graph) The neighbouring information graph $G(t) = \{V, E(t), W(t)\}$ is a time-varying undirected graph *consisting of a set of* N *vertices* $V = \{n_1, n_2, ..., n_N\}$, *indexed by the different agents of the group and a set of edges* $E(t) = \{(n_i, n_j) \mid \|x_{ij}(t)\| < R\}$, which represents the position neighbouring relations. $W(t) = [w_{ij}(t)]^{N \times N}$ is the weight matrix which consists of the coefficients of interaction between the agents, where w_{ij} is the weight of arc (n_i, n_j) [20], [21].

Let $N_i(t) \triangleq \{n_j \mid (n_i, n_j) \in E(t)\}$ denote all the communication neighbours of agent i. Since self loops are not allowed ($n_i \notin N_i(t)$).

Definition 3.2 (Potential Function) Potential V_{ij} is a continuous, differentiable and nonnegative function of the distance between agents i and j, such that $V_{ij}(\|x_{ij}\|) \to \infty$ as $x_{ij} \to 0$. V_{ij} attains its unique minimum when agents i and j are located at a desired distance, and V_{ij} is a constant for $\|x_{ij}\| \geq R$.

Functions $V_{ij}, i, j = 1, 2, ..., N$ *are the artificial potential functions that govern the mutual interactions. One example of such smooth potential functions is the following.*

$$V_{ij}(\|x_{ij}\|) = \begin{cases} a\ln\|x_{ij}\|^2 + \dfrac{b}{\|x_{ij}\|^2} & 0 < \|x_{ij}\| \leq \sqrt{\dfrac{b}{a}} \\ a\ln\|x_{ij}\|^2 + \dfrac{b}{\|x_{ij}\|^2} + \cos\left(1 + \dfrac{\|x_{ij}\|^2 - \frac{b}{a}}{R^2 - \frac{b}{a}}\right)\pi + 1 & \sqrt{\dfrac{b}{a}} < \|x_{ij}\| < R \\ a\ln\|x_{ij}\|^2 + \dfrac{b}{\|x_{ij}\|^2} + 2 & \textit{otherwise} \end{cases}$$

where a, b and R are positive constants such that $b > \frac{e}{a}$, and $R > \sqrt{\frac{b}{a}}$. Note that the potential function V_{ij} *is everywhere continuous differentiable in the domain (see Fig. 1), which could avoid the nonsmooth switching of the controllers brought by dynamically changing neighbouring relations due to the motion of agents.*

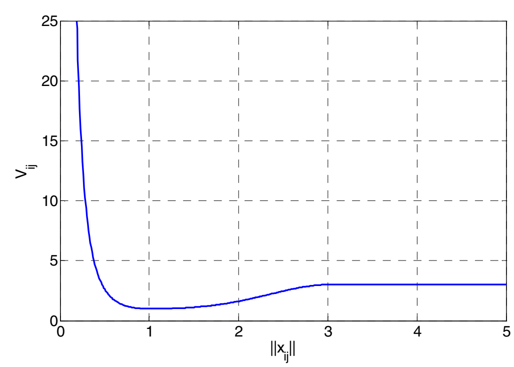

Fig.1 Smooth potential function V_{ij} for a=1, b=1

By the definition of V_{ij}, the total potential of agent i can be expressed as

$$V_i = \sum_{j \notin N_i} V_{ij}(R) + \sum_{j \in N_i} V_{ij}(\| x_{ij} \|) \quad (3)$$

Note that during the course of motion, each agent regulates its position and velocity based on the external signal and the state information of its neighbours. However, it is known that, in reality, because of the influence of some external factors, the reference signal is not always detected by all agents in the group. In this paper, the case where the signal is sent continuously at any time is considered and we assume that there exists at least one agent in the group who can detect it.

Before presenting the main results of this paper, it needs to analyze the eigenstructure of Laplacian matrix of $G \Leftrightarrow$ which is defined as follows

Definition 3.3 (Graph Laplacian) The graph Laplacian of $G(t)$ is defined by $L(G) = \Delta(t) - A(t)$,where $A(t) = [a_{ij}(t)]_{N \times N}$ is the adjacency matrix of $G(t)$ defined by $a_{ij}(t) = a_{ji}(t) = w_{ij}$ if $(v_i, v_j) \in E(G)$ and 0 otherwise; $\Delta(t) = [d_i(t)]_{N \times N}$ is the degree matrix of $G(t)$ which is a diagonal matrix with the *i*-th diagonal element being $d_i(t) = \sum_{j \in N_i(t)} a_{ij}(t)$.

It is known that $L(G)$ has several well-studied properties, which are listed below:

1) $L(G)$ is always symmetric and positive semi-definite.

2) For a connected graph $G(t)$, it has a single eigenvalue 0, the associated eigenvector is 1_N and null $(L(G))$ =span { 1_N }. Where null $(\cdot)$ denotes the null space and 1_N denotes the N -dimensional vector with all entries equal to one.

The explicit control protocol is in the following form

$$u_i = -\sum_{j \in N_i} \nabla_{x_i} V_{ij} - \sum_{j \in N_i} w_{ij}(v_i - v_j) + m_i(v_i - v_l) \quad (4)$$

where $v_l \in R^n$ is the desired common velocity and is a constant vector, $w_{ij} \geq 0, w_{ij} = w_{ji}$ and $w_{ii} = 0$, $i, j = 1, 2, ..., N$ represent the interaction coefficients.□Thus, the matrix W is symmetric, and by the connectivity of the neighbouring graph, W is irreducible.

B. Stability Analysis

Before presenting the main results of this paper, an important lemma is given below

Lemma 3.4 Let $A \in R^{n \times n}$ be any diagonal matrix with positive diagonal entries. Then $(A\text{span}\{1\}^{\perp}) \bigcap \text{span}\{1\}=0$, where $1 = (1, 1, ..., 1)^T$ and $span\{1\}^{\perp}$ denotes the orthogonal complement space of $span\{1\}$.

Proof Let $p \in Aspan\{1\}^{\perp} \bigcap span\{1\}$. Then $p \in span\{1\}$ and there exists a vector $q \in span\{1\}^{\perp}$ such that $p = Aq$. It follows that $q^T p = q^T Aq = 0$. Since A is positive definite by assumption, we have $q = 0$ and hence $p = 0$

Theorem 3.5 By taking the control law in (4), all agent velocities in the group described in (1) will asymptotically approach the same, collision avoidance is ensured between neighbouring agents, and the group final configuration minimizes all agent potentials.

Proof Define the following error vectors

$$\hat{x}_i = x_i - v_l t \quad (5)$$

$$\hat{v}_i = v_i - v_l \quad (6)$$

where t is the time variable and v_l is the desired constant common velocity. $\hat{x}_i$ represents the relative position vector between the actual position of agent i and its desired position; $\hat{v}_i$ represents the velocity difference vector between the actual velocity and the desired velocity of agent i . Hence, the error dynamics is given by below:

$$\begin{aligned} \dot{\hat{x}}_i &= \hat{v}_i \\ \dot{\hat{v}}_i &= \frac{u_i}{m_i} \quad i = 1, 2, ..., N \end{aligned} \quad (7)$$

By the definition of V_{ij} , it follows that

$$V_{ij}(\| x_{ij} \|) = V_{ij}(\| \hat{x}_{ij} \|) \triangleq \tilde{V}_{ij} \quad (8)$$

where $\hat{x}_{ij} = \hat{x}_i - \hat{x}_j$, and hence $\tilde{V}_i = V_i, \nabla_{\hat{x}_i} \tilde{V}_i = \nabla_{x_i} V_i$. Thus, the control input for agent i in the error system has the following form:

$$u_i = -\sum_{j \in N_i(t)} \nabla_{\hat{x}_{ij}} \tilde{V}_{ij}(\| \hat{x}_{ij} \|) - \sum_{j \in N_i(t)} w_{ij}(t)(\hat{v}_i - \hat{v}_j) + m_i \hat{v}_i \quad (9)$$

Consider the following positive semi-definite function

$$H = \frac{1}{2} \sum_{i=1}^{N} \left(\sum_{j \in N_i(t)} \tilde{V}_{ij}(\| \hat{x}_{ij} \|) + m \hat{v}_i^T \hat{v}_i \right) \quad (10)$$

It is easy to see that H is the sum of the total artificial potential energy and the total kinetic energy of all agents in the error system. Define the level set of H in the space of agent velocities and relative distances in the error system.

$$\Omega = \{(\hat{x}_{ij}, \hat{v}_i) \mid H \leq c\} \quad (11)$$

In what follows, we will prove that the set is compact. In fact, the set Ω is closed by continuity.□ Moreover, boundedness can be proved by connectivity. More specifically, from $H \leq c$, we have $\tilde{V}_{ij} \leq c$. Moreover, since the potential function□ V_{ij} is radically unbounded, $\tilde{V}_{ij}$ is also radically unbounded, and there is a positive constant l_{ij} such that $\| \hat{x}_{ij} \| \leq l_{ij}$. Denote $l_{\max} = \max_{j \in N_i} \{l_{ij}\}$. Since the neighbouring graph is connected, there must be a path connecting any two agents i and j, and its length does not exceed $N - 1$. Hence, we have $\| \hat{x}_{ij} \| \leq (N-1) l_{\max}$.□By similar analysis, we have $\hat{v}_i^T \hat{v}_i \leq \frac{2c}{m_i}$, thus $\| \hat{v}_i \| \leq \sqrt{\frac{2c}{m_i}}$. Hence, Ω is compact. Therefore by LaSalle□s invariant principle, if the initial solutions of the system lie in Ω, their trajectories will converge to the largest invariant set inside the region of $\bar{\Omega} = \{v \mid \dot{H} = 0\}$. Then taking the time derivative of H, we have

$$\begin{aligned}\dot{H} &= \sum_{i=1}^{N} \hat{v}_i^T \cdot \Big(\sum_{j\in N_i(t)} \nabla_{\hat{x}_i} \tilde{V}_{ij}\Big) + \sum_{i=1}^{N} m_i \hat{v}_i^T \dot{\hat{v}}_i \\ &= \sum_{i=1}^{N} \hat{v}_i^T \Big(\sum_{j\in N_i(t)} \nabla_{\hat{x}_i} \tilde{V}_{ij}\Big) - \sum_{i=1}^{N} \hat{v}_i^T \Big(\sum_{j\in N_i(t)} \nabla_{\hat{x}_i} \tilde{V}_{ij}\Big) \\ &\quad - \sum_{j\in N_i(t)} \hat{v}_i^T w_{ij}(t)(\hat{v}_i - \hat{v}_j) - \sum_{i=1}^{N} m_i \hat{v}_i^T \hat{v}_i \\ &= -\hat{v}^T (L(t)\otimes I_n)\hat{v} - \hat{v}^T (M \otimes I_n)\hat{v}\end{aligned} \tag{12}$$

where $v = (v_1, v_2, ..., v_N)^T$ is the stack vector of system velocity and $\otimes$ denotes the Kronecker product. $M = diag(m_1, m_2, ..., m_N)$. By the positive-definiteness of $L(t)$ and M, $\dot{H} = 0$ implies that $\hat{v}_1 = \hat{v}_2 = ... = \hat{v}_N$. This occurs only $v_1 = v_2 = ... = v_l$, hence $v_1, v_2, ..., v_N \in span\{1\}$. Meanwhile, note that at this time, the agent velocity dynamics becomes:

$$\dot{\hat{v}}_i = \frac{u_i}{m_i} = \frac{-\sum_{j\in N_i(t)} \nabla_{\hat{x}_{ij}} \tilde{V}_{ij}(\| \hat{x}_{ij} \|)}{m_i} \tag{13}$$

and therefore it follows that

$$\dot{\hat{v}} = (-M^{-1}B \otimes I_n)\begin{bmatrix} \vdots \\ \nabla_{\hat{x}_{ij}} \tilde{V}_{ij}(\| \hat{x}_{ij} \|) \\ \vdots \end{bmatrix} \tag{14}$$

Where $M^{-1} = diag(m_1^{-1}, m_2^{-1}, ..., m_N^{-1})$ and matrix B is the incidence matrix of the neighbouring graph. Hence

$$\dot{\hat{v}}_{i_k} = -(M^{-1}B)[\nabla_{\hat{x}_{ij}} \tilde{V}_{ij}(\| \hat{x}_{ij} \|)]_k, k = 1, 2, .., n \tag{15}$$

Thus $\dot{\hat{v}}_{i_k} \in range(M^{-1}B), k = 1, 2, ..., n$. By matrix theory and by the connectivity of the neighbouring graph G, we have

$$range(M^{-1}B) = M^{-1} range(B) = M^{-1} range(BB^T) = M^{-1} span\{1\}^{\perp} \tag{16}$$

Therefore, by lemma 3.4

$$\dot{\hat{v}}_{i_k} \in span\{1\} \cap (M^{-1} span\{1\})^{\perp} = \{0\}, k = 1, 2, ..., n \tag{17}$$

Thus, in steady state, all agent velocities in the error system no longer change and equal zero, and moreover, from (17), the potential $\tilde{V}_i$ of each agent i is minimized. Collision avoidance can be ensured between neighbouring agents since otherwise it will result in $\tilde{V}_i \to \infty$.

IV. Simulations and Experiments

In this section, various kinds of simulations and experiments which are on the basis of the flocking algorithm are conducted to validate the stability, convergence and robustness of the algorithm. The simulation is based on MobileSim.

In simulations and experiments, the robots keep the distance $d = 1.5m$ between each other, and they form the stabilized formation of quasi α lattice [18], [19]. In (3) and Fig. 1, $a = 1$, $b = 2.25$. In our experiments, a 54 Mbps wireless router of NETGEAR WGR614 is utilized, and the effective communication range is 500 meters, so $R = 10.0m$ chosen here is possible. However, in our simulations and experiments, the distances between robots are no more than 10*m.* The admissible errors of distances and headings are 0.15 meters and 3 degrees respectively.

A. Simulations

At the beginning, the robots on one line are waiting for the command to start at the same time. See Fig. 2.

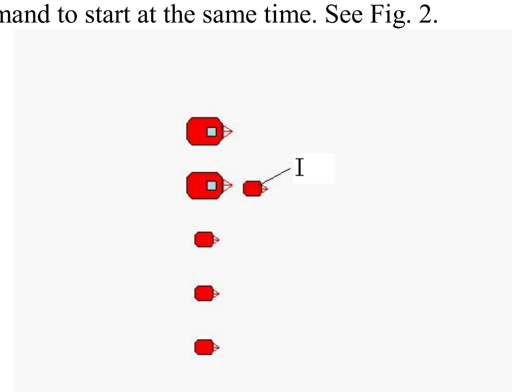

Fig.2 Robots at the beginning in simulations

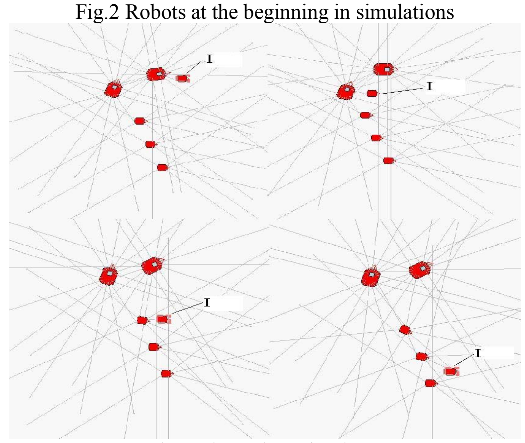

Fig.3 The action of Robot I

In Fig. 2, the bigger agent represents robot Pioneer 3-At, and the smaller represents robot Amigo in Experiments.

Fig. 4 shows that with the controller (4) designed by the flocking algorithm in section 2 the robots diverse and then their velocities and directions converge to the same. When the simulation begins, robot I in Fig. 2 moves to anywhere to disturb the formation, and it is illustrated in Fig. 3. Then we will see their positions in Picture A. When the distances between Robot I and other robots are less than 1.5*m*, the repulsive forces make them go away and then the robots are in new positions and directions. Robot I will leave the simulation after its assignment, so it no longer exists in Fig.4. (3) tells us that each robot regulates its position and velocity from Robot I and partners.

Picture A and B show that all the robots are communicating with each other for the destination to form a new formation. At this time, the attractive force is in dominant. Picture C suggests that robots have assembled and maintain the distance $d = 1.5m$ while they have different velocities. Picture D explains that all the robots are moving in the same velocities as well as the fixed distance.

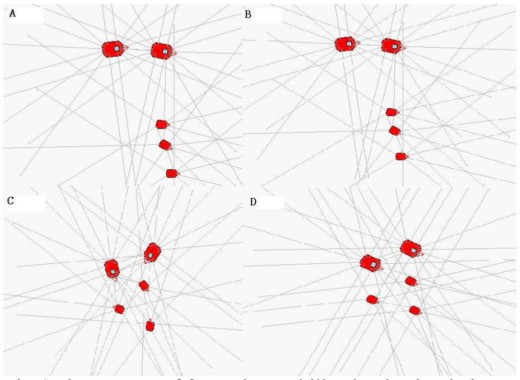

Fig.4 The process of formation stabilization in simulations

B. Experiments


Fig.5 Robots at the beginning in experiments

Fig.6 The process of formation stabilization in experiments

In Fig. 5 and Fig. 6, we see that there is not a robot similar to robot I in Fig. 2 and 3. Pictures A, B, C and D depict the same scenarios as those in simulations.

Picture A depicts that all the robots have the new distances and they are wondering for a little while. During this time, the robots are computing the distance between them. Then the smooth artificial potential functions V_{ij} will play an important role here. In general, the distance between two robots is more than *d*, so the attractive forces are in dominates at this moment.

That the robots are trying to maintain the distance of 1.5*m* is described in Picture B.

In picture C, all the robots' velocities in the group will asymptotically approach the same and be in the appropriate positions at last. The attractive or repulsive forces will make the robots more stabilized in *quasi* α-*lattices with a little alignment until they are in the same velocities as shown in Picture D.*

On the basis of the simulations and experiments above, the effectiveness of the flocking algorithm, and especially the gradient-based protocol and velocity consensus protocol are illustrated.

IV. Conclusion

This paper proposes a new method for formation control based on the flocking algorithm. The new smooth artificial potential functions and controllers are proposed to cope with the problem of shape generation. On the basis of the Lyapunov stability analysis, the multi-agent system mentioned in Section 2 is proved to be stabilized. Finally, the applications to the formation control in simulations and experiments with a team of nonholonomic mobile robots are given to demonstrate the effectiveness of our strategies.

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