Phase space coordinate translation between PTC, MAD-X and COSY Infinity 9.1

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The coordinates in the 6D phase space of COSY Infinity 9.1 are defined as follows.

$r_1 = x$,
$r_2 = p_x/p_0$,
$r_3 = y$,
$r_4 = p_y/p_0$,
$r_5 = l = -(t - t_0)v_0\gamma/(1 + \gamma)$,
$r_6 = \delta_k = (K - K_0)/K_0$,

where p_0, K_0, v_0, t_0 and γ are the momentum, kinetic energy, velocity, time of flight, and total energy over m_0c^2, respectively, of the reference particle. The six variables form three canoically conjungate pairs.

The coordintes of MAD-X are defined as follows.

$X = x$,
$PX = p_x/p_0$,
$Y = y$,
$PY = p_y/p_0$,
$T = -c \cdot \delta t$,
$PT = \delta E/cp_0$,

where δt and ΔE are the time difference and energy difference with respect to the reference particle. $\delta t = t - t_0$ and $\delta E = K - K_0$.

PTC uses the following coordinates: (PTC can use other coordinates. But we assume it uses the following ones for our convenience.)

$X1 = x$,
$X2 = p_x/p_0$,
$X3 = y$,
$X4 = p_y/p_0$,
$X5 = \delta E/cp_0$,
$X6 = c \cdot \delta t$

Comparing the above three group of coordinates, the first four are exactly the same. But the last two coordinates are defined defferently. Using the relation $\beta = v_0/c = \sqrt{1 - 1/\gamma^2}$, one can find the relation between r_5 and T.

$$r_5 = -\delta t \frac{v_0\gamma}{1+\gamma} = \frac{T}{c}\frac{v_0\gamma}{1+\gamma} = T\frac{\beta\gamma}{1+\gamma} = T\sqrt{\frac{\gamma-1}{\gamma+1}}.$$

Finding the relation between r_6 and PT is also straightforward. Using

$$K_0 = cp_0\sqrt{\frac{\gamma-1}{\gamma+1}},$$

We get

$$r_6 = \frac{\delta E}{K_0} = \frac{\delta E}{cp_0}\sqrt{\frac{\gamma+1}{\gamma-1}} = PT\sqrt{\frac{\gamma+1}{\gamma-1}}.$$

The above relations allow us to convert from MAD-X coordinates to COSY Infnity coordinates. It is easy to see the relation between PTC coordinates and MAD-X coordinates is:

$$X5 = PT, \quad X6 = -T.$$

And the relation between COSY Infinity 9.x is

$$r_5 = -X6\sqrt{\frac{\gamma-1}{\gamma+1}}, \quad r_6 = X5\sqrt{\frac{\gamma+1}{\gamma-1}}.$$

It is good to see the translation is linear, which means the trancated map using one group of coordinates can be converted into a map using the other group of coordinates without introducing errors.