



2025年湍流与噪声和CFD方法暑期高级讲习班

基于间断反馈因子的任意高阶有限 体积格式

Mach 10 rocket by CGKS

Strive for originality

报告人：姬兴

西安交通大学 航天航空学院

Jul 9 2025



刘宏宇 (HKUST)

张红

王逸骁 (HKUST)

穆俊磊

高行健

张悦 (HKUST)

赵丰祥博士 (HKUST)

徐昆教授 (HKUST)

...

目录



1

研究背景

2

基于间断反馈因子的有限体积格式

3

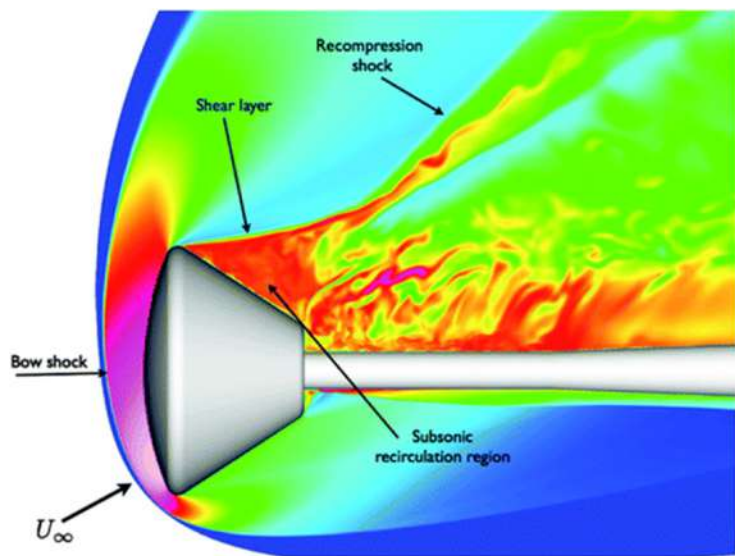
典型场景验证及应用



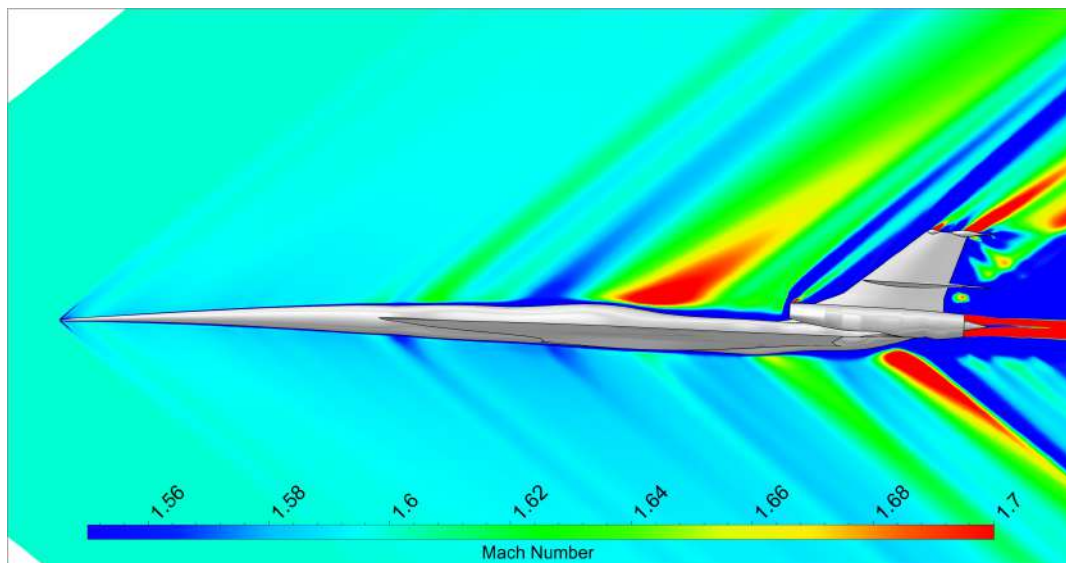
1

研究背景

含间断流体仿真属于战略前沿领域的基础科学问题



高超声速再入飞行器湍流仿真



超声速飞行器气动噪声仿真

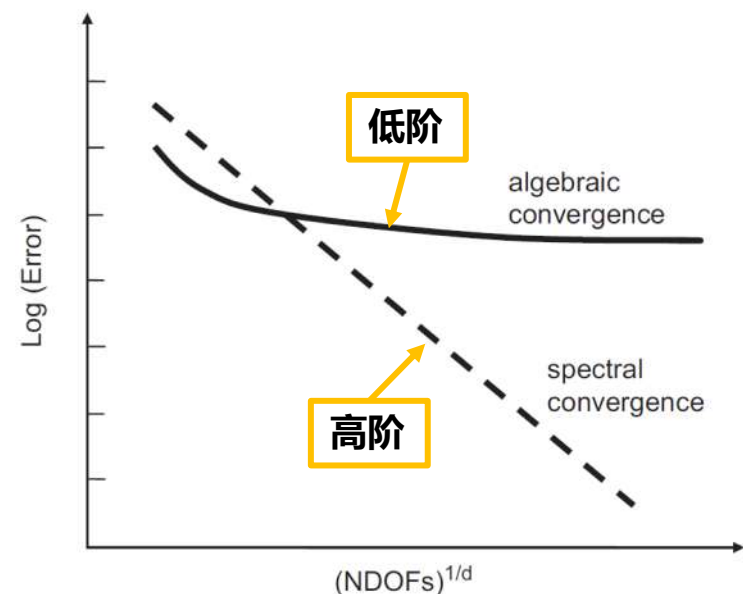
对应计算流体力学中的关键问题

1. 格式如何高精度模拟小尺度非定常流动结构？
2. 格式如何保证间断捕捉的鲁棒性？

1.研究背景

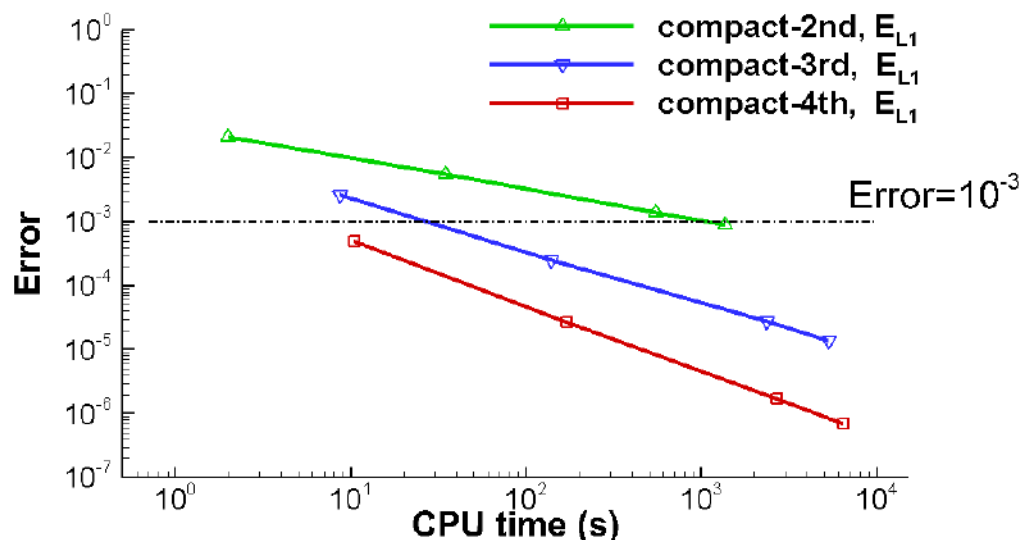
高阶格式具备对流体精细结构高效捕捉的显著潜力

高阶格式（大于2阶精度）具有**高分辨率**、**高效率**等优势，能够更准确地解析**湍流**、**噪声**等流动中的复杂结构和流动分离现象



随着计算自由度增长，相同计算时间高阶格式的误差逐渐小于低阶格式

[ZJ Wang, 2007]



相同计算时间，不同阶数紧致气体动力学格式在三维正弦波输运计算中的实际误差表现

[FX ZHAO et al., 2023]

随着阶数提升，高阶格式中精度和鲁棒性的矛盾越来越突出

Very High-order
Scheme
(极高阶格式 ?)

→ FDM

1. **WENO9** (I Vallet et. al. J. Comput. Phys. 2009)
2. **WENO9-AO**(CW Shu et. al. J. Comput. Phys.2016)
3. **TENO10-AA**(L Fu. Comput. Methods Appl. Mech. Eng. 2021)
4. **WCNS9**(S Zhang. J. Comput. Phys. 2008)

→ FVM

1. **MOOD** (S Diot et. al. Int. J. Numer. Meth. Fluids. 2013)
2. **8th order CGKS**(K Xu et. al. Adv. Aerodyn. 2019)
3. **KFVM-WENO**(D Lee et. al. Astrophys. J. 2024)

→ DG/FR

1. **ADER-DG** (Isan S. Popov. Int. J. Sci. Comput. 2024)
2. **High-order FR/CPR**(ZJ Wang et. al.)



2

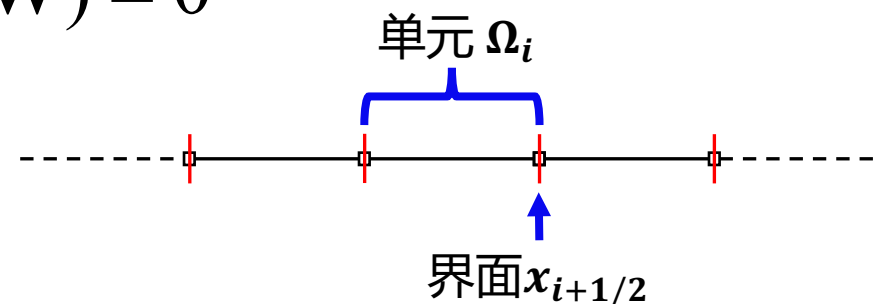
基于间断反馈因子的 有限体积格式

双曲守恒律求解有限体积框架

1维Euler方程为例

双曲守恒律：
$$\mathbf{W}_t + \nabla \cdot \mathbf{F}(\mathbf{W}) = 0$$

以封闭单元
体为基本离
散自由度

$$\frac{\partial}{\partial t} \int_{\Omega} \mathbf{W} d\Omega + \int_{\partial\Omega} (\mathbf{F}(\mathbf{W})) dS = 0$$


利用散度定理可得

$$\frac{d\bar{\mathbf{W}}_i}{dt} = -\frac{1}{\Delta x} \left(\boxed{\mathbf{F}_{i+1/2}(\mathbf{W})} - \mathbf{F}_{i-1/2}(\mathbf{W}) \right) := L(\mathbf{W})$$

Runge-Kutta
时间推进

$$\begin{aligned} \bar{\mathbf{W}}_i^{(0)} &= \bar{\mathbf{W}}_i^n \\ \bar{\mathbf{W}}_i^{(k)} &= \bar{\mathbf{W}}_i^n + \sum \alpha_k \Delta t L(\bar{\mathbf{W}}^{(k-1)}), \quad k=1, \dots, s \\ \bar{\mathbf{W}}_i^{n+1} &= \bar{\mathbf{W}}_i^{(s)} \end{aligned}$$

三阶RK
案例

$$\begin{aligned} \bar{\mathbf{W}}_i^{(1)} &= \bar{\mathbf{W}}_i^n + \Delta t L(\bar{\mathbf{W}}^n) \\ \bar{\mathbf{W}}_i^{(2)} &= \bar{\mathbf{W}}_i^n + \frac{1}{4} \Delta t (L(\bar{\mathbf{W}}^n) + L(\bar{\mathbf{W}}^{(1)})) \\ \bar{\mathbf{W}}_i^{n+1} &= \bar{\mathbf{W}}_i^n + \frac{1}{6} \Delta t L(\bar{\mathbf{W}}^n) + \frac{1}{6} \Delta t L(\bar{\mathbf{W}}^{(1)}) + \frac{2}{3} \Delta t L(\bar{\mathbf{W}}^{(2)}) \end{aligned}$$

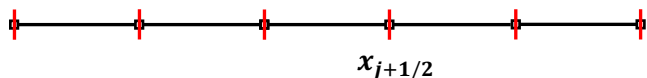
$$\mathbf{F}_{i+\frac{1}{2}}(\mathbf{W}) = ? \longrightarrow \mathbf{F}_{i+\frac{1}{2}}(\mathbf{W}) \approx \mathbf{F}(\boxed{\mathbf{W}_{i+\frac{1}{2}}^l, \mathbf{W}_{i+\frac{1}{2}}^r}) \longrightarrow \mathbf{W}_{i+\frac{1}{2}}^{l,r} = R(\dots, \bar{\mathbf{W}}_i, \bar{\mathbf{W}}_{i+1}, \dots)$$

黎曼求解器

重构

高阶WENO类型非线性重构

5th-order WENO-AO

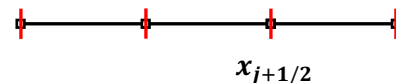


重构模板

$$S_0 = \{I_{i-2}, I_{i-1}, I_i\}, S_1 = \{I_{i-1}, I_i, I_{i+1}\}, S_2 = \{I_i, I_{i+1}, I_{i+2}\}$$

$$S_3 = \{S_0, S_1, S_2\}$$

5th-order HWENO-AO



重构模板

$$S_0 = \{I_{i-1}, I_i\}, S_1 = \{I_i, I_{i+1}\}, S_2 = \{I_{i-1}, I_i, I_{i+1}\}$$

$$S_3 = \{S_0, S_1, S_2\}.$$

强激波、强稀疏波、强剪切层往往失效！
是高阶格式工业化应用的长期瓶颈！

$$\frac{1}{\Delta x} \int_{I_{i-j-k-1}} p_k^{r3}(x) dx = \bar{Q}_{i-j-k-1}, j = -1, 0, 1,$$

$$p_0^{r3}(x_{i+1/2}) = \frac{1}{3} \bar{Q}_{i-2} - \frac{7}{6} \bar{Q}_{i-1} + \frac{11}{6} \bar{Q}_i,$$

$$p_1^{r3}(x_{i+1/2}) = -\frac{1}{6} \bar{Q}_{i-1} + \frac{5}{6} \bar{Q}_i + \frac{1}{3} \bar{Q}_{i+1},$$

$$p_2^{r3}(x_{i+1/2}) = \frac{1}{3} \bar{Q}_i + \frac{5}{6} \bar{Q}_{i+1} - \frac{1}{6} \bar{Q}_{i+2}.$$

$$p_3^{r5}(x_{i+1/2}) = \frac{1}{60} (47 \bar{Q}_i - 13 \bar{Q}_{i-1} + 2 \bar{Q}_{i-2} + 27 \bar{Q}_{i+1} - 3 \bar{Q}_{i+2}).$$

$$\frac{1}{\Delta x} \int_{I_{i+j}} p_3^{r5}(x) dx = \bar{Q}_{i+j}, j = -1, 0, 1,$$

$$\frac{1}{\Delta x} \int_{I_{i+j}} (p_3^{r5})_x(x) dx = (\bar{Q}_x)_{i+j}, j = -1, 0, 1.$$

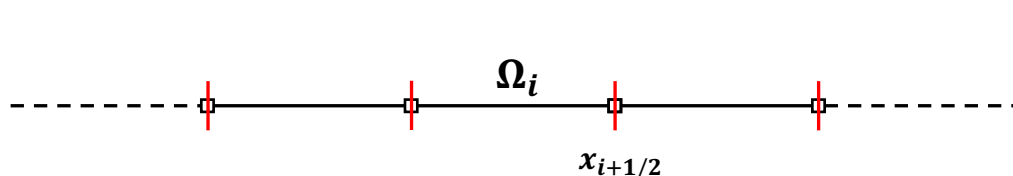
$$\frac{1}{\Delta x} \int_{I_{i+j}} p_3^{r5}(x) dx = \bar{Q}_{i+j}, j = -1, 0, 1,$$

$$\frac{1}{\Delta x} \int_{I_{i+j}} (p_3^{r5})_x(x) dx = (\bar{Q}_x)_{i+j}, j = -1, 0, 1.$$

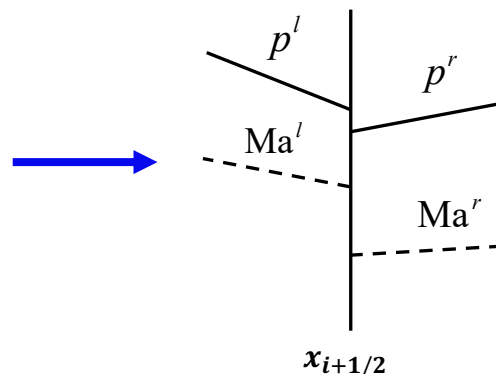
$$P^{AO(5,3)}(x) = \bar{\omega}_3 \left(\frac{1}{\gamma_3} p_3^{r5}(x) - \sum_0^2 \frac{\gamma_k}{\gamma_3} p_k^{r3}(x) \right) + \sum_0^2 \bar{\omega}_k p_k^{r3}(x).$$

界面间断反馈因子的定义

一维情况, 界面 $x_{i+1/2}$ 处间断强度记为标量 $\sigma_{i+1/2}$ $\sigma_{i+1/2} \rightarrow \begin{cases} 0, & \text{当界面处初值光滑} \\ +\infty, & \text{当界面处初值间断} \end{cases}$

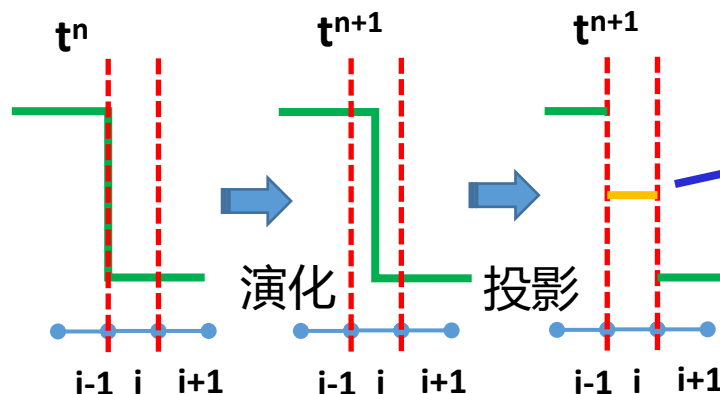


$$\sigma_{i+1/2,m} = \frac{|p^l - p^r|}{p^l} + \frac{|p^l - p^r|}{p^r} + (Ma^l - Ma^r)^2$$



DF 是单元界面的间断强度一种表征

依据CFL数稳定性条件：在 $n+1$ 时刻，间断面运动至左/右两侧的单元中



间断落在单元内部，
如何正确重构？

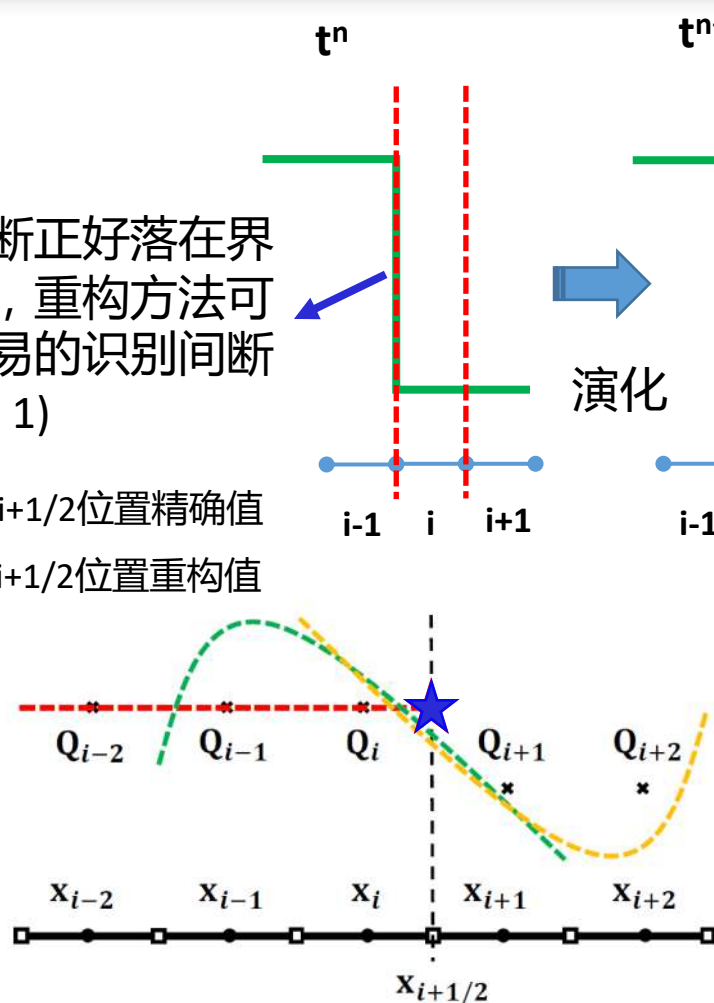
2. 基于间断反馈因子的有限体积格式

仅基于n时刻单元数据的重构方法无法对单元内间断准确限制

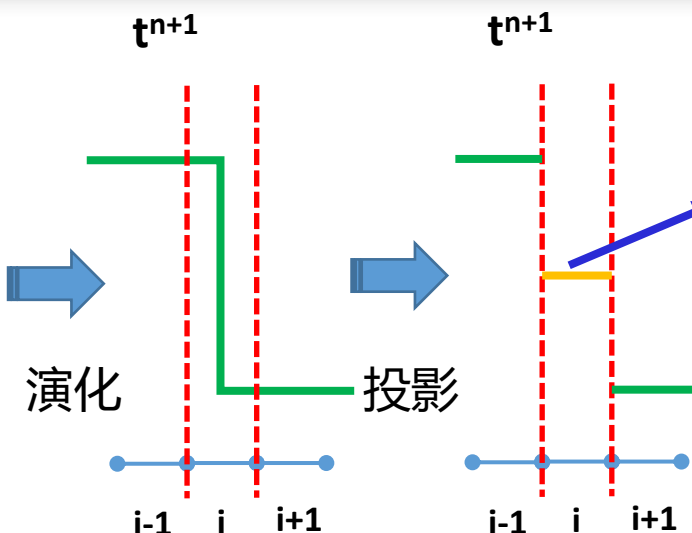
当间断正好落在界面时，重构方法可以轻易的识别间断 (案例 1)

★ 代表 $i+1/2$ 位置精确值

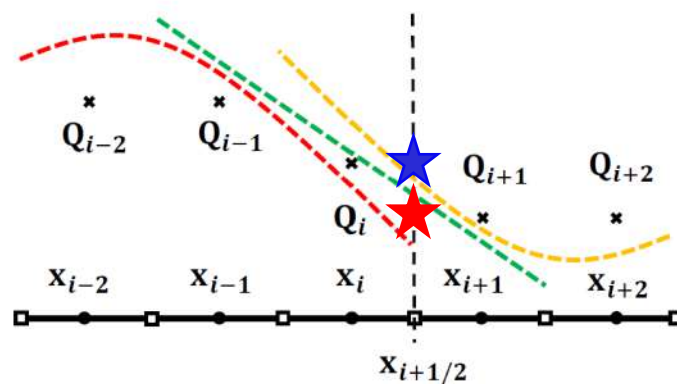
★ 代表 $i+1/2$ 位置重构值



案例 1 t^n



基于这个单元平均值的重构办法都会失效 (案例 2)



案例 2 t^{n+1}

怎么办？

注意到：虽然 $n+1$ 时刻单元内部间断无法知道，但 n 时刻间断位于界面时候重构是有效的

流体基本特性：

1. 对于我们关注的双曲守恒律问题， n 时刻界面上间断，会演化出向界面左右两侧的多个间断
2. 依据CFL数稳定性条件：在 $n+1$ 时刻是间断面运动至左右两侧的单元中
3. 特例：对于1维Euler方程来说， n 时刻的界面间断会演化出激波、稀疏波、接触间断等

利用 n 时刻界面间断强度信息，反馈作用于 $n+1$ 时刻重构

定义： **$n+1$ 时刻模板间断反馈因子 α_S** ：重构模版中所有界面的 n 时刻间断强度 σ 的度量

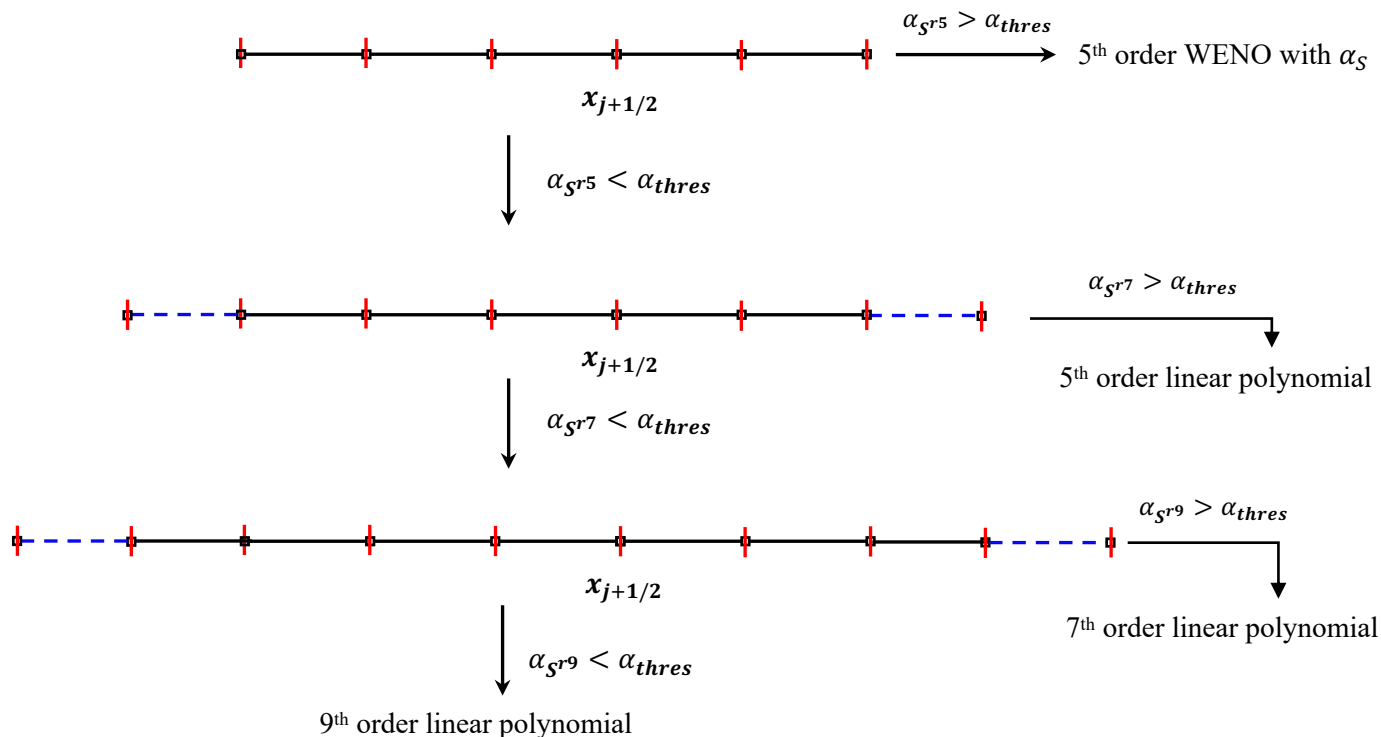
$$A = \cdots + \sigma_{i-3/2,j} + \sigma_{i-1/2,j} + \sigma_{i+1/2,j} + \sigma_{i+3/2,j} + \cdots,$$

$$\alpha_S = \begin{cases} 1.0 & \text{if } A < \sigma_{thres}, \\ \frac{\sigma_{thres}}{\cdots + \sigma_{i-3/2,j} + \sigma_{i-1/2,j} + \sigma_{i+1/2,j} + \sigma_{i+3/2,j} + \cdots} & \text{otherwise.} \end{cases}$$

- A ：重构模板的总间断强度
- σ_{thres} ：间断强度阈值参数（默认1.0）
- $\alpha_S \rightarrow 1$ ：判断为光滑模板
- $\alpha_S \rightarrow 0$ ：判断为强间断模板

间断反馈因子与非线性重构的结合

非紧致模板 S 的DF值记为 $\alpha_S(W:)$



DF修正重构多项式

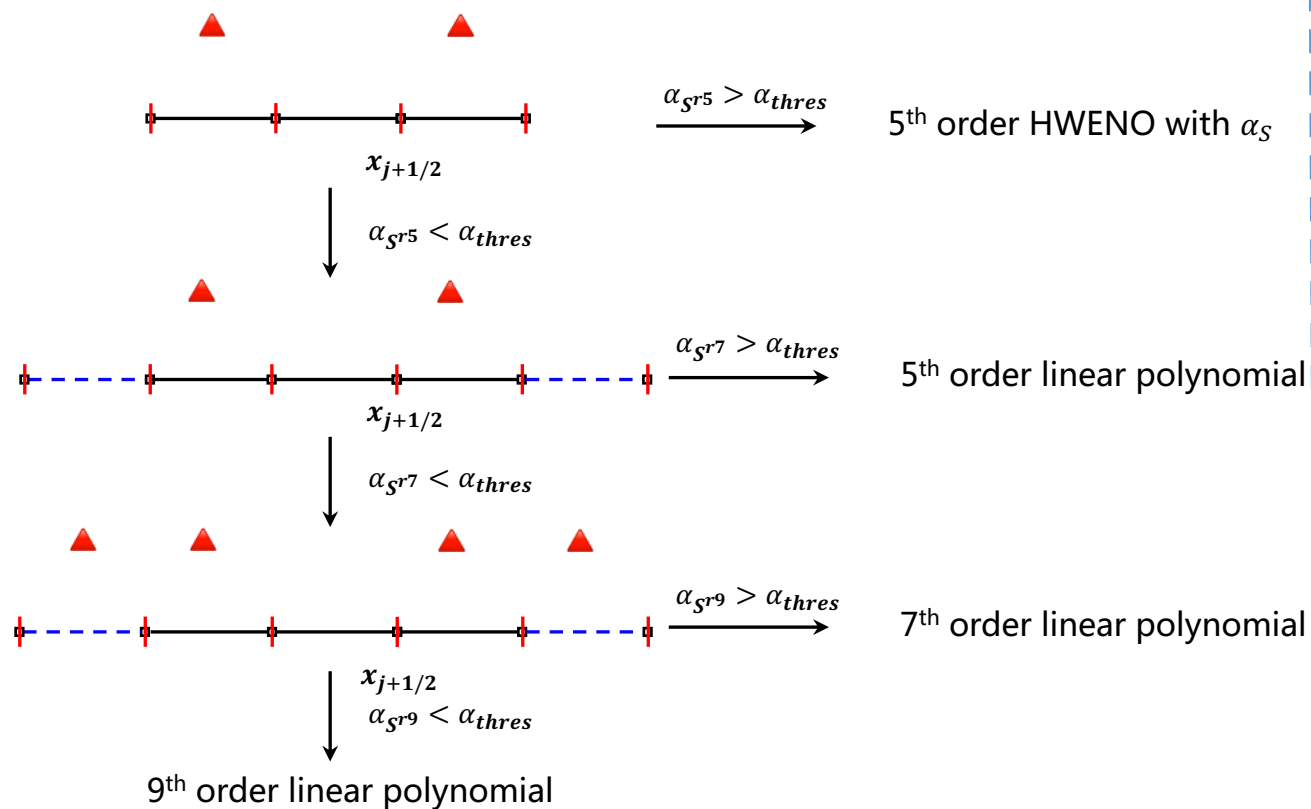
$$P^{DF} = W_i + \alpha_S (P^r(x) - W_i), P_x^{DF} = \alpha_S P_x^r(x)$$

$$W_{i+1/2}^l = P^{DF}(x_{i+1/2})$$

2.基于间断反馈因子的有限体积格式

间断反馈因子与非线性重构的结合

紧致模板 S 的DF值记为 $\alpha_S(W; W_x; \blacktriangle)$



DF修正重构多项式 $P^{DF} = W_i + \alpha_S (P^r(x) - W_i)$, $P_x^{DF} = \alpha_S P_x^r(x)$, $P_{xx}^{DF} = \alpha_S P_{xx}^r(x)$

$$W_{i+1/2}^l = P^{DF}(x_{i+1/2}), (W_x^l)_{i+1/2} = P_x^{DF}(x_{i+1/2}), (W_{xx}^l)_{i+1/2} = P_{xx}^{DF}(x_{i+1/2})$$

Compact Gas-kinetic scheme

$$f_t + \mathbf{u} \cdot \nabla f = \frac{g - f}{\tau}$$

$$W(x, t) = \int \psi f(x, t, \mathbf{u}, \xi) d\Xi$$

$$\mathbf{F}(x, t) = \int \mathbf{u} \psi f(x, t, \mathbf{u}, \xi) d\Xi$$

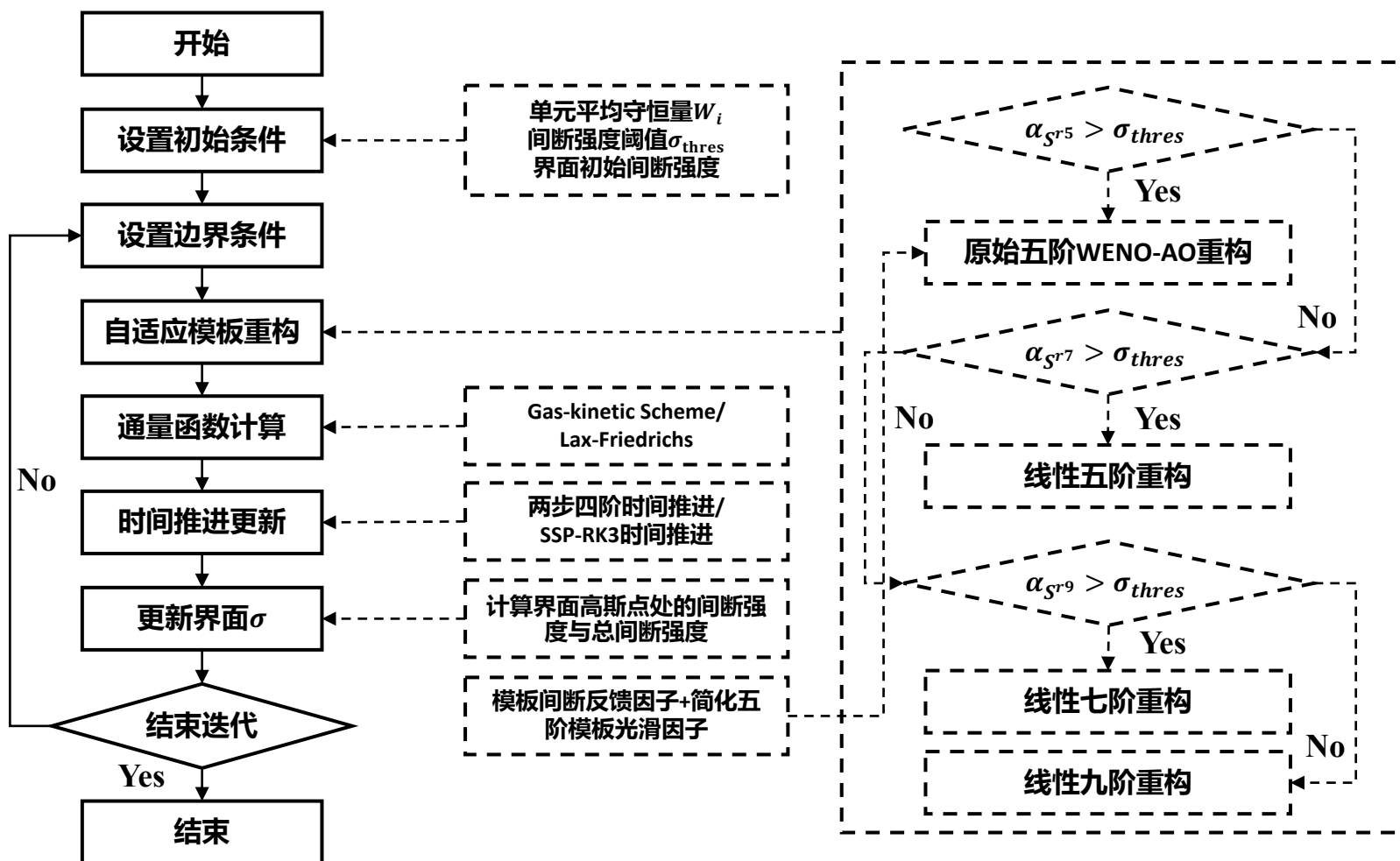
$$\mathbf{F}(x, t) = \int \mathbf{u} \psi f(x, t, \mathbf{u}, \xi) d\Xi$$

$$W(x, t) = \int \psi f(x, t, \mathbf{u}, \xi) d\Xi$$

$$\frac{dW_i}{dt} = -\frac{1}{\Delta x} (\mathbf{F}_{i+1/2}(t) - \mathbf{F}_{i-1/2}(t))$$

$$(W_x)_i = \frac{1}{\Delta x} \int_{I_i} \frac{\partial W}{\partial x} dx = \frac{1}{\Delta x} (W_{i+1/2} - W_{i-1/2})$$

基于间断反馈因子的算法流程

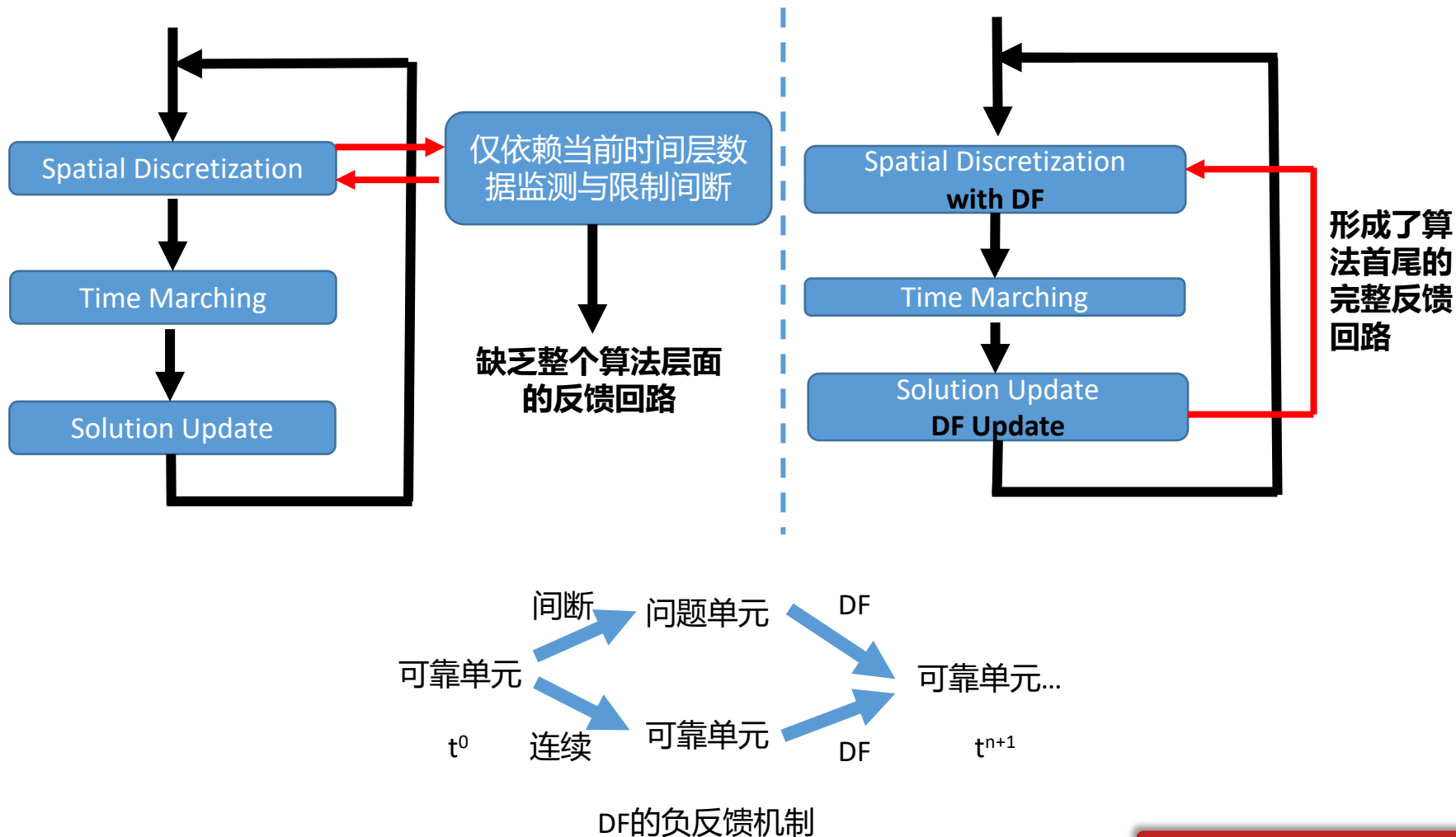


间断反馈因子的优势

1. 负反馈调节机制使得自由参数调节不敏感
2. 物理意义明确
3. 精度保证且易做到更高阶
4. 计算量小
5. 存储量小

间断反馈因子的优势

1. 负反馈调节机制使得自由参数调节不敏感



间断反馈因子的优势

2. 物理意义明确

$$\sigma_{i+1/2,m} = \underbrace{\frac{|p^l - p^r|}{p^l}}_{\text{激波}} + \underbrace{\frac{|p^l - p^r|}{p^r}}_{\text{稀疏波}} + \underbrace{\left(\text{Ma}_n^l - \text{Ma}_n^r\right)^2}_{\text{剪切层}} + \underbrace{\left(\text{Ma}_\tau^l - \text{Ma}_\tau^r\right)^2}_{\text{剪切层}}$$

间断反馈因子的优势

3. 精度保证且易做到更高阶

$$\sigma_{i+1/2,m} = \boxed{\frac{|p^l - p^r|}{p^l}} + \frac{|p^l - p^r|}{p^r} + \left(\text{Ma}_n^l - \text{Ma}_n^r\right)^2 + \left(\text{Ma}_\tau^l - \text{Ma}_\tau^r\right)^2$$



记压力真值为 p , 重构阶数为 k

$$\frac{|p^l - p^r|}{p^l} = \frac{|p + O(\Delta x^k) - (p + O(\Delta x^k))|}{p + O(\Delta x^k)} = O(\Delta x^{k+1})$$

自动满足任意 k 阶重构精度要求！

间断反馈因子的优势

4. 计算量小

5. 存储量小

每个单元/界面仅增加1个数据

计算复杂度分析（假设 k 阶重构）

1. 多项式计算复杂度：最终的重构多项式是对每个Stencil点的非线性组合，子Stencil的计算复杂度为 $O(k)$ ，多项式个数为 $O(k)$ ，总计算复杂度为 $O(k^2)$
2. 平滑指标计算复杂度：每个子模板的多项式是 $(k+1)/2$ 阶，对其求导并平方，最多涉及 $(k-1)/2$ 次导数的计算，自模板为 $O(k)$ 个，**总计算复杂度为 $O(k^3)$**

WENO-Z

1. 多项式计算复杂度：最终的重构多项式是对每个Stencil点的非线性组合，子Stencil的计算复杂度为 $O(k)$ ，多项式个数为 $O(k)$ ，总计算复杂度为 $O(k^2)$
2. 平滑指标计算复杂度：每个子模板的多项式是 $(m+1)/2$ 阶, $m=2,3,\dots,k$ ，对其求导并平方，最多涉及 $(m-1)/2$ 次导数的计算，自模板为 $O(k)$ 个，**总计算复杂度为 $O(k^3)$**

Multi-resolution WENO

1. DF计算复杂度：DF在不断增加阶数过程中需要持续计算，计算结果可以继承，但涉及到除法，**复杂度 $O(k)$**
2. 无平滑因子计算

DF-WENO



3

典型场景验证及应用

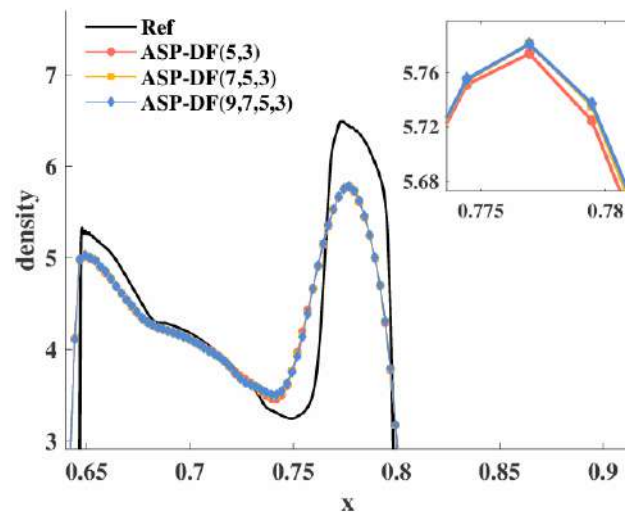
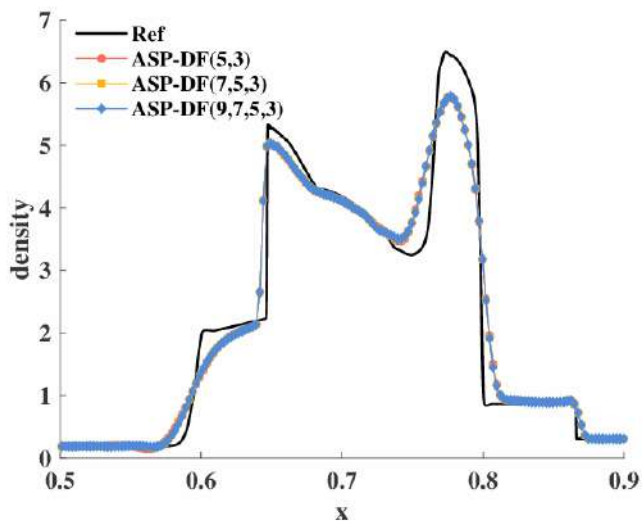
3. 典型场景验证及应用

一维任意高阶有限体积格式算例验证

Accuracy test

N	ASP-DF(5,3)	order	ASP-DF(7,5,3)	order	ASP-DF(9,7,5,3)	order
20	2.585904e-05		6.366042e-07		1.424192e-08	
40	8.337553e-07	4.95	5.108345e-09	6.96	2.848478e-11	8.97
80	2.680325e-08	4.95	4.042055e-11	6.98	9.085371e-14	8.29
160	8.585536e-10	4.96	3.343673e-13	6.92	2.544534e-16	8.48

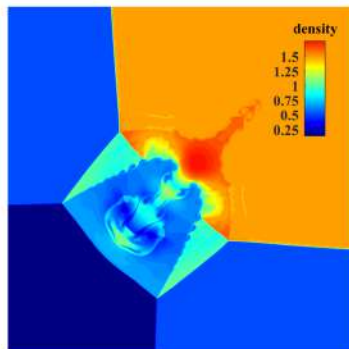
Blast wave problem



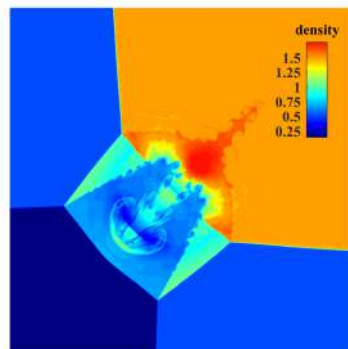
3. 典型场景验证及应用

二维任意高阶有限体积格式算例验证

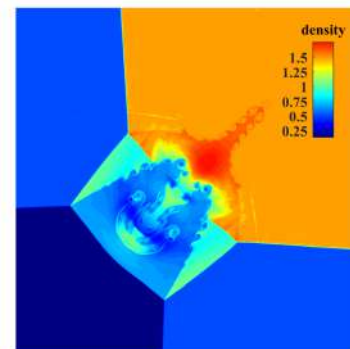
Two-dimensional Riemann problem



(a) ASE-DF(5,3)

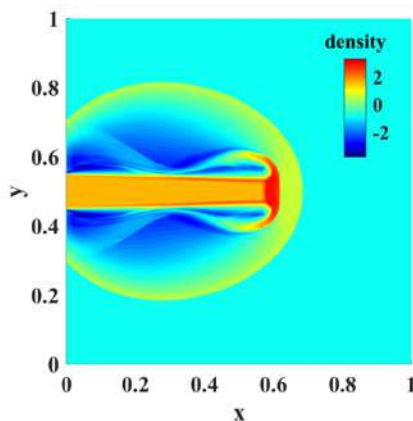


(b) ASE-DF(7,5,3)

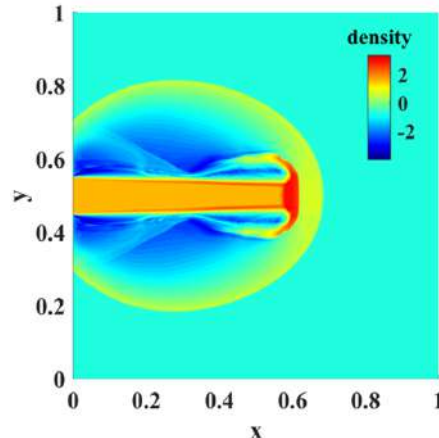


(c) ASE-DF(9,7,5,3)

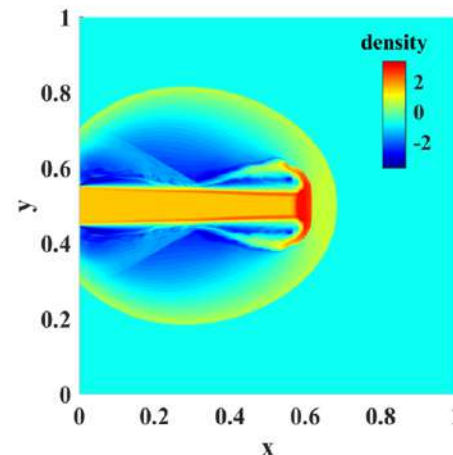
Ma = 20000 astrophysical jet



(a) ASE-DF(5,3)



(b) ASE-DF(7,5,3)

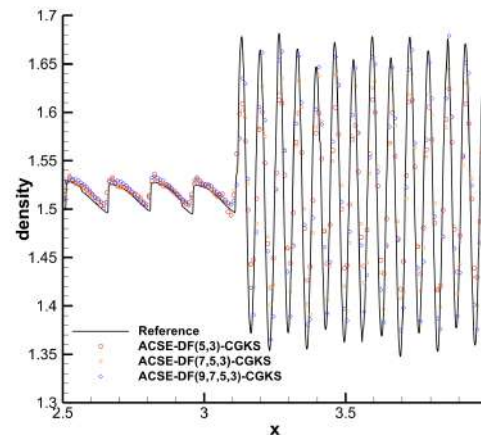
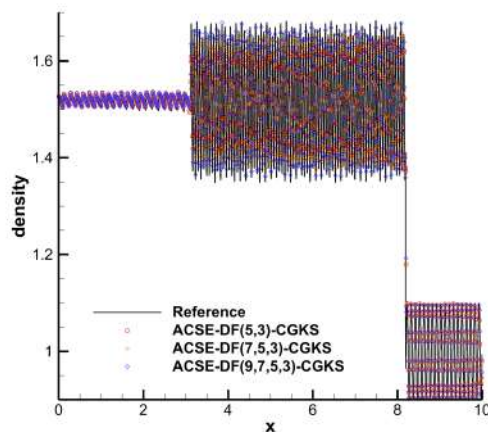


(c) ASE-DF(9,7,5,3)

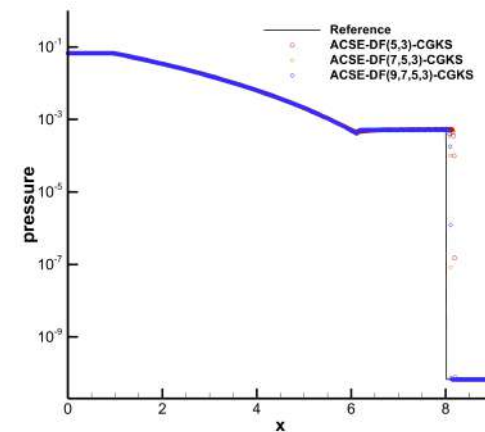
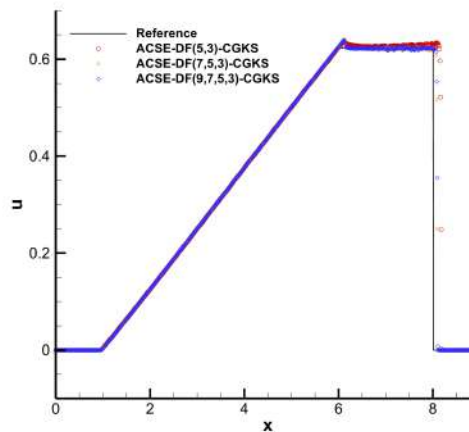
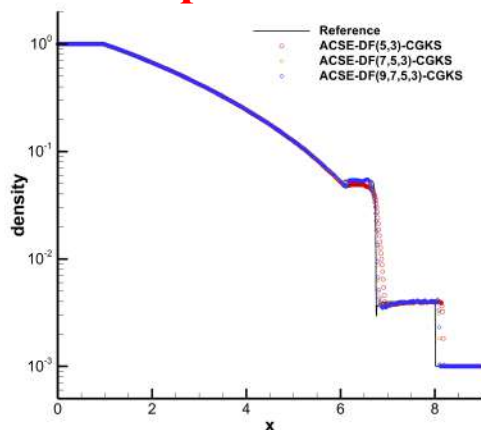
3. 典型场景验证及应用

一维任意紧致高阶气体动理学格式算例验证

Titarev-Toro problem



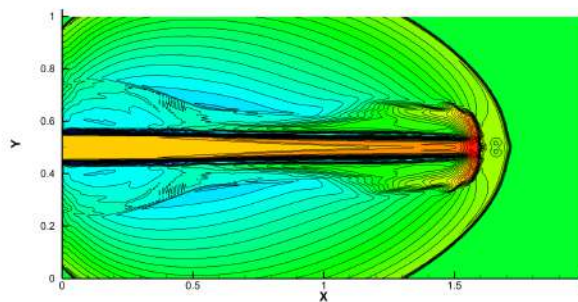
Le Blanc problem



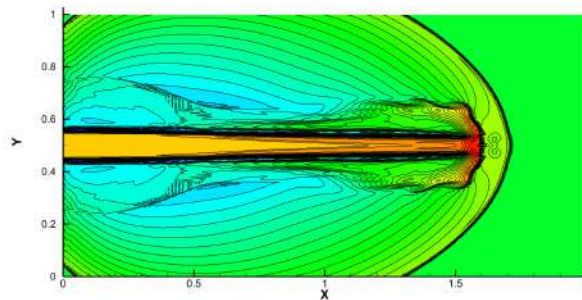
3. 典型场景验证及应用

二维任意紧致高阶气体动力学格式算例验证

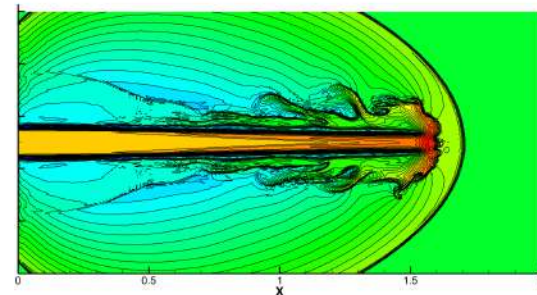
High Mach number astrophysical jet(Ma=80)



(a) ACSE-DF(5,3)-CGKS

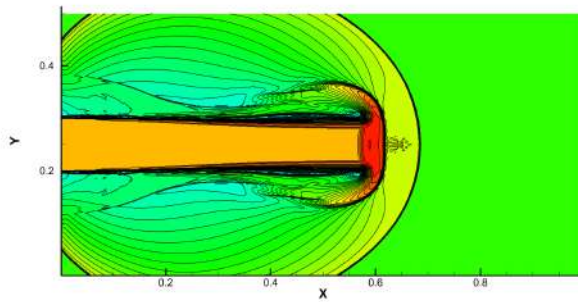


(b) ACSE-DF(7,5,3)-CGKS

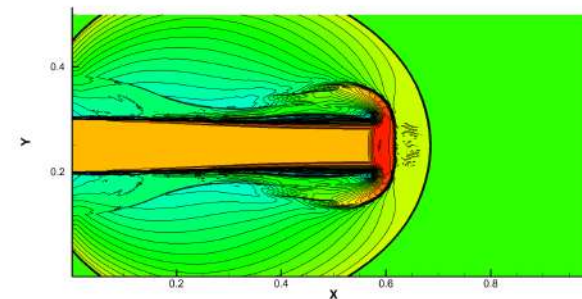


(c) ASE-DF(9,7,5,3)-CGKS

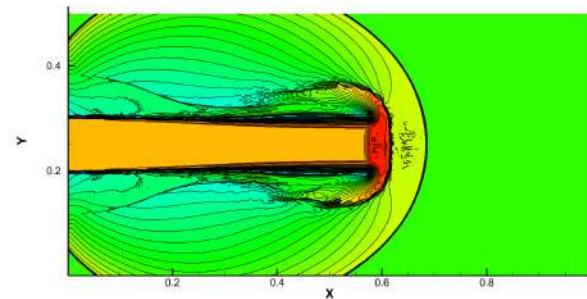
High Mach number astrophysical jet(Ma=2000)



(a) ACSE-DF(5,3)-CGKS



(b) ACSE-DF(7,5,3)-CGKS



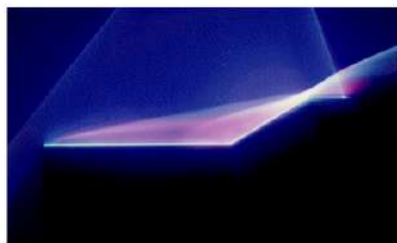
(c) ASE-DF(9,7,5,3)-CGKS

3. 典型场景验证及应用

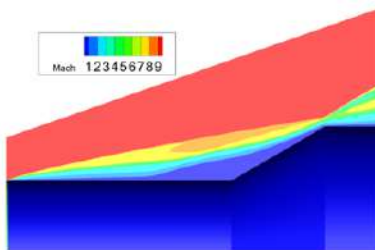
三维非结构网格高阶计算

高超声速空心圆柱裙

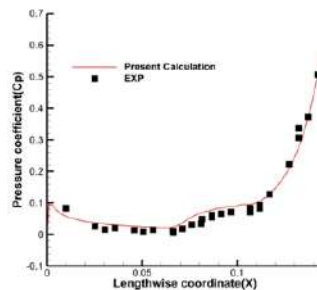
Ma	$T_\infty (k)$	$P_\infty (Pa)$	$L(m)$	$T_w (k)$	$\rho (kg/m^3)$	$Re_x (/m)$	Re
9.91	51	6.32	0.1017	293	0.43×10^{-3}	1.86×10^5	1.891×10^4



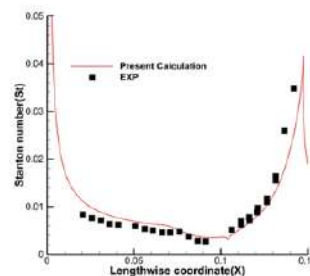
地面试验设置



局部流场马赫数云图



表面压力分布



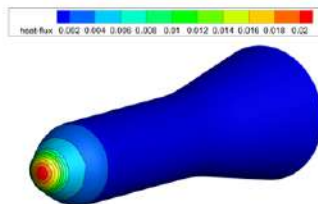
表面热流分布

高超声速弹道导弹模型 HB-2

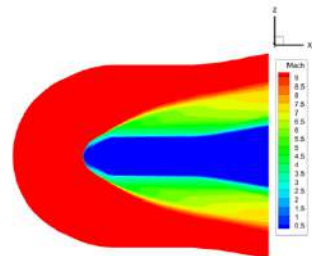
Ma	$\alpha(^{\circ})$	$\beta(^{\circ})$	$Re_x (/m)$	$T_\infty (k)$	$P_\infty (Pa)$
9.46	0	0	9.4×10^5	50.4	32.8



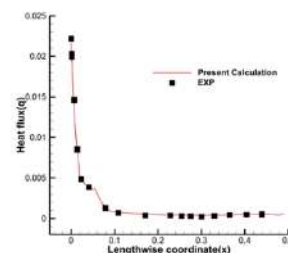
地面试验设置



热流量云图



二维界面马赫数云图



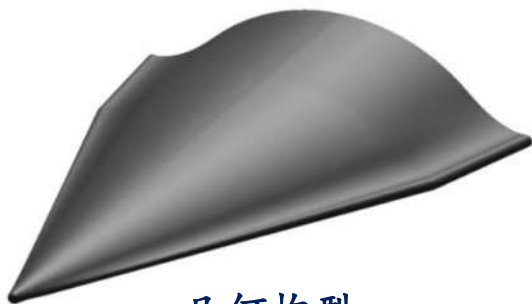
表面热流分布对比

3. 典型场景验证及应用

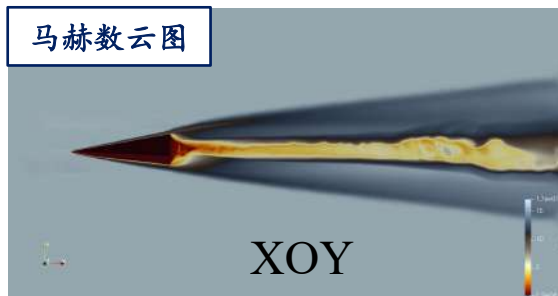
三维非结构网格高阶计算

HTV-2高超声速底部流动

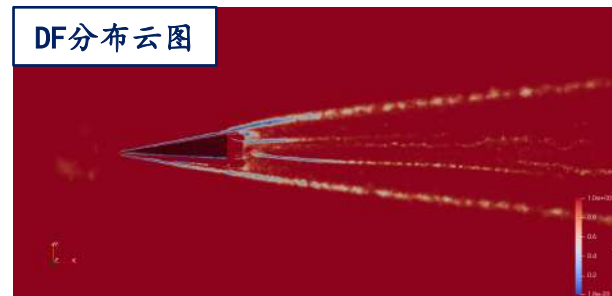
$$Ma = 16.38, Re = 8.7 \times 10^5$$



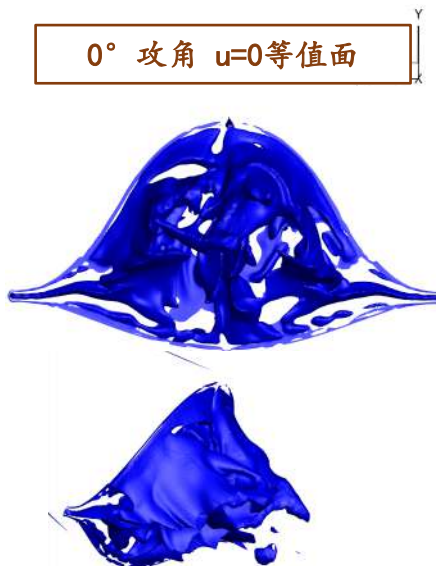
马赫数云图



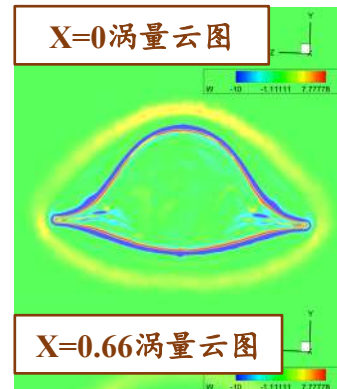
DF分布云图



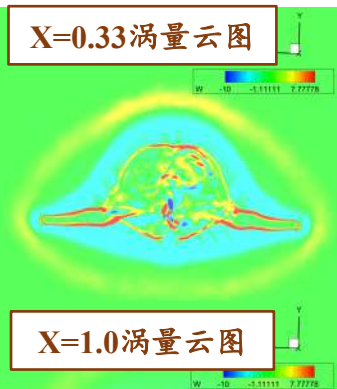
0° 攻角 $u=0$ 等值面



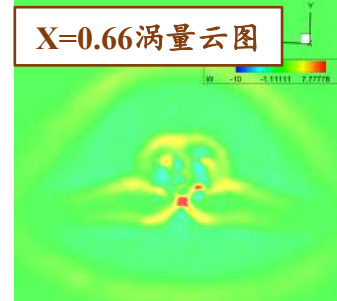
X=0 涡量云图



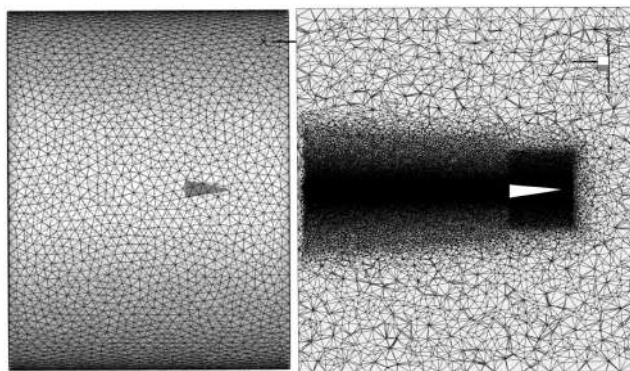
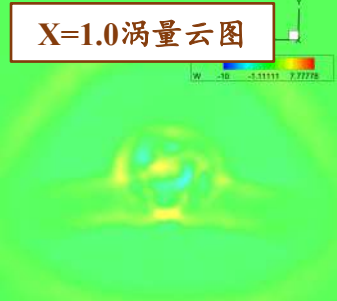
X=0.33 涡量云图



X=0.66 涡量云图



X=1.0 涡量云图

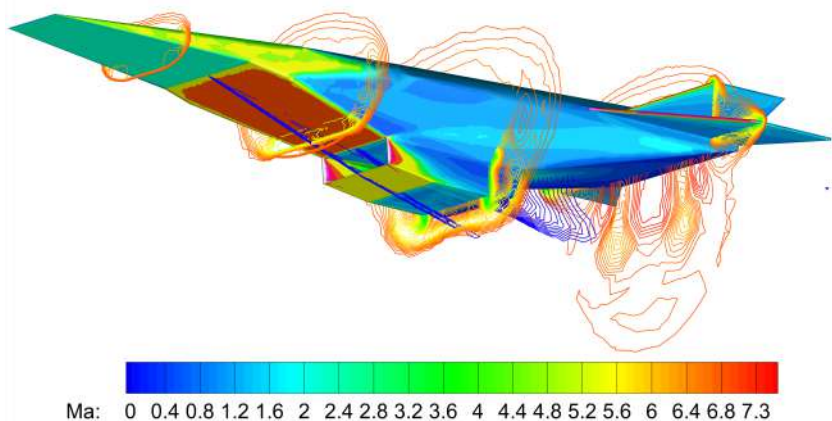


全局及局部网格拓扑

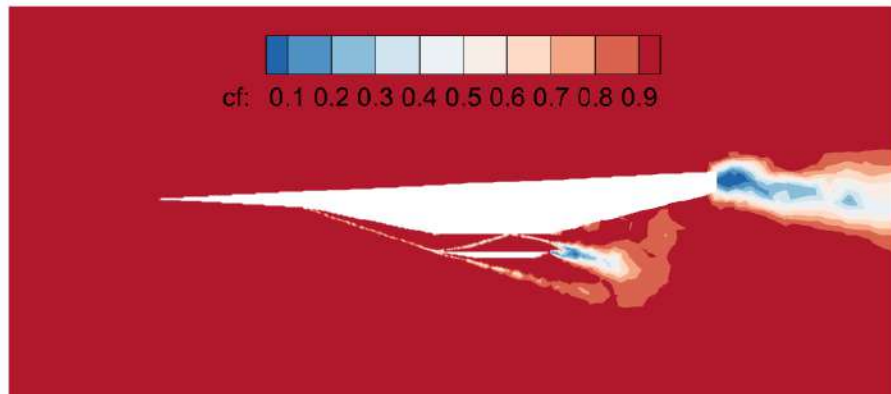
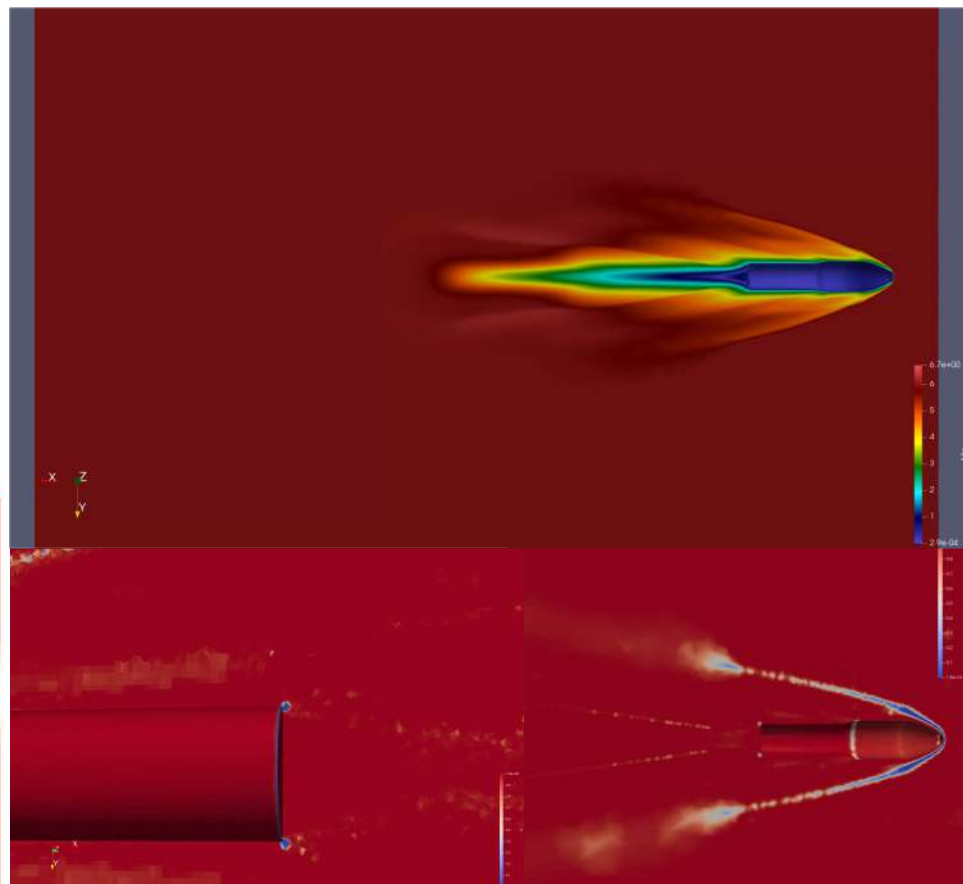
3. 典型场景验证及应用

三维非结构网格高阶计算

高超声速X-43算例



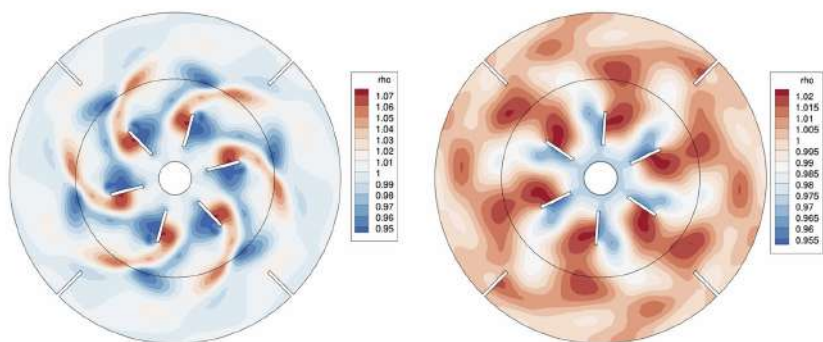
高超声速整流罩算例



3. 典型场景验证及应用

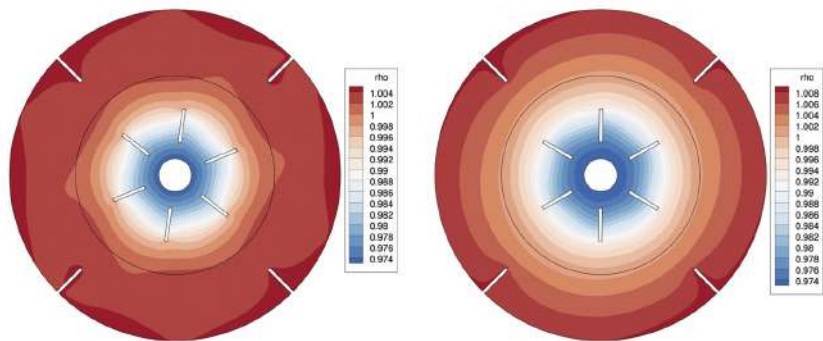
高阶滑移网格计算

层流搅拌罐



(a)

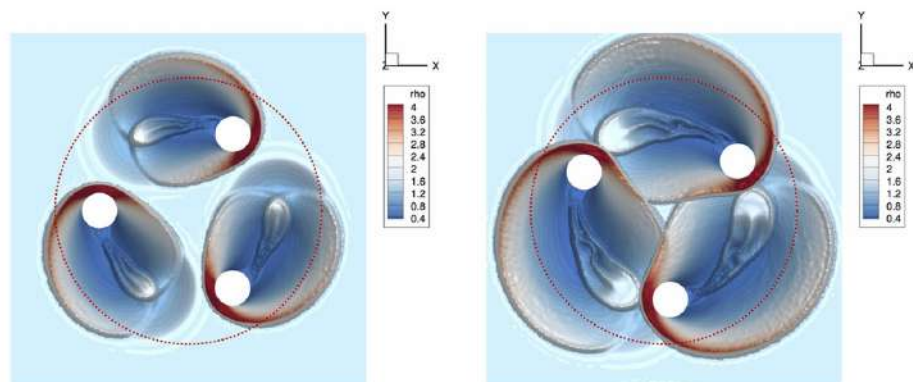
(b)



(c)

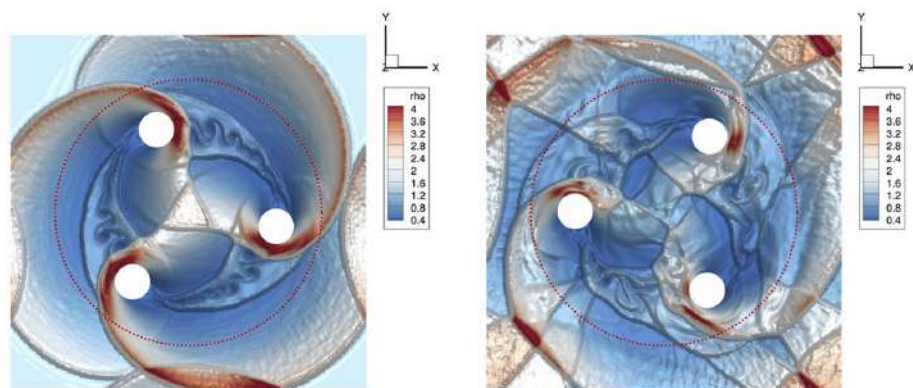
(d)

超声速三旋转圆柱



(a)

(b)



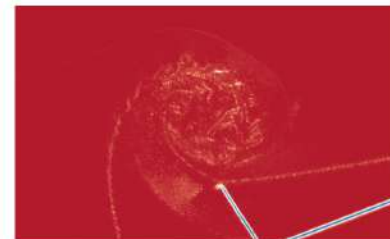
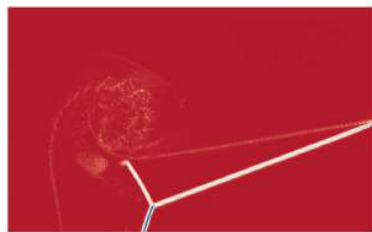
(c)

(d)

3. 典型场景验证及应用

高阶两组分流动仿真

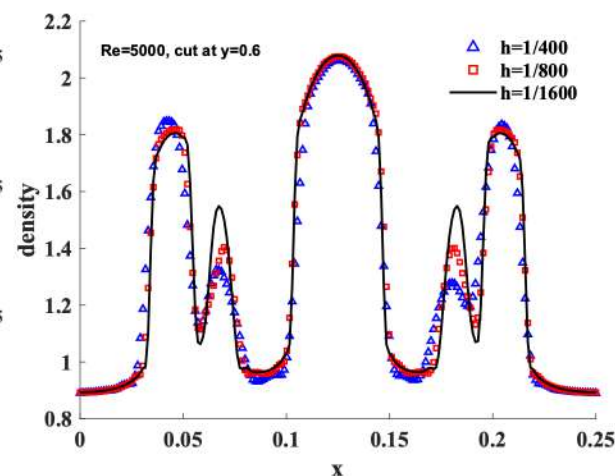
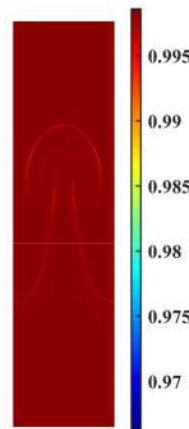
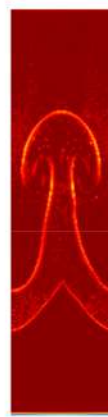
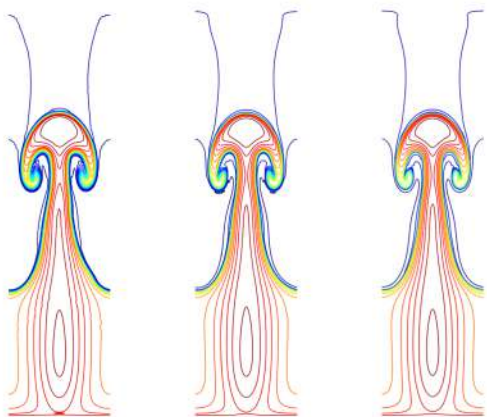
Two-component Triple-Point Problem



(a) $t=3.5s$. Left: total density distribution. Right: df factor distribution

(b) $t=5.0s$. Left: total density distribution. Right: df factor distribution

Two-component Rayleigh-Taylor problem



(a) $h = \frac{1}{400}, \frac{1}{800}, \frac{1}{1600}$ from left to right

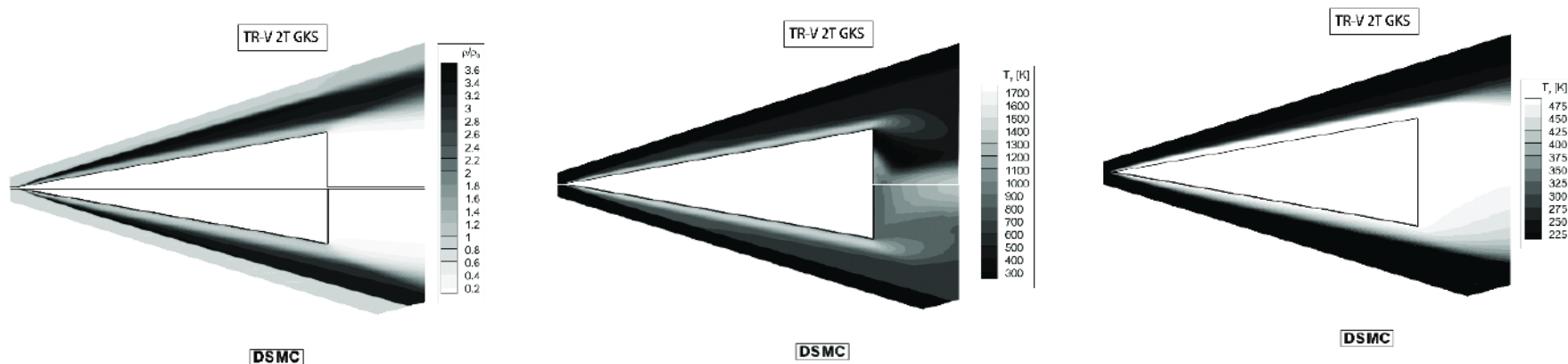
(b) DF factor distribution

(c) Mesh refinement

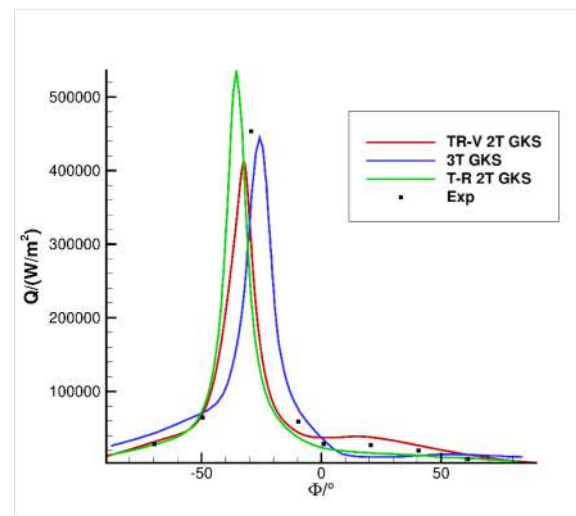
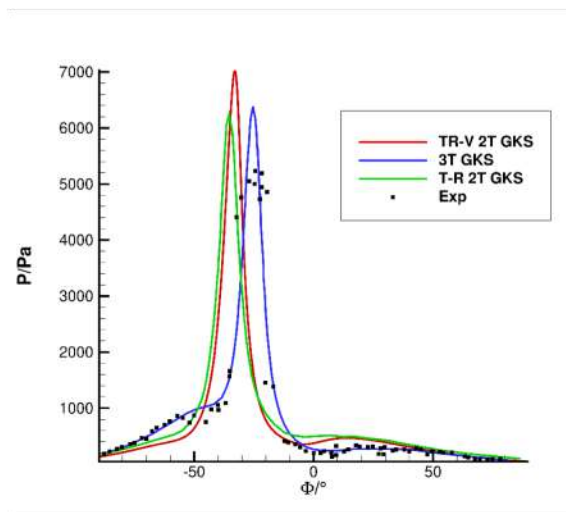
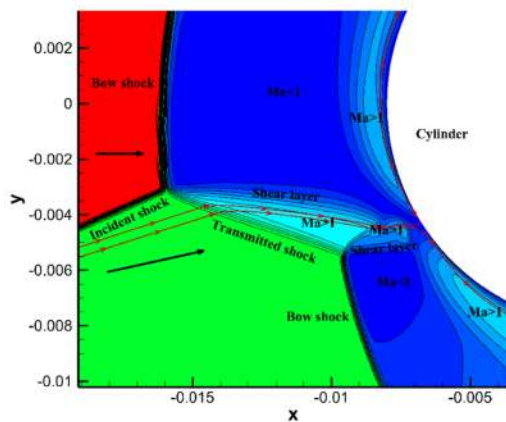
3. 典型场景验证及应用

高阶多温度真实气体效应仿真

高超声速楔体绕流



Edney第IV类激波/激波干扰



GPU高阶异构计算

实验平台

使用三种 GPU 集群对 GKS-LES 进行了详细的性能测试，分别是 AMD-MI50 集群，NVIDIA-V100 集群与 DCU 集群。MI50集群与V100集群均是单节点八卡机，DCU集群则是“东方”超算计算环境，采用的配置是单节点四卡机。三个集群中每张GPU加速卡的显存均是16GB。

GPU集群配置

集群	AMD-MI50	NVIDIA-V100	DCU
CPU	Intel® Xeon® Gold 5117	Intel® Xeon® Gold 5117	Hygon C86 7185
内存	8*16GB DDR4 2666MHz	8*16GB DDR4 2666MHz	8*16GB DDR4 2666MHz
网络	Infiniband 100Gb/s 1 vports	Infiniband 100Gb/s 1 vports	Infiniband 200Gb/s 4 vports
操作系统	Ubuntu 20.04	Ubuntu 20.04	CentOs 7.6
MPI 环境	OpenMPI 5.0.5	OpenMPI 5.0.5	OpenMPI 4.1.4

测试指标

加速比= $\frac{\text{实际计算墙钟时间}}{\text{参考墙钟时间}}$

并行效率= $\frac{\text{加速比} \times \text{参考进程数}}{\text{实际计算进程数}}$

测试方法

对主体迭代演化部分的墙钟时间进行测试，不计入前处理如网格读入等与后处理流场输出等时间，计入主体计算时间与数据通信时间。为验证GPU加速算法较原始CPU算法的加速效果，使用Intel® Xeon® Gold 5117 CPU处理器计算的结果进行对比。

3. 典型场景验证及应用

GPU高阶异构计算

GPU加速算法与串行算法比较

串行算法：Intel® Xeon® Gold 5117 14核CPU处理器，单进程运行

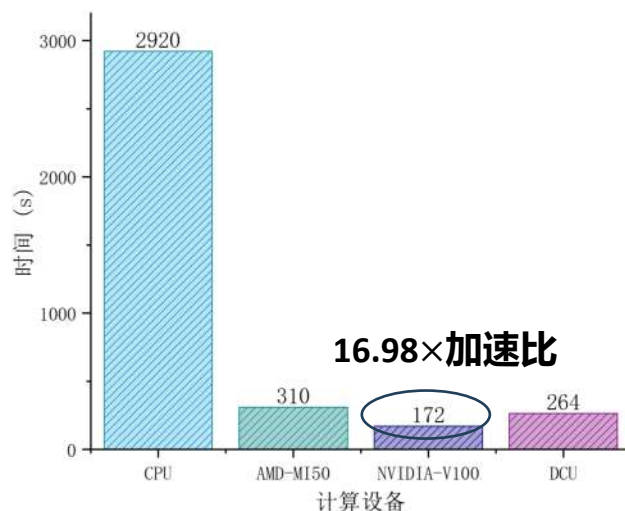
三种GPU加速下相较于串行算法的加速比

计算设备	计算时间	加速比
CPU（串行）	30813s	-
MI50	310s	99.40×
V100	172s	179.15×
DCU	264s	116.71×

不同GPU双精度浮点计算能力和内存带宽存在差异，实现的加速比也各不相同。不同GPU架构的硬件设计差异亦会对加速效果产生显著影响。如基于AMD Vega 20架构的MI50缺乏专用的常量缓存硬件，常量内存在实际分配时仍被映射至全局内存，影响了加载速度和整体性能表现。

GPU加速算法与单CPU并行算法比较

单CPU并行算法：Intel® Xeon® Gold 5117 14核CPU处理器，14个MPI进程并行计算



多GPU加速与单CPU并行效率比较

计算设备	4 卡计算时间	4 卡加速比	8 卡计算时间	8 卡加速比
MI50	83s	35.18×	45s	64.89×
V100	50s	58.40×	26s	112.31×
DCU	71s	41.12×	38s	76.84×

82.7%并行效率！

3. 典型场景验证及应用

GPU高阶异构计算

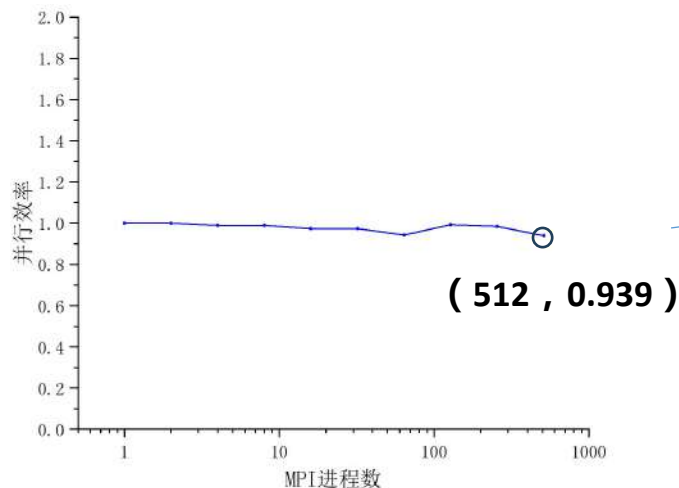
强拓展性测试

测试算例：一千万非结构混合网格三维圆柱绕流，以4卡并行测试速度为基准。强拓展性指在固定问题规模不变的情况下，随着计算资源（这里等同于使用的DCU卡数）的增加，程序运行时间是否显著减少。

DCU 卡数	4	8	16	32	64	128	256	512
时间/s	2613	1315	668	349	180	101	64	39
加速比	1.0	1.987	3.912	7.487	14.517	25.871	40.828	67
并行效率	1.0	0.994	0.978	0.936	0.907	0.808	0.638	0.523

弱拓展性测试

测试算例：三维正弦波精度测试算例，保持每个进程/加速卡处理1024000个计算单元，逐步扩大进程数。弱拓展性是指当问题规模与计算资源同步增加时，程序的运行效率是否能保持基本稳定。



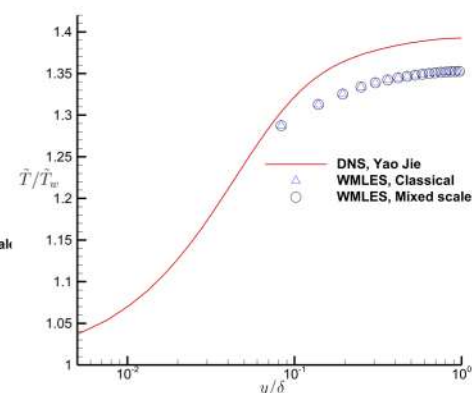
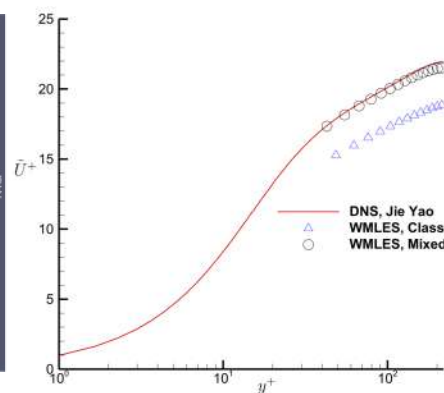
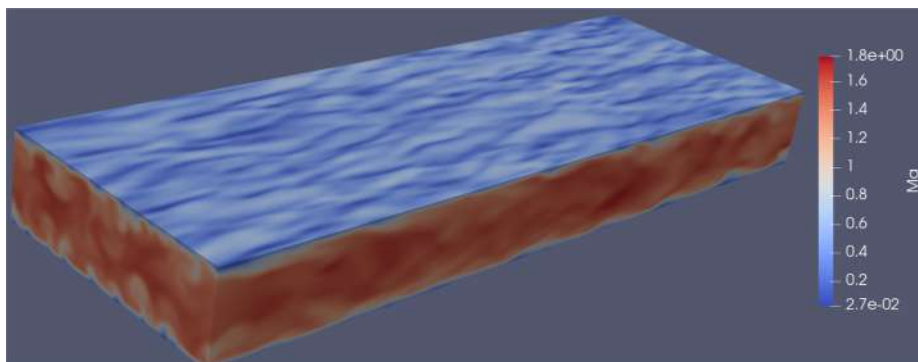
程序扩展性良好，具备大规模并行能力！

对应物理空间5.24亿
的非结构网格

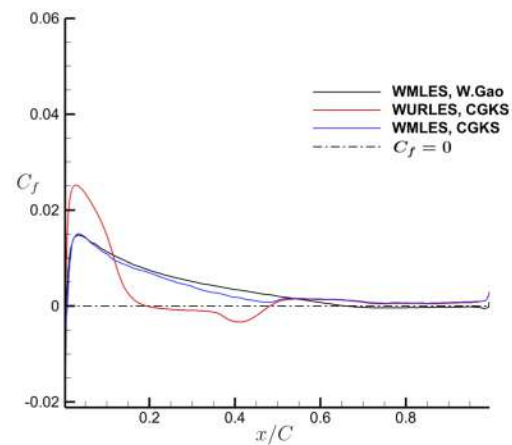
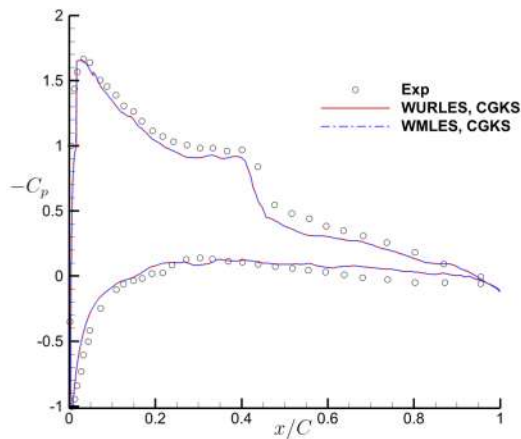
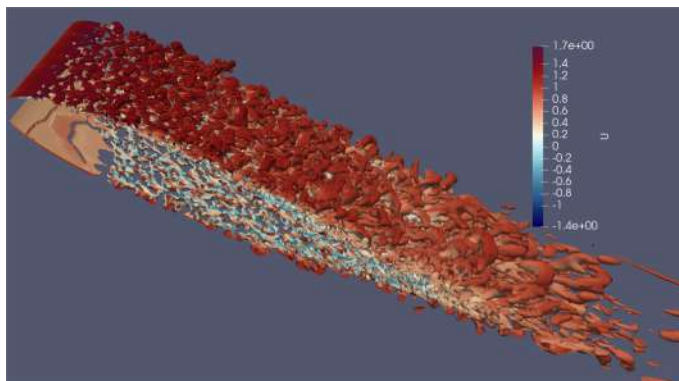
3. 典型场景验证及应用

复杂构型的壁面模化湍流模拟

可压缩槽道湍流



NACA0012翼型 $Re=10^5$



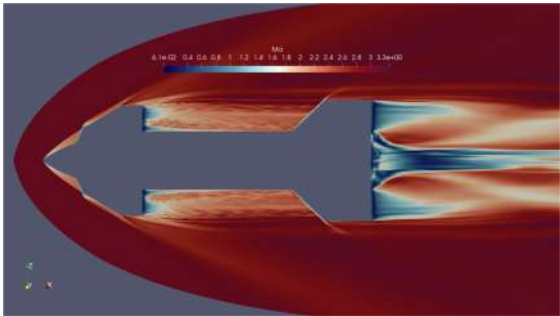
复杂构型的壁面模化湍流模拟

高超声速星舰构型数值模拟

超声速计算状态

Case	Ma	H	A
Case 2	3	40	0°

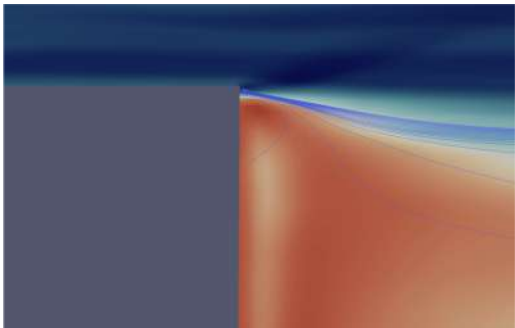
间断反馈因子在多激波干扰区域表现良好，准确地识别并捕捉到多层激波结构



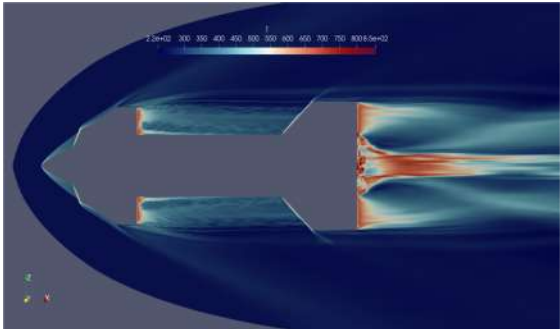
俯视马赫数分布



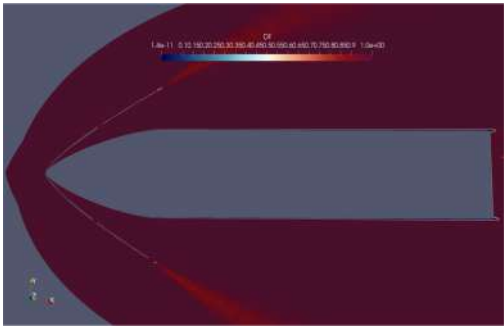
正面间断反馈因子分布



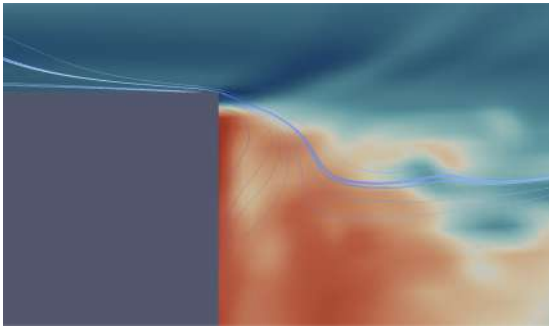
前翼根部区局部流线图



俯视温度分布



侧面间断反馈因子分布



后翼与底部交界处局部流线图

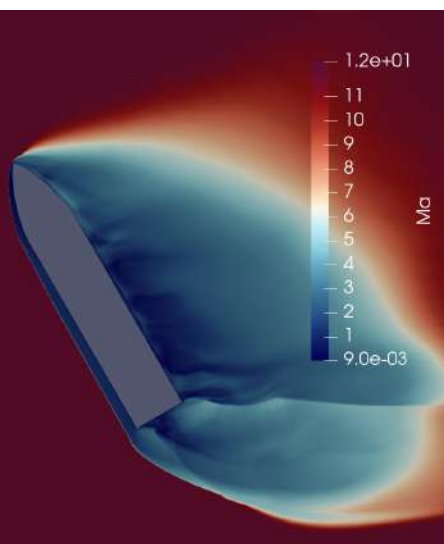
3. 典型场景验证及应用

复杂构型的壁面模化湍流模拟

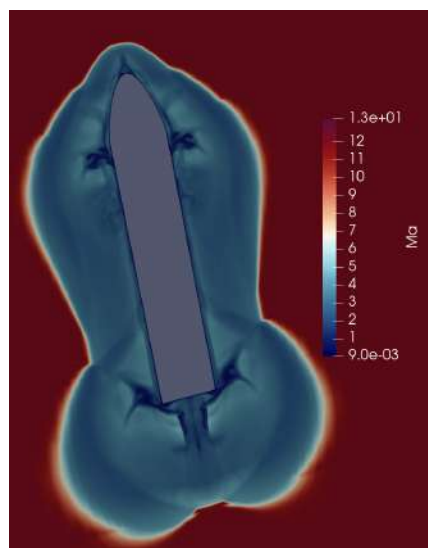
高超声速星舰构型数值模拟

飞行高度	空气密度/ $\text{kg} \cdot \text{m}^{-3}$	空气温度/ K	空气粘性/ $\text{Pa} \cdot \text{s}$	飞行马赫数
10km	4.14×10^{-1}	223.3	1.46×10^{-5}	0.2/0.7
40km	4.00×10^{-3}	250.4	1.59×10^{-5}	3
70km	8.77×10^{-5}	218.6	1.44×10^{-5}	12

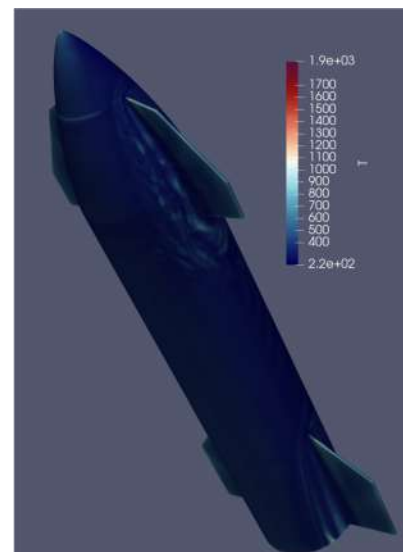
60°攻角飞行，高超声速飞行阶段



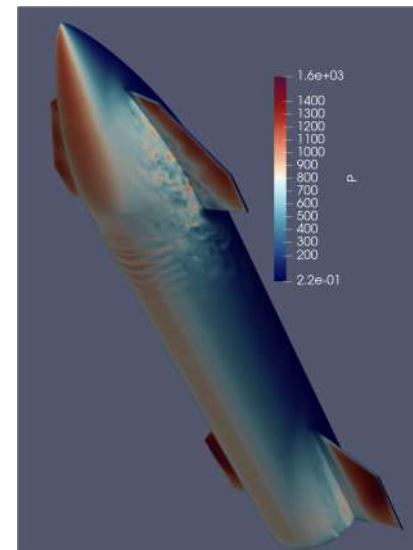
侧视截面马赫数分布



俯仰面剖面马赫数分布



星舰表面温度分布

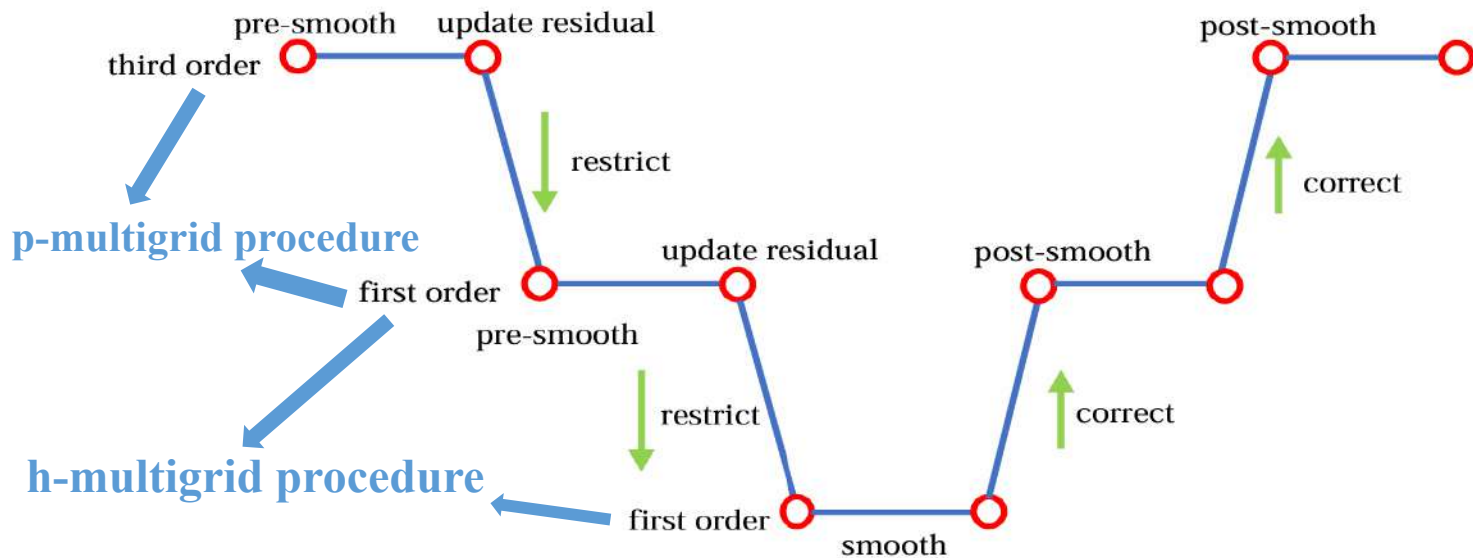


星舰表面压力分布

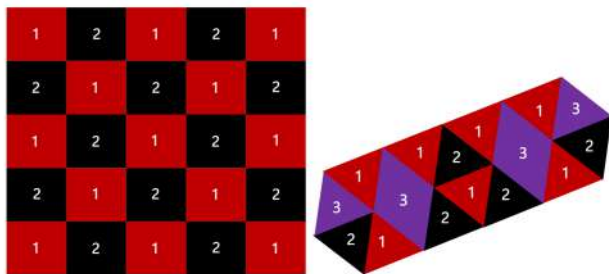
3. 典型场景验证及应用

基于GPU的高阶格式稳态加速算法

■ Flowchart of the hp-multigrid on GPU



■ Multi-Color LU-SGS to Solve Parallel Issues in Implicit Methods



Algorithm 1: The procedure of coloring method.

Input: The original mesh cells $\{C_i \mid i \in \Omega\}$ and a start cell v_0 .

Output: The array of cell colors colorArray (:) and total number of colors N_{color} .

```
1 initialize colorArray (:) = 0;
```

- 2 choose a cell v_0 as the start point of the traversal, and set $\text{color}(v_0) = 1$;

3 repeat

4 **for each colored cell** $\{v \mid \text{color}(v) > 0\}$ **do**5 **for each** uncolored cell $\{w \mid w \in C_v\}$ **do**6 $\text{paint color}(w) = \min \{k > 0 \mid k \neq \text{color}(j), \forall j \in C_w\};$

```
7      end
```

8 end

```

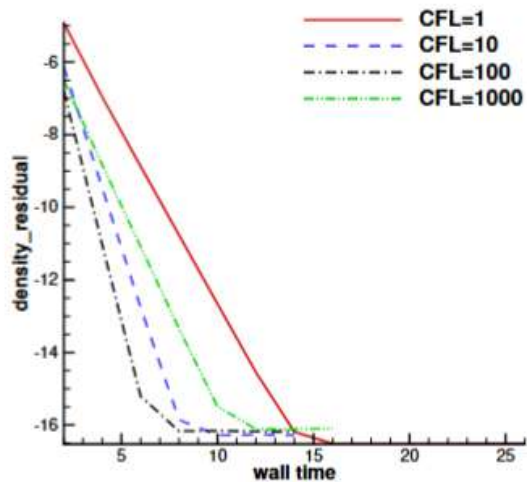
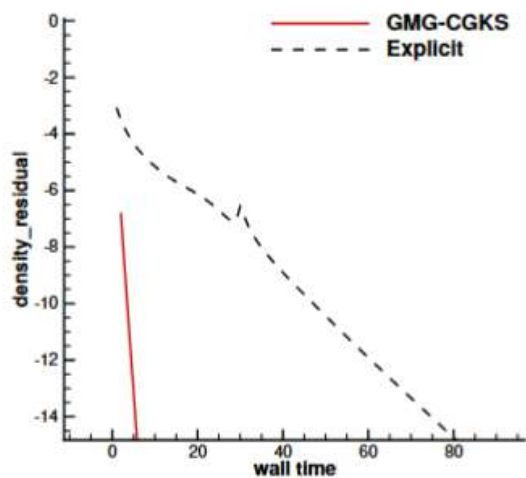
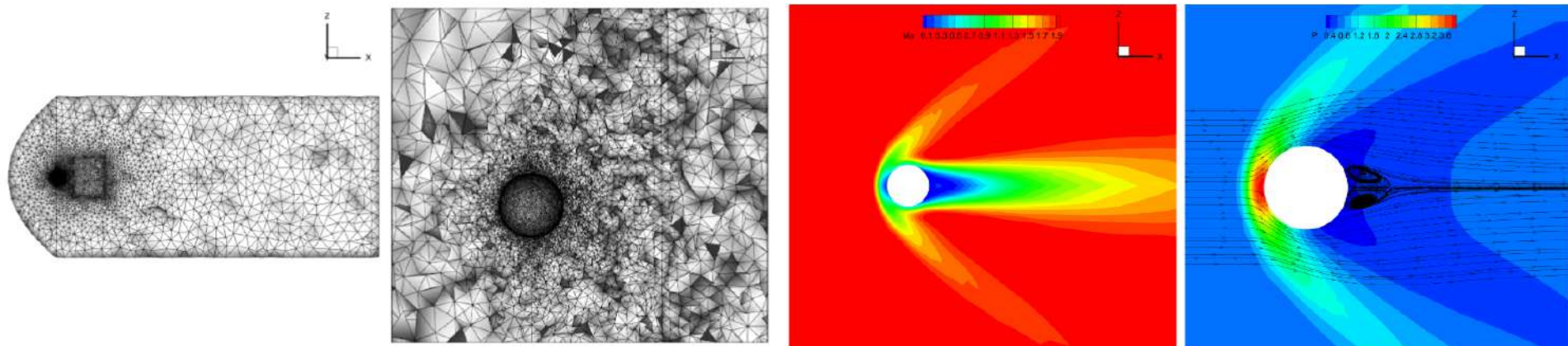
9 until all cells are painted;

```

3. 典型场景验证及应用

高阶格式稳态加速算法

超声速圆球绕流, $Ma=200$, $Re=300$

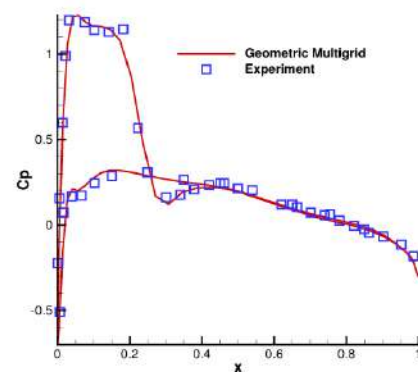
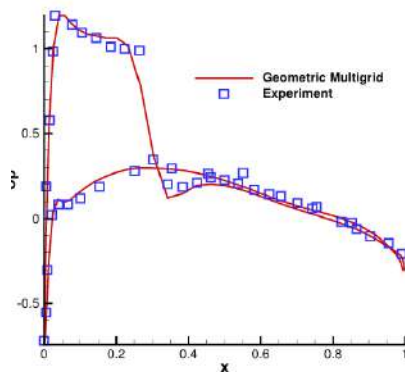
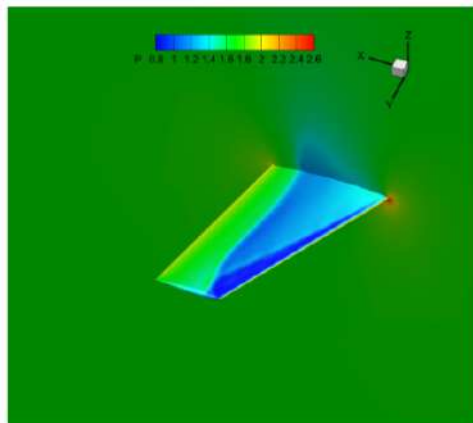
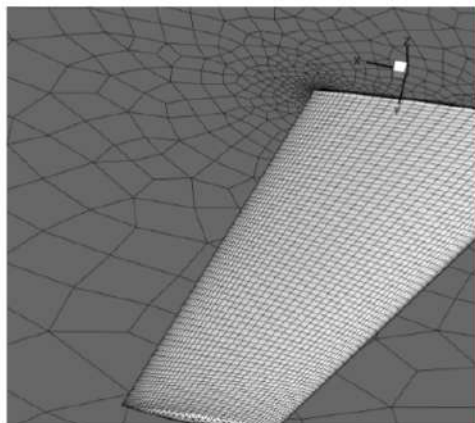


8张GPU卡仅需要
6s到达机器误差

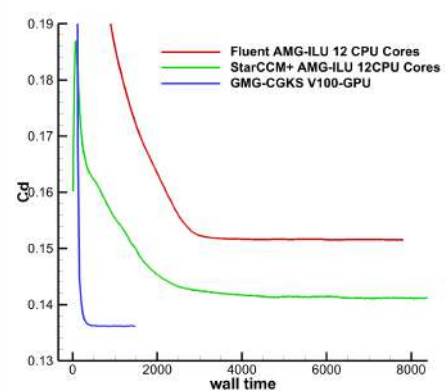
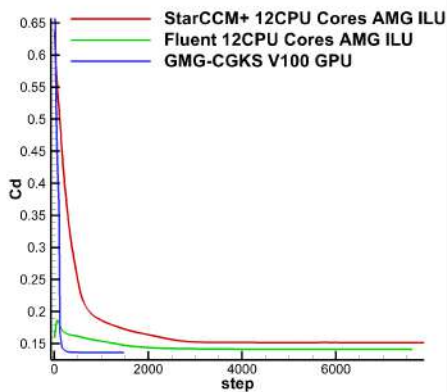
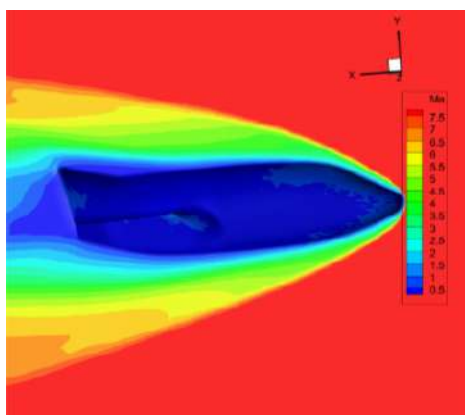
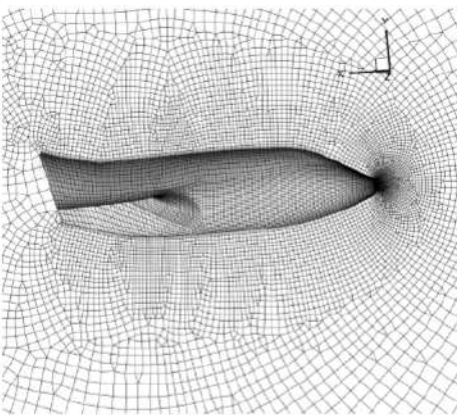
3. 典型场景验证及应用

高格式稳态加速算法

跨声速M6机翼流动, $Ma=0.8395$



高超声速X-38飞行器, $Ma=8.0$, $Re=14289$





2025年湍流与噪声和CFD方法暑期高级讲习班

谢谢！

July 9 2025

姬兴

西安交通大学 航天航空学院

合作：香港科技大学徐昆教授课题组

<https://osredm.com/p35462178/gks2d-str>



相关文献

1. Zhang Hong, Ji Xing, Xu Kun. (2024). A robustness-enhanced reconstruction based on discontinuity feedback factor for high-order finite volume scheme[J]. Journal of Scientific Computing, 2024, 101(1): 20.
2. Liu Hongyu, Ji Xing, Mao Yunpeng et al. (2024). A Compact Gas-Kinetic Scheme with Scalable Geometric Multigrid Acceleration for Steady-State Computation on 3D Unstructured Meshes[J/OL]. arXiv preprint
3. Zhang Yue, Ji Xing, Xu Kun. (2023). A High-Order Compact Gas-Kinetic Scheme in a Rotating Coordinates Frame and on Sliding Mesh[J]. International Journal of Computational Fluid Dynamics, 37(3), 181-200.
4. Ji Xing, Shyy Wei, Xu Kun. (2021). A gradient compression-based compact high-order gas-kinetic scheme on 3D-hybrid unstructured meshes[J]. International Journal of Computational Fluid Dynamics, 35(7), 485-509.
5. Zhang Hong, Ji Xing, Xu Kun . (2024). An Adaptive Reconstruction Method for Arbitrary High-Order Accuracy Using Discontinuity Feedback[J/OL]. arXiv preprint.
6. Liu Hongyu, Ji Xing, Mao Yunpeng, et al. (2024). A Memory Reduction Compact Gas Kinetic Scheme on 3D Unstructured Meshes[J/OL]. arXiv preprint.

