



第二十一届流体力学数值方法研讨会

基于间断反馈因子的自适应高阶紧致/非紧致气体动理学格式

Mach 10 rocket by CGKS

Strive for originality

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目录



- 1 研究背景
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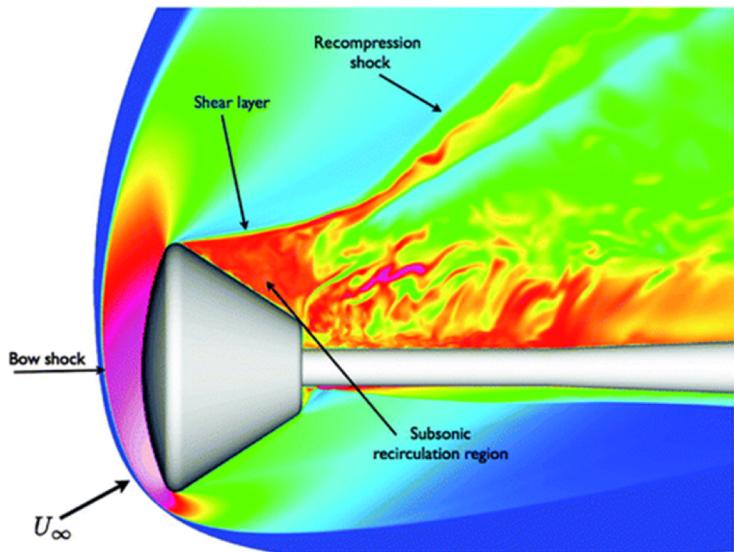


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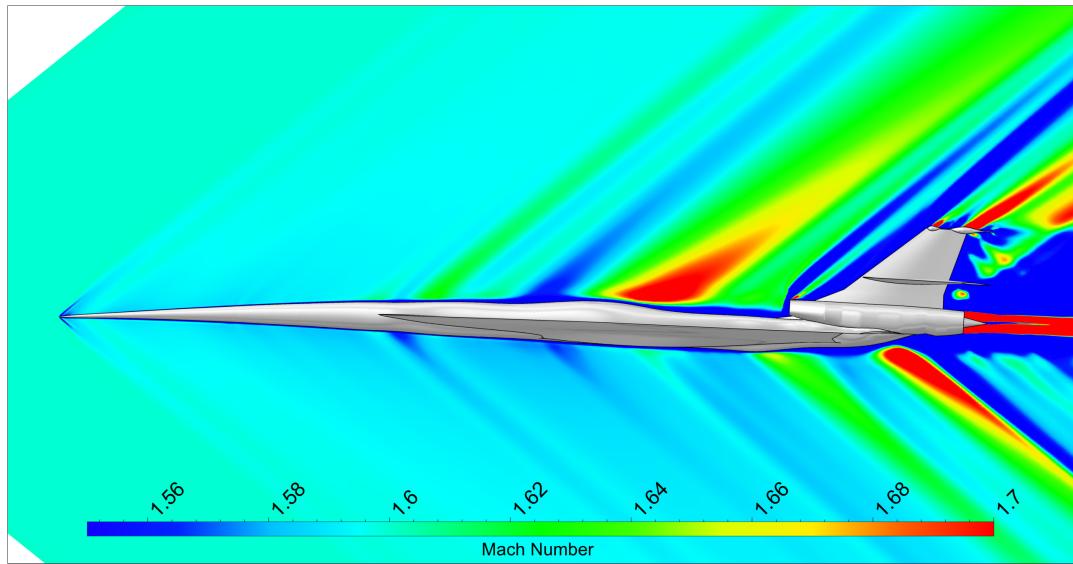
研究背景

1.研究背景

含间断流体仿真属于战略前沿领域的基础科学问题



高超声速再入飞行器湍流仿真



超声速飞行器气动噪声仿真

对应计算流体力学中的关键问题

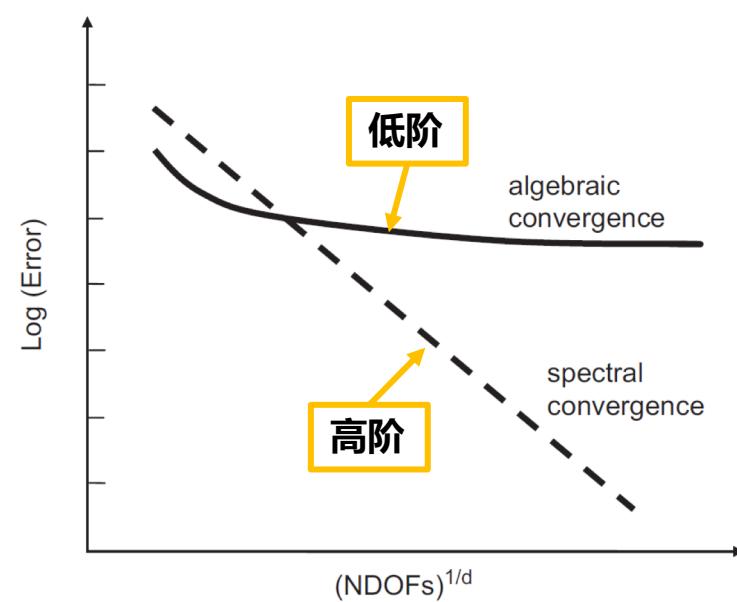
1. 格式如何高精度模拟小尺度非定常流动结构？

2. 格式如何保证间断捕捉的鲁棒性？

1.研究背景

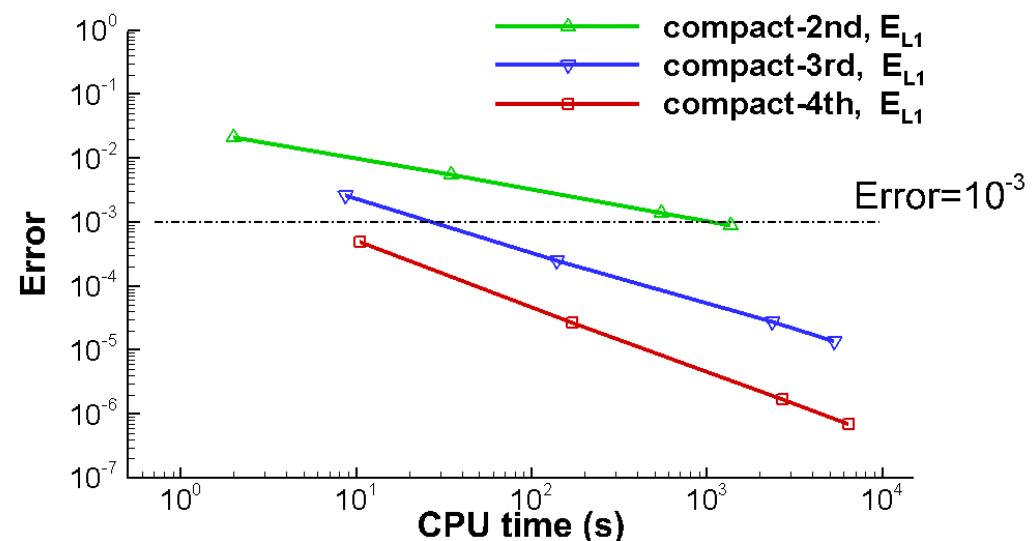
高阶格式具备对流体精细结构高效捕捉的显著潜力

高阶格式（大于2阶精度）具有**高分辨率、高效率**等优势，能够更准确地解析**湍流、噪声**等流动中的复杂结构和流动分离现象



随着计算自由度增长，相同计算时间高阶格式的误差逐渐小于低阶格式

[ZJ Wang, 2007]

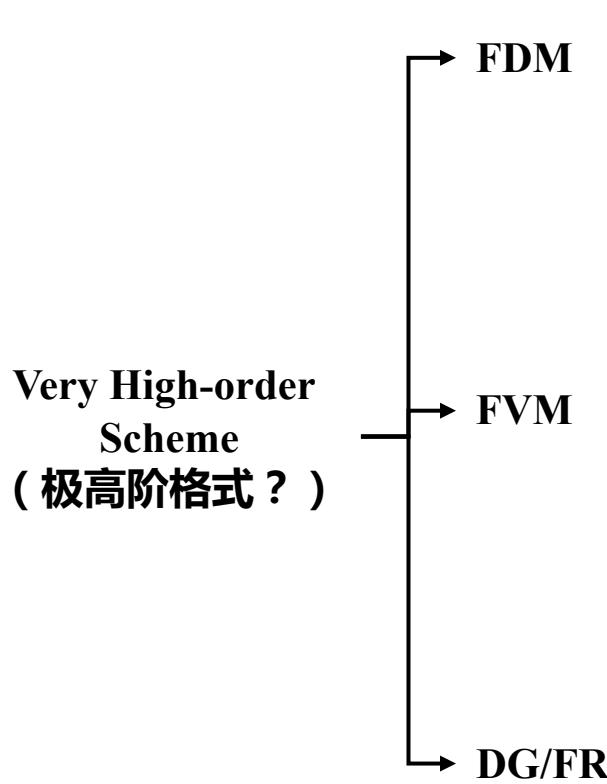


相同计算时间，不同阶数紧致气体动
理学格式在三维正弦波输运计算中的
实际误差表现

[FX ZHAO et al., 2023]

1.研究背景

随着阶数提升，高阶格式中精度和鲁棒性的矛盾越来越突出



1. **WENO9** (I Vallet et. al. J. Comput. Phys. 2009)
2. **WENO9-AO**(CW Shu et. al. J. Comput. Phys.2016)
3. **TENO10-AA**(L Fu. Comput. Methods Appl. Mech. Eng. 2021)
4. **WCNS9**(S Zhang. J. Comput. Phys. 2008)

1. **MOOD** (S Diot et. al. Int. J. Numer. Meth. Fluids. 2013)
2. **8th order CGKS**(K Xu et. al. Adv. Aerodyn. 2019)
3. **KFVM-WENO**(D Lee et. al. Astrophys. J. 2024)

1. **ADER-DG** (Isan S. Popov. Int. J. Sci. Comput. 2024)
2. **High-order FR/CPR**(ZJ Wang et. al.)



2

基于间断反馈因子的 有限体积格式

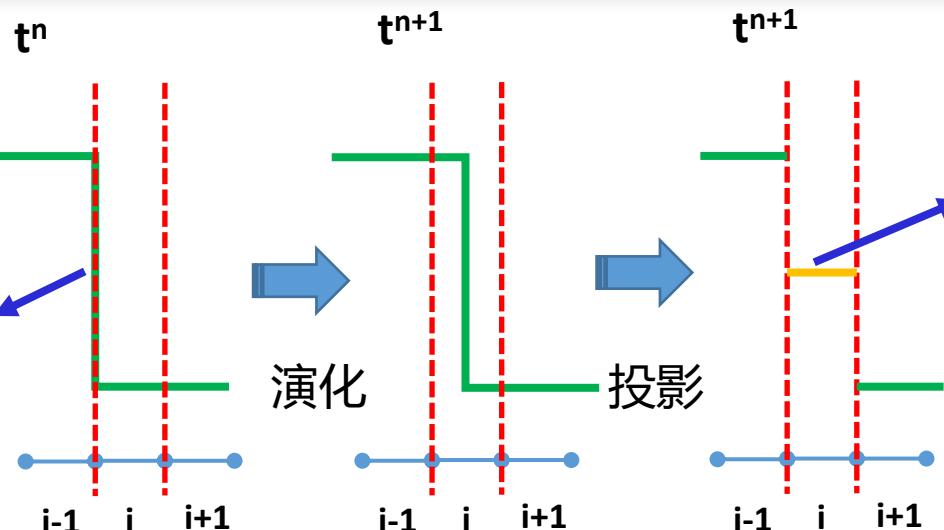
2. 基于间断反馈因子的有限体积格式

仅基于n时刻单元数据的重构方法无法对单元内间断准确限制

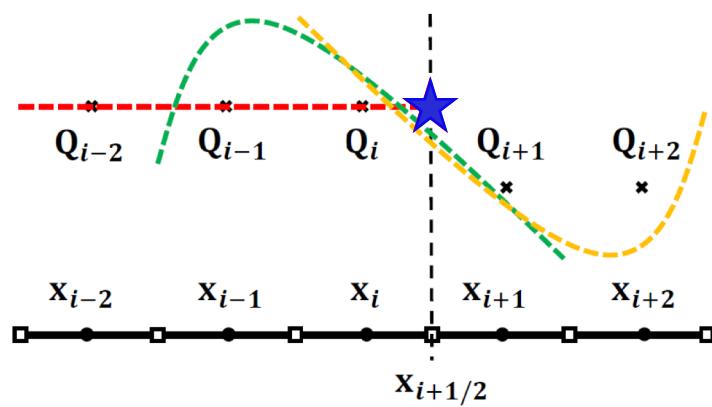
当间断正好落在界面时，重构方法可以轻易的识别间断
(案例 1)

★ 代表 $i+1/2$ 位置重构值

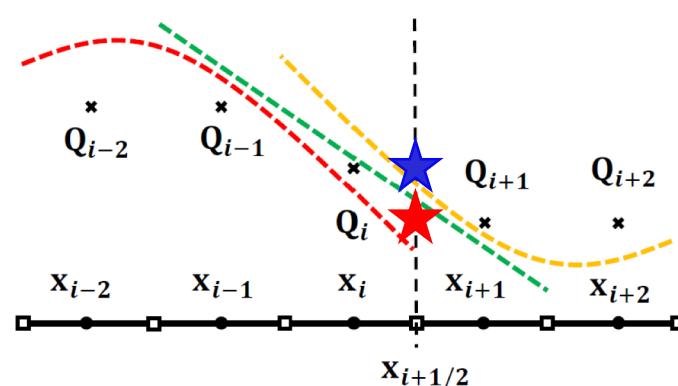
★ 代表 $i+1/2$ 位置精确值



基于这个单元平均值的重构办法都会失效 (案例 2)



案例 1 t^n



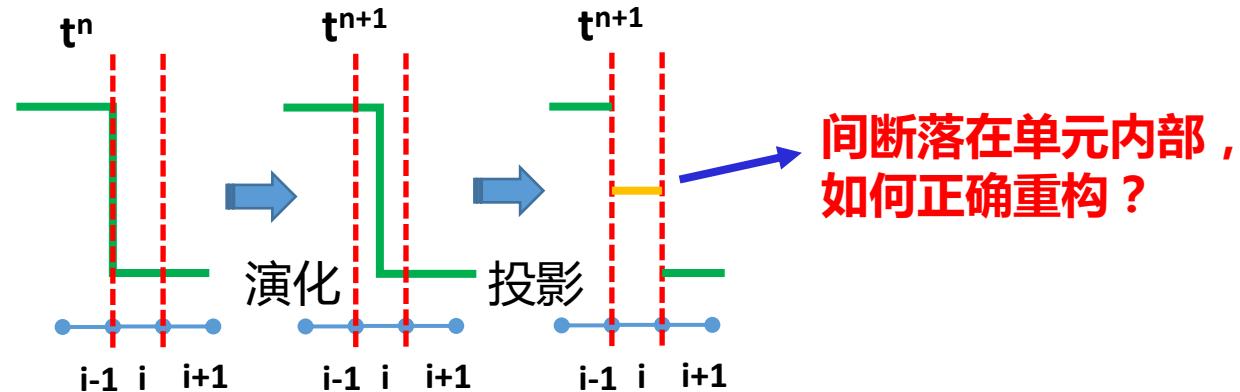
案例 2 t^{n+1}

2. 基于间断反馈因子的有限体积格式

流体基本特性：

1. 对于我们关注的双曲守恒律问题， n 时刻界面上间断，会演化出向界面左右两侧的多个间断
2. 依据CFL数稳定性条件：在 $n+1$ 时刻是间断面运动至左右两侧的单元中
3. 特例：对于1维Euler方程来说， n 时刻的界面间断会演化出激波、稀疏波、接触间断等

依据CFL数稳定性条件：在 $n+1$ 时刻，间断面运动至左/右两侧的单元中



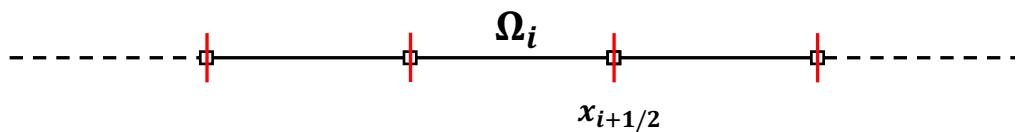
怎么办？

注意到：虽然 $n+1$ 时刻单元内部间断无法知道，但 n 时刻间断位于界面时候重构是有效的

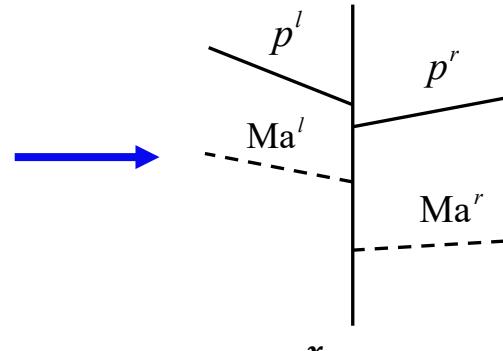
2. 基于间断反馈因子的有限体积格式

界面间断反馈因子的定义

一维情况, 界面 $x_{i+1/2}$ 处间断强度记为标量 $\sigma_{i+1/2}$ $\sigma_{i+1/2} \rightarrow \begin{cases} 0, & \text{当界面处初值光滑} \\ +\infty, & \text{当界面处初值间断} \end{cases}$



$$\sigma_{i+1/2,m} = \frac{|p^l - p^r|}{p^l} + \frac{|p^l - p^r|}{p^r} + (\text{Ma}^l - \text{Ma}^r)^2$$



DF是单元界面的间断强度一种表征

利用n时刻界面间断强度信息，反馈作用于n+1时刻重构

定义：n+1时刻模板间断反馈因子 α_s ：重构模版中所有界面的n时刻间断强度 σ 的度量

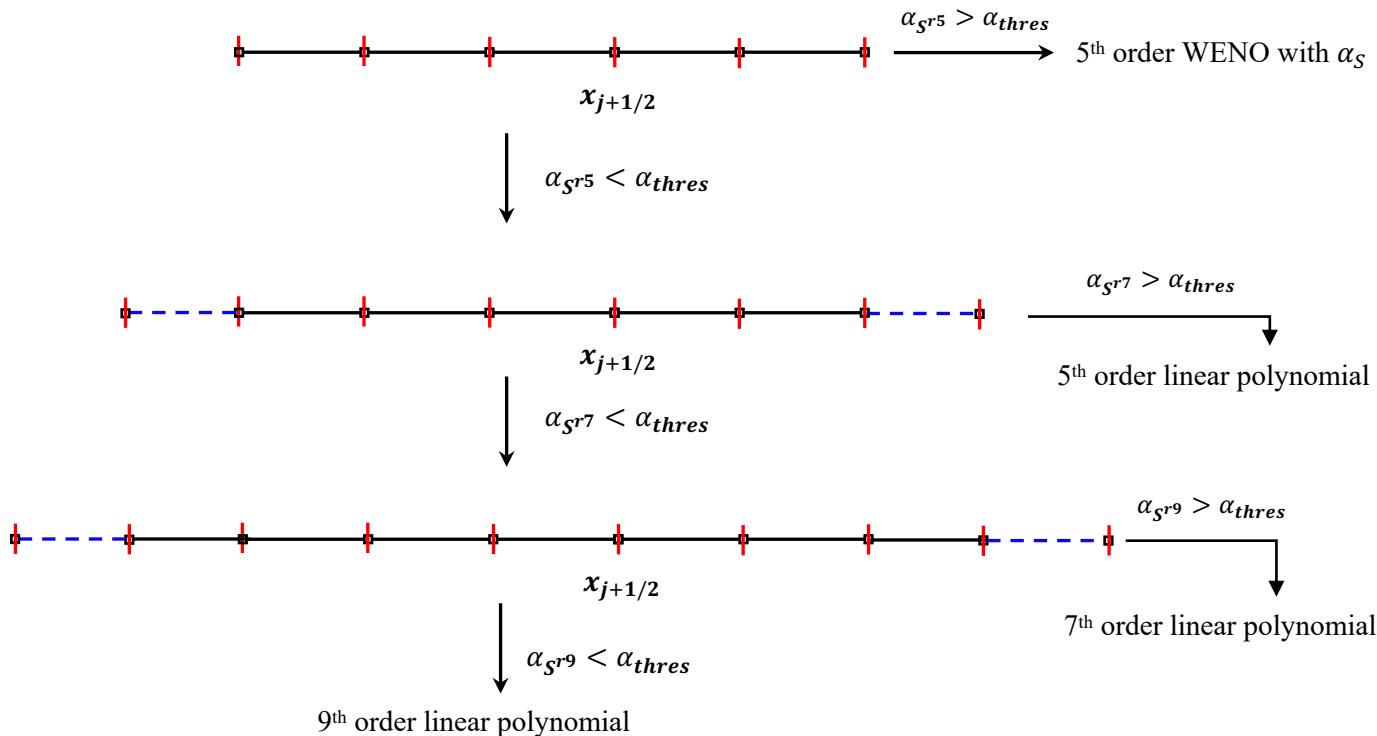
$$A = \cdots + \sigma_{i-3/2,j} + \sigma_{i-1/2,j} + \sigma_{i+1/2,j} + \sigma_{i+3/2,j} + \cdots,$$

$$\alpha_s = \begin{cases} 1.0 & \text{if } A < \sigma_{\text{thres}}, \\ \frac{\sigma_{\text{thres}}}{\cdots + \sigma_{i-3/2,j} + \sigma_{i-1/2,j} + \sigma_{i+1/2,j} + \sigma_{i+3/2,j} + \cdots} & \text{otherwise.} \end{cases}$$

- A ：重构模板的总间断强度
- σ_{thres} ：间断强度阈值参数（默认1.0）
- $\alpha_s \rightarrow 1$ ：判断为光滑模板
- $\alpha_s \rightarrow 0$ ：判断为强间断模板

间断反馈因子与非线性重构的结合

非紧致模板 S 的DF值记为 $\alpha_S(W)$



DF修正重构多项式

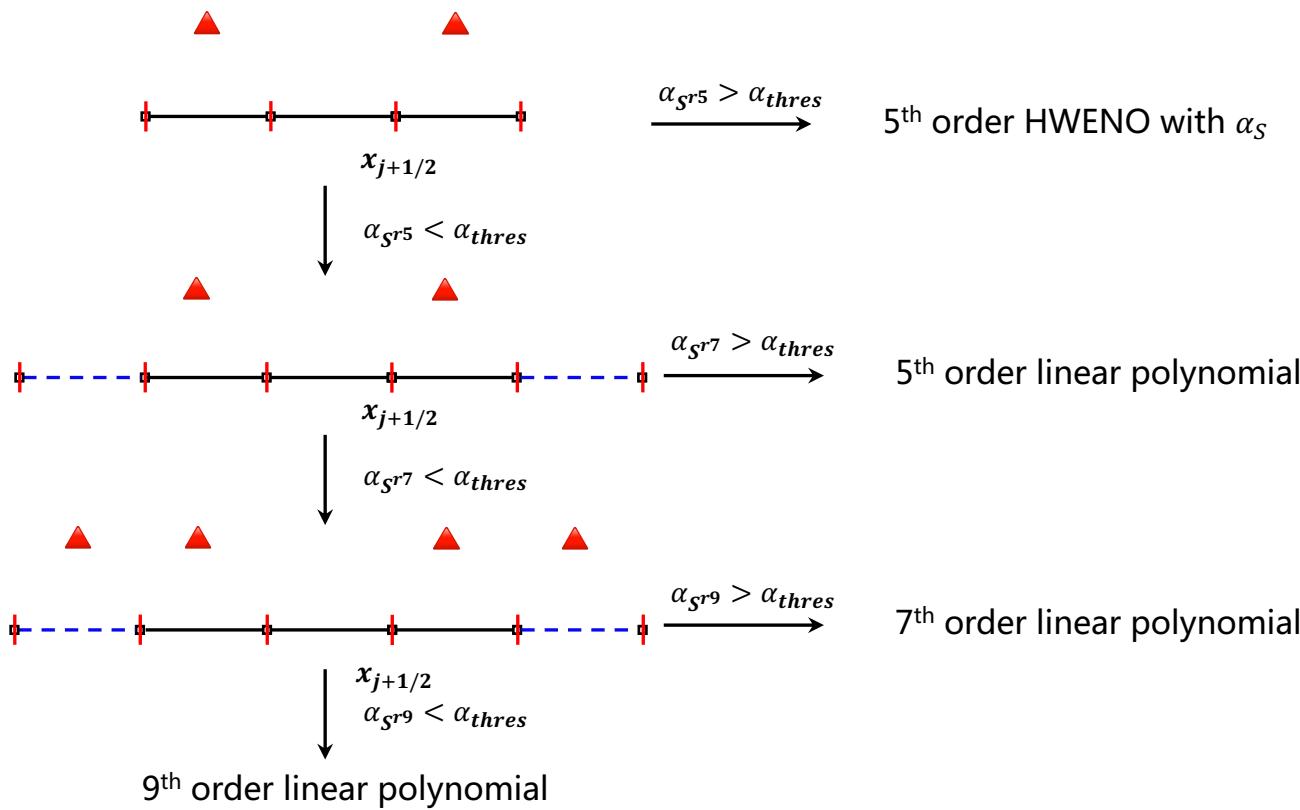
$$P^{DF} = W_i + \alpha_S (P^r(x) - W_i), P_x^{DF} = \alpha_S P_x^r(x)$$

$$W_{i+1/2}^l = P^{DF}(x_{i+1/2})$$

2. 基于间断反馈因子的有限体积格式

间断反馈因子与非线性重构的结合

紧致模板S的DF值记为 $\alpha_s(W: \cdot, W_x: \Delta)$



DF修正重构多项式 $P^{DF} = W_i + \alpha_s (P^r(x) - W_i), P_x^{DF} = \alpha_s P_x^r(x), P_{xx}^{DF} = \alpha_s P_{xx}^r(x)$

$W_{i+1/2}^l = P^{DF}(x_{i+1/2}), (W_x^l)_{i+1/2} = P_x^{DF}(x_{i+1/2}), (W_{xx}^l)_{i+1/2} = P_{xx}^{DF}(x_{i+1/2})$

Compact Gas-kinetic scheme

$$f_t + \mathbf{u} \cdot \nabla f = \frac{g - f}{\tau}$$

$$W(x, t) = \int \psi f(x, t, \mathbf{u}, \xi) d\xi$$

$$\mathbf{F}(\mathbf{x}, t) = \int \mathbf{u} \psi f(\mathbf{x}, t, \mathbf{u}, \xi) d\xi$$

$$\mathbf{F}(\mathbf{x}, t) = \int \mathbf{u} \psi f(\mathbf{x}, t, \mathbf{u}, \xi) d\xi$$

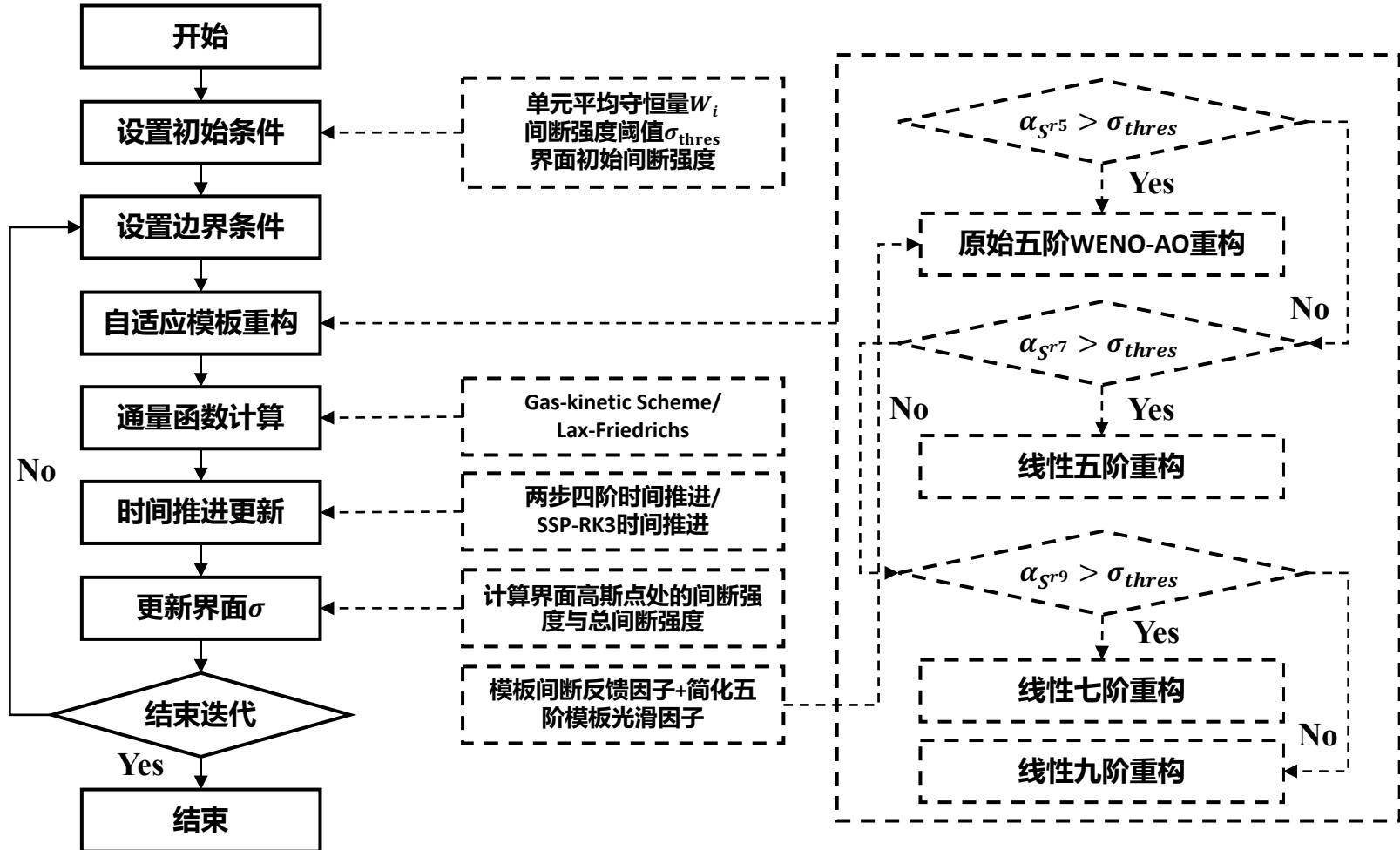
$$W(x, t) = \int \psi f(x, t, \mathbf{u}, \xi) d\xi$$

$$\frac{dW_i}{dt} = -\frac{1}{\Delta x} (\mathbf{F}_{i+1/2}(t) - \mathbf{F}_{i-1/2}(t))$$

$$(W_x)_i = \frac{1}{\Delta x} \int_{I_i} \frac{\partial W}{\partial x} dx = \frac{1}{\Delta x} (W_{i+1/2} - W_{i-1/2})$$

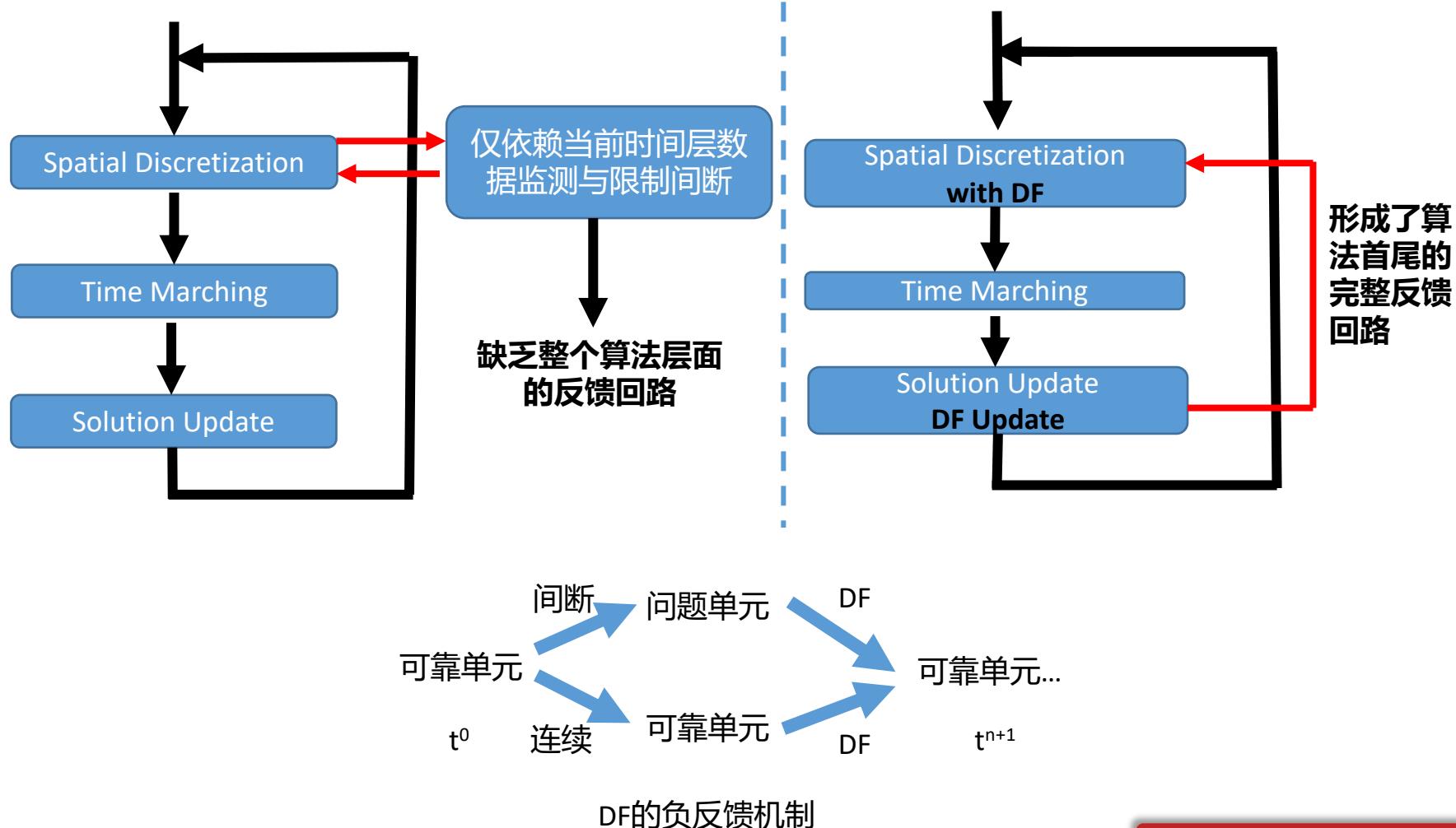
2. 基于间断反馈因子的有限体积格式

基于间断反馈因子的算法流程



间断反馈因子的优势

1. 负反馈调节机制使得自由参数调节不敏感



间断反馈因子的优势

2. 物理意义明确

$$\sigma_{i+1/2,m} = \frac{|p^l - p^r|}{p^l} + \frac{|p^l - p^r|}{p^r} + (\text{Ma}_n^l - \text{Ma}_n^r)^2 + (\text{Ma}_\tau^l - \text{Ma}_\tau^r)^2$$

激波

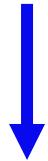
稀疏波

剪切层

间断反馈因子的优势

3. 精度保证且易做到更高阶

$$\sigma_{i+1/2,m} = \frac{|p^l - p^r|}{p^l} + \frac{|p^l - p^r|}{p^r} + (\text{Ma}_n^l - \text{Ma}_n^r)^2 + (\text{Ma}_\tau^l - \text{Ma}_\tau^r)^2$$



记压力真值为 p , 重构阶数为 k

$$\frac{|p^l - p^r|}{p^l} = \frac{|p + O(\Delta x^k) - (p + O(\Delta x^k))|}{p + O(\Delta x^k)} = O(\Delta x^{k+1})$$

自动满足任意 k 阶重构精度要求 !

间断反馈因子的优势

- 4. 计算量小
- 5. 存储量小

每个单元/界面仅增加1个数据

计算复杂度分析 (假设k阶重构)

1. 多项式计算复杂度：最终的重构多项式是对每个 Stencil点的非线性组合，子Stencil的计算复杂度为 $O(k)$ ，多项式个数为 $O(k)$ ，总计算复杂度为 $O(k^2)$
2. 平滑指标计算复杂度：每个子模板的多项式是 $(k + 1)/2$ 阶，对其求导并平方，最多涉及 $(k - 1)/2$ 次导数的计算，自模板为 $O(k)$ 个，
总计计算复杂度为 $O(k^3)$

WENO-Z

1. 多项式计算复杂度：最终的重构多项式是对每个 Stencil点的非线性组合，子Stencil的计算复杂度为 $O(k)$ ，多项式个数为 $O(k)$ ，总计算复杂度为 $O(k^2)$
2. 平滑指标计算复杂度：每个子模板的多项式是 $(m + 1)/2$ 阶, $m=2,3,\dots,k$ ，对其求导并平方，最多涉及 $(m - 1)/2$ 次导数的计算，自模板为 $O(k)$ 个，
总计计算复杂度为 $O(k^3)$

Multi-resolution WENO

1. DF计算复杂度：DF在不断增加阶数过程中需要持续计算，计算结果可以继承，但涉及到除法，**复杂度 $O(k)$**
2. 无平滑因子计算

DF-WENO



3

典型场景验证及应用

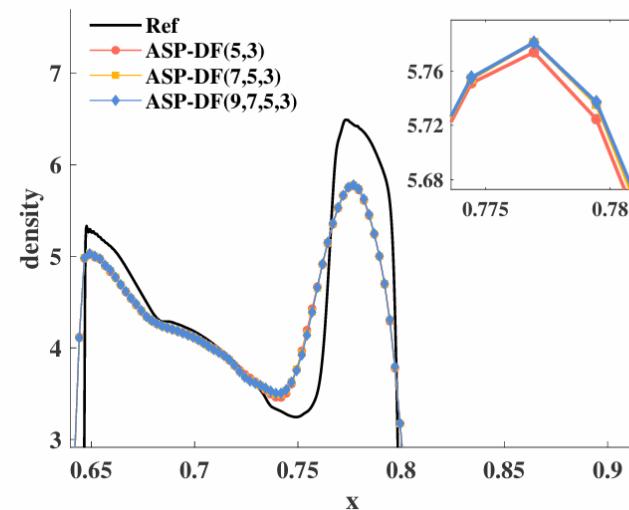
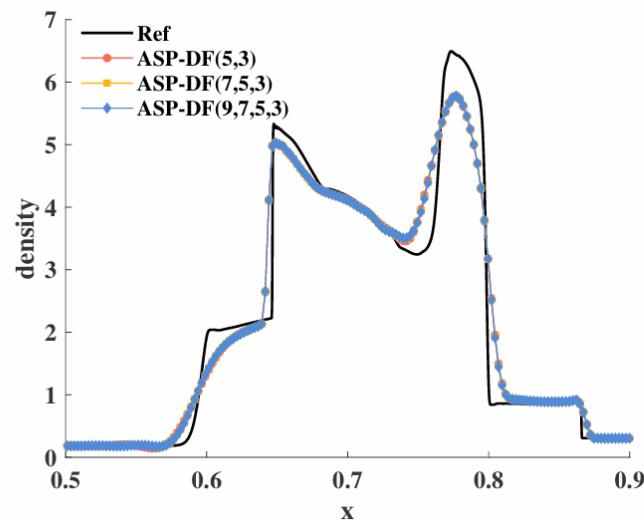
3. 典型场景验证及应用

一维任意高阶有限体积格式算例验证

Accuracy test

N	ASP-DF(5,3)	order	ASP-DF(7,5,3)	order	ASP-DF(9,7,5,3)	order
20	2.585904e-05		6.366042e-07		1.424192e-08	
40	8.337553e-07	4.95	5.108345e-09	6.96	2.848478e-11	8.97
80	2.680325e-08	4.95	4.042055e-11	6.98	9.085371e-14	8.29
160	8.585536e-10	4.96	3.343673e-13	6.92	2.544534e-16	8.48

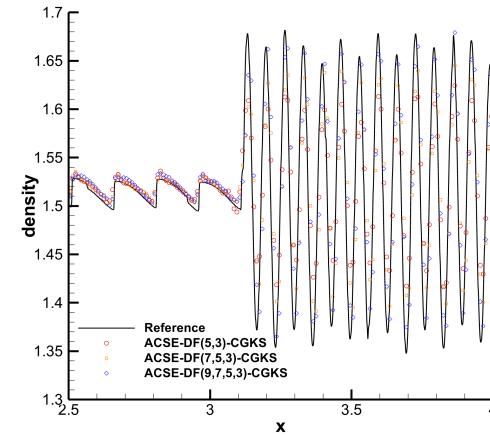
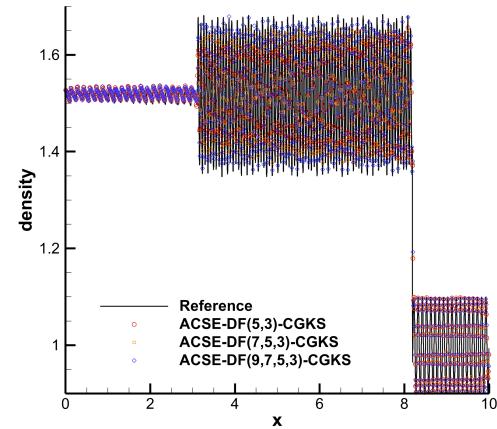
Blast wave problem



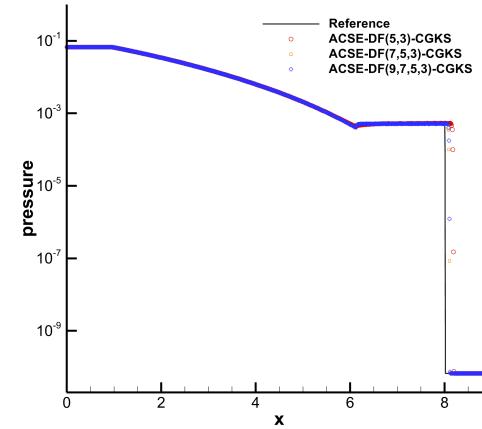
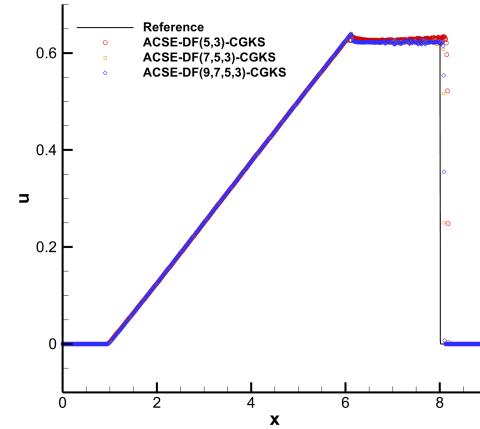
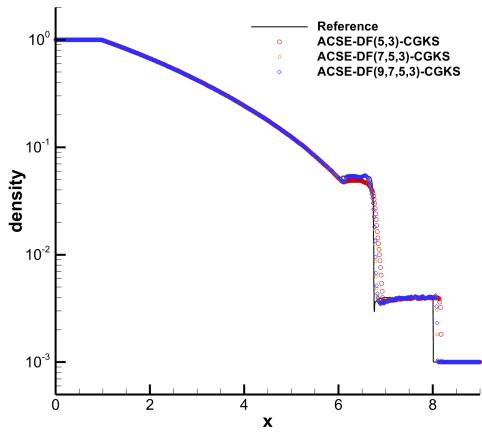
3. 典型场景验证及应用

一维任意紧致高阶气体动理学格式算例验证

Titarev-Toro problem



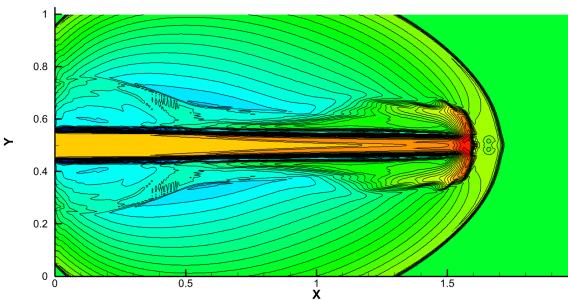
Le Blanc problem



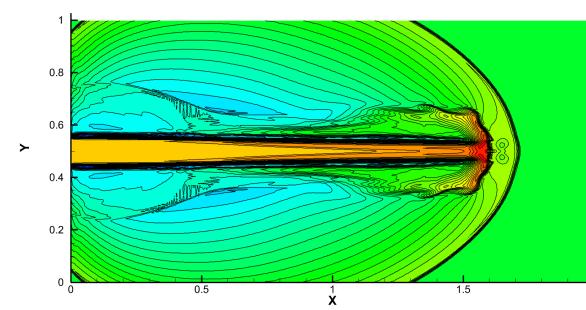
3. 典型场景验证及应用

二维任意紧致高阶气体动理学格式算例验证

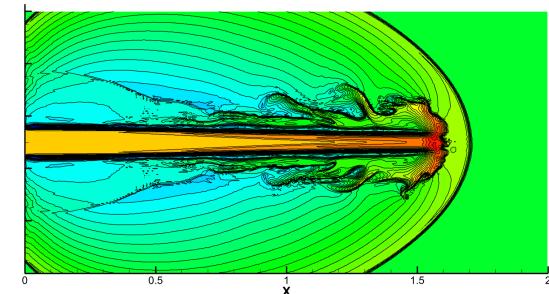
High Mach number astrophysical jet($\text{Ma}=80$)



(a) ACSE-DF(5,3)-CGKS

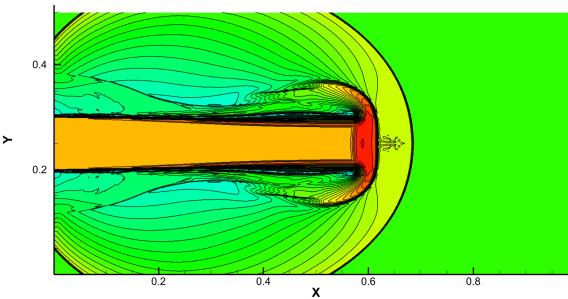


(b) ACSE-DF(7,5,3)-CGKS

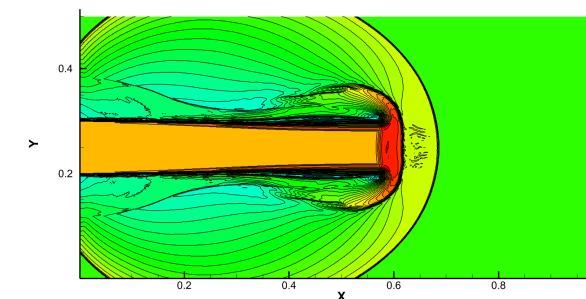


(c) ASE-DF(9,7,5,3)-CGKS

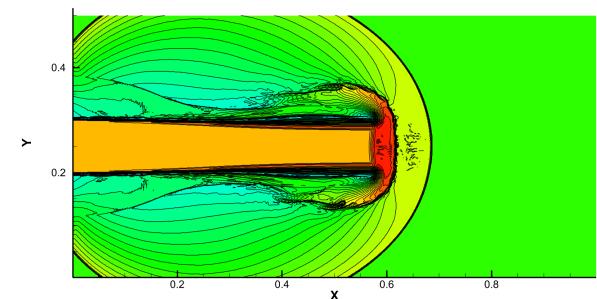
High Mach number astrophysical jet($\text{Ma}=2000$)



(a) ACSE-DF(5,3)-CGKS



(b) ACSE-DF(7,5,3)-CGKS

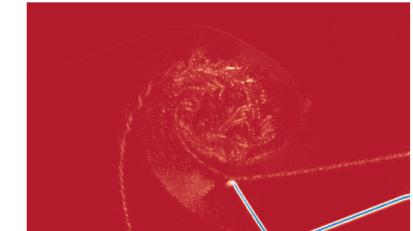
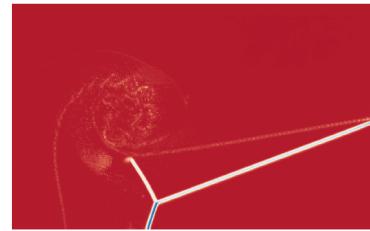


(c) ASE-DF(9,7,5,3)-CGKS

3. 典型场景验证及应用

高阶两组分流动仿真

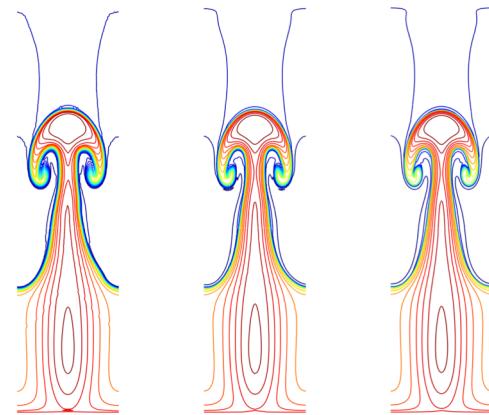
Two-component Triple-Point Problem



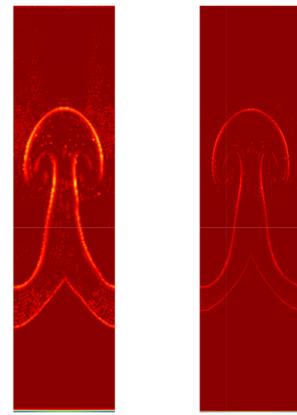
(a) t=3.5s. Left: total density distribution. Right: df factor distribution

(b) t=5.0s. Left: total density distribution. Right: df factor distribution

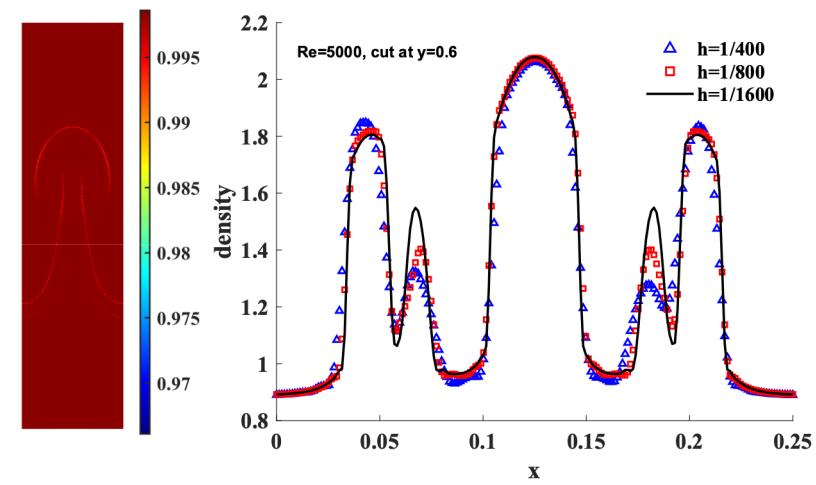
Two-component Rayleigh-Taylor problem



(a) $h = \frac{1}{400}, \frac{1}{800}, \frac{1}{1600}$ from left to right



(b) DF factor distribution



(c) Mesh refinement



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谢谢！

Aug 8 2025

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相关文献

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2. Liu Hongyu, Ji Xing, Mao Yunpeng et al. (2024). A Compact Gas-Kinetic Scheme with Scalable Geometric Multigrid Acceleration for Steady-State Computation on 3D Unstructured Meshes[J/OL]. arXiv preprint
3. Zhang Yue, Ji Xing, Xu Kun. (2023). A High-Order Compact Gas-Kinetic Scheme in a Rotating Coordinates Frame and on Sliding Mesh[J]. International Journal of Computational Fluid Dynamics, 37(3), 181-200.
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5. Zhang Hong, Ji Xing, Xu Kun . (2024). An Adaptive Reconstruction Method for Arbitrary High-Order Accuracy Using Discontinuity Feedback[J/OL]. arXiv preprint.
6. Liu Hongyu, Ji Xing, Mao Yunpeng, et al. (2024). A Memory Reduction Compact Gas Kinetic Scheme on 3D Unstructured Meshes[J/OL]. arXiv preprint.