



2025年湍流与噪声和CFD方法暑期高级讲习班

基于间断反馈因子的任意高阶有限 体积格式

Mach 10 rocket by CGKS

Strive for originality

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Jul 9 2025



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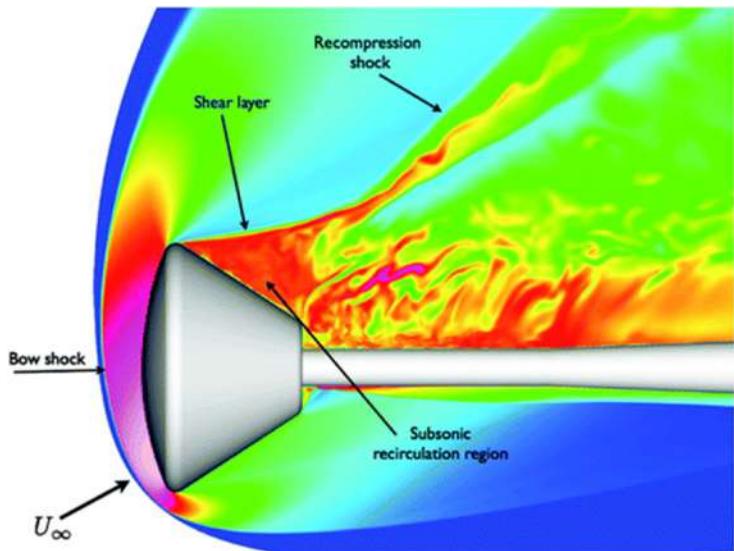


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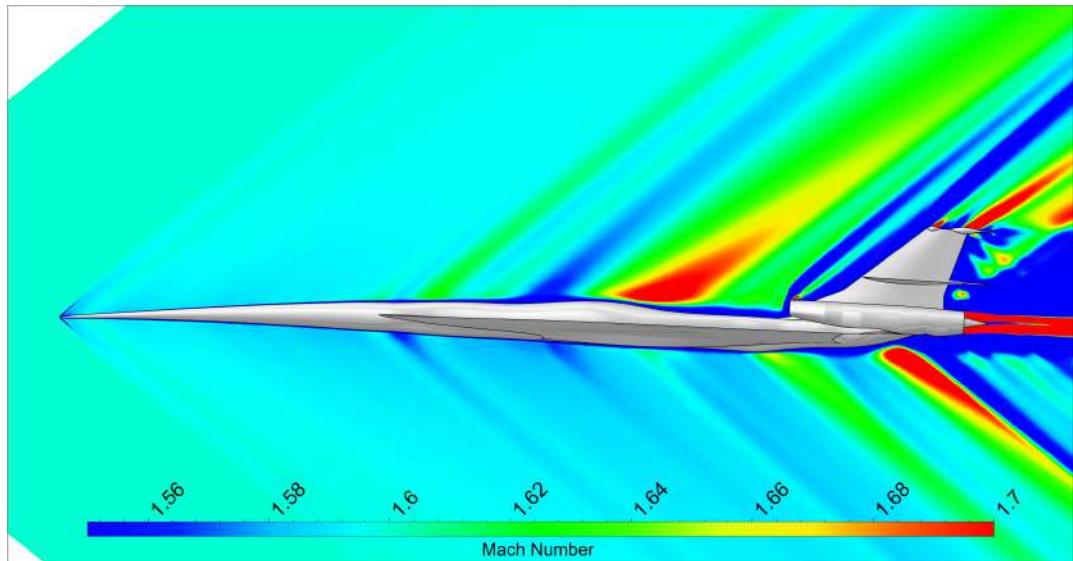
研究背景

1.研究背景

含间断流体仿真属于战略前沿领域的基础科学问题



高超声速再入飞行器湍流仿真



超声速飞行器气动噪声仿真

对应计算流体力学中的关键问题

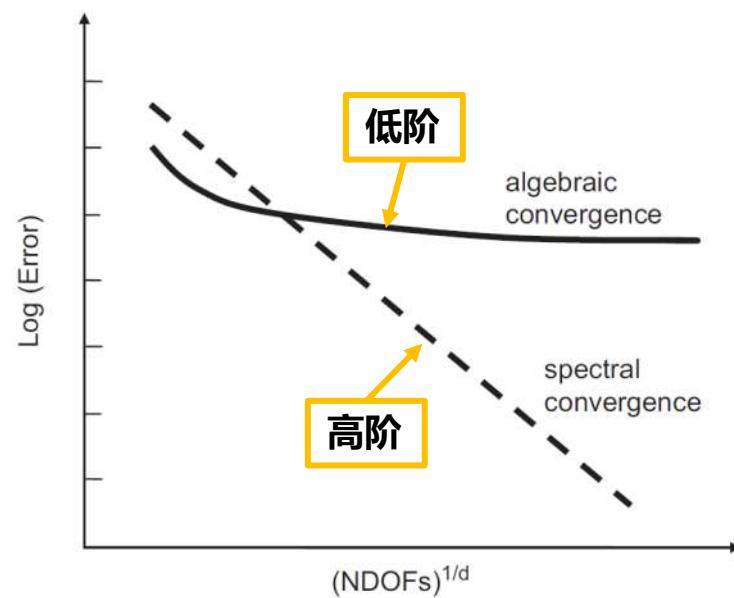
1. 格式如何高精度模拟小尺度非定常流动结构？

2. 格式如何保证间断捕捉的鲁棒性？

1.研究背景

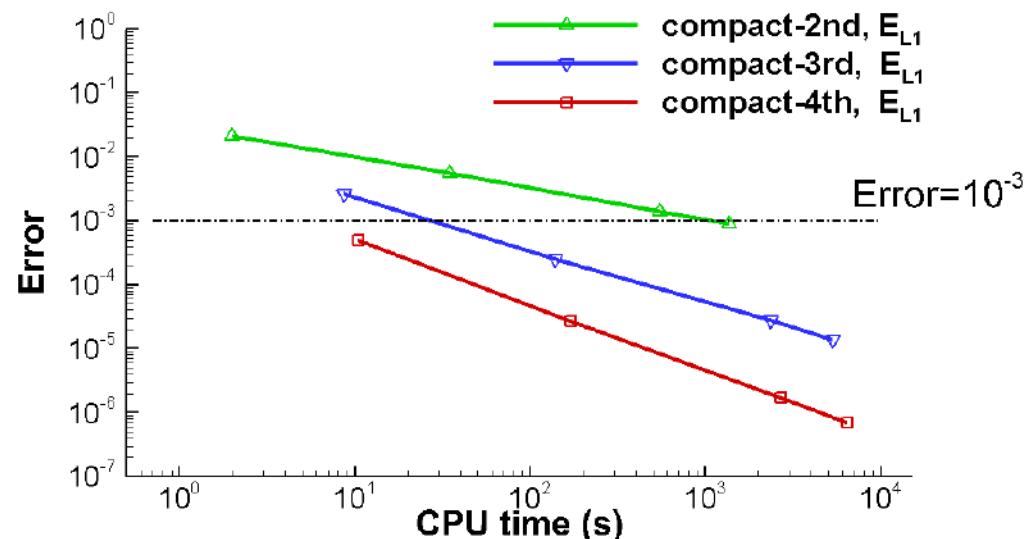
高阶格式具备对流体精细结构高效捕捉的显著潜力

高阶格式（大于2阶精度）具有高分辨率、高效率等优势，能够更准确地解析湍流、噪声等流动中的复杂结构和流动分离现象



随着计算自由度增长，相同计算时间高阶格式的误差逐渐小于低阶格式

[ZJ Wang, 2007]

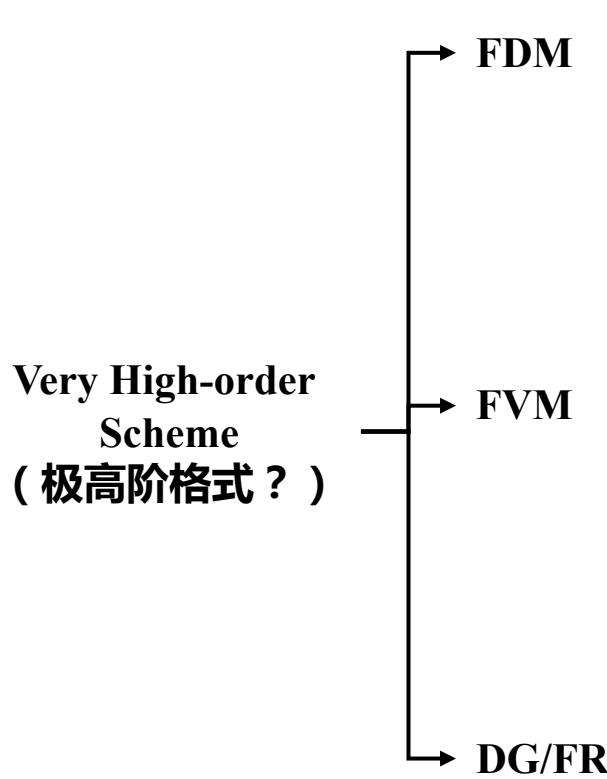


相同计算时间，不同阶数紧致气体动
理学格式在三维正弦波输运计算中的
实际误差表现

[FX ZHAO et al., 2023]

1.研究背景

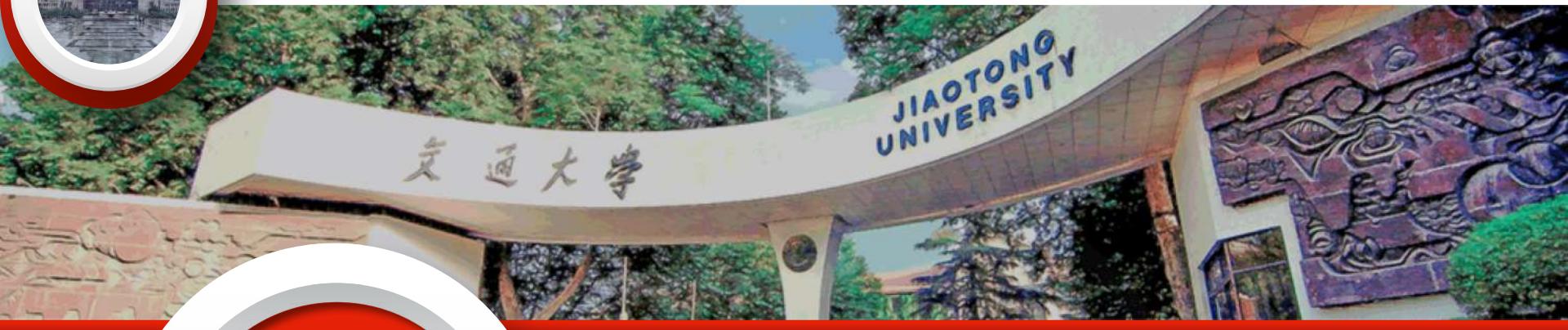
随着阶数提升，高阶格式中精度和鲁棒性的矛盾越来越突出



1. **WENO9** (I Vallet et. al. J. Comput. Phys. 2009)
2. **WENO9-AO**(CW Shu et. al. J. Comput. Phys.2016)
3. **TENO10-AA**(L Fu. Comput. Methods Appl. Mech. Eng. 2021)
4. **WCNS9**(S Zhang. J. Comput. Phys. 2008)

1. **MOOD** (S Diot et. al. Int. J. Numer. Meth. Fluids. 2013)
2. **8th order CGKS**(K Xu et. al. Adv. Aerodyn. 2019)
3. **KFVM-WENO**(D Lee et. al. Astrophys. J. 2024)

1. **ADER-DG** (Isan S. Popov. Int. J. Sci. Comput. 2024)
2. **High-order FR/CPR**(ZJ Wang et. al.)



2

基于间断反馈因子的 有限体积格式

2. 基于间断反馈因子的有限体积格式

双曲守恒律求解有限体积框架

1维Euler方程为例

双曲守恒律：

$$\mathbf{W}_t + \nabla \cdot \mathbf{F}(\mathbf{W}) = 0$$

以封闭单元
体为基本离
散自由度

$$\frac{\partial}{\partial t} \int_{\Omega} \mathbf{W} d\Omega + \int_{\partial\Omega} (\mathbf{F}(\mathbf{W})) dS = 0$$

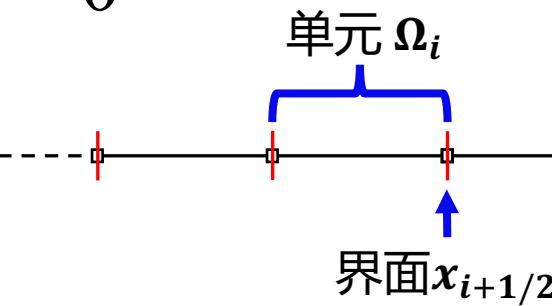
利用散度定理可得

$$\frac{d\bar{\mathbf{W}}_i}{dt} = -\frac{1}{\Delta x} (\mathbf{F}_{i+1/2}(\mathbf{W}) - \mathbf{F}_{i-1/2}(\mathbf{W})) := L(\mathbf{W})$$

Runge- $\bar{\mathbf{W}}_i^{(0)} = \bar{\mathbf{W}}_i^n$

Kutta 时间推进 $\bar{\mathbf{W}}_i^{(k)} = \bar{\mathbf{W}}_i^n + \sum \alpha_k \Delta t L(\bar{\mathbf{W}}^{(k-1)}), \quad k = 1, \dots, s$

$$\bar{\mathbf{W}}_i^{n+1} = \bar{\mathbf{W}}_i^{(s)}$$



$$\bar{\mathbf{W}}_i^{(1)} = \bar{\mathbf{W}}_i^n + \Delta t L(\bar{\mathbf{W}}^n)$$

三阶RK 案例

$$\bar{\mathbf{W}}_i^{(2)} = \bar{\mathbf{W}}_i^n + \frac{1}{4} \Delta t (L(\bar{\mathbf{W}}^n) + L(\bar{\mathbf{W}}^{(1)}))$$

$$\bar{\mathbf{W}}_i^{n+1} = \bar{\mathbf{W}}_i^n + \frac{1}{6} \Delta t L(\bar{\mathbf{W}}^n) + \frac{1}{6} \Delta t L(\bar{\mathbf{W}}^{(1)}) + \frac{2}{3} \Delta t L(\bar{\mathbf{W}}^{(2)})$$

$$\mathbf{F}_{\frac{i+1}{2}}(\mathbf{W}) = ? \longrightarrow \mathbf{F}_{\frac{i+1}{2}}(\mathbf{W}) \approx \mathbf{F}(\mathbf{W}_{\frac{i+1}{2}}^l, \mathbf{W}_{\frac{i+1}{2}}^r) \longrightarrow \mathbf{W}_{\frac{i+1}{2}}^{l,r} = R(\dots, \bar{\mathbf{W}}_i, \bar{\mathbf{W}}_{i+1}, \dots)$$

黎曼求解器

重构

2. 基于间断反馈因子的有限体积格式

高阶WENO类型非线性重构

5th-order WENO-AO



重构模板

$$\begin{aligned} S_0 &= \{I_{i-2}, I_{i-1}, I_i\}, S_1 = \{I_{i-1}, I_i, I_{i+1}\}, S_2 = \{I_i, I_{i+1}, I_{i+2}\} \\ S_3 &= \{S_0, S_1, S_2\} \end{aligned}$$

5th-order HWENO-AO



重构模板

$$\begin{aligned} S_0 &= \{I_{i-1}, I_i\}, S_1 = \{I_i, I_{i+1}\}, S_2 = \{I_{i-1}, I_i, I_{i+1}\} \\ S_3 &= \{S_0, S_1, S_2\}. \end{aligned}$$

**强激波、强稀疏波、强剪切层往往失效！
是高阶格式**工业化应用**的长期瓶颈！**

$$\frac{1}{\Delta x} \int_{I_{i-j-k-1}} p_k^{r^3}(x) dx = \bar{Q}_{i-j-k-1}, j = -1, 0, 1,$$

$$p_0^{r^3}(x_{i+1/2}) = \frac{1}{3} \bar{Q}_{i-2} - \frac{7}{6} \bar{Q}_{i-1} + \frac{11}{6} \bar{Q}_i,$$

$$p_1^{r^3}(x_{i+1/2}) = -\frac{1}{6} \bar{Q}_{i-1} + \frac{5}{6} \bar{Q}_i + \frac{1}{3} \bar{Q}_{i+1},$$

$$p_2^{r^3}(x_{i+1/2}) = \frac{1}{3} \bar{Q}_i + \frac{5}{6} \bar{Q}_{i+1} - \frac{1}{6} \bar{Q}_{i+2}.$$

$$p_3^{r^5}(x_{i+1/2}) = \frac{1}{60} (47 \bar{Q}_i - 13 \bar{Q}_{i-1} + 2 \bar{Q}_{i-2} + 27 \bar{Q}_{i+1} - 3 \bar{Q}_{i+2}).$$

$$\frac{1}{\Delta x} \int_{I_{i+j}} p_3^{r^5}(x) dx = \bar{Q}_{i+j}, j = -1, 0, 1$$

$$\frac{1}{\Delta x} \int_{I_{i+j}} (p_3^{r^5})_x(x) dx = (\bar{Q}_x)_{i+j}, j = -1, 0, 1.$$

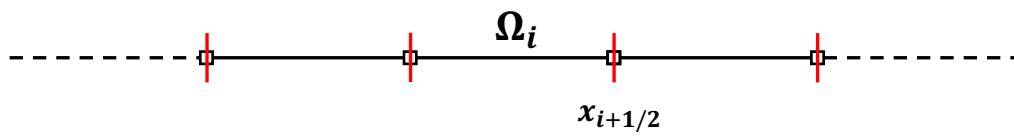
$$\frac{1}{\Delta x} \int_{I_{i+j}} p_3^{r^5}(x) dx = \bar{Q}_{i+j}, j = -1, 0, 1$$

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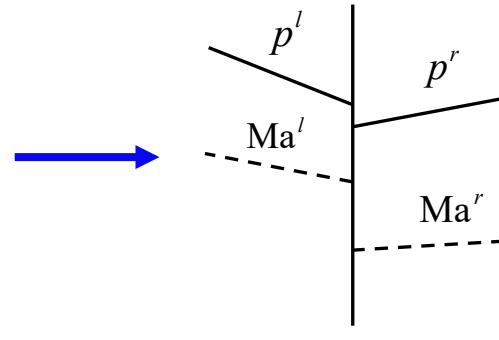
$$P^{AO(5,3)}(x) = \overline{\omega}_3 \left(\frac{1}{\gamma_3} p_3^{r^5}(x) - \sum_0^2 \frac{\gamma_k}{\gamma_3} p_k^{r^3}(x) \right) + \sum_0^2 \overline{\omega}_k p_k^{r^3}(x).$$

界面间断反馈因子的定义

一维情况, 界面 $x_{i+1/2}$ 处间断强度记为标量 $\sigma_{i+1/2}$ $\sigma_{i+1/2} \rightarrow \begin{cases} 0, & \text{当界面处初值光滑} \\ +\infty, & \text{当界面处初值间断} \end{cases}$

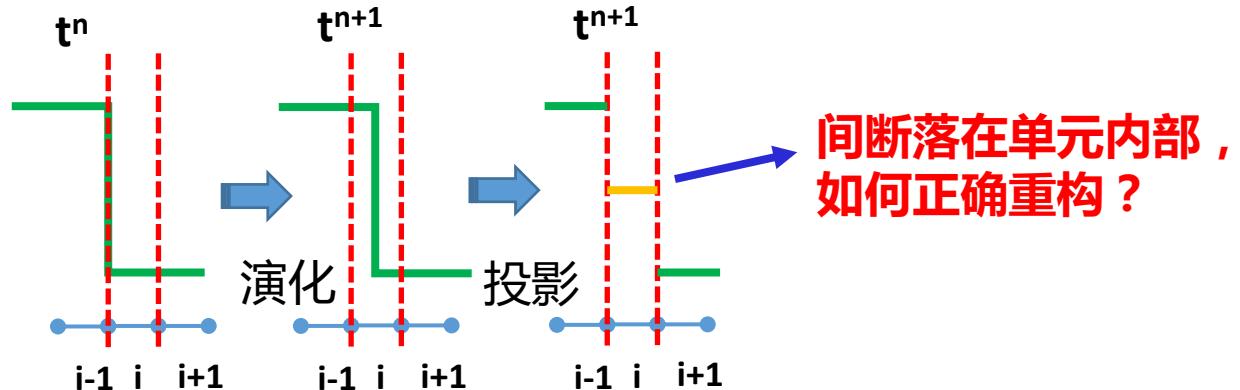


$$\sigma_{i+1/2,m} = \frac{|p^l - p^r|}{p^l} + \frac{|p^l - p^r|}{p^r} + (\text{Ma}^l - \text{Ma}^r)^2$$



DF是单元界面的间断强度一种表征

依据CFL数稳定性条件：在n+1时刻，间断面运动至左/右两侧的单元中



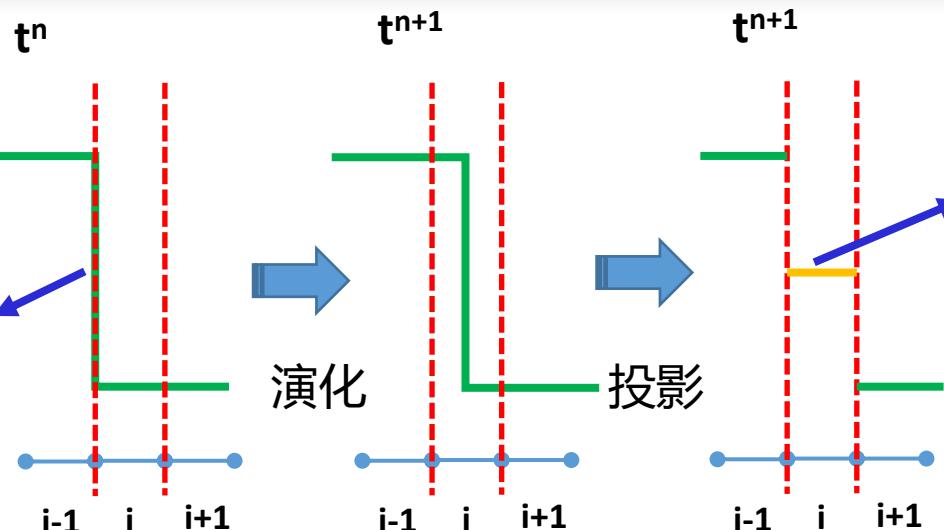
2. 基于间断反馈因子的有限体积格式

仅基于n时刻单元数据的重构方法无法对单元内间断准确限制

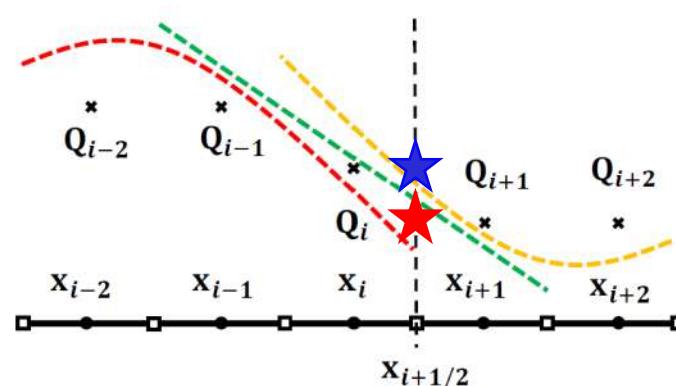
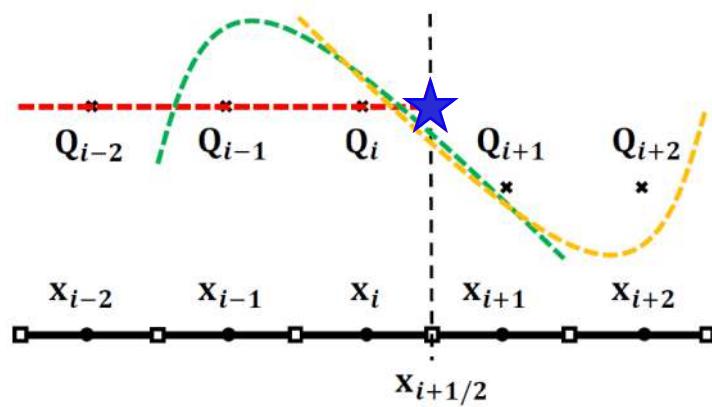
当间断正好落在界面时，重构方法可以轻易的识别间断
(案例 1)

★ 代表 $i+1/2$ 位置精确值

☆ 代表 $i+1/2$ 位置重构值



基于这个单元平均值的重构办法都会失效 (案例 2)



2. 基于间断反馈因子的有限体积格式

怎么办？

注意到：虽然 $n+1$ 时刻单元内部间断无法知道，但 n 时刻间断位于界面时候重构是有效的

流体基本特性：

1. 对于我们关注的双曲守恒律问题， n 时刻界面上间断，会演化出向界面左右两侧的多个间断
2. 依据CFL数稳定性条件：在 $n+1$ 时刻是间断面运动至左右两侧的单元中
3. 特例：对于1维Euler方程来说， n 时刻的界面间断会演化出激波、稀疏波、接触间断等

利用 n 时刻界面间断强度信息，反馈作用于 $n+1$ 时刻重构

定义： **$n+1$ 时刻模板间断反馈因子 α_s** ：重构模版中所有界面的 n 时刻间断强度 σ 的度量

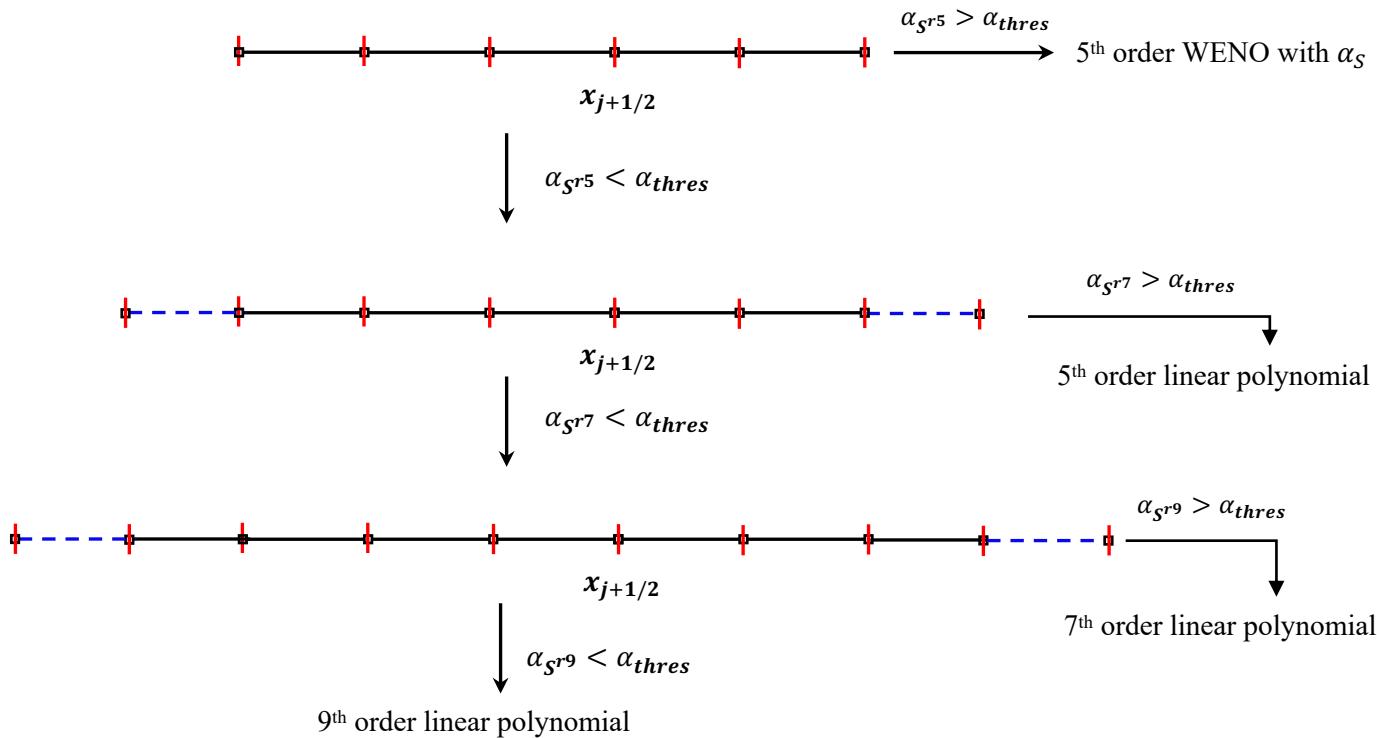
$$A = \cdots + \sigma_{i-3/2,j} + \sigma_{i-1/2,j} + \sigma_{i+1/2,j} + \sigma_{i+3/2,j} + \cdots,$$

$$\alpha_s = \begin{cases} 1.0 & \text{if } A < \sigma_{\text{thres}}, \\ \frac{\sigma_{\text{thres}}}{\cdots + \sigma_{i-3/2,j} + \sigma_{i-1/2,j} + \sigma_{i+1/2,j} + \sigma_{i+3/2,j} + \cdots} & \text{otherwise.} \end{cases}$$

- A ：重构模版的总间断强度
- σ_{thres} ：间断强度阈值参数（默认1.0）
- $\alpha_s \rightarrow 1$ ：判断为光滑模版
- $\alpha_s \rightarrow 0$ ：判断为强间断模版

间断反馈因子与非线性重构的结合

非紧致模板 S 的DF值记为 $\alpha_S(W)$



DF修正重构多项式

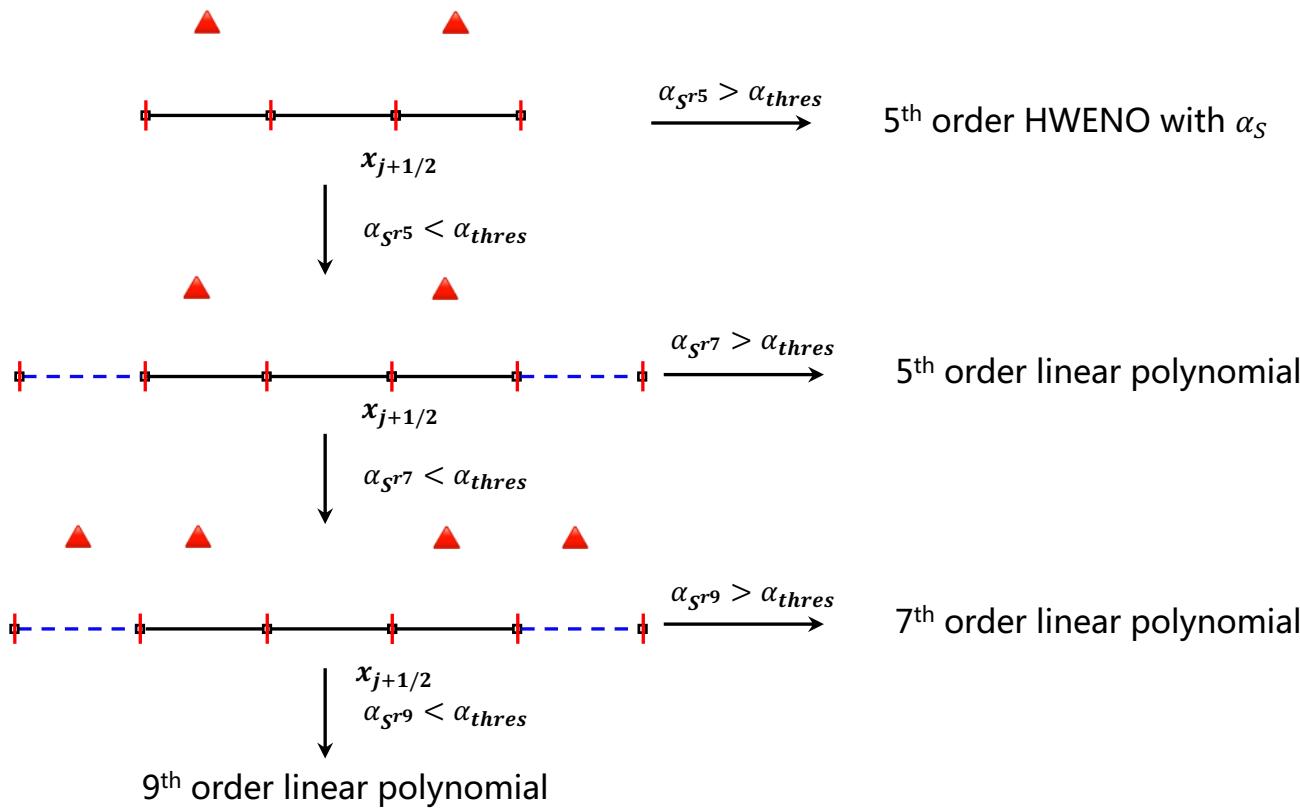
$$P^{DF} = W_i + \alpha_S (P^r(x) - W_i), P_x^{DF} = \alpha_S P_x^r(x)$$

$$W_{i+1/2}^l = P^{DF}(x_{i+1/2})$$

2. 基于间断反馈因子的有限体积格式

间断反馈因子与非线性重构的结合

紧致模板S的DF值记为 $\alpha_s(W: \cdot, W_x: \Delta)$



$$\text{DF修正重构多项式 } P^{DF} = W_i + \alpha_s (P^r(x) - W_i), P_x^{DF} = \alpha_s P_x^r(x), P_{xx}^{DF} = \alpha_s P_{xx}^r(x)$$

$$W_{i+1/2}^l = P^{DF}(x_{i+1/2}), (W_x^l)_{i+1/2} = P_x^{DF}(x_{i+1/2}), (W_{xx}^l)_{i+1/2} = P_{xx}^{DF}(x_{i+1/2})$$

Compact Gas-kinetic scheme

$$f_t + \mathbf{u} \cdot \nabla f = \frac{g - f}{\tau}$$

$$W(x, t) = \int \psi f(x, t, \mathbf{u}, \xi) d\xi$$

$$\mathbf{F}(\mathbf{x}, t) = \int \mathbf{u} \psi f(\mathbf{x}, t, \mathbf{u}, \xi) d\xi$$

$$\mathbf{F}(\mathbf{x}, t) = \int \mathbf{u} \psi f(\mathbf{x}, t, \mathbf{u}, \xi) d\xi$$

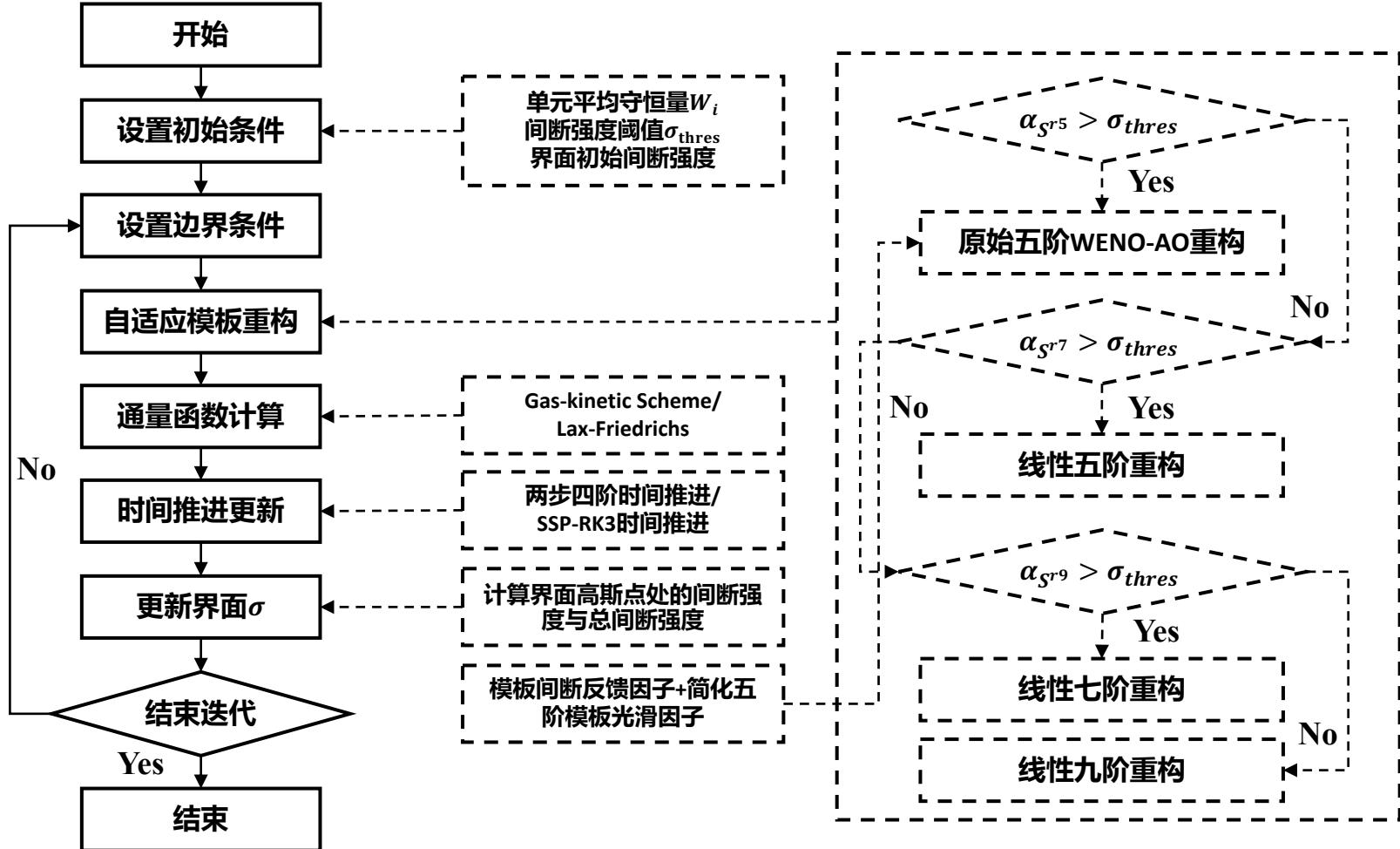
$$W(x, t) = \int \psi f(x, t, \mathbf{u}, \xi) d\xi$$

$$\frac{dW_i}{dt} = -\frac{1}{\Delta x} (\mathbf{F}_{i+1/2}(t) - \mathbf{F}_{i-1/2}(t))$$

$$(W_x)_i = \frac{1}{\Delta x} \int_{I_i} \frac{\partial W}{\partial x} dx = \frac{1}{\Delta x} (W_{i+1/2} - W_{i-1/2})$$

2. 基于间断反馈因子的有限体积格式

基于间断反馈因子的算法流程

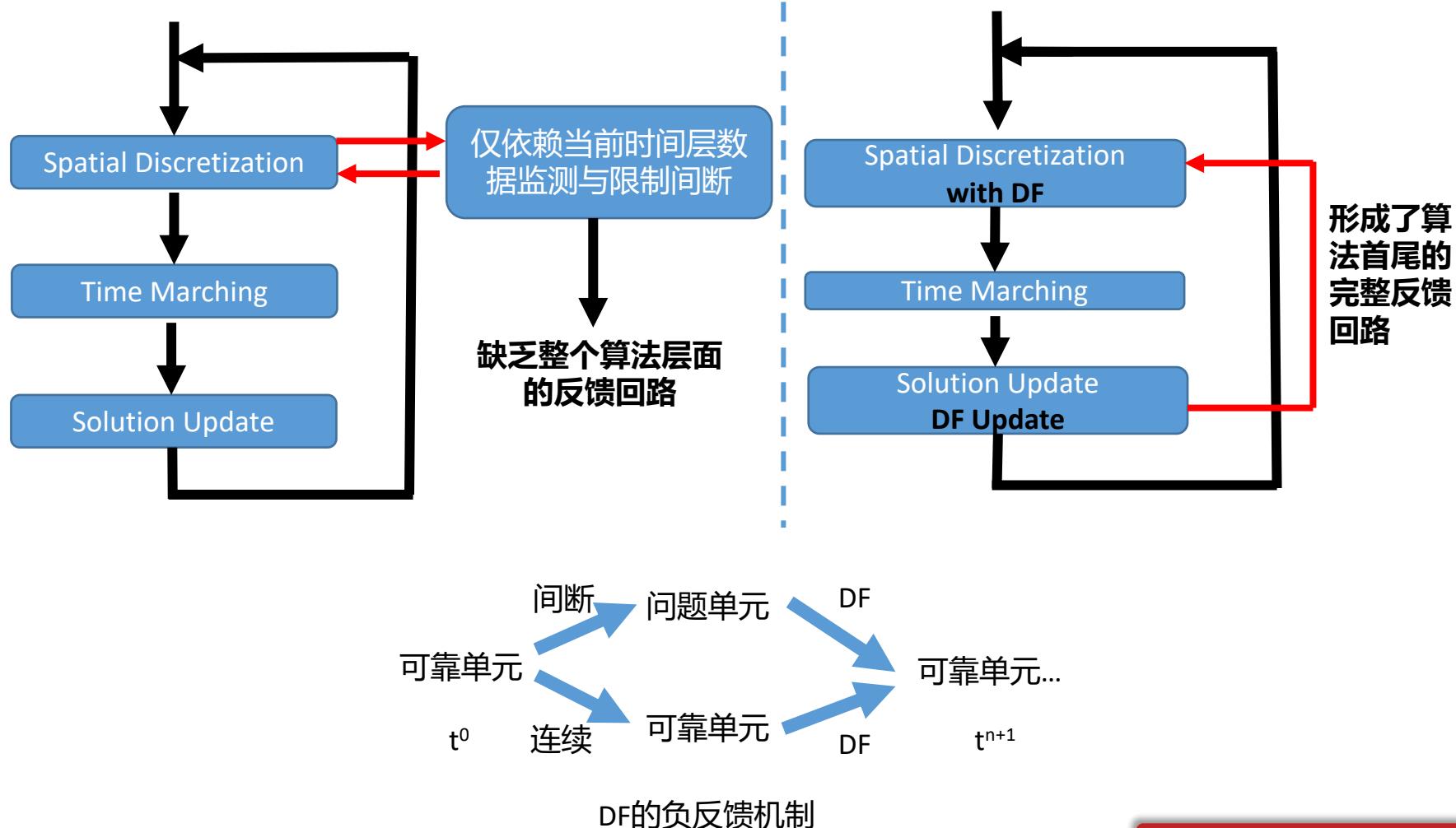


间断反馈因子的优势

1. 负反馈调节机制使得自由参数调节不敏感
2. 物理意义明确
3. 精度保证且易做到更高阶
4. 计算量小
5. 存储量小

间断反馈因子的优势

1. 负反馈调节机制使得自由参数调节不敏感



间断反馈因子的优势

2. 物理意义明确

$$\sigma_{i+1/2,m} = \frac{|p^l - p^r|}{p^l} + \frac{|p^l - p^r|}{p^r} + (\text{Ma}_n^l - \text{Ma}_n^r)^2 + (\text{Ma}_\tau^l - \text{Ma}_\tau^r)^2$$

激波

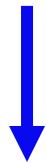
稀疏波

剪切层

间断反馈因子的优势

3. 精度保证且易做到更高阶

$$\sigma_{i+1/2,m} = \frac{|p^l - p^r|}{p^l} + \frac{|p^l - p^r|}{p^r} + (\text{Ma}_n^l - \text{Ma}_n^r)^2 + (\text{Ma}_\tau^l - \text{Ma}_\tau^r)^2$$



记压力真值为 p , 重构阶数为 k

$$\frac{|p^l - p^r|}{p^l} = \frac{|p + O(\Delta x^k) - (p + O(\Delta x^k))|}{p + O(\Delta x^k)} = O(\Delta x^{k+1})$$

自动满足任意 k 阶重构精度要求 !

间断反馈因子的优势

- 4. 计算量小
- 5. 存储量小

每个单元/界面仅增加1个数据

计算复杂度分析 (假设k阶重构)

- 1. 多项式计算复杂度：最终的重构多项式是对每个 Stencil点的非线性组合，子Stencil的计算复杂度为 $O(k)$ ，多项式个数为 $O(k)$ ，总计算复杂度为 $O(k^2)$
- 2. 平滑指标计算复杂度：每个子模板的多项式是 $(k + 1)/2$ 阶，对其求导并平方，最多涉及 $(k - 1)/2$ 次导数的计算，自模板为 $O(k)$ 个，
总计计算复杂度为 $O(k^3)$

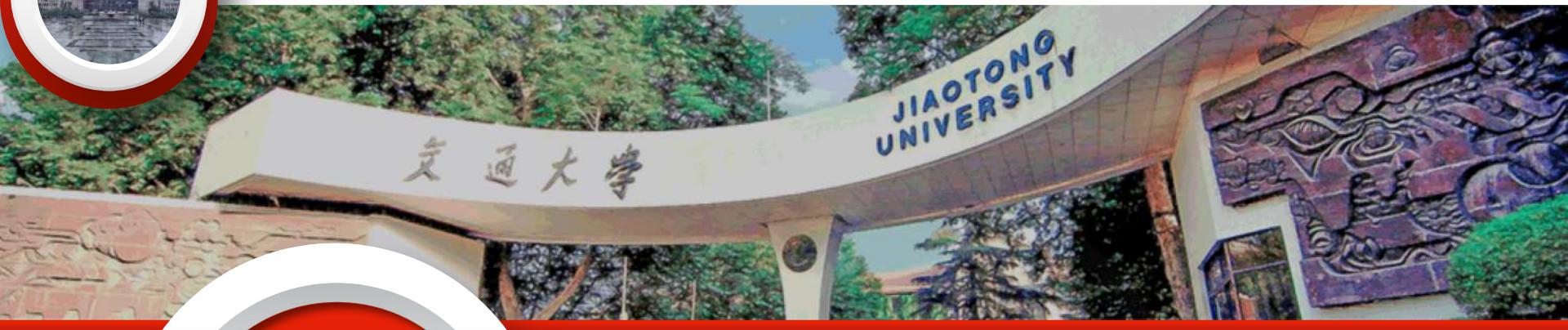
WENO-Z

- 1. 多项式计算复杂度：最终的重构多项式是对每个 Stencil点的非线性组合，子Stencil的计算复杂度为 $O(k)$ ，多项式个数为 $O(k)$ ，总计算复杂度为 $O(k^2)$
- 2. 平滑指标计算复杂度：每个子模板的多项式是 $(m + 1)/2$ 阶, $m=2,3,\dots,k$ ，对其求导并平方，最多涉及 $(m - 1)/2$ 次导数的计算，自模板为 $O(k)$ 个，
总计计算复杂度为 $O(k^3)$

Multi-resolution WENO

- 1. DF计算复杂度：DF在不断增加阶数过程中需要持续计算，计算结果可以继承，但涉及到除法，**复杂度 $O(k)$**
- 2. 无平滑因子计算

DF-WENO



3

典型场景验证及应用

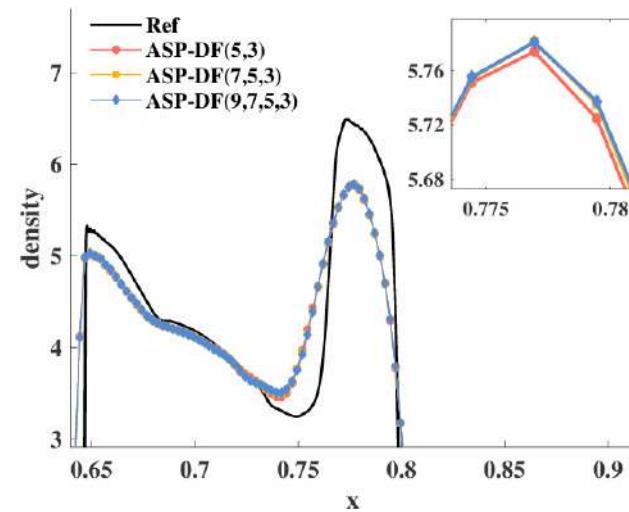
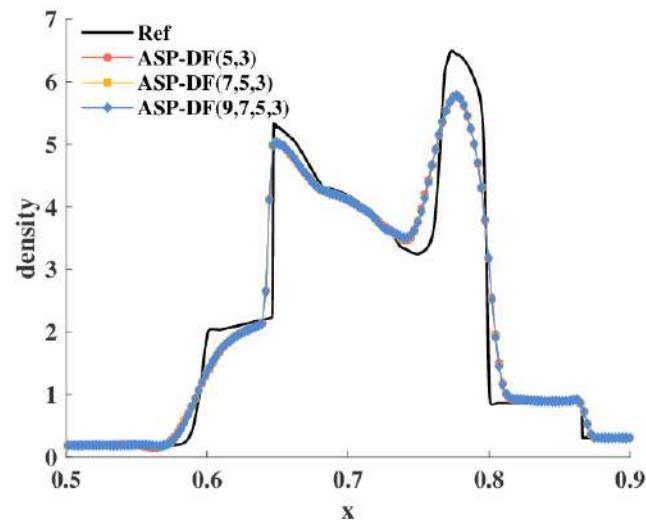
3. 典型场景验证及应用

一维任意高阶有限体积格式算例验证

Accuracy test

N	ASP-DF(5,3)	order	ASP-DF(7,5,3)	order	ASP-DF(9,7,5,3)	order
20	2.585904e-05		6.366042e-07		1.424192e-08	
40	8.337553e-07	4.95	5.108345e-09	6.96	2.848478e-11	8.97
80	2.680325e-08	4.95	4.042055e-11	6.98	9.085371e-14	8.29
160	8.585536e-10	4.96	3.343673e-13	6.92	2.544534e-16	8.48

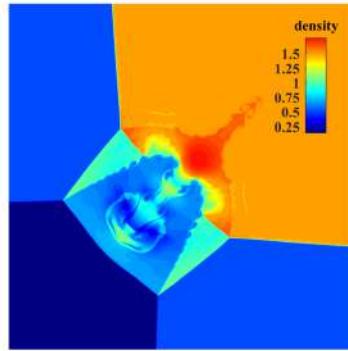
Blast wave problem



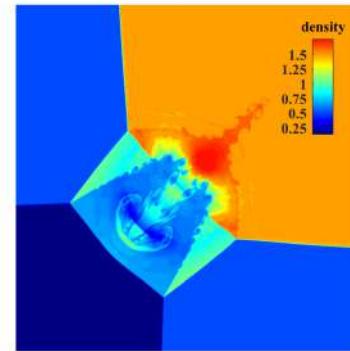
3. 典型场景验证及应用

二维任意高阶有限体积格式算例验证

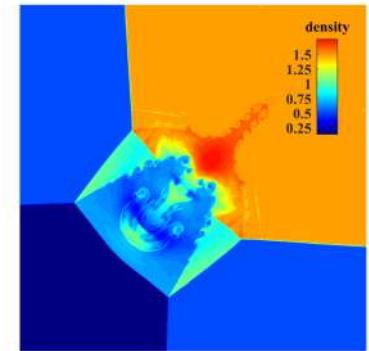
Two-dimensional Riemann problem



(a) ASE-DF(5,3)

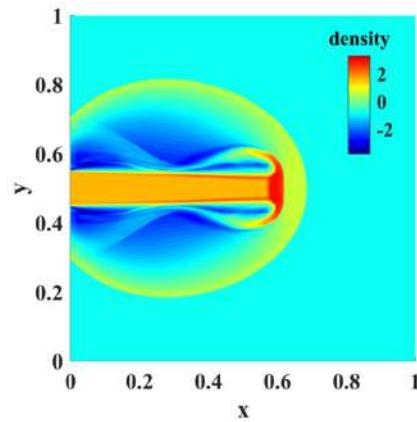


(b) ASE-DF(7,5,3)

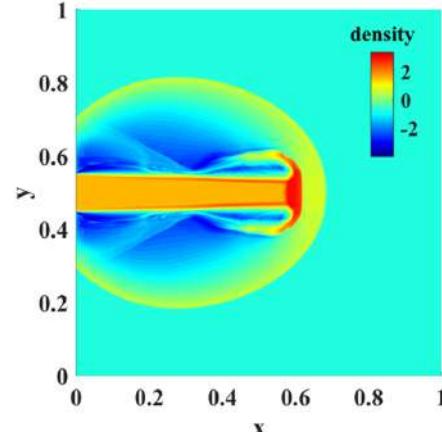


(c) ASE-DF(9,7,5,3)

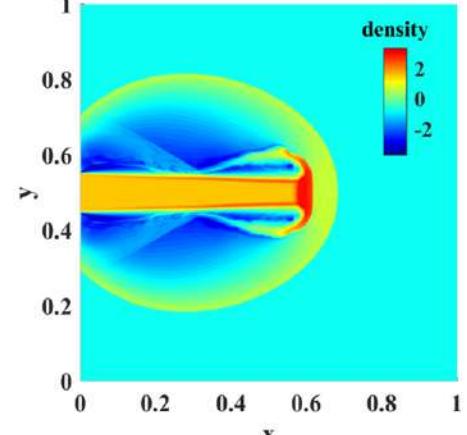
Ma = 20000 astrophysical jet



(a) ASE-DF(5,3)



(b) ASE-DF(7,5,3)

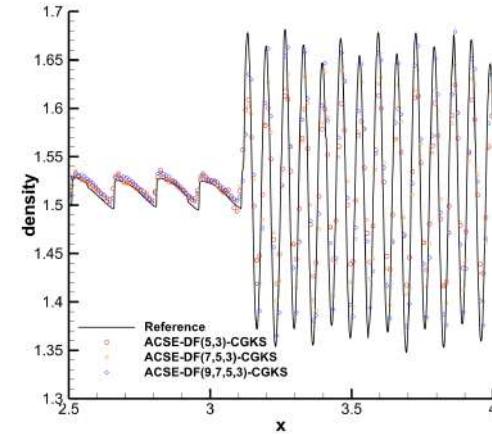
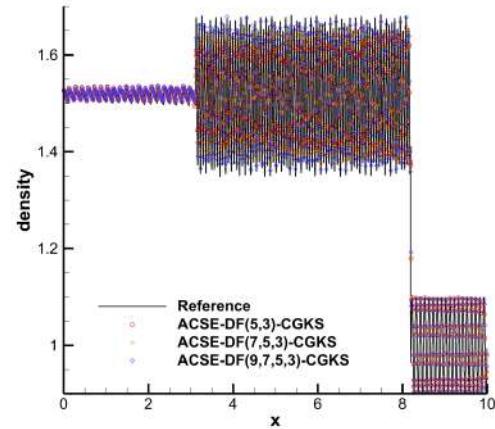


(c) ASE-DF(9,7,5,3)

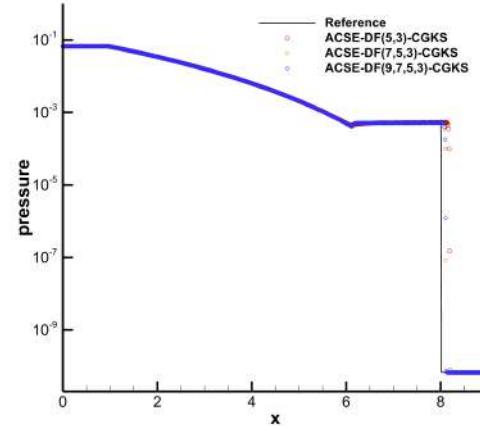
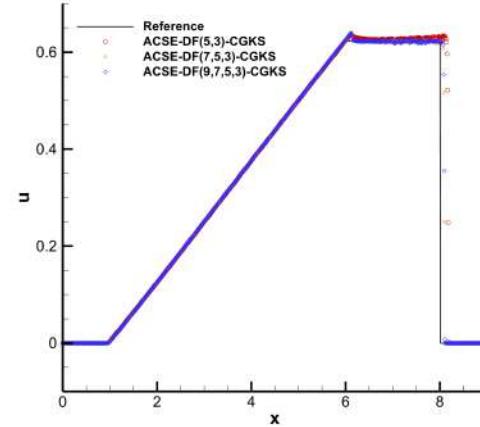
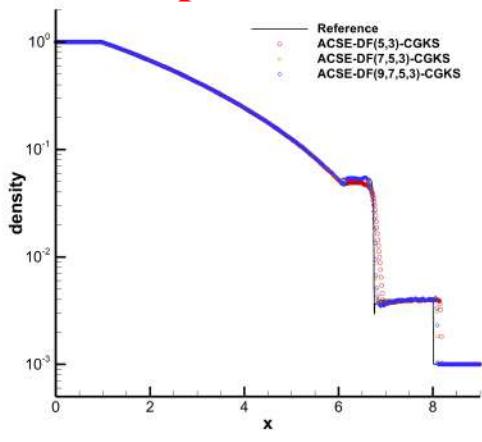
3. 典型场景验证及应用

一维任意紧致高阶气体动理学格式算例验证

Titarev-Toro problem



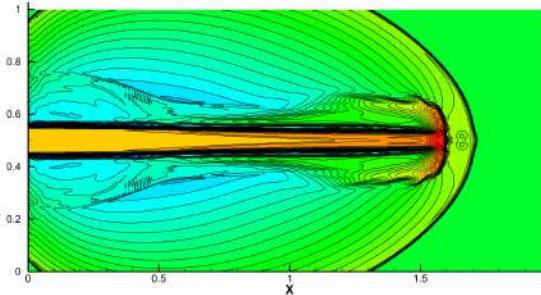
Le Blanc problem



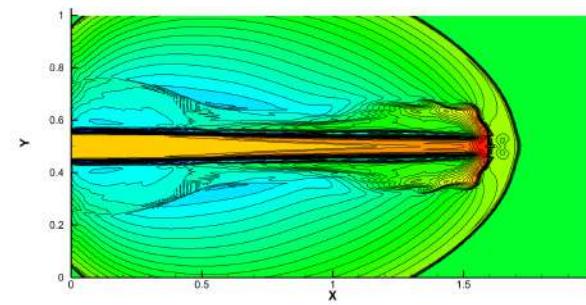
3. 典型场景验证及应用

二维任意紧致高阶气体动理学格式算例验证

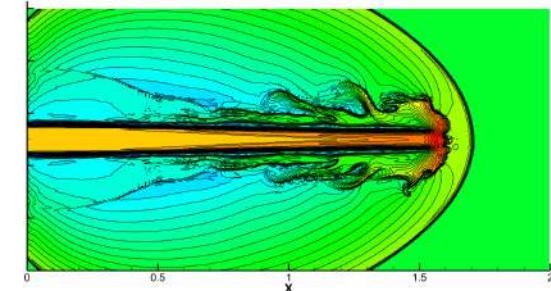
High Mach number astrophysical jet($\text{Ma}=80$)



(a) ACSE-DF(5,3)-CGKS

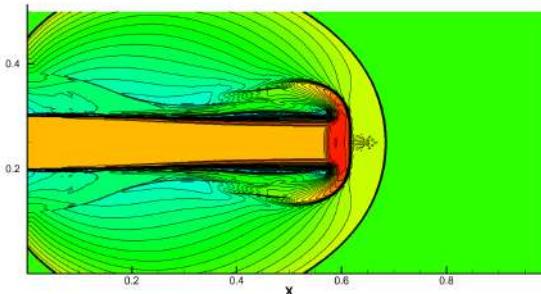


(b) ACSE-DF(7,5,3)-CGKS

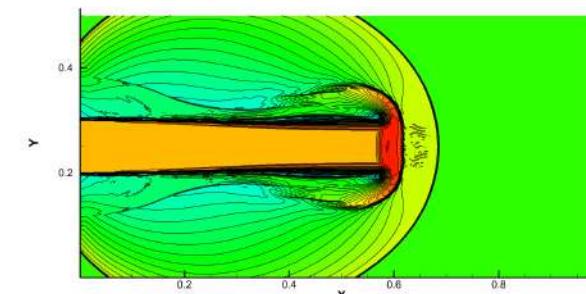


(c) ASE-DF(9,7,5,3)-CGKS

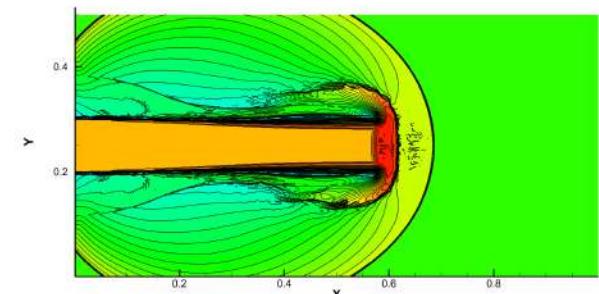
High Mach number astrophysical jet($\text{Ma}=2000$)



(a) ACSE-DF(5,3)-CGKS



(b) ACSE-DF(7,5,3)-CGKS



(c) ASE-DF(9,7,5,3)-CGKS

3. 典型场景验证及应用

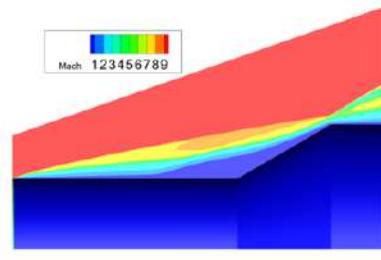
三维非结构网格高阶计算

高超声速空心圆柱裙

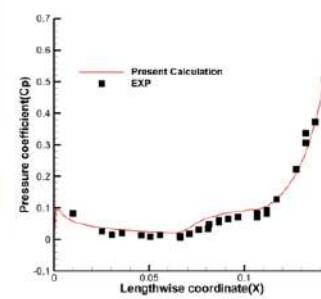
Ma	$T_{\infty}(k)$	$P_{\infty}(Pa)$	$L(m)$	$T_w(k)$	$\rho(kg / m^3)$	$Re_x(/m)$	Re
9.91	51	6.32	0.1017	293	0.43×10^{-3}	1.86×10^5	1.891×10^4



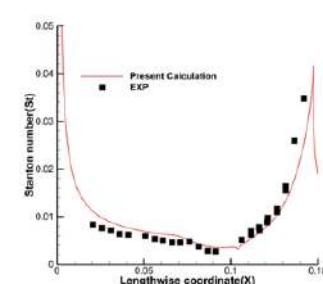
地面对试验设置



局部流场马赫数云图



表面压力分布



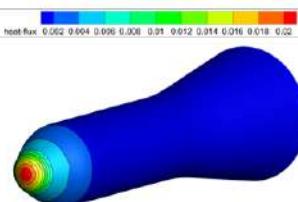
表面热流分布

高超声速弹道导弹模型 HB-2

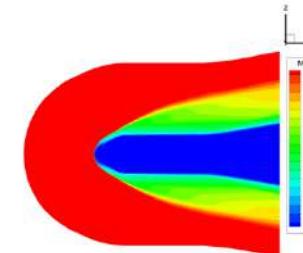
Ma	$\alpha(^{\circ})$	$\beta(^{\circ})$	$Re_x(/m)$	$T_{\infty}(k)$	$P_{\infty}(Pa)$
9.46	0	0	9.4×10^5	50.4	32.8



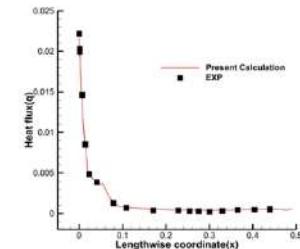
地面对试验设置



热流通量云图



二维界面马赫数云图



表面热流分布对比

3. 典型场景验证及应用

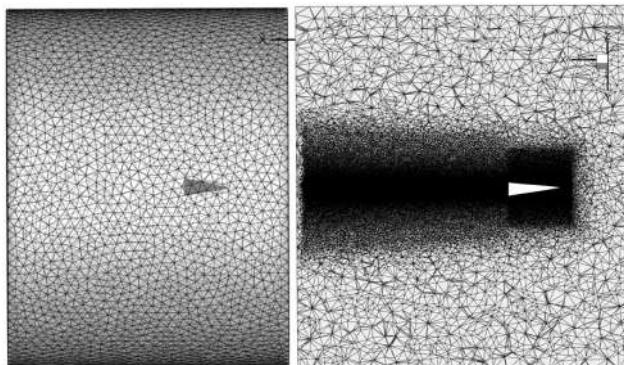
三维非结构网格高阶计算

HTV-2高超声速底部流动

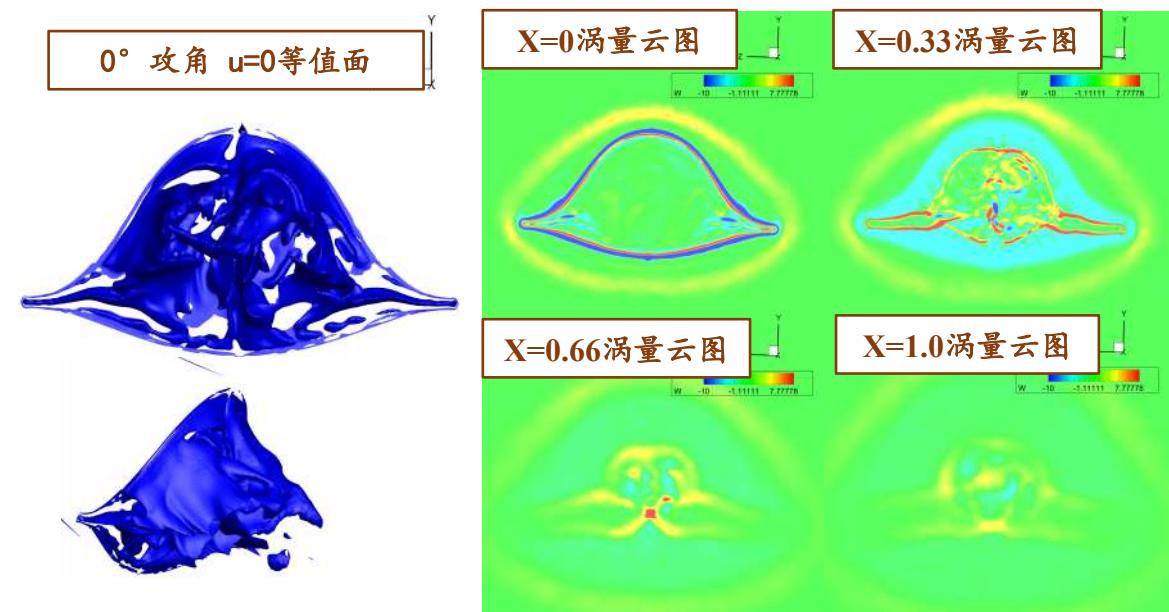
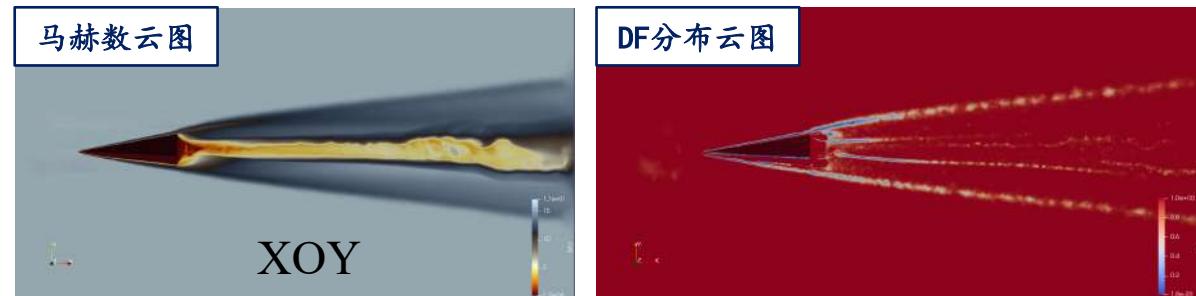
$Ma = 16.38, Re = 8.7 \times 10^5$



几何构型



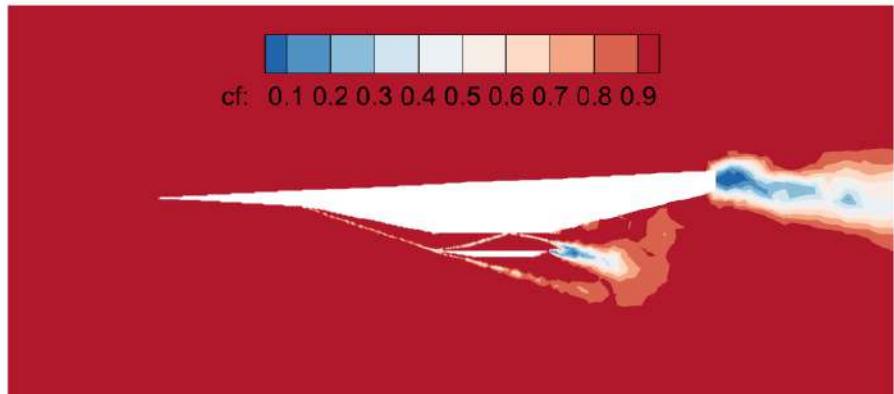
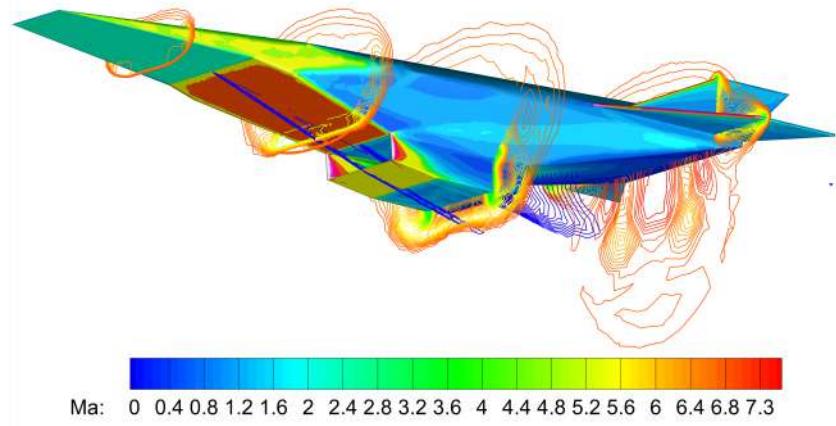
全局及局部网格拓扑



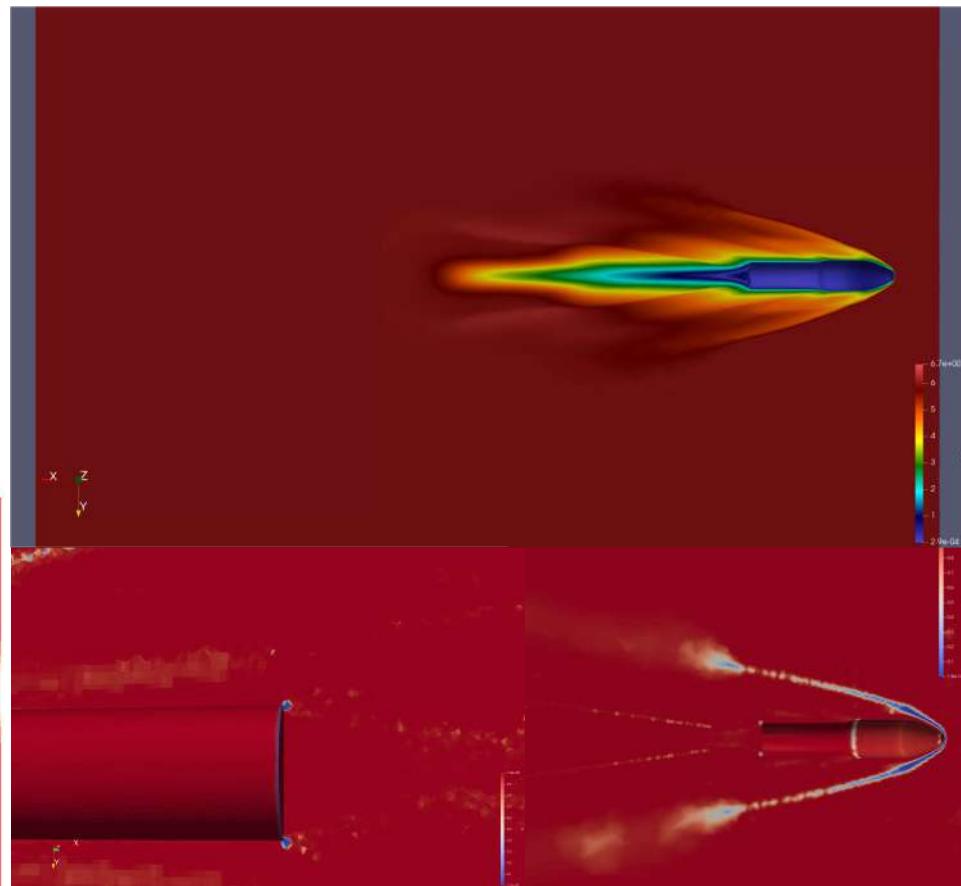
3. 典型场景验证及应用

三维非结构网格高阶计算

高超声速X-43算例



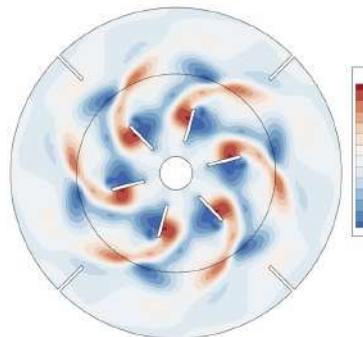
高超声速整流罩算例



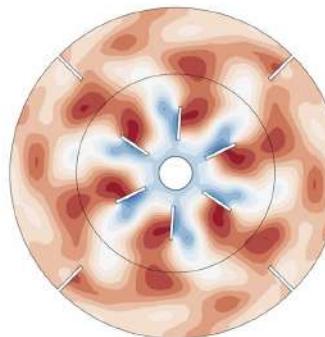
3. 典型场景验证及应用

高阶滑移网格计算

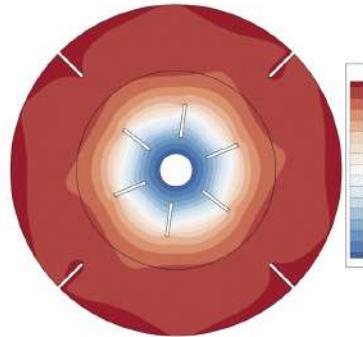
层流搅拌罐



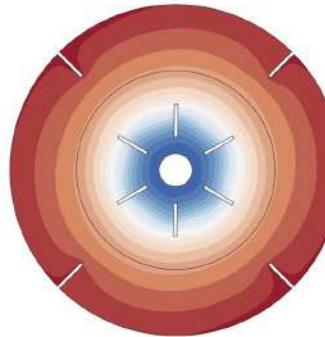
(a)



(b)

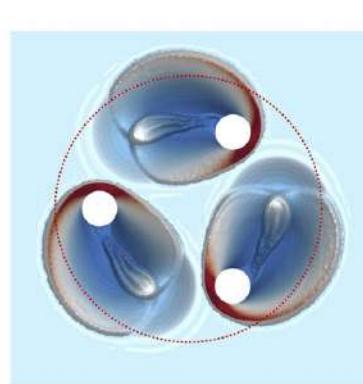


(c)

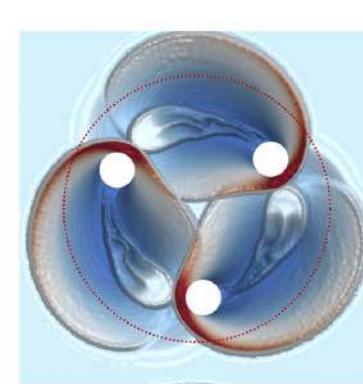


(d)

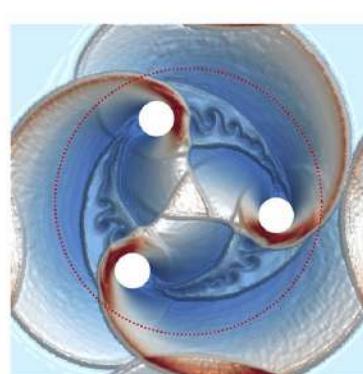
超声速三旋转圆柱



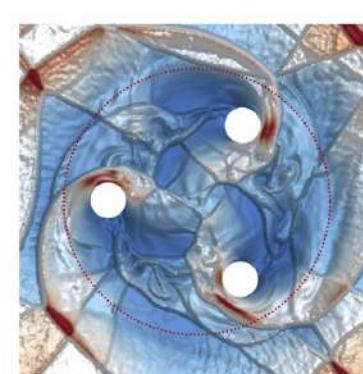
(a)



(b)



(c)

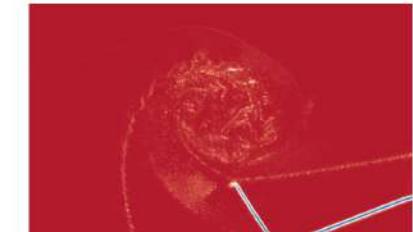
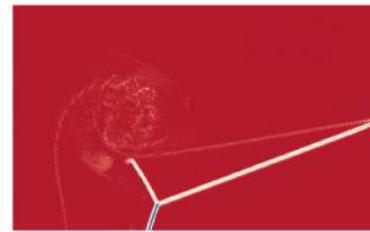


(d)

3. 典型场景验证及应用

高阶两组分流动仿真

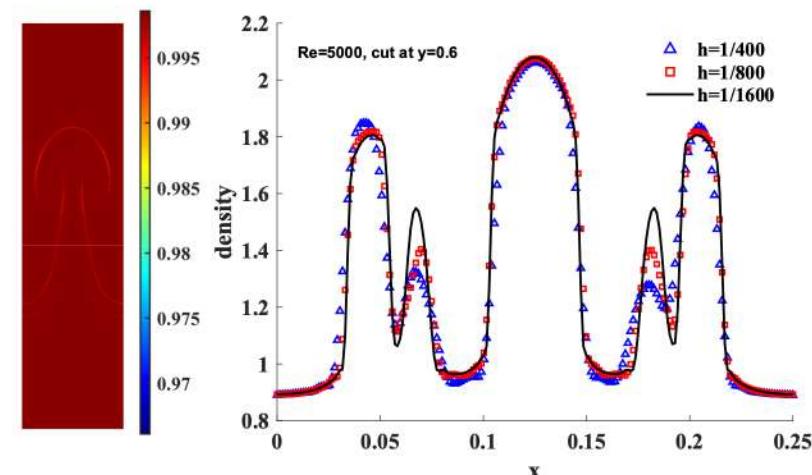
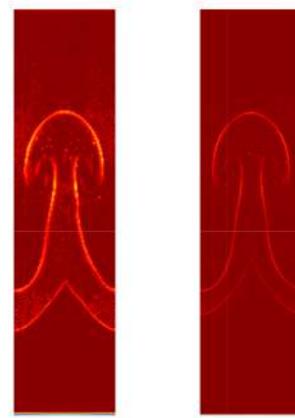
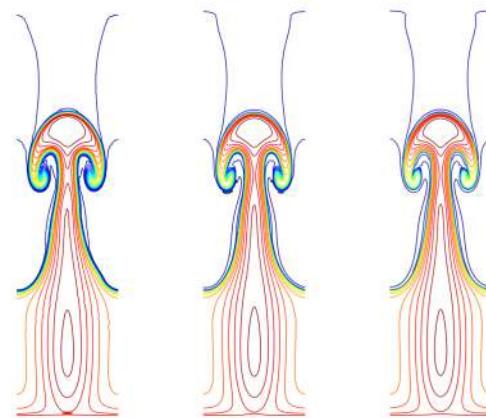
Two-component Triple-Point Problem



(a) $t=3.5s$. Left: total density distribution. Right: df factor distribution

(b) $t=5.0s$. Left: total density distribution. Right: df factor distribution

Two-component Rayleigh-Taylor problem



(a) $h = \frac{1}{400}, \frac{1}{800}, \frac{1}{1600}$ from left to right

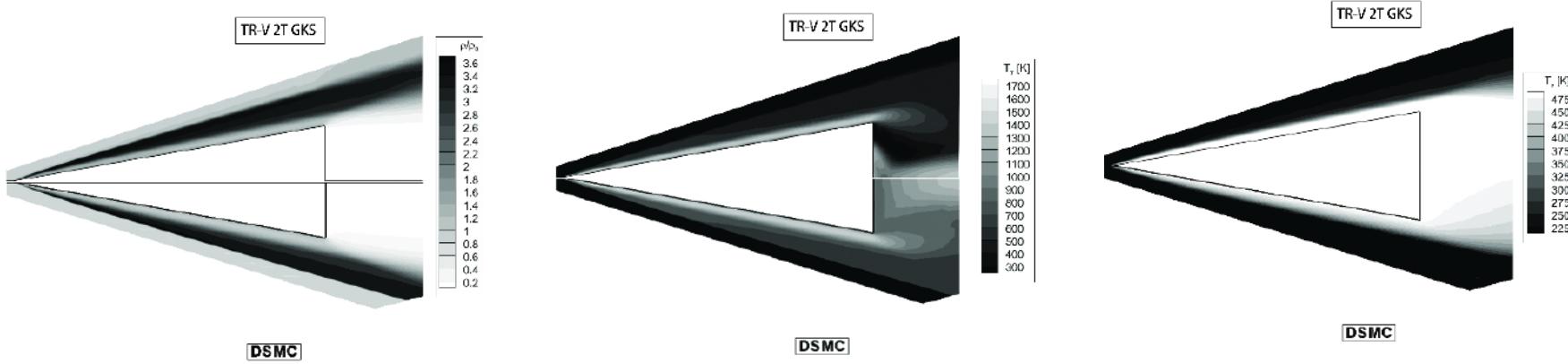
(b) DF factor distribution

(c) Mesh refinement

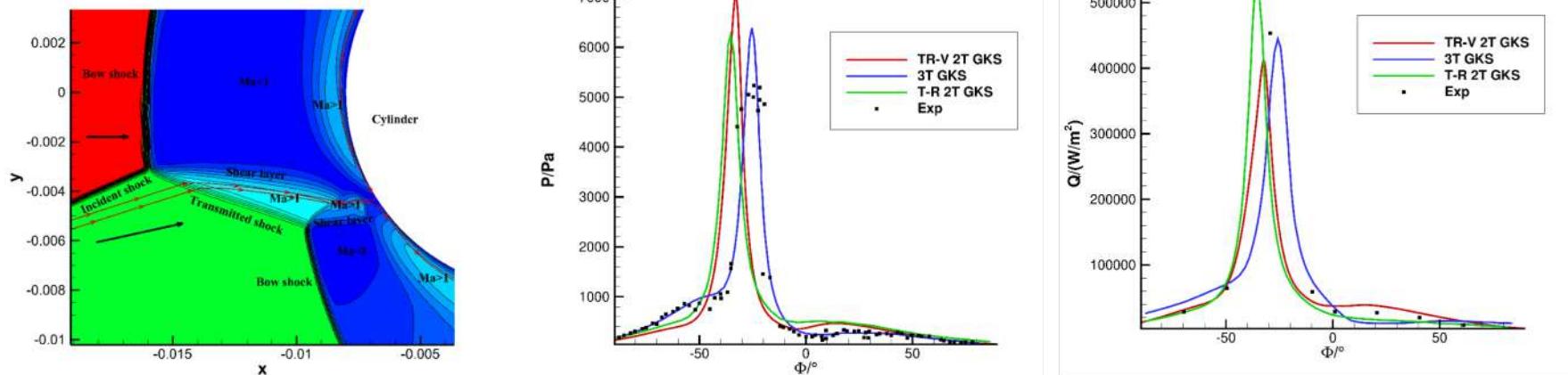
3. 典型场景验证及应用

高阶多温度真实气体效应仿真

高超声速楔体绕流



Edney第IV类激波/激波干扰



3. 典型场景验证及应用

GPU高阶异构计算

实验平台

使用三种 GPU 集群对 GKS-LES 进行了详细的性能测试，分别是 AMD-MI50 集群，NVIDIA-V100 集群与 DCU 集群。MI50集群与V100集群均是单节点八卡机，DCU集群则是“东方”超算计算环境，采用的配置是单节点四卡机。三个集群中每张GPU加速卡的显存均是16GB。

GPU集群配置

集群	AMD-MI50	NVIDIA-V100	DCU
CPU	Intel® Xeon® Gold 5117	Intel® Xeon® Gold 5117	Hygon C86 7185
内存	8*16GB DDR4 2666MHz	8*16GB DDR4 2666MHz	8*16GB DDR4 2666MHz
网络	Infiniband 100Gb/s 1 vports	Infiniband 100Gb/s 1 vports	Infiniband 200Gb/s 4 vports
操作系统	Ubuntu 20.04	Ubuntu 20.04	CentOs 7.6
MPI 环境	OpenMPI 5.0.5	OpenMPI 5.0.5	OpenMPI 4.1.4

测试指标

$$\text{加速比} = \frac{\text{实际计算墙钟时间}}{\text{参考墙钟时间}}$$

$$\text{并行效率} = \frac{\text{加速比} \times \text{参考进程数}}{\text{实际计算进程数}}$$

测试方法

对主体迭代演化部分的墙钟时间进行测试，不计入前处理如网格读入等与后处理流场输出等时间，计入主体计算时间与数据通信时间。为验证GPU加速算法较原始CPU算法的加速效果，使用Intel® Xeon® Gold 5117 CPU处理器计算的结果进行对比。

3. 典型场景验证及应用

GPU高阶异构计算

GPU加速算法与串行算法比较

串行算法：Intel® Xeon® Gold 5117 14核CPU处理器，单进程运行

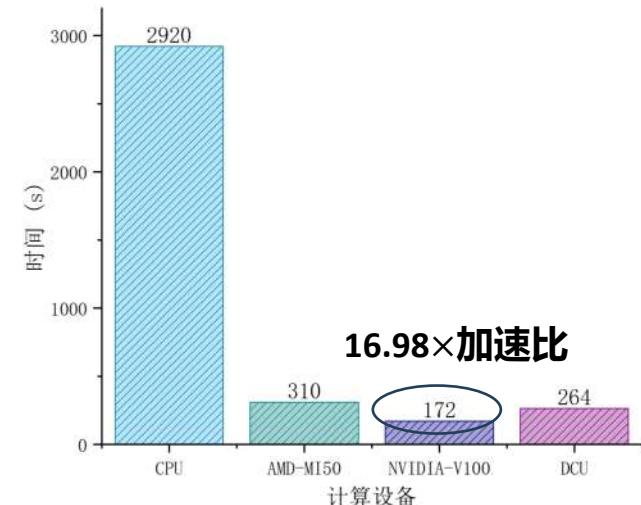
三种GPU加速下相较于串行算法的加速比

计算设备	计算时间	加速比
CPU（串行）	30813s	-
MI50	310s	99.40×
V100	172s	179.15×
DCU	264s	116.71×

不同GPU双精度浮点计算能力和内存带宽存在差异，实现的加速比也各不相同。不同GPU架构的硬件设计差异亦会对加速效果产生显著影响。如基于AMD Vega 20架构的MI50缺乏专用的常量缓存硬件，常量内存在实际分配时仍被映射至全局内存，影响了加载速度和整体性能表现。

GPU加速算法与单CPU并行算法比较

单CPU并行算法：Intel® Xeon® Gold 5117 14核CPU处理器，14个MPI进程并行计算



多GPU加速与单CPU并行效率比较

计算设备	4 卡计算时间	4 卡加速比	8 卡计算时间	8 卡加速比
MI50	83s	35.18×	45s	64.89×
V100	50s	58.40×	26s	112.31×
DCU	71s	41.12×	38s	76.84×

3. 典型场景验证及应用

GPU高阶异构计算

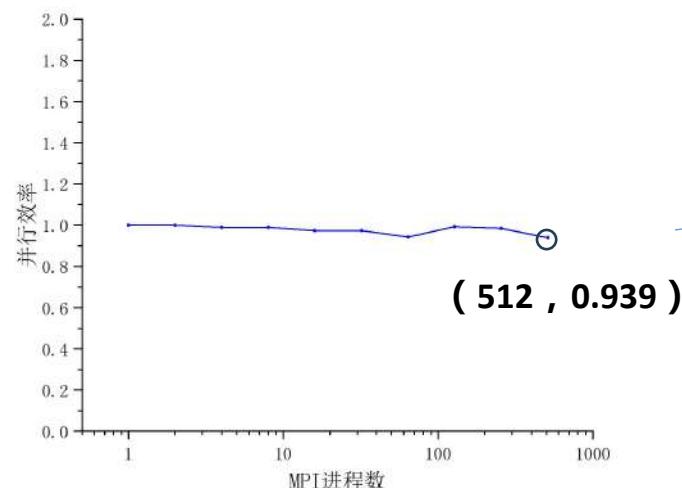
强拓展性测试

测试算例：一千万非结构混合网格三维圆柱绕流，以4卡并行测试速度为基准。强拓展性指在固定问题规模不变的情况下，随着计算资源（这里等同于使用的DCU卡数）的增加，程序运行时间是否显著减少。

DCU 卡数	4	8	16	32	64	128	256	512
时间/s	2613	1315	668	349	180	101	64	39
加速比	1.0	1.987	3.912	7.487	14.517	25.871	40.828	67
并行效率	1.0	0.994	0.978	0.936	0.907	0.808	0.638	0.523

弱拓展性测试

测试算例：三维正弦波精度测试算例，保持每个进程/加速卡处理1024000个计算单元，逐步扩大进程数。弱拓展性是指当问题规模与计算资源同步增加时，程序的运行效率是否能保持基本稳定。



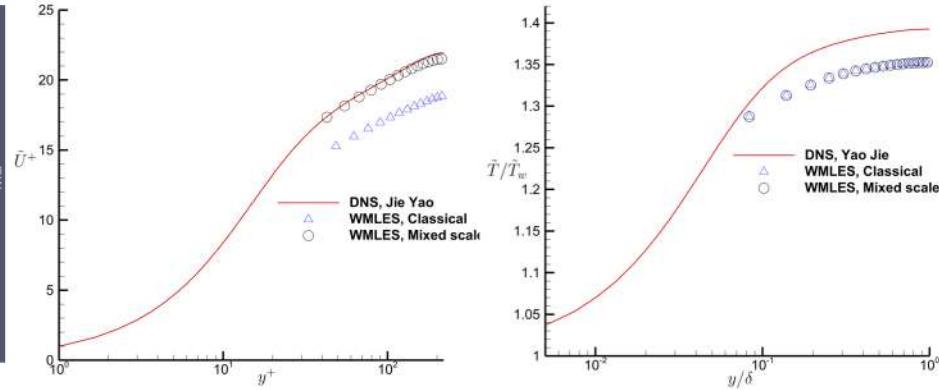
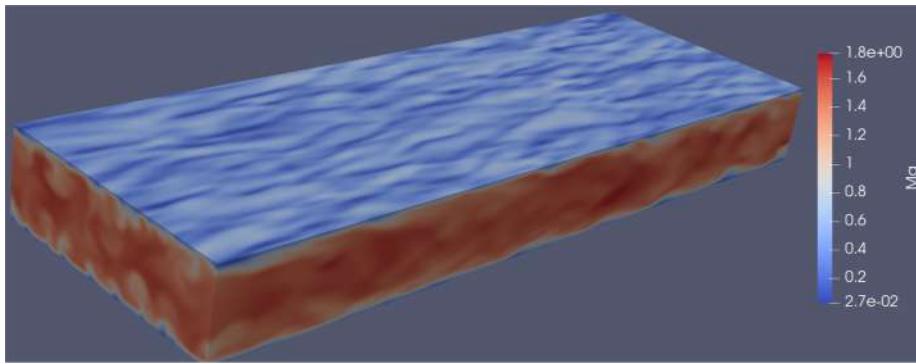
程序扩展性良好，具备大规模并行能力！

对应物理空间5.24亿
的非结构网格

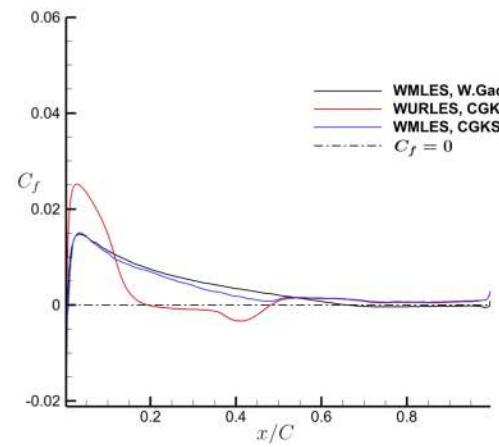
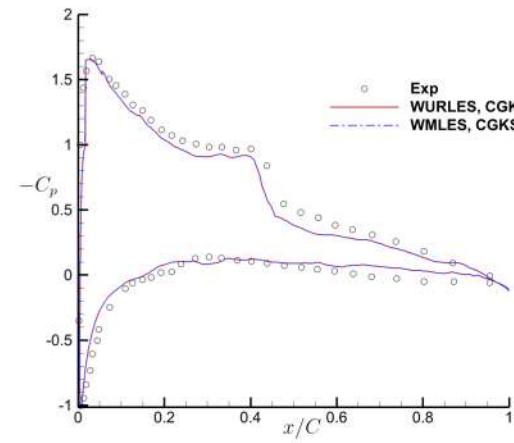
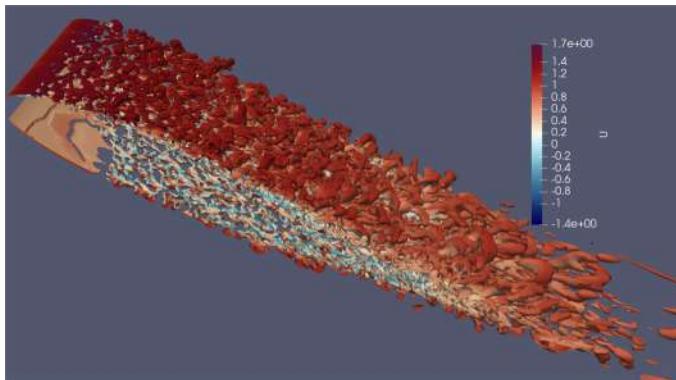
3. 典型场景验证及应用

复杂构型的壁面模化湍流模拟

可压缩槽道湍流



NACA0012翼型 $Re = 10^5$



3. 典型场景验证及应用

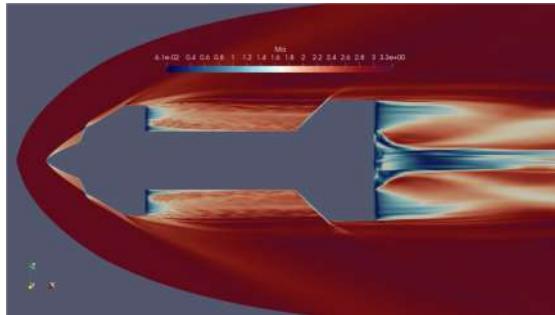
复杂构型的壁面模化湍流模拟

高超声速星舰构型数值模拟

超声速计算状态

Case	Ma	H	A
Case 2	3	40	0°

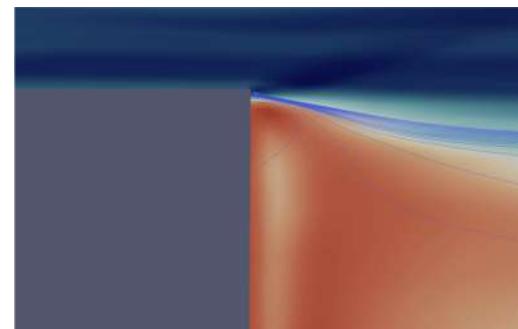
间断反馈因子在多激波干扰区域表现良好，准确地识别并捕捉到多层激波结构



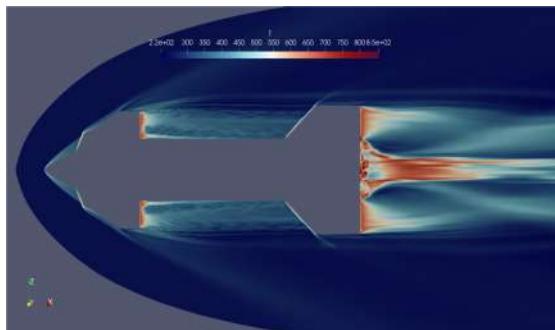
俯视马赫数分布



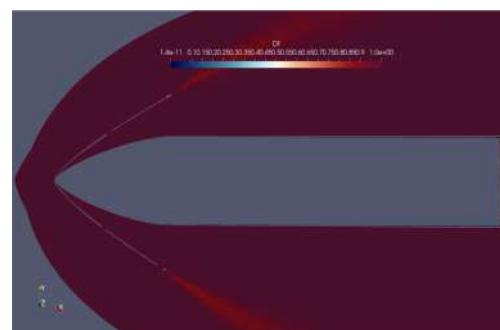
正面间断反馈因子分布



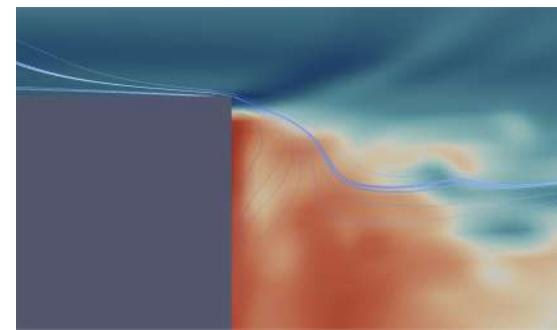
前翼根部区局部流线图



俯视温度分布



侧面间断反馈因子分布



后翼与底部交界处局部流线图

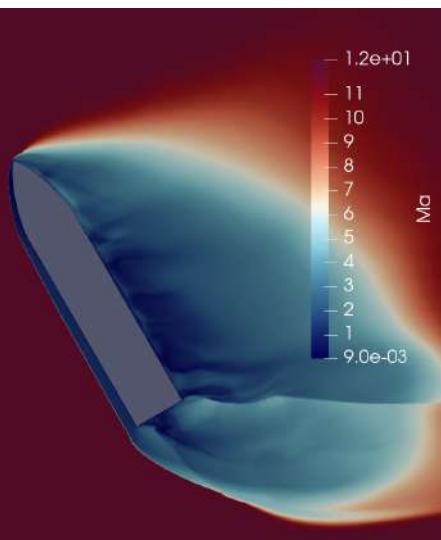
3. 典型场景验证及应用

复杂构型的壁面模化湍流模拟

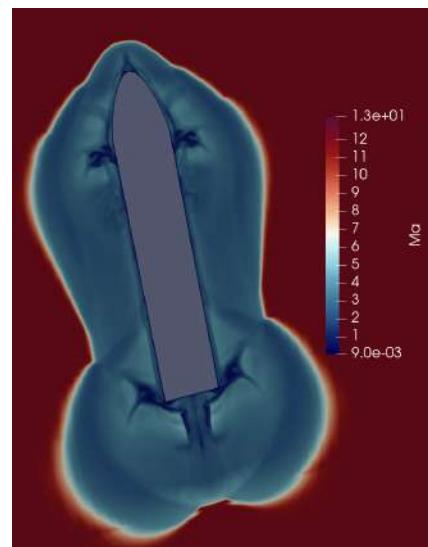
高超声速星舰构型数值模拟

飞行高度	空气密度/ $kg \cdot m^{-3}$	空气温度/K	空气粘性/Pa·s	飞行马赫数
10km	4.14×10^{-1}	223.3	1.46×10^{-5}	0.2/0.7
40km	4.00×10^{-3}	250.4	1.59×10^{-5}	3
70km	8.77×10^{-5}	218.6	1.44×10^{-5}	12

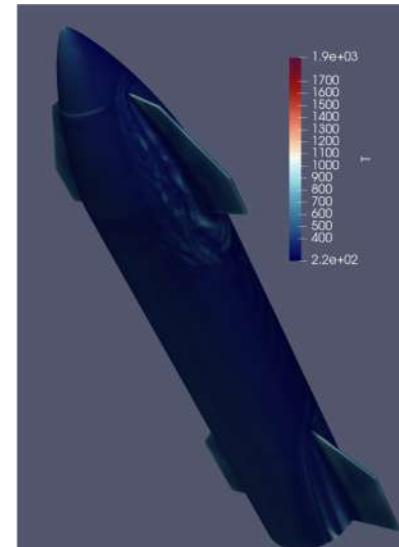
60°攻角飞行，高超声速飞行阶段



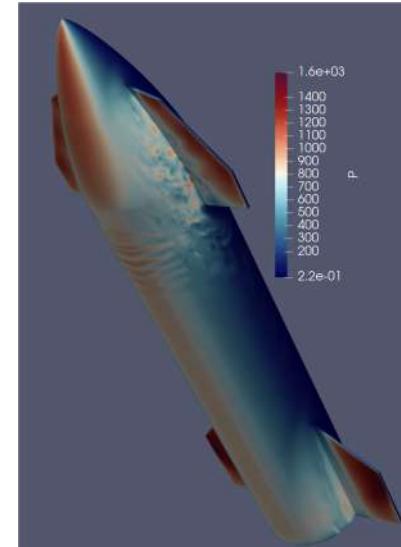
侧视截面马赫数分布



俯仰面剖面马赫数分布



星舰表面温度分布

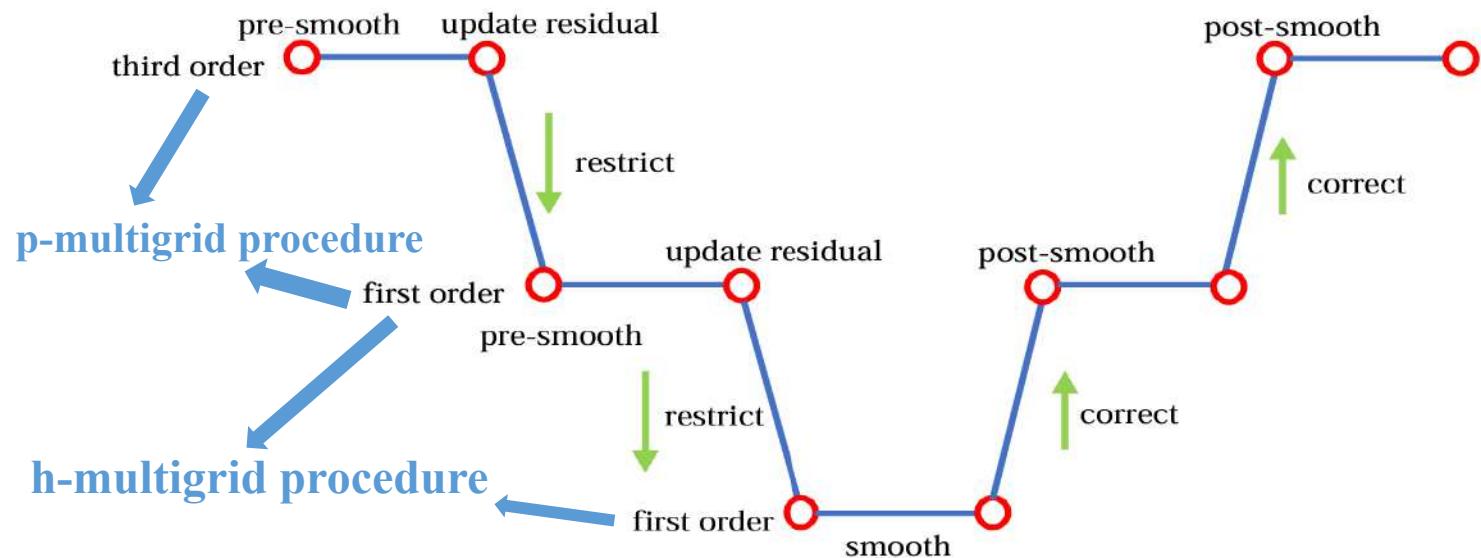


星舰表面压力分布

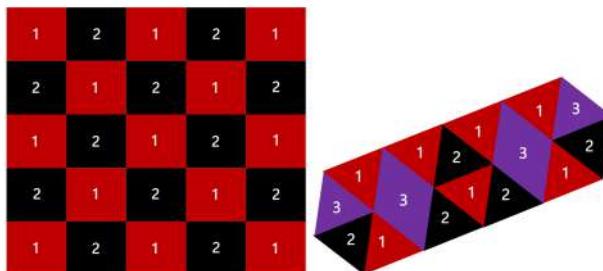
3. 典型场景验证及应用

基于GPU的高阶格式稳态加速算法

■ Flowchart of the hp-multigrid on GPU



■ Multi-Color LU-SGS to Solve Parallel Issues in Implicit Methods



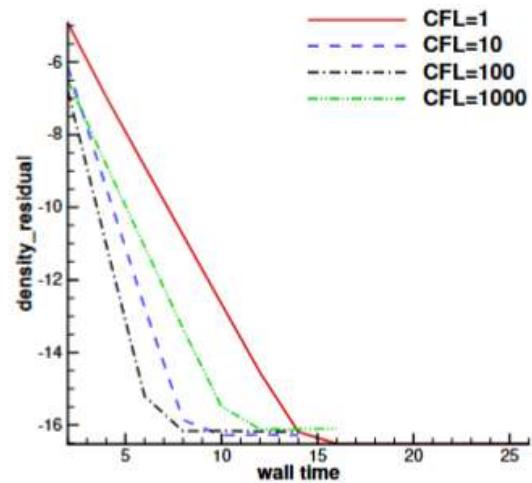
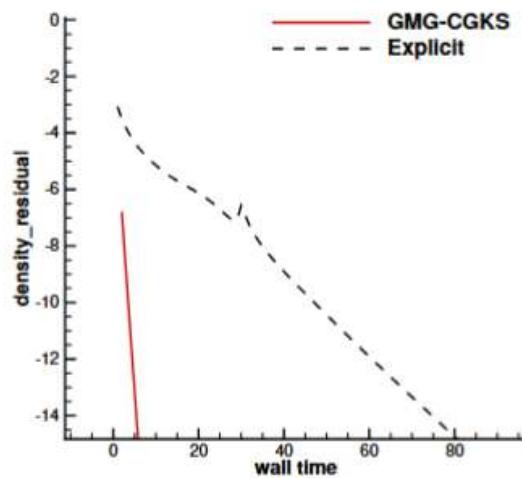
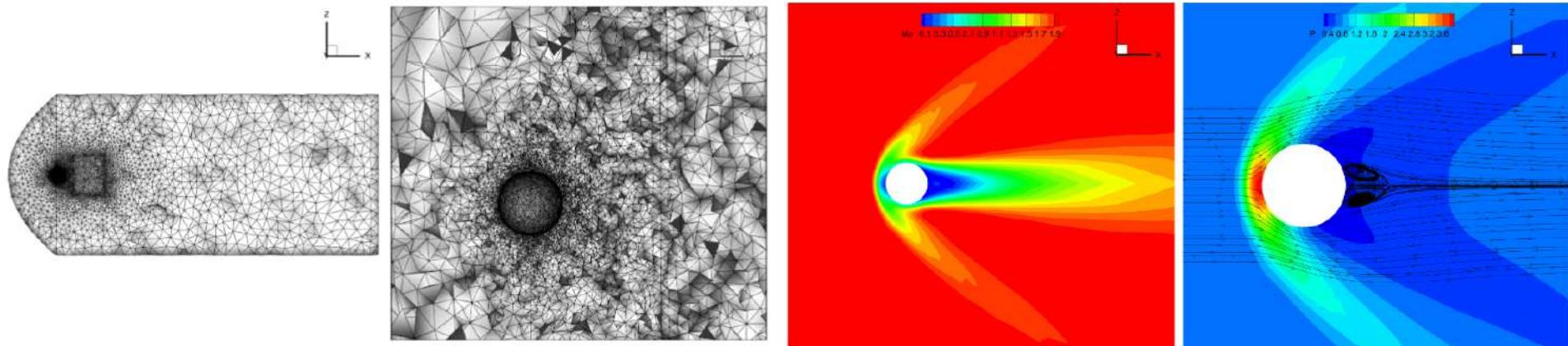
Algorithm 1: The procedure of coloring method.

```
Input: The original mesh cells  $\{C_i \mid i \in \Omega\}$  and a start cell  $v_0$ .  
Output: The array of cell colors  $\text{colorArray}(\cdot)$  and total number of colors  $N_{\text{color}}$ .  
1 initialize  $\text{colorArray}(\cdot) = 0$ ;  
2 choose a cell  $v_0$  as the start point of the traversal, and set  $\text{color}(v_0) = 1$ ;  
3 repeat  
4   for each colored cell  $\{v \mid \text{color}(v) > 0\}$  do  
5     for each uncolored cell  $\{w \mid w \in C_v\}$  do  
6       paint  $\text{color}(w) = \min \{k > 0 \mid k \neq \text{color}(j), \forall j \in C_w\}$ ;  
7     end  
8   end  
9 until all cells are painted;
```

3. 典型场景验证及应用

高阶格式稳态加速算法

超声速圆球绕流， $Ma=200$ ， $Re=300$

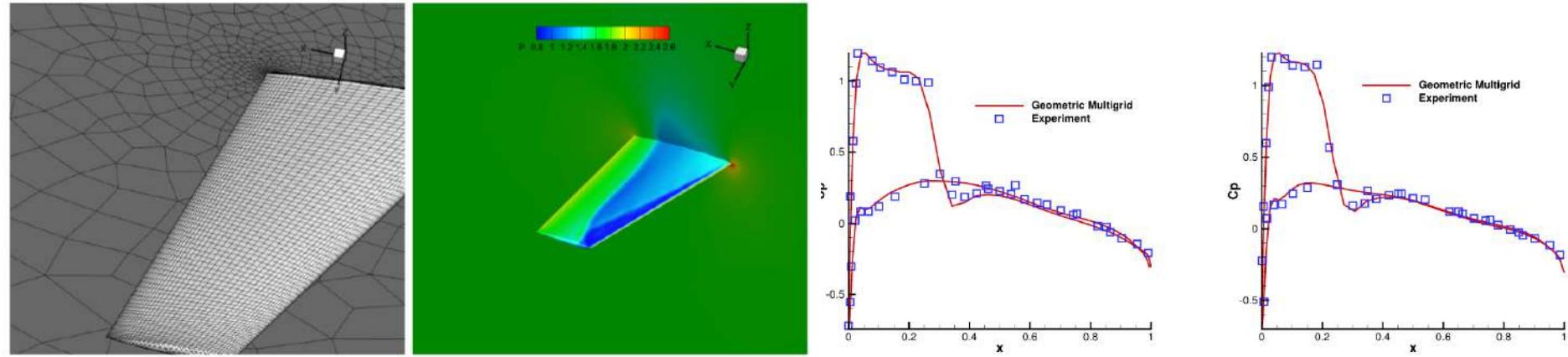


8张GPU卡仅需要
6s到达机器误差

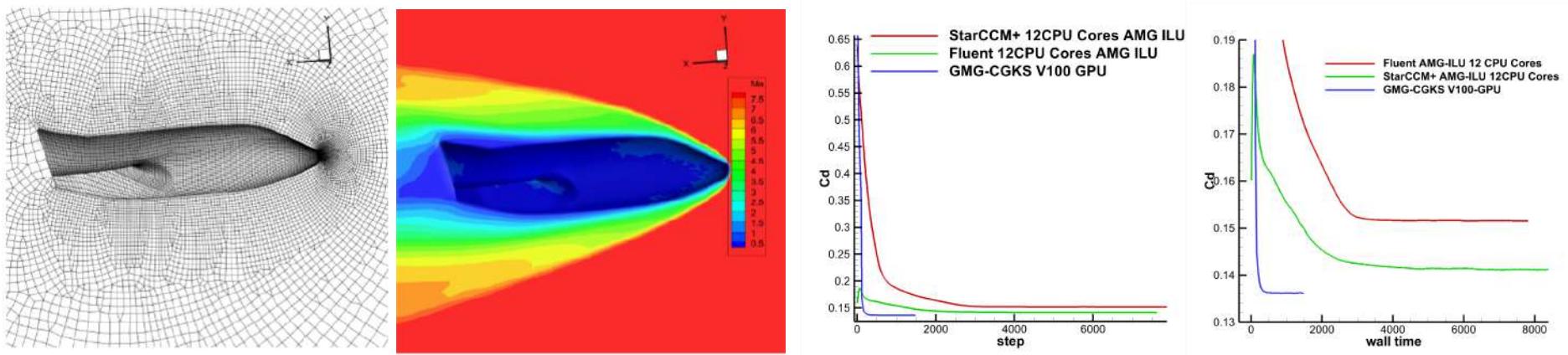
3. 典型场景验证及应用

高阶格式稳态加速算法

跨声速M6机翼流动， $Ma=0.8395$



高超声速X-38飞行器， $Ma=8.0$ ， $Re=14289$





2025年湍流与噪声和CFD方法暑期高级讲习班

谢谢！

July 9 2025

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<https://osredm.com/p35462178/gks2d-str>



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