Fitting Volatility Surfaces of Index Options Using Variational Autoencoders (VAE)

COS 513 – Spring 2025 – Project Proposal

Zhuolin Xiang, Haoqian Zhang, Siqing Zou
Princeton University
{zx4614,hz0531,sz6982}@princeton.edu

Abstract

This project aims to explore the application of Variational Autoencoders (VAEs) in modeling and fitting the implied volatility surface of SPX index options. Traditional parametric models of stochastic volatility such as Heston and SABR impose rigid structural assumptions that may not fully capture market nuances. A VAE-based approach offers a flexible, data-driven alternative that can adapt to diverse market conditions, interpolate missing volatility data, and provide a generative framework for synthetic volatility surfaces. Our project will investigate the accuracy, robustness, and computational efficiency of VAEs compared to traditional models, with the goal of improving volatility estimation in options pricing and risk management.

1 Background and Motivations

Volatility modeling is crucial in financial markets, as it underpins option pricing, risk management, and trading strategies. The implied volatility surface, which represents the market's expectations of future volatility across different strike prices and maturities, is fundamental to these applications. However, existing modeling methodologies have limitations. Parametric models like Heston and SABR rely on specific assumptions that may not generalize well across varying market conditions. Moreover, market-implied volatilities often contain missing or noisy data, requiring robust interpolation and extrapolation techniques. Given the advancements in deep learning, VAEs provide a compelling alternative by learning a structured, low-dimensional representation of volatility surfaces without explicit assumptions about their shape. Our project seeks to evaluate how VAEs can enhance volatility modeling by capturing latent structures and improving both interpolation and extrapolation capabilities.

2 Methodologies

Variational Autoencoder (VAE) This study will employ a VAE framework(see fig.1) to model the SPX implied volatility surface. The VAE will be trained with an encoder-decoder structure, where the encoder maps the volatility surface into a latent space distribution, and the decoder reconstructs the surface from sampled latent variables. The training objective will balance reconstruction loss and Kullback-Leibler (KL) divergence to enforce meaningful latent representations.

We follow the structure in Bergeron et al. [2021] to construct the VAE in two ways: the **grid-based** approach and the **pointwise** approach(see fig.2).

Evaluation Once trained, the model's performance will be evaluated through reconstruction error, interpolation accuracy, and generalization ability across different market conditions. The VAE's

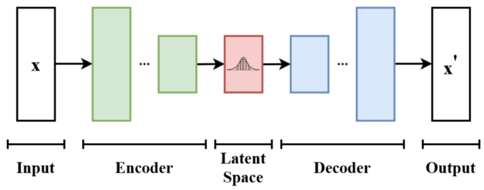
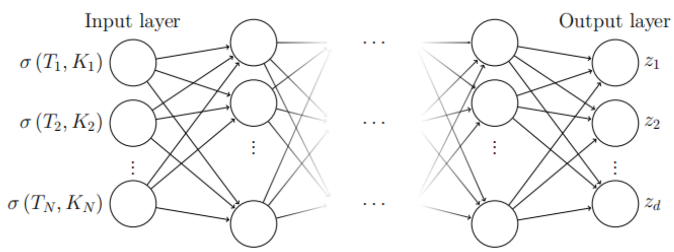
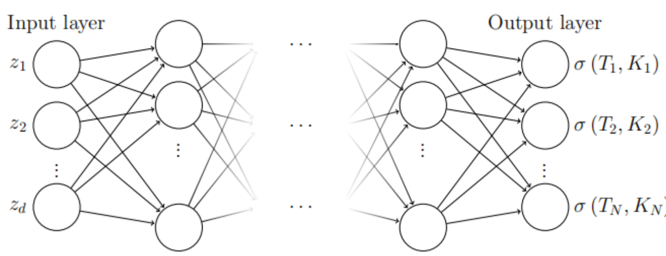


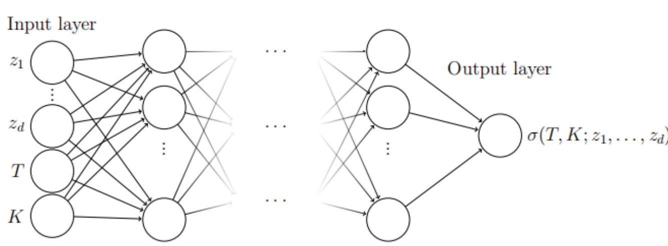
Figure 1: VAE structure Source:wikipedia



(a) The encoder architecture



(b) The decoder architecture for the grid-based training approach.



(c) The decoder architecture for the pointwise training approach.

Figure 2: Architectures of grid-based and pointwise method

output will be benchmarked against traditional stochastic models like Heston Solganik [2011] and SABR Mazzoni [2011] to assess its viability as an alternative approach.

3 Data

Source Wharton Research Data Services (WRDS) **Detailed description**

Statistics **Add statistics here**

4 Expected Challenges

Several challenges are anticipated in this project.

Ensuring data quality One of the primary difficulties is ensuring data quality, particularly in handling missing or noisy volatility values while maintaining an arbitrage-free surface. Prices of illiquid options may not reflect the true market expectation of future volatility. Also, restricted by data source availability, our data does not contain trade prices, while only providing best bid, best ask, and last trade date, which may bring difficulties in the accuracy of estimation.

Hyperparameter tuning Hyperparameter tuning is another challenge, as optimizing the trade-off between reconstruction loss and KL divergence is critical for meaningful latent space representation. Also, we need to choose the distribution and dimension of the latent space in VAEs carefully to achieve best performance with lower computation cost.

Generalization across various market conditions As financial markets exhibit regime shifts that may not be fully captured by historical data, our VAEs model may not generalize to new regimes and it can be computationally intensive to fit new models as market changes instantly.

References

Maxime Bergeron, Nicholas Fung, John Hull, and Zissis Poulos. Variational autoencoders: A hands-off approach to volatility, 2021. URL `https://arxiv.org/abs/2102.03945`.

Thomas Mazzoni. Implied and local volatility dynamics in the sabr model, 2011. URL `demonstrations.wolfram.com/ImpliedAndLocalVolatilityDynamicsInTheSABRModel/`.

Slava Solganik. Volatility surface in the heston model, 2011. URL `https://demonstrations.wolfram.com/VolatilitySurfaceInTheHestonModel/`.