张鹤潇 2018011365 Quantum Computation

1. $e^{-iHt}\cdot(e^{-iHt})^{*}=e^{i(H^{*}-H)t}=1$，因H是Hermitian阵.

故 e^{-iHt} 是unitary operator.

2. $A=\begin{bmatrix} z & x-iy \\ x+iy & -z \end{bmatrix}$ $\det(A-\lambda I)=\lambda^2-(x^2+y^2+z^2)=\lambda^2-1$

故A的特征值是 ±1，记对应特征向量为 $|e_1\rangle, |e_2\rangle$.

$$f(\theta A)=f(\theta)\,|e_1\rangle\langle e_1|+f(-\theta)\,|e_2\rangle\langle e_2|$$

$$|e_1\rangle\langle e_1|=\frac{I+A}{2},\quad |e_2\rangle\langle e_2|=\frac{I-A}{2}$$

$$\therefore f(\theta A)=\frac{f(\theta)+f(-\theta)}{2}I+\frac{f(\theta)-f(-\theta)}{2}A$$

3. X, Y, Z 都满足 $A^2=I$ $(A=X,Y,Z)$

$$R_x(\theta)=\exp(-i\theta X/2)=\cos(\tfrac{\theta}{2})I-i\sin(\tfrac{\theta}{2})X=\begin{bmatrix}\cos\frac{\theta}{2} & -i\sin\frac{\theta}{2}\\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{bmatrix}$$

$$R_y(\theta)=\exp(-i\theta Y/2)=\cos(\tfrac{\theta}{2})I-i\sin(\tfrac{\theta}{2})Y=\begin{bmatrix}\cos\frac{\theta}{2} & -\sin\frac{\theta}{2}\\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{bmatrix}$$

$$R_z(\theta)=\begin{bmatrix}e^{-\frac{i\theta}{2}} & 0\\ 0 & e^{\frac{i\theta}{2}}\end{bmatrix}$$

4. $HXH=\frac{1}{2}\begin{pmatrix}1&1\\-1&1\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}=\begin{pmatrix}1&\\&-1\end{pmatrix}=Z$

$HYH=\frac{1}{2}\begin{pmatrix}i&-i\\-i&-i\end{pmatrix}\begin{pmatrix}1&+1\\1&-1\end{pmatrix}=\begin{pmatrix}&+i\\-i&\end{pmatrix}=-Y$

$HZH=\frac{1}{2}\begin{pmatrix}1&-1\\1&1\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}=\begin{pmatrix}&1\\1&\end{pmatrix}=X$

$HTH=\frac{1}{2}\begin{pmatrix}1&e^{i\pi/4}\\1&-e^{i\pi/4}\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}=\frac{1}{2}\begin{pmatrix}1+e^{\frac{i\pi}{4}}&1-e^{\frac{i\pi}{4}}\\1-e^{\frac{i\pi}{4}}&1+e^{\frac{i\pi}{4}}\end{pmatrix}=e^{i\theta}\begin{pmatrix}\frac{1}{2}\sqrt{2+\sqrt{2}}&\frac{-i}{2}\sqrt{2-\sqrt{2}}\\\frac{-i}{2}\sqrt{2-\sqrt{2}}&\frac{1}{2}\sqrt{2+\sqrt{2}}\end{pmatrix}=R_x(\frac{\pi}{4})$

其中，$e^{i\theta}=\frac{(\frac{\sqrt{2}}{2}+1+\frac{\sqrt{2}}{2}i)}{\sqrt{2+\sqrt{2}}}=\frac{\frac{1}{2}+(\frac{\sqrt{2}}{2}-1)i}{\sqrt{2-\sqrt{2}}}$

5. V应当也是 Unitary 的，记等式右边电路为 U_f.

U, U_f 显然不改变前2个 qubit.

$|00\rangle U_f|t\rangle = |00t\rangle = |00\rangle U|t\rangle$

$|01\rangle U_f|t\rangle = |01\rangle VV^{-1}|t\rangle = |00t\rangle$

$|10\rangle U_f|t\rangle = |10\rangle V^{+}V|t\rangle = |00t\rangle$

$|11\rangle U_f|t\rangle = |11\rangle V^{2}|t\rangle = |11\rangle U|t\rangle$

经验证，两个电路是相等的.

6. 记 $P_i = |i\rangle\langle i|$, $\rho' = \sum_{i=0,1} p_i\rho = \sum_{i=0,1} tr(P_iP_i\rho)\dfrac{P_i\rho P_i}{tr(P_iP_i\rho)} = \sum_{i=0,1} P_i\rho P_i$

$tr_B(\rho') = tr_B(P_1\rho P_1) + tr_B(P_2\rho P_2)$

$= (a_{00,00} + a_{01,01})|0\rangle\langle 0| + (a_{00,10} + a_{01,11})|0\rangle\langle 1| + (a_{10,00} + a_{11,01})|1\rangle\langle 0|$

$+ (a_{10,10} + a_{11,11})|1\rangle\langle 1|$

$= tr_B(\rho)$

7. $|000\rangle \longrightarrow |+0t\rangle$

$\longrightarrow \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \otimes |t\rangle$

$\longrightarrow \frac{1}{2}(|00\rangle + |10\rangle + i|01\rangle - i|11\rangle) \otimes |t\rangle$

$\longrightarrow \frac{\sqrt{2}}{4}(|000\rangle + |001\rangle + i|010\rangle + i|011\rangle + |100\rangle + |101\rangle - i|110\rangle - i|111\rangle)$

$\longrightarrow \frac{\sqrt{2}}{4}(|000\rangle - |001\rangle - i|010\rangle + i|011\rangle + |100\rangle - |101\rangle + i|110\rangle - i|111\rangle)$

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以下算法来源于 https://arxiv.org/pdf/quant-ph/9605034.pdf, Section 4.

```
m = 1
while m <= \sqrt{N}:
   pick k in {1 ... m}
   apply the Grover iterate k times to the superposition state
   measure the outcome;
   if a solution, exit and return
   else: m = lambda * m
```

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不会啊，太难了。

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实验结果如下:

```
{'0111': 10}
```

设置 4 个输入 Qubit，1 个附加 Qubit，手动编写 32 * 32 的 multi-controlled-T 门和 16 * 16 的 multi-controlled-Z 门。

> isQ 平台的报错支持很不友好，莫名其妙的 pending，体验不佳。

(1). 初始化 Qubit 如下：, p5 初始化为 |->, p1-p4 初始化为 |+>.

```
   p1 = |0>;
   p2 = |0>;
   p3 = |0>;
   p4 = |0>;
   p5 = |0>;
   H<p1>;
   H<p2>;
   H<p3>;
   H<p4>;
   X<p5>;
   H<p5>;
```

(2). Oracle 代码如下，使得 |0111>|-> -> -|0111>|->:

```
   X<p1>;
   MCT4<p1, p2, p3, p4, p5>;
   X<p1>;
```

(3). 再对q1-q4做 Householder 变换:

```
   H<p1>;
   H<p2>;
   H<p3>;
```

```
    H<p4>;
    X<p1>;
    X<p2>;
    X<p3>;
    X<p4>;
    MCZ3<p1, p2, p3, p4>;
    X<p1>;
    X<p2>;
    X<p3>;
    X<p4>;
    H<p1>;
    H<p2>;
    H<p3>;
    H<p4>;
```

重复执行步骤 (2) 和 (3) 两次，最后测量 q1-q4 的输出：

```
    x1 = M[p1];
    x2 = M[p2];
    x3 = M[p3];
    x4 = M[p4];
    print x1;
    print x2;
    print x3;
    print x4;
```

汇总源代码如下：

```
Defgate MCT4 = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
```

```
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0];
Defgate MCZ3 = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1
];

qbit p1, p2, p3, p4, p5;
int x1, x2, x3, x4;
procedure main(){
    p1 = |0>;
```

```
    p2 = |0>;
    p3 = |0>;
    p4 = |0>;
    p5 = |0>;
    H<p1>;
    H<p2>;
    H<p3>;
    H<p4>;
    X<p5>;
    H<p5>;

    X<p1>;
    MCT4<p1, p2, p3, p4, p5>;
    X<p1>;

    H<p1>;
    H<p2>;
    H<p3>;
    H<p4>;
    X<p1>;
    X<p2>;
    X<p3>;
    X<p4>;
    MCZ3<p1, p2, p3, p4>;
    X<p1>;
    X<p2>;
    X<p3>;
    X<p4>;
    H<p1>;
    H<p2>;
    H<p3>;
    H<p4>;

    X<p1>;
    MCT4<p1, p2, p3, p4, p5>;
    X<p1>;

    H<p1>;
    H<p2>;
    H<p3>;
    H<p4>;
    X<p1>;
    X<p2>;
    X<p3>;
    X<p4>;
    MCZ3<p1, p2, p3, p4>;
    X<p1>;
    X<p2>;
    X<p3>;
    X<p4>;
    H<p1>;
    H<p2>;
    H<p3>;
    H<p4>;

    x1 = M[p1];
    x2 = M[p2];
    x3 = M[p3];
```

```
    x4 = M[p4];
    print x1;
    print x2;
    print x3;
    print x4;
}
```