Problem 4.18

$$\mu_{mle} = \bar{X} = (4, 6)^T, \Sigma_{mle} = \frac{1}{4}\sum_{i=1}^{4}(X - \bar{X})(X - \bar{X})^T = \begin{bmatrix} 0.5 & 0.25 \\ 0.25 & 1.5 \end{bmatrix}$$

Problem 4.19

(a).$(X_1 - \mu)^T \Sigma^{-1}(X_1 - \mu) \sim \chi_6^2$.

(b).$\bar{X} \sim N_6(\mu, \frac{1}{20}\Sigma), \sqrt{n}(\bar{X} - \mu) \sim N_6(0, \Sigma)$.

(c).$(n-1)S \sim W_6(19, \Sigma)$

Problem 4.20(b)

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, B(19S)B^T = 19\begin{bmatrix} s_{11} & s_{13} \\ s_{31} & s_{33} \end{bmatrix}$$

记 $\Sigma_{13} = \begin{bmatrix} \sigma_{11} & \sigma_{13} \\ \sigma_{31} & \sigma_{33} \end{bmatrix}$, 则 $B(19S)B^T \sim W_2(19, \Sigma_{13})$.

Problem 5.1

(a).

$$\bar{X} = (6, 10)^T, s = \begin{bmatrix} 8 & -10/3 \\ -10/3 & 2 \end{bmatrix}$$

$$T^2 = 4(\bar{X} - \mu)^T s^{-1}(\bar{X} - \mu) = 13.6$$

(b).$T^2 \sim \frac{2(4-1)}{4-2}F_{2,4-2} = 3F_{2,2}$

(c).$3F_{2,2}(0.05) = 57 > T^2$, 无法拒绝 H_0.

Problem 5.2

$$Y = XC^T = \begin{bmatrix} -10 & 14 \\ -1 & 17 \\ -3 & 15 \\ -2 & 18 \end{bmatrix}, \mu_Y = C\mu = (-4, 16)^T$$

计算可得 $T^2 = 4(\bar{Y} - \mu_Y)S_Y^{-1}(\bar{Y} - \mu_Y)^T = 13.6$, 保持不变。

Problem 5.5

$T^2 = 42(\bar{X} - \mu)S^{-1}(\bar{X} - \mu)^T = 1.17$

而 $\frac{n(n-1)}{n-p} = 2.05, 2.05F_{p,n-p}(\alpha) = 2.05F_{2,40}(0.05) = 6.62 > T^2$.

无法拒绝 H_0. 而由图 5.1，可知 μ 处于椭圆内部，也即 μ 在置信域内。