

Problem 1

Since X is orthonormal, $XX^T = I$, $\hat{\beta}^{ols} = (X^T X)^{-1} X^T y = X^T y$.

$$\begin{aligned}(y - X\beta)^T (y - X\beta) &= (X^T y)^T X^T y + \beta^T X^T X \beta - 2\beta^T X^T y \\&= (\hat{\beta}^{ols})^T \hat{\beta}^{ols} + \beta^T \beta - 2\beta^T \hat{\beta}^{ols} \\&= \sum_{j=1}^p (\beta_j - \hat{\beta}_j^{ols})^2 + (\beta_0 - \hat{\beta}_0^{ols})^2\end{aligned}$$

So we can optimize β componentwise.

Problem 2

See [attachment](#).