CSIT 5500 Advanced Algorithms
2022 Fall Semester
Written Assignment 1
Handed out: September 9, 2022
Due: 23:59 on September 26, 2022
Please submit a soft copy via the canvas system by the due date and time shown above. Late assignments will not be graded.

1. (10 points) This question is about red-black tree.

   (a) (7 points) Starting from an initially empty red-back tree T, insert the numbers 1, 4, 6, 2, 7, 5, 10, 8, 9, 3 in this order into T. Show the tree T after inserting each number. You do not need to show other intermediate steps.

   (b) (3 points) Starting from the final T that you obtained in (a) above containing the numbers 1 to 10, delete the numbers 5, 8, 2, in this order. Show the tree T after deleting each number. You do not need to show other intermediate steps.

2. (10 points) Let $A[1..n]$ be an array of n distinct integers for some $n \geq 1$.

   (a) (7 points) Describe a *recursive* algorithm that accepts a parameter $k \in [1, n]$ returns a list of permutations of subsets of A of size k. You can either describe your algorithm in text or in a documented pseudocode. Explain your output representation. Make sure that your algorithm is recursive. For example, if $A = [3, 0, 1, 4]$ and $k = 2$, your output should be $(3, 0), (0, 3), (3, 1), (1, 3), (3, 4), (4, 3), (0, 1), (1, 0), (0, 4), (4, 0), (1, 4), (4, 1)$.

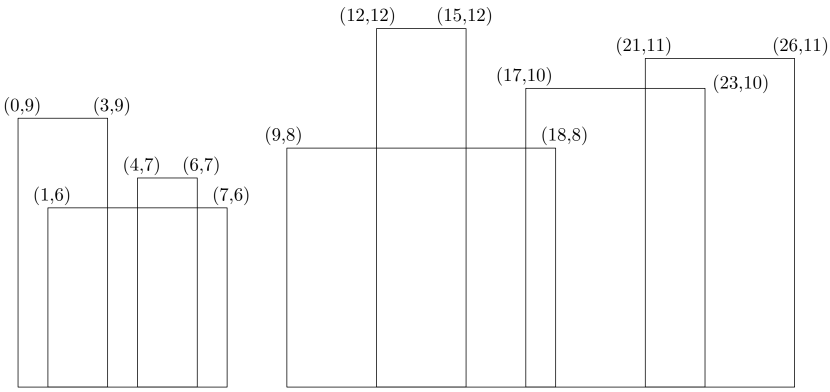
   (b) (3 points) Write down the recurrence for the running time of your recursive algorithm in (a) with the boundary conditions. Explain your notations. Solve your recurrence to obtain the the running time of your algorithm.

3. (10 points) Let $A[1..n]$ be an unordered array of n distinct integers. Let k be an integer from $[2, n]$ such that n is divisible by k.

   (a) (5 points) Let L denote the the list of the elements of A in increasing order. Imagine that we partition L into contiguous sublists of n/k elements each. For any $i \in [1, k]$, the ith segment of A refers to the elements in the ith sublist in the partition of L; however, it is unnecessary for the ith segment of A in sorted order. Describe an $O(n)$-time algorithm that returns the ith segment of A for any given i, k such that k divides n and $i \in [1, k]$. You can return the elements in the ith segment of A in any order that you find convenient. Explain the correctness of your algorithm and its running time.

   (b) (5 points) Describe an algorithm that returns the smallest element in the ith segment of A for all $i \in [1, k]$. Note that i is no longer part of the input, and your algorithm should output k numbers. Your algorithm should run in $O(n \log k)$ time. Explain the correctness of your algorithm and its running time.

4. (10 points) Let $\mathcal{S}$ be a set of n rectangles that sit on the real line $\mathbb{R}$. Each rectangle $R \in \mathcal{S}$ is specified by the coordinates of its four vertices $(a, 0), (b, 0), (a, c), (b, c)$ for some positive a, b, c such that $b > a$. We assume that the top edges of the rectangles in $\mathcal{S}$ have distinct y-coordinates. The skyline of $\mathcal{S}$ is the list of vertices at the top of the union of the rectangles in $\mathcal{S}$ from left to right. In the following example, the skyline is (0,9), (3,9), (3,6), (4,6), (4,7), (6,7), (6,6), (7,6), (9,8), (12,8), (12,12), (15,12), (15,8), (17,8),(17,10), (21,10), (21,11), (26,11). Describe a divide-and-conquer algorithm that returns the skyline of $\mathcal{S}$. Your algo-



rithm must use the divide-and-conquer strategy, and it must run in $O(n \log n)$ time. Explain the correctness of your algorithm and its running time.