CSIT 5500 Advanced Algorithms
2022 Fall Semester
Written Assignment 2
Handed out: September 30, 2022
Due: 23:59 on October 17, 2022
Please submit a soft copy via the canvas system by the due date and time shown above. Late assignments will not be graded.

1. (10 points) Consider the following string

   abbabbbaabbcab

   (a) Treat the above string as a patten and compute the next(·) table for it that supports the KMP string matching algorithm. Show your steps.

   (b) Treat the above string as the text. Consider the following pattern

   bbc

   Run the KMP string matching algorithm to find the occurrence of bbc in the above string. Show the next(·) table for bbc. You do not need to show the intermediate steps. Show the intermediate steps of running the KMP algorithm as follows:

   - Initially, draw the pattern and the text so that their leftmost symbols are aligned.
   - Whenever the KMP algorithm accesses the next(·) table,
     - indicate which two symbols in the pattern and the text were just compared by the KMP algorithm right before it accesses the next(·) table, and
     - draw the alignment of the pattern and the text to show the effect of accessing the next(·) table.
   - You can stop the KMP algorithm immediately after finding the occurrence of the pattern.

2. (10 points) Consider the following string

   alotofklokkokko

   (a) Construct the suffix array for the above string by following the construction algorithm in the lecture notes which consists of $O(\log n)$ stages. For each stage $j = 0, 1, 2, \ldots$, show the following:

   - The ordered pairs of substrings to be processed, as well as their corresponding ordered pairs of ranks.
   - The output table R for this stage j: $R[i]$ gives the rank of the substring that starts at position i and has length 2^j.

   (b) Draw a full binary tree to represent possible execution paths a binary search would take when searching for a pattern using the suffix array that you construct for (a). In your drawing, each node of the full binary tree should show the following information:

   - The starting and ending indices of the range of the suffix array represented by that node.
   - The middle index of the range represented by that node.
   - The values of L_j and R_j for that node as defined in the lecture nodes.

3. (10 points) Let $A[1..n]$ be an array of n integers. A sequence of indices $i_1 < i_2 < \cdots < i_k$ is called an *increasing subsequence* of A if $A[i_1] < A[i_2] < \cdots < A[i_k]$. Such a sequence is maximum if it is not shorter than any other increasing subsequence. Design a dynamic programming algorithm to determine a maximum increasing subsequence of A. Your algorithm should return the sequence of indices, and it should run in $O(n^2)$ time. Explain the correctness of your algorithm and its running time.

4. (10 points) Let R be a straight road. You want to drive your car from point s to point t on R. Assume that there are n gas stations $\{g_i : i \in [1, n]\}$ on R such that:

   - All n gas stations lie between s and t.
   - The g_i's are at increasing distances from s as i increases from 1 to n
   - The distance between g_1 and s is d_0, the distance between g_i and g_{i+1} is d_i for $i \in [1, n-1]$, and the distance between g_n and t is d_n.

   Gas station g_i charges c_i dollars for one unit of fuel. One unit of fuel allows your car to move one unit of distance. Suppose that your car can travel L units of distance on full tank, and your car has a full tank when you begin your journey. Assume that the parameters L, d_i, and c_i are integers. Design a dynamic programming algorithm that determines the lowest cost for you to drive from s to t. Explain your design and the running time of your algorithm.