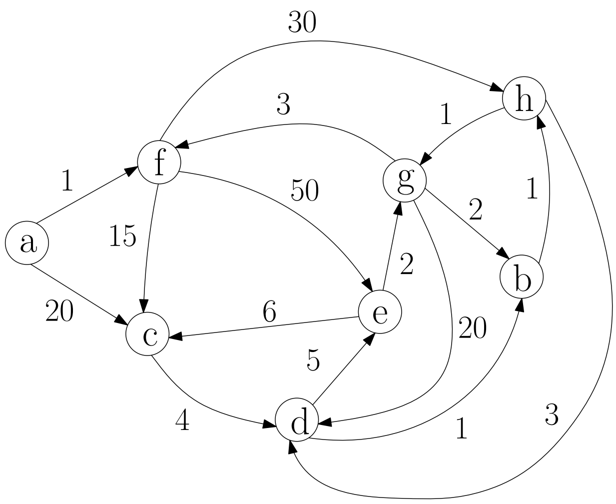
CSIT 5500 Advanced Algorithms
2022 Fall Semester
Written Assignment 3
Handed out: October 26, 2022
Due: 23:59 on November 9, 2022
Please submit a soft copy via the canvas system by the due date and time shown above. Late assignments will not be graded.

1. (10 points) There is another algorithm for finding a minimum spanning tree (MST) of an undirected graph $G = (V, E)$. Assume that all edge weights in G are distinct. Let s be an arbitrary vertex of G. Unlike Kruskal's algorithm, this algorithm initializes a partial MST T to contain the vertex s only and then iterates until T contains all vertices of G. In each iteration, the algorithm grows T by a new vertex (and hence a new edge to this new vertex as well).

   (a) Let T be the partial MST obtained by the algorithm at the end of some iteration. Assume that T is a subgraph of the MST of G. Among the edges that connect vertices in T to vertices not in T, let e be the one with the minimum weight. Show that the MST of G must contain e.

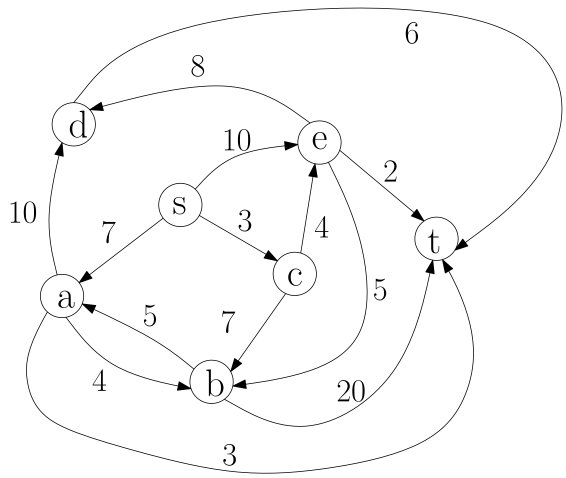
   (b) Describe an algorithm based on the idea in (a) that finds the MST of G in $O(m \log n)$ time, where n and m are the numbers of vertices and edges in G, respectively. Explain why the running time is $O(m \log n)$.

2. (10 points) Run Dijkstra's algorithm on the following directed graph G. Use vertex a as the source. Use the same convention and notation as in the lecture notes to show the values of $D[\cdot]$ and pred$[\cdot]$ for the nodes of G. Show the graph G and the values of $D[\cdot]$ and pred$[\cdot]$ after removing and processing each vertex from Q as in the lecture notes. Either color the pointer pred$[\cdot]$ red as in the lecture notes or show the pointer pred$[\cdot]$ as a dashed arrow. A vertex should be shown shaded if it no longer belongs to Q.



3. (10 points) Run Ford-Fulkerson's maximum flow algorithm on the following directed graph G. Use s as the source and t as the sink. Use the same convention and notation as in the lecture notes to show:

   - The residual graph G_f and the augmenting path selected in G_f.
   - The flow values on the edges of G and G_f after using the augmenting path selected to update the flow.



4. (10 points) Let G be a directed graph with n vertices and m edges.

   (a) Fix two distinct vertices s and t of G. The st edge connectivity of G is the minimum number of edges that are to be deleted so that the remaining subgraph of G does not contain any path from s to t. Describe an algorithm that determines the st edge connectivity of G by using a maximum flow algorithm as a subroutine. Explain the correctness of your algorithm.

   (b) The edge connectivity of G is the minimum number of edges that must be deleted so that, in the remaining subgraph of G, there exist vertices u and v for which there is no path from u to v. Describe an algorithm that determines the edge connectivity of G by using a maximum flow algorithm as a subroutine. Your algorithm should find the maximum flow in at most n flow networks, each having $O(n)$ vertices and $O(m)$ edges. Explain the correctness of your algorithm.