# problem1

February 19, 2022

## 1 Problem 1

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
```

### 1.1 Part A

Two reasonable test statistics are

$$T_1 = \frac{1}{n}\sum_{i=1}^{n} X_i$$

and

$$T_2 = \frac{1}{n}\sum_{i=1}^{n} (X_i - 6)^2$$

where $n = 6115$

[2]:
```
sample = np.array([7, 45, 181, 478, 829, 1112, 1343, 1033,
                670, 286, 104, 24, 3])

t1_sample = np.sum(np.arange(13)*sample)/6115

t2_sample = np.sum(((np.arange(13)-6)**2) * sample)/6115

print(f"T_1 sample value: {t1_sample}")
print(f"T_2 sample value: {t2_sample}")
```

```
T_1 sample value: 5.769419460343418
T_2 sample value: 3.542436631234669
```

### 1.2 Part B

[3]:
```
null_samples = np.random.binomial(12, .5, (1000, 6115))
```

The exterior of the red lines in the two following histograms is the rejection region for a two-sided $\alpha = 0.05$ significance test. The green line is the sample value of the test statistic

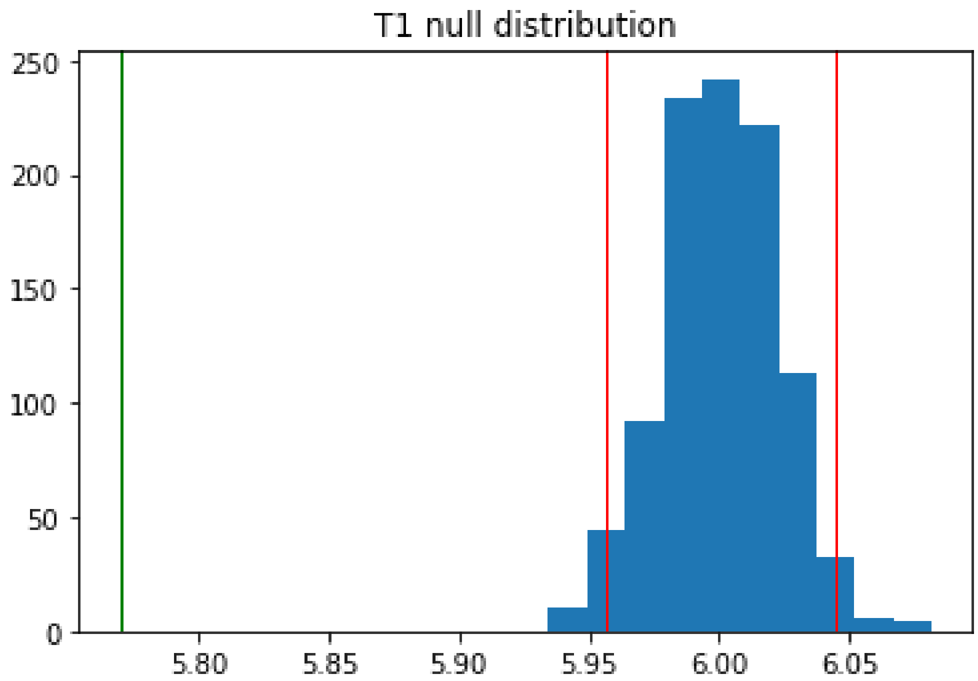
[4]:
```
# Histogram and rejection region
t1_null = np.sort(np.mean(null_samples, axis=1))
plt.hist(t1_null)
```

```
plt.title("T1 null distribution")

t1_lower_reject = (t1_null[25]+t1_null[24])/2
plt.axvline(t1_lower_reject, color='r', linestyle='solid', linewidth=1)

t1_upper_reject = (t1_null[975]+t1_null[974])/2
plt.axvline(t1_upper_reject, color='r', linestyle='solid', linewidth=1)


# p value computation
plt.axvline(t1_sample, color='g', linestyle='solid', linewidth=1)
insert_idx = np.searchsorted(t1_null, t1_sample)
p_value = (min(insert_idx, 1000-insert_idx)*2)/1000
plt.show()
print(f"p value is: {p_value}")
```




```
p value is: 0.0
```

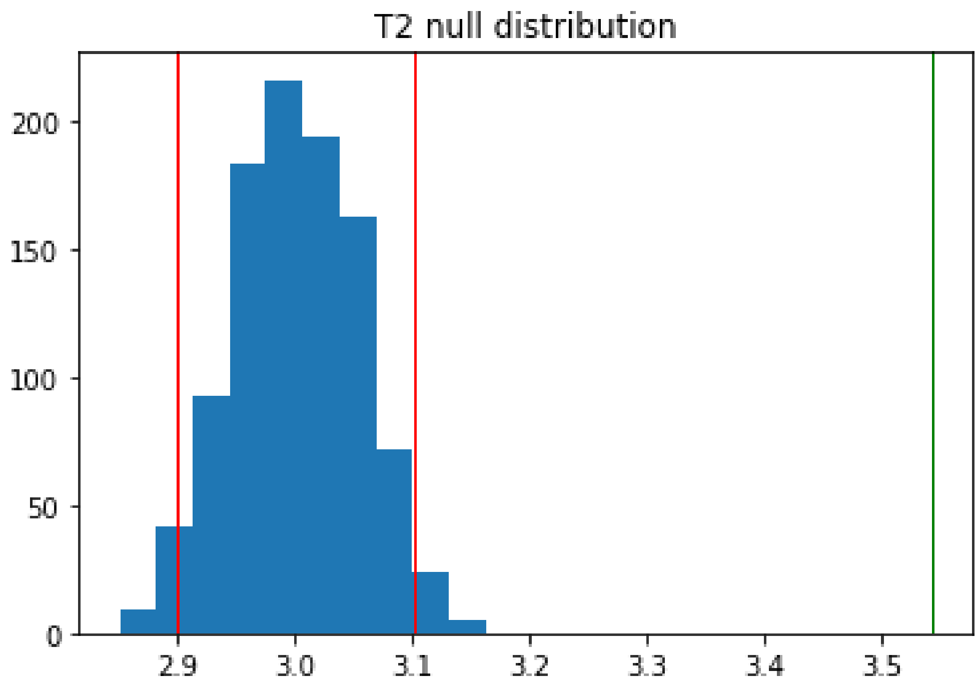
[5]:
```
# T2
t2_null = np.sort(np.mean((null_samples - 6)**2, axis=1))
plt.hist(t2_null)
plt.title("T2 null distribution")

t2_lower_reject = (t2_null[25]+t2_null[24])/2
```

```
plt.axvline(t2_lower_reject, color='r', linestyle='solid', linewidth=1)

t2_upper_reject = (t2_null[975]+t2_null[974])/2
plt.axvline(t2_upper_reject, color='r', linestyle='solid', linewidth=1)

# p value computation
plt.axvline(t2_sample, color='g', linestyle='solid', linewidth=1)
insert_idx = np.searchsorted(t2_null, t2_sample)
p_value = (min(insert_idx, 1000-insert_idx)*2)/1000
plt.show()
print(f"p value is: {p_value}")
```




```
p value is: 0.0
```

For both test statistics, we get approximate p-values of 0. This means in both cases we can reject $H_0$ at significance level $\alpha = 0.05$.

### 1.3 Part C

One reason the null hypothesis may not hold is because of selection bias introduced by only considering families that have 12 children. For example, suppose each family $i$ adopts the strategy of having children until they reach $n_i$ boys where $n_i$ is typically less than six. Then the families that reach twelve children will have less than 50% boys.