Lecture Two: What to Learn

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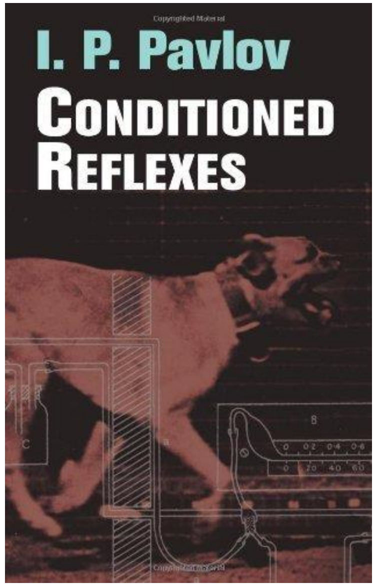
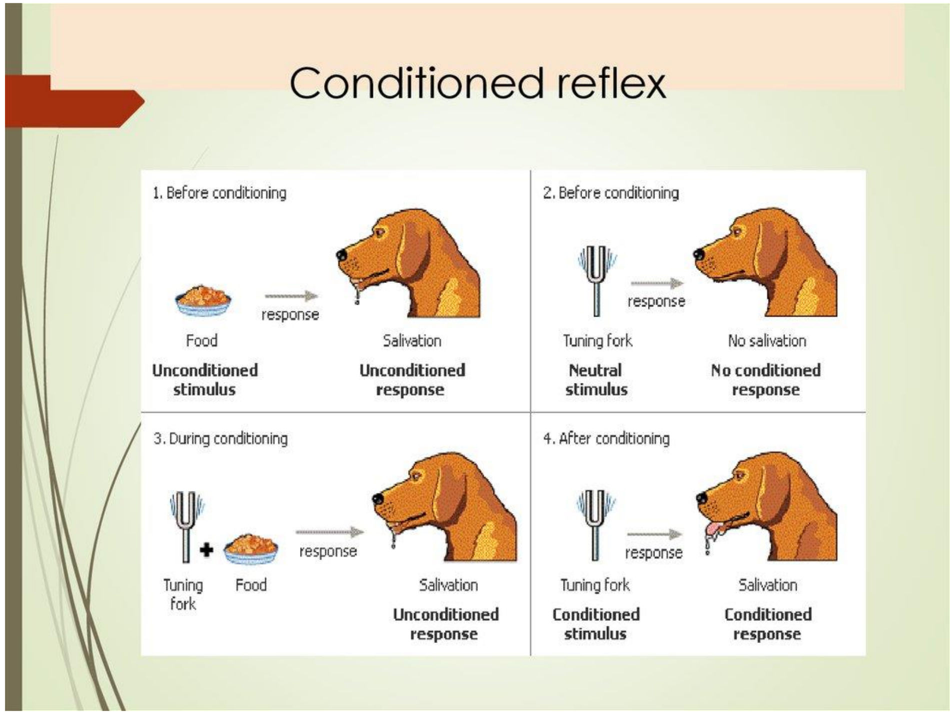
What to Learn?

Predictable information from data sensed of the external world

(Humans, including animals, have learned a model of the world)

Prediction based on correlation among things

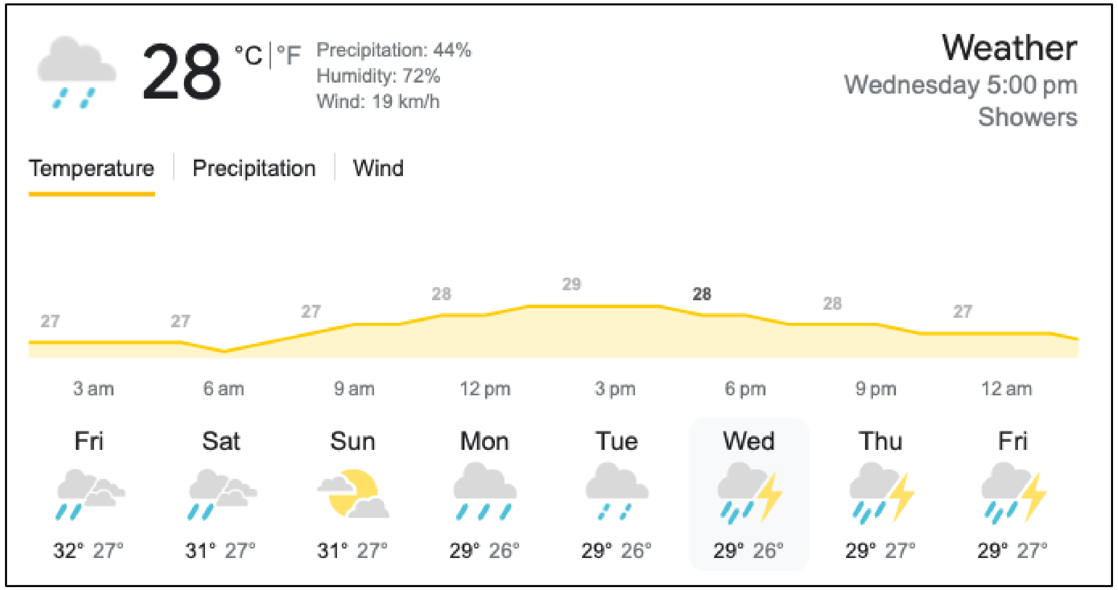




What to Learn?

Predictable information from data sensed of the external world

(Humans, including animals, have learned a model of the world)

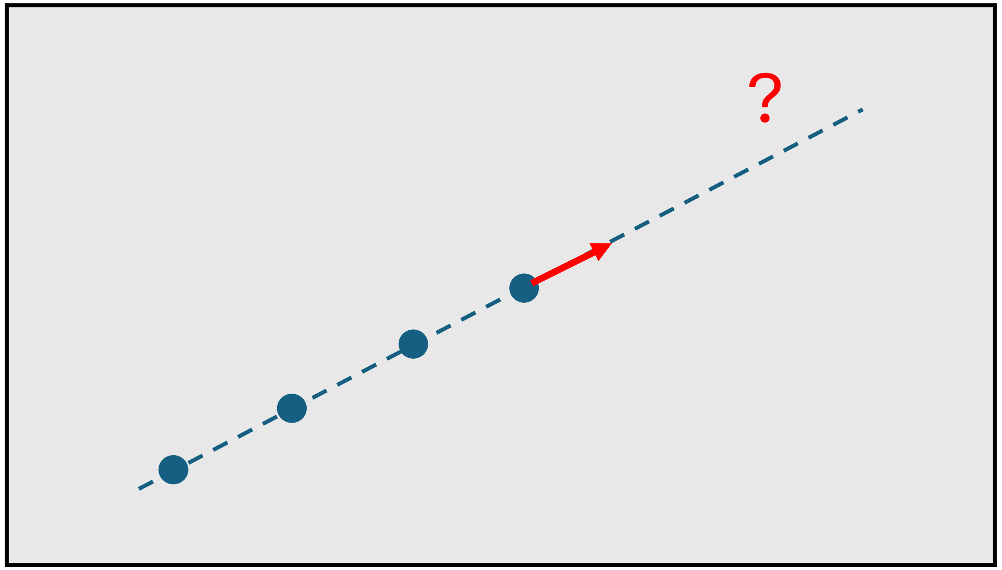


But correlation is not causality.

What to Learn? **Predictable information** from data sensed of the external world

(Human starts to develop physical and mathematical models of the world)

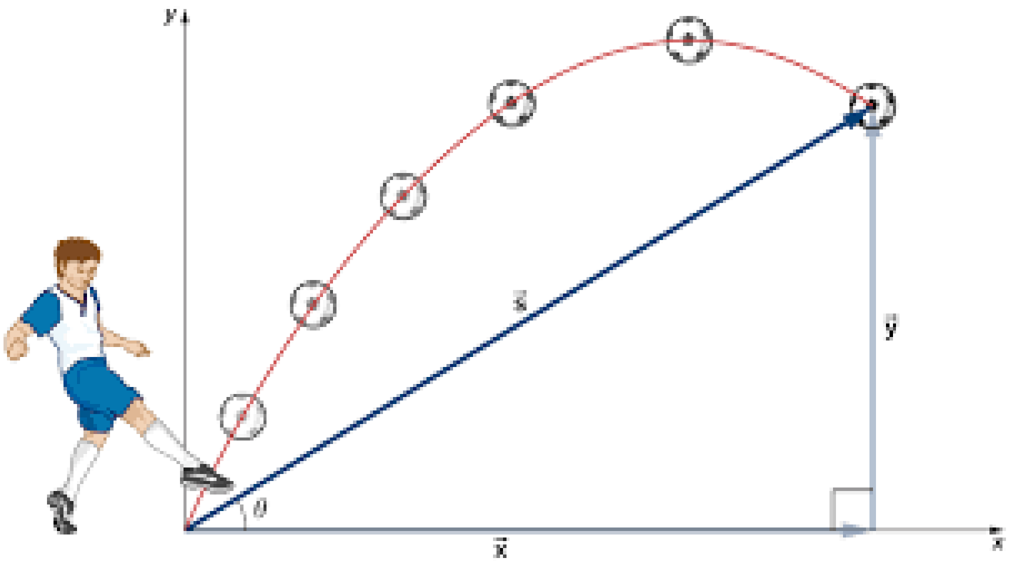
Newton’s first law of Motion

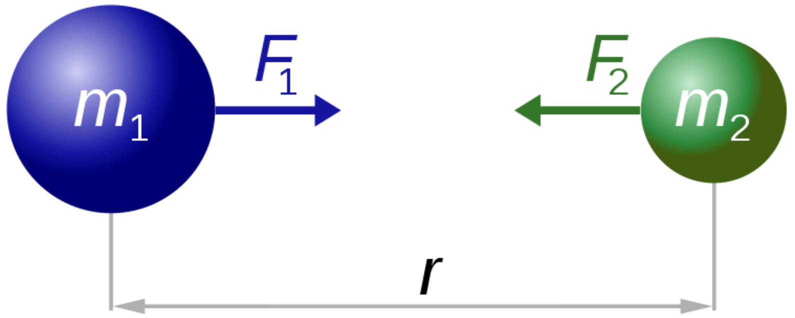


What to Learn? **Predictable information** from data sensed of the external world

(Human starts to develop physical and mathematical models of the world)

Newton's second law of motion and law of gravity





$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

What to Learn? **Predictable information** from data sensed of the external world

(Human starts to develop physical and mathematical models of the world)

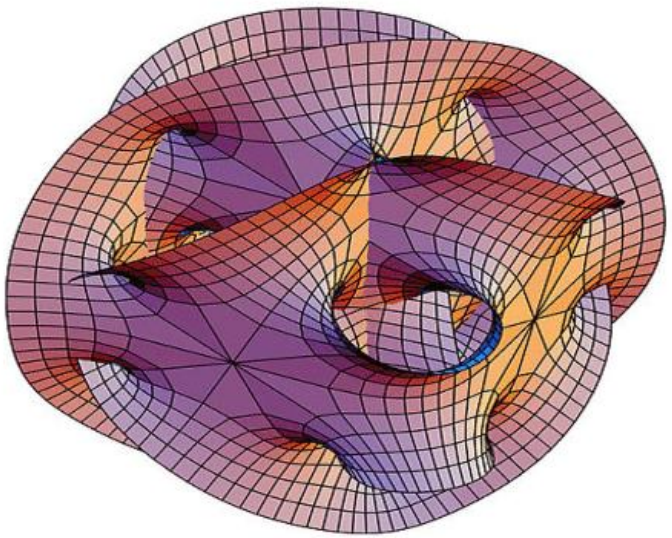
Why equations?

17 Equations That Changed the World
by Ian Stewart

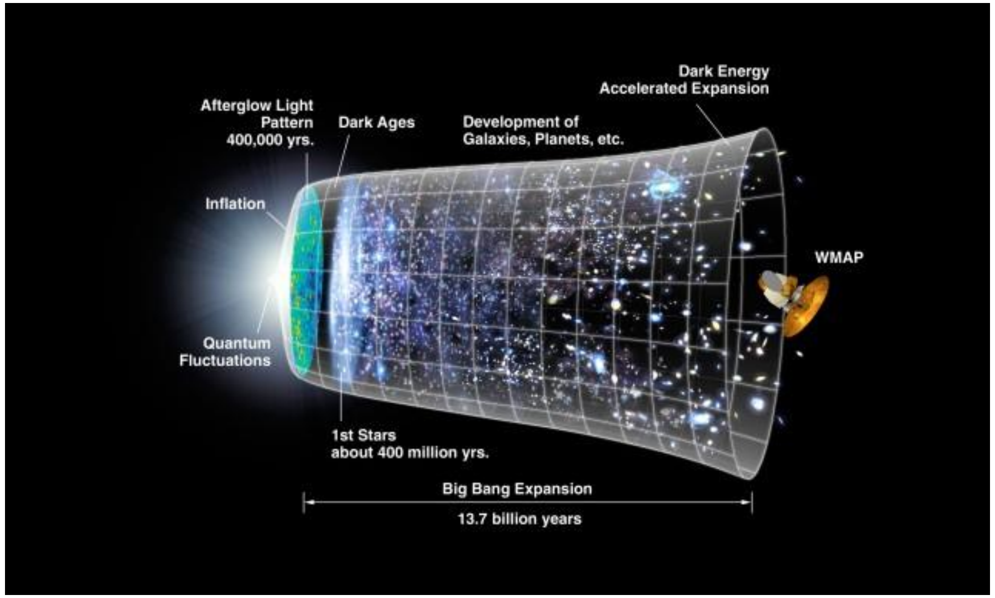
1.	**Pythagoras's Theorem**	$a^2 + b^2 = c^2$	Pythagoras,530 BC
2.	**Logarithms**	$\log xy = \log x + \log y$	John Napier, 1610
3.	**Calculus**	$\frac{\mathrm{d}f}{\mathrm{d}t} = \lim_{h \to 0} = \frac{f(t+h) - f(t)}{h}$	Newton, 1668
4.	**Law of Gravity**	$F = G\frac{m_1 m_2}{r^2}$	Newton, 1687
5.	**The Square Root of Minus One**	$i^2 = -1$	Euler, 1750
6.	**Euler's Formula for Polyhedra**	$V - E + F = 2$	Euler, 1751
7.	**Normal Distribution**	$\Phi(x) = \frac{1}{\sqrt{2\pi\rho}} e^{\frac{(x-\mu)^2}{2\rho^2}}$	C.F. Gauss, 1810
8.	**Wave Equation**	$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$	J. d'Almbert, 1746
9.	**Fourier Transform**	$f(\omega) = \int_{\infty}^{\infty} f(x) e^{-2\pi i x \omega} \mathrm{d}x$	J. Fourier, 1822
10.	**Navier-Stokes Equation**	$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}\right) = -\nabla p + \nabla \cdot \mathbf{T} + \mathbf{f}$	C. Navier, G. Stokes, 1845
11.	**Maxwell's Equations**	$\nabla \cdot \mathbf{E} = 0$, $\nabla \cdot \mathbf{H} = 0$, $\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}$, $\nabla \times \mathbf{H} = \frac{1}{c}\frac{\partial E}{\partial t}$	J.C. Maxwell, 1865
12.	**Second Law of Thermodynamics**	$\mathrm{d}S \geq 0$	L. Boltzmann, 1874
13.	**Relativity**	$E = mc^2$	Einstein, 1905
14.	**Schrodinger's Equation**	$i\hbar\frac{\partial}{\partial t}\Psi = H\Psi$	E. Schrodinger, 1927
15.	**Information Theory**	$H = -\sum p(x) \log p(x)$	C. Shannon, 1949
16.	**Chaos Theory**	$x_{t+1} = kx_t(1 - x_t)$	Robert May, 1975
17.	**Black-Scholes Equation**	$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} - rV = 0$	F. Black, M. Scholes, 1990

What to Learn? **Predictable information** from data sensed of the external world

The Calabi-Yau manifold
(the standard model & string theory)



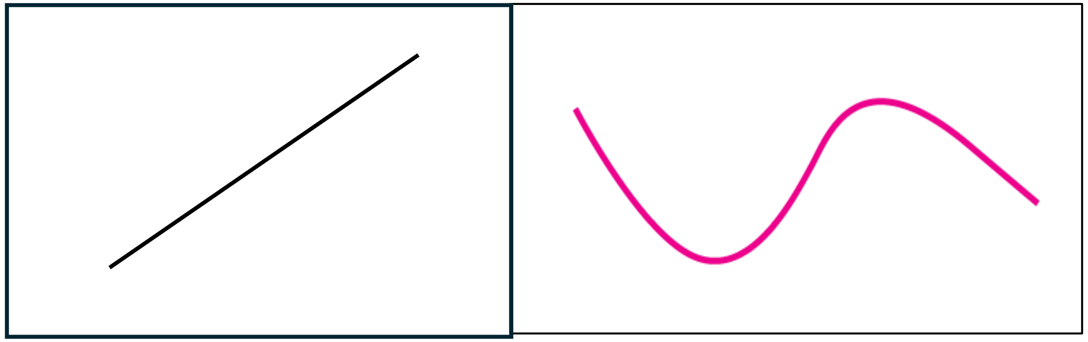
9+1-dimensional universe



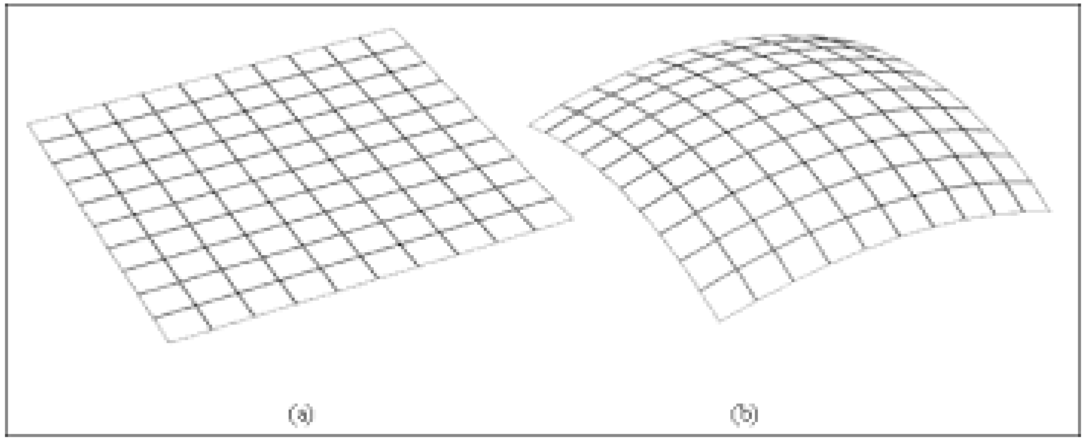
What to Learn?

How to model predictability mathematically or computationally?

$$x_{n+1} = f(x_n)$$



$$x_{n+1} = f(x_n, x_{n-1})$$



What to Learn?

How to model predictability mathematically or computationally?

$$x_{n+1} = f(x_n, x_{n-1}, \dots, x_{n-d+1})$$

What to Learn?

How to model predictability mathematically or computationally?

$$x_{n+1} = f(x_n, u_n)$$

What to Learn? How to model predictability mathematically or computationally?

$$x_{n+1} = f(x_n, x_{n-1}, \ldots, x_{n-d+1})$$

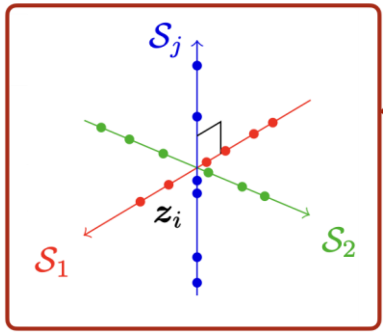
$$\vec{x} = [x_i, x_{i+1}, \ldots, x_{i+D-1}]$$

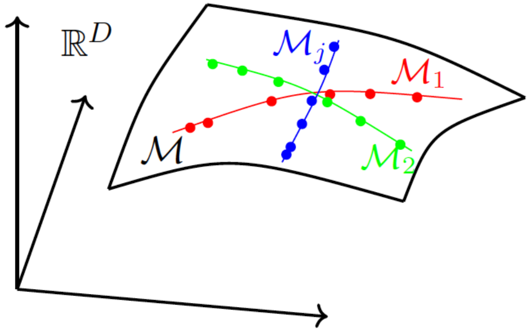
What to Learn? How to model predictability mathematically or computationally?

$$x_{n+1} = f_1(x_n, x_{n-1}, \ldots, x_{n-d+1})$$

$$x_{n+1} = f_2(x_n, x_{n-1}, \ldots, x_{n-d+1})$$

$$x_{n+1} = f_3(x_n, x_{n-1}, \ldots, x_{n-d+1})$$

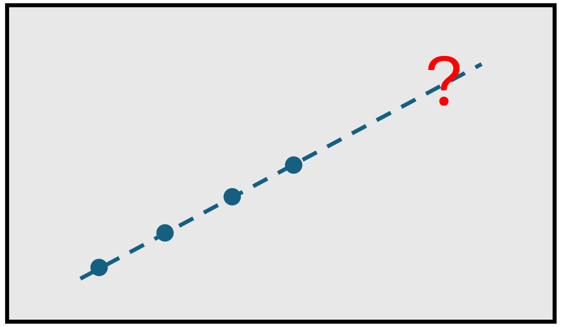


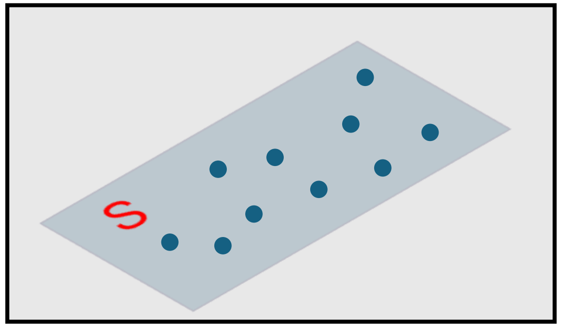


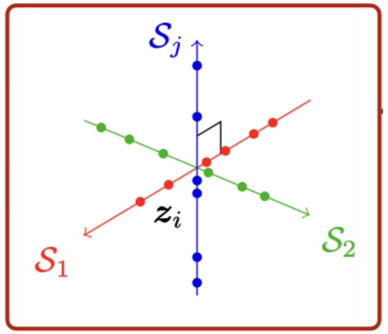
$$\vec{x} = [x_i, x_{i+1}, \ldots, x_{i+D-1}]$$

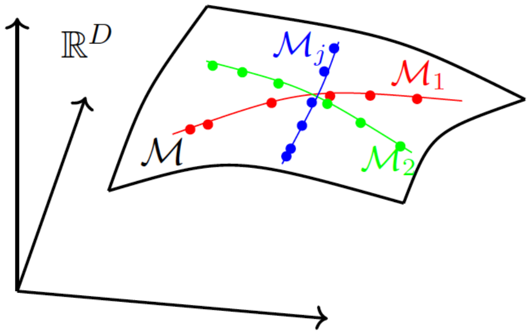
What to Learn? **Predictable information** from data sensed of the external world

Mathematically, all predictable information can be modeled as certain low-dimensional structures in the high-dimensional data



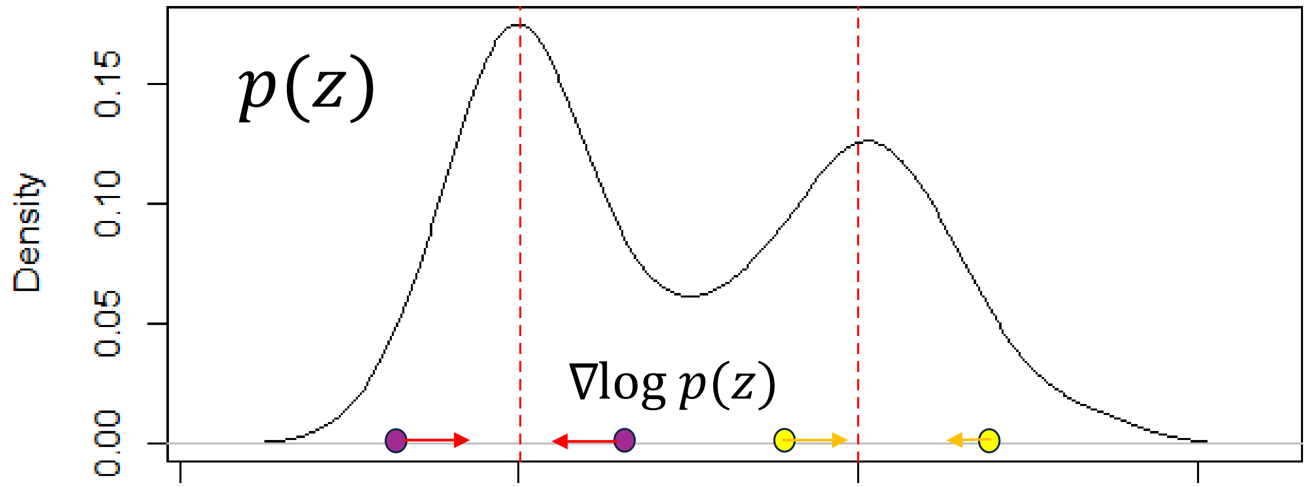


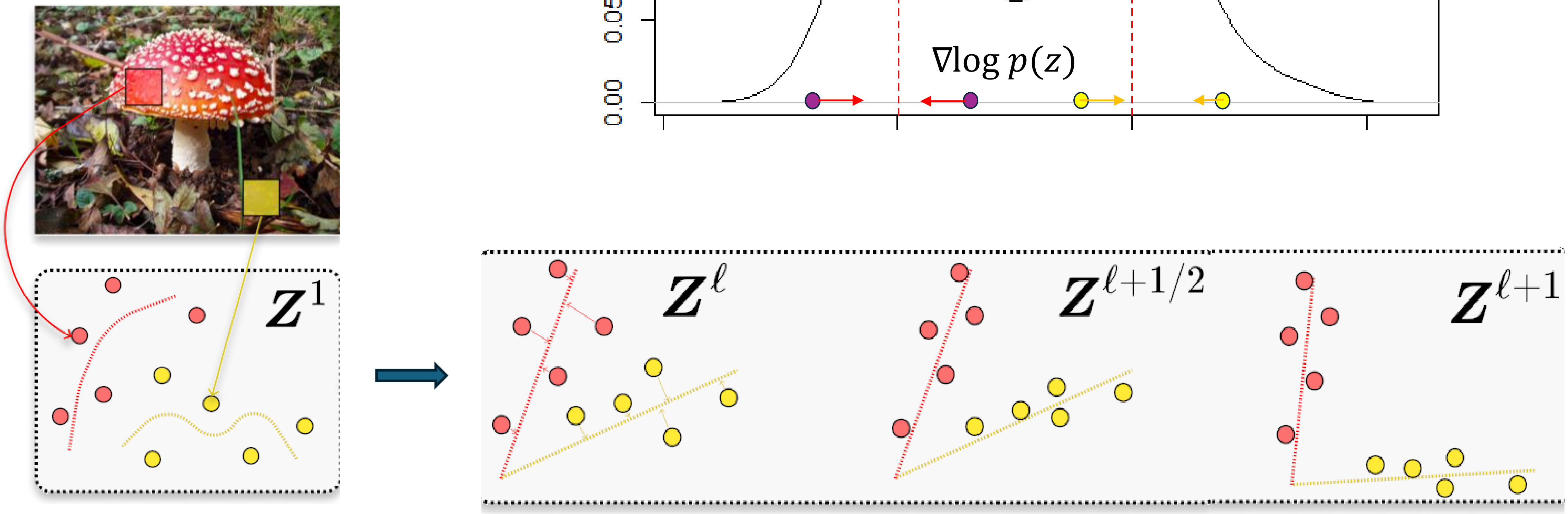




What to Learn?

low-dimensional probabilistic distributions





What to Learn?

Properties of low-dimensional structures:

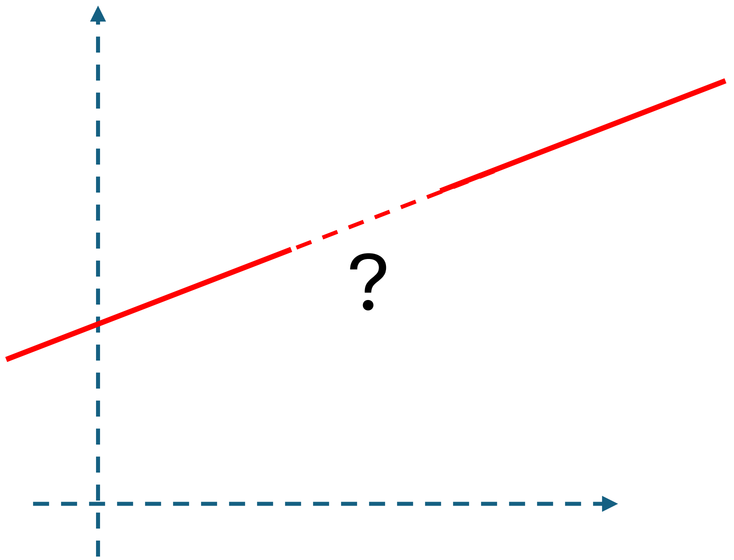
1. Completion

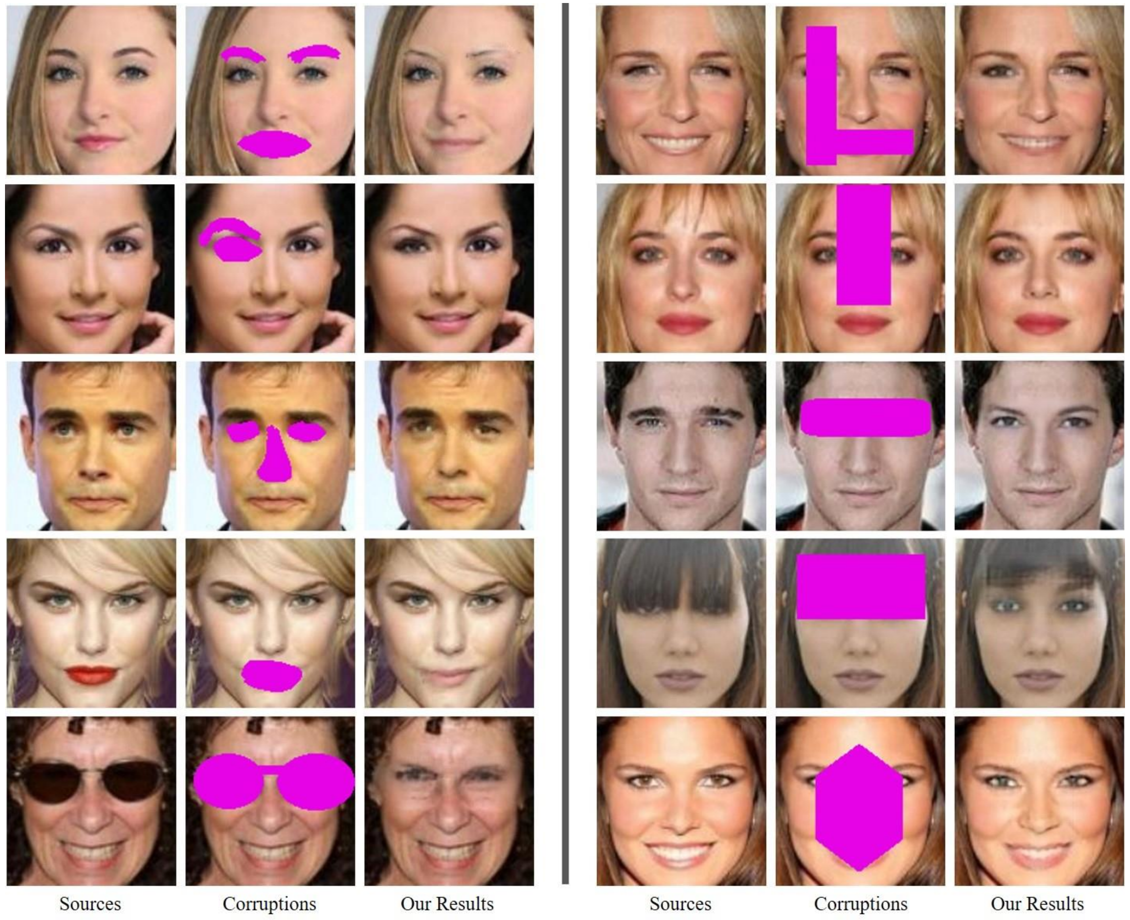
2. Denoising

3. Error Correction

What to Learn?

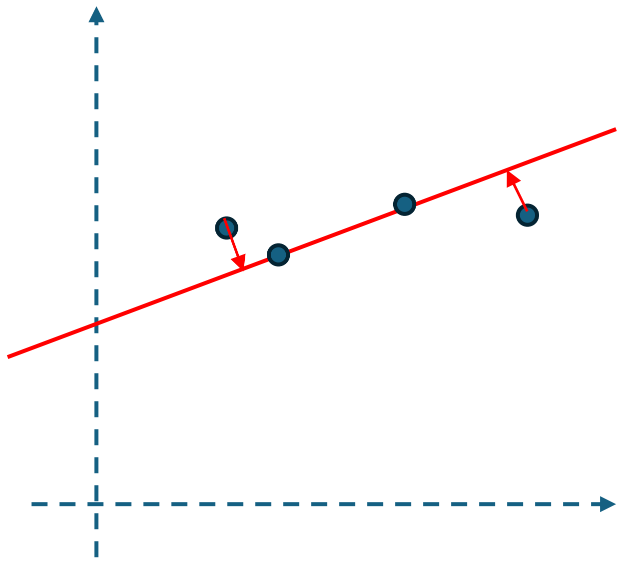
Properties of low-dimensional structures: Completion

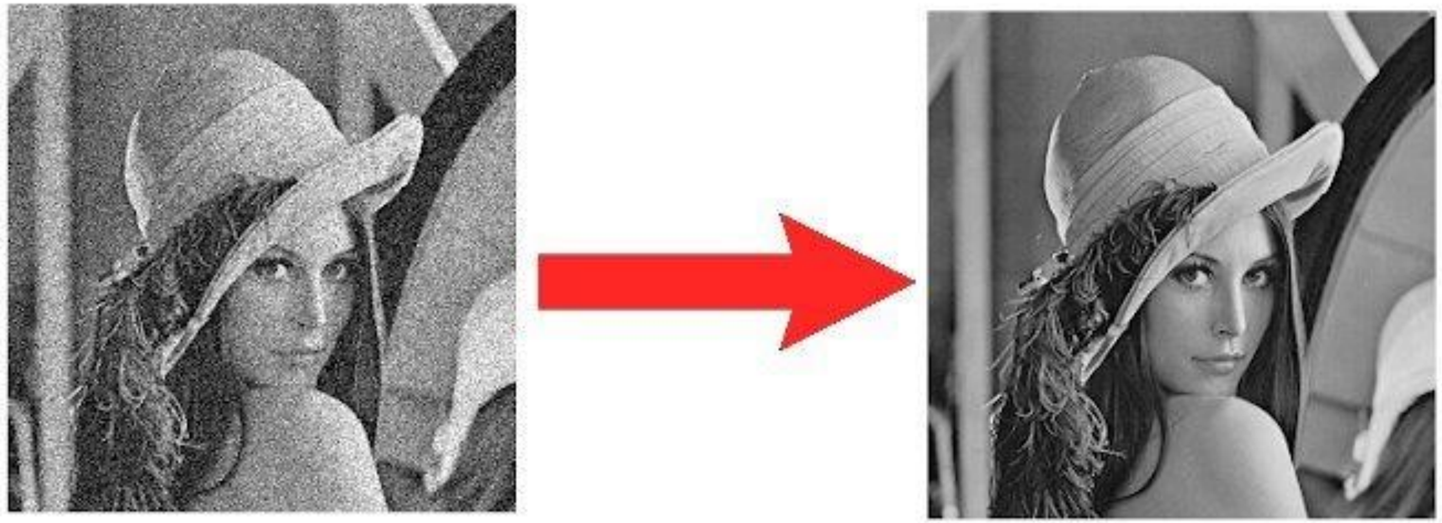




What to Learn?

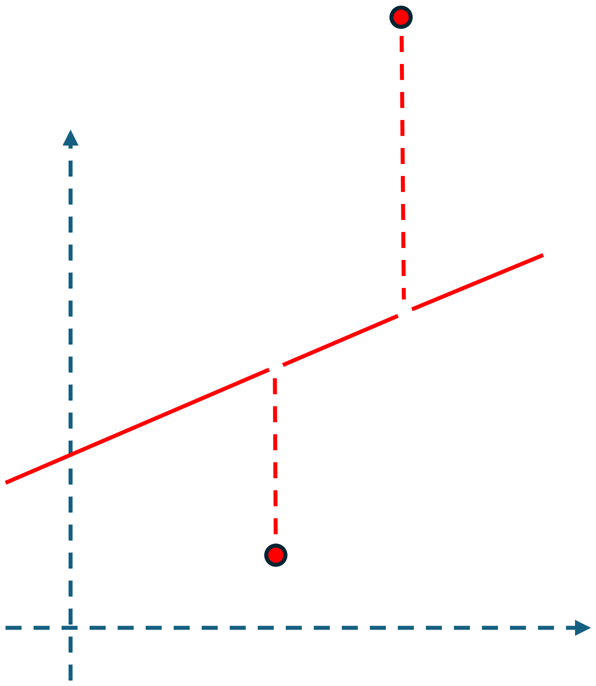
Properties of low-dimensional structures: Denoising

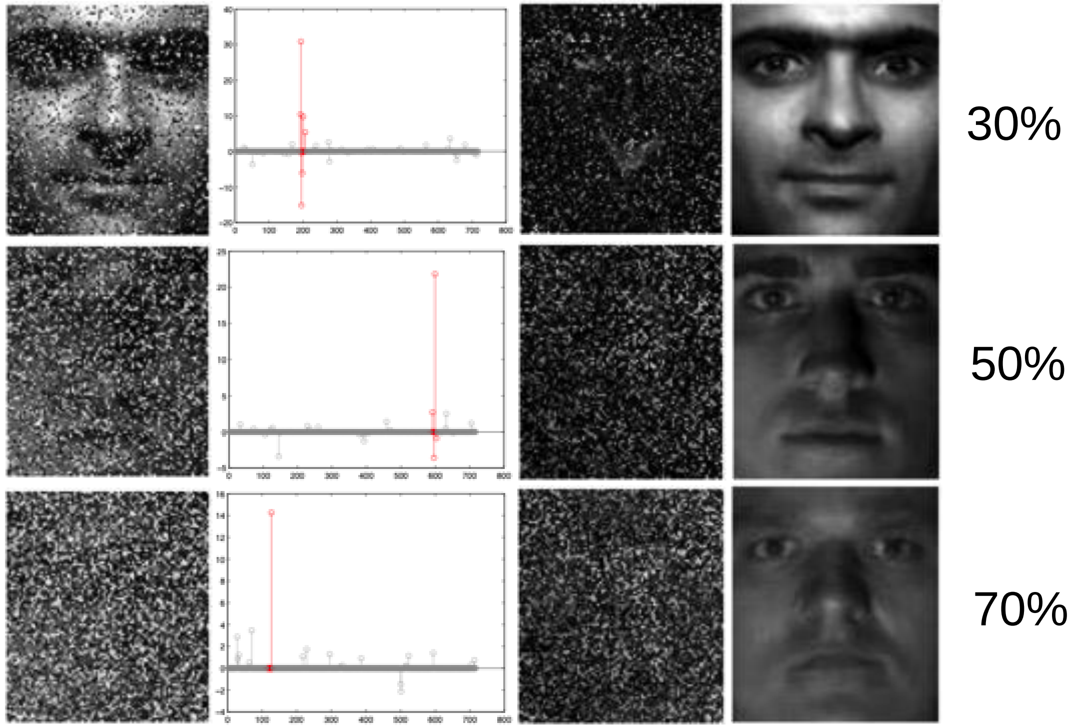


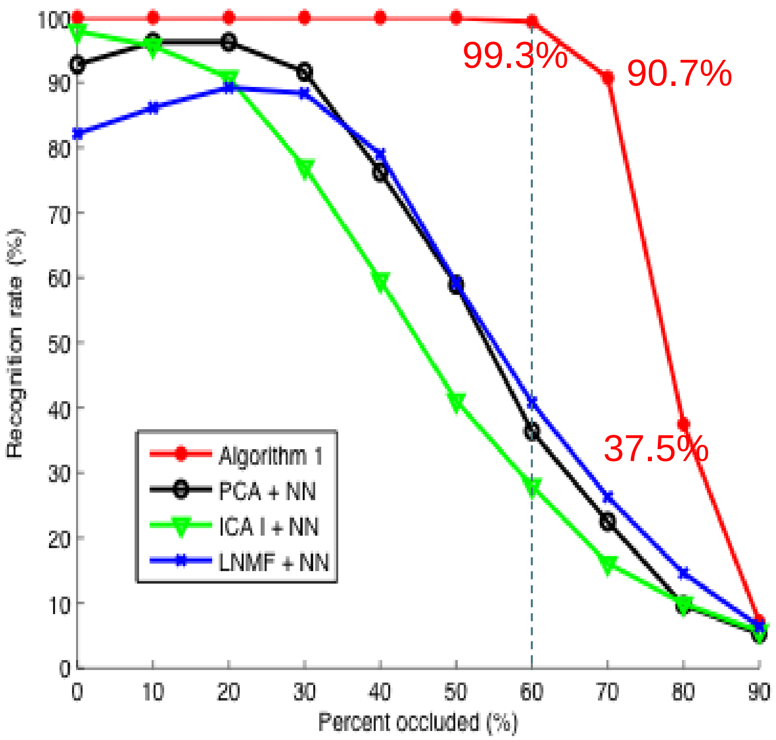


What to Learn?

Properties of low-dimensional structures: Error correction

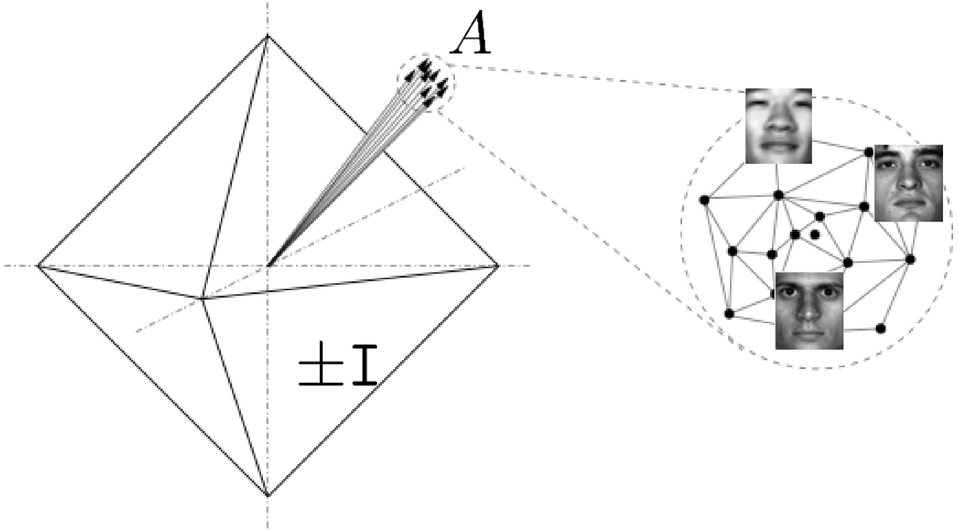






What to Learn?

Properties of low-dimensional structures: Error correction



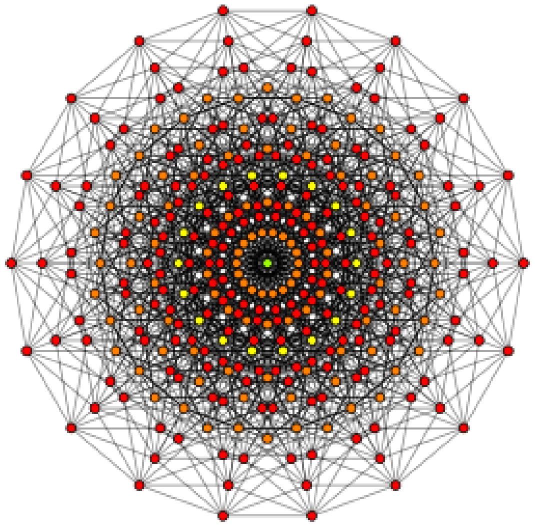
Highly coherent

(volume $\leq 1.5 \times 10^{-229}$)

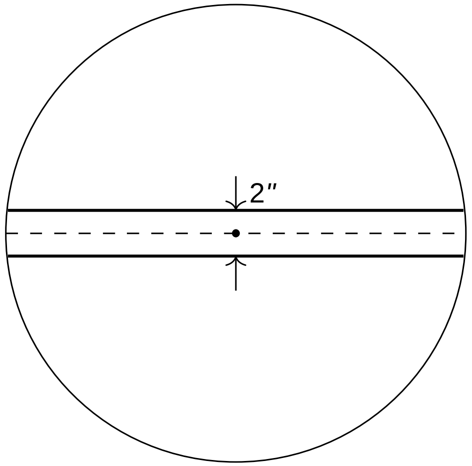
What to Learn?

Completely different geometric and statistical phenomena in high-dimensional spaces

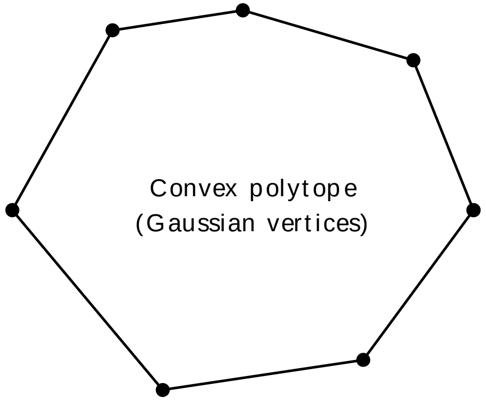
a 9-dimensional cube



a high-dim sphere



a high-dim Gaussian



How to Learn?