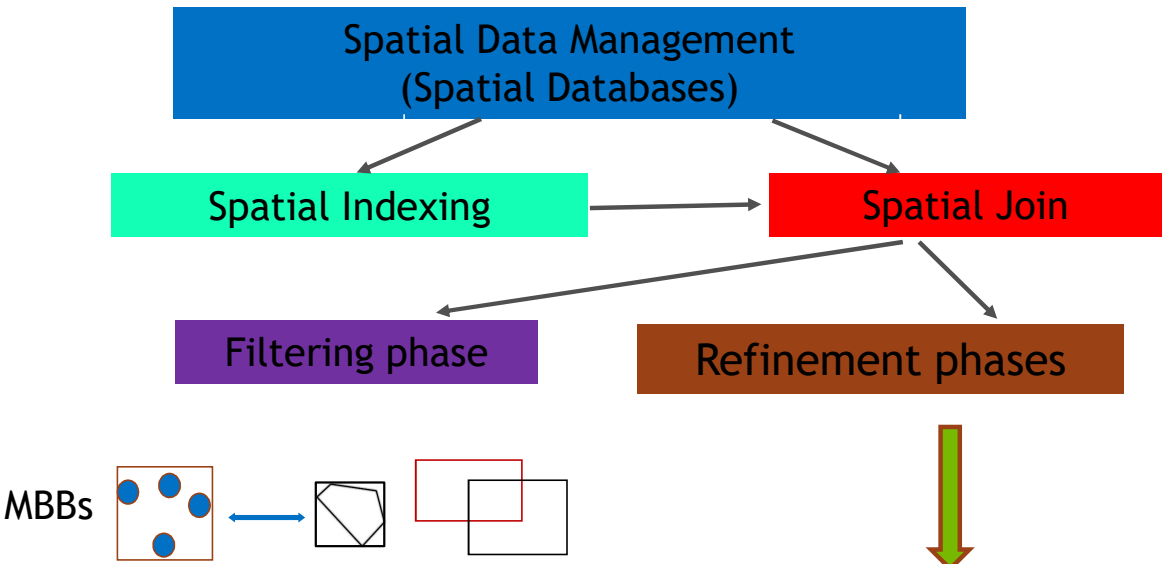


GPU-Accelerated Spatial and Trajectory Data Management



Point-in-Polygon Test $P \bowtie_{PIP} Q$

A blue polygon with a red point inside. A dashed line with an arrow points from the point to the left.

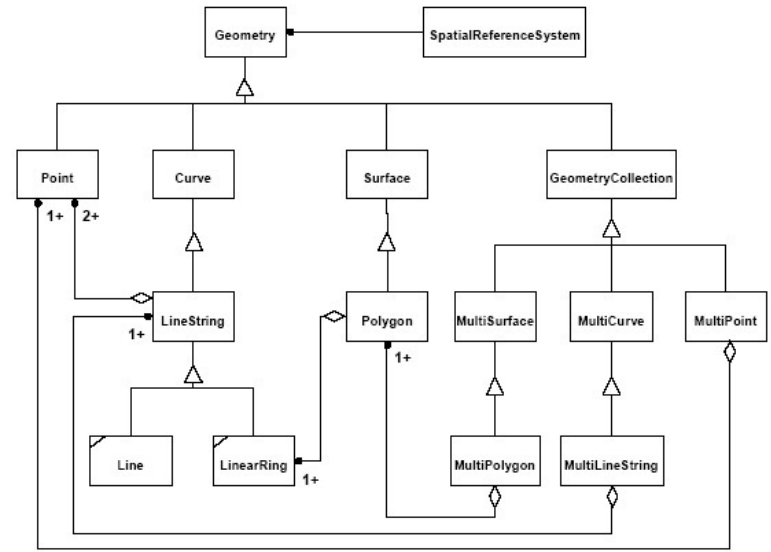
A map showing a black polygon on a grid. A red point is located inside the polygon.

GDAL/OGR: `OGRGeometry.Contains(OGRGeometry)`

SQL: `ST_WITHIN(Point.geometry, Polygon.Geometry)`

GEOS/JTS, GDAL/OGR, GRASS, PostGIS/PostgreSQL
ArcMap/ArcGIS, Oracle, SQLServer

OGC Simple Features Specification (SFS) for SQL



Methods for Geometry:

Basic Methods

Dimension, GeometryType, SRID, Envelope, AsText, AsBinary, IsEmpty, IsSimple, Boundary

Methods for testing Spatial Relations

Equals, Disjoint, **Intersects**, Touches, Crosses, **Within**, **Contains**, Overlaps, Relate [more general]

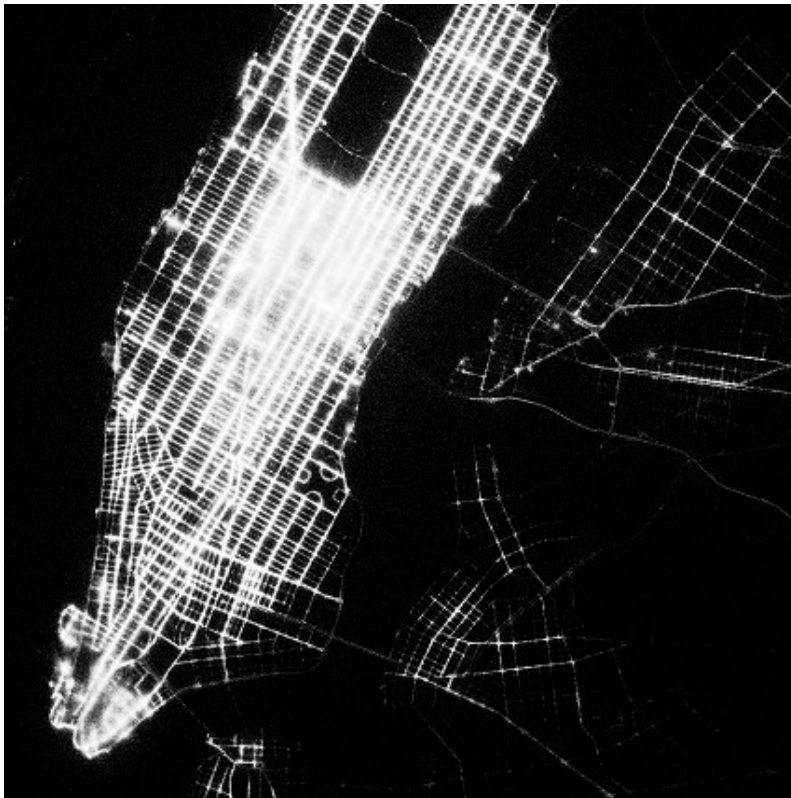
Methods that support Spatial Analysis

Distance, Buffer, ConvexHull, Intersection, Union, Difference, SymDifference

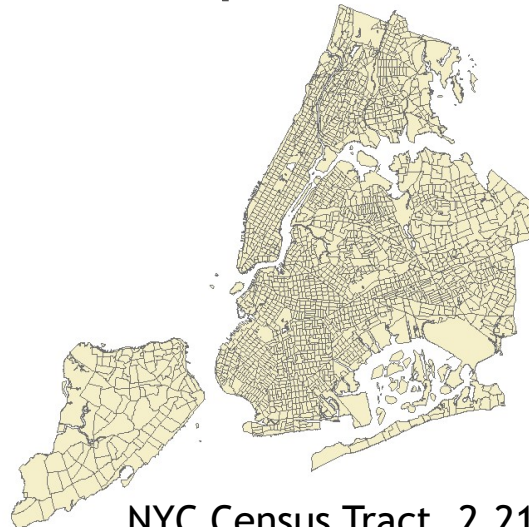
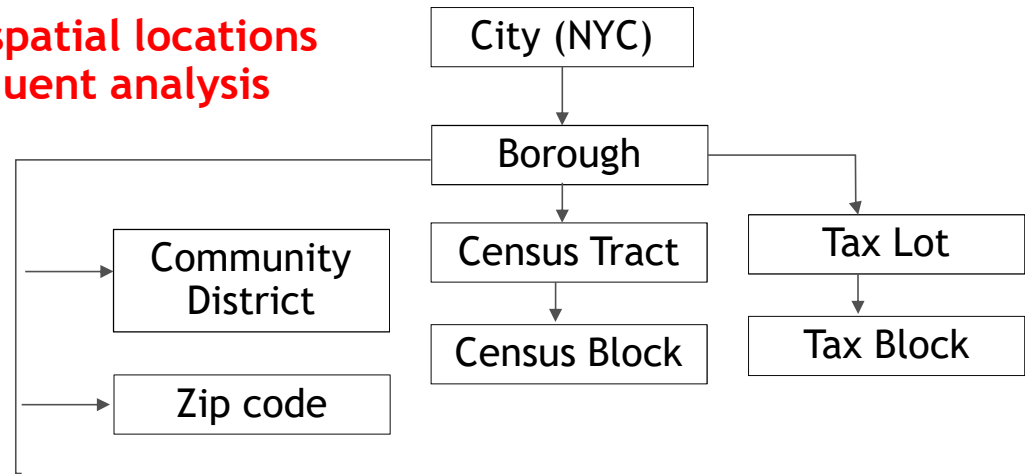


Point-In-Polygon Tests from real world applications

- Polygons are the semantic framework of geospatial locations
- PIP gives points semantic meaning for subsequent analysis



NYC Taxi Trip Pickup Locations
168,898,952 in 2009



NYC Census Tract, 2,216 polygons
(2,411 rings, 170,329 vertices)

For more NYC polygon datasets, see

<https://www1.nyc.gov/site/planning/data-maps/open-data/districts-download-metadata.page>

And

<https://opendata.cityofnewyork.us/>

Existing PIP Functionality in cuSpatial

```
{cuspatial}/python/cuspatial/cuspatial/tests/test_pip.py
```

```
def test_one_point_in():  
    result = cuspatial.point_in_polygon_bitmap(  
        cudf.Series([0]),  
        cudf.Series([0]),  
        cudf.Series([1]),  
        cudf.Series([3]),  
        cudf.Series([-1, 0, 1]),  
        cudf.Series([-1, 1, -1]) )
```

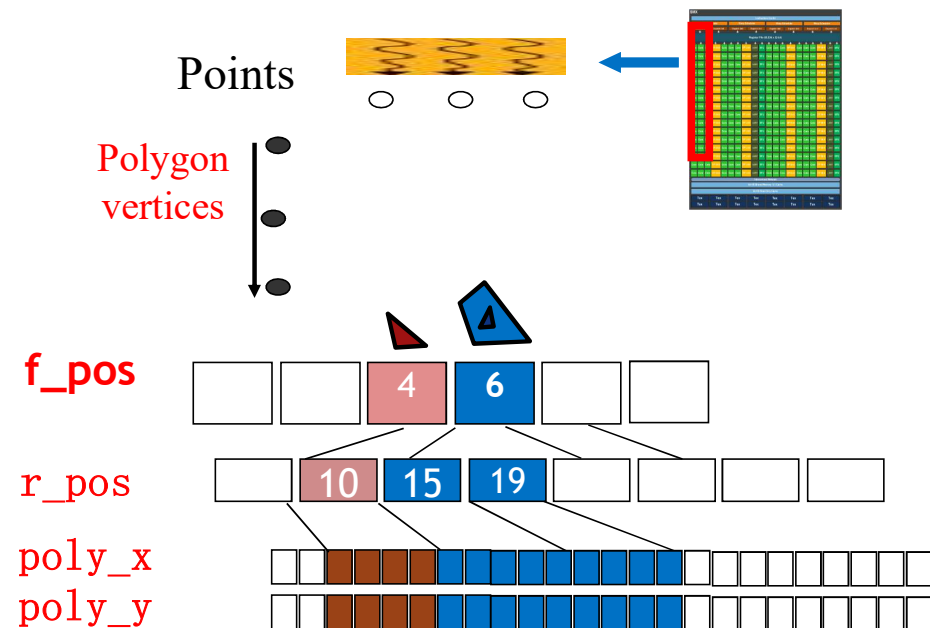
```
{cuspatial}/python/cuspatial/cuspatial/core/gis.py  
cpp_point_in_polygon_bitmap
```

```
{cuspatial}/python/cuspatial/cuspatial/_lib/spatial.pyx  
cpp_point_in_polygon_bitmap
```

```
{cuspatial}/cpp/src/spatial/point_in_polygon.cu  
gdf_column point_in_polygon_bitmap(  
    const gdf_column& points_x, const gdf_column& points_y,  
    const gdf_column& poly_fpos, const gdf_column& poly_rpos,  
    const gdf_column& poly_x, const gdf_column& poly_y)
```

Assumptions:

- A point can be in any of the polygons → use integer as bitvector for bitmap rep.
- The number of polygons is no more than sizeof(uint8_t/uint16_t/**uint32_t**)
- A thread processes a point and loops over multiple polygons
- No indexing on points
- no polygon BBox approximation
- Considered as a spatial function



Motivations for Spatial Indexing and PIP-based Spatial join

- Bitmap-based PIP result rep. is not suitable for large numbers of polygons
- Limiting a point to be in no more than one polygon is not always practical
- Brute force PIP test between every point and every polygon is not efficient

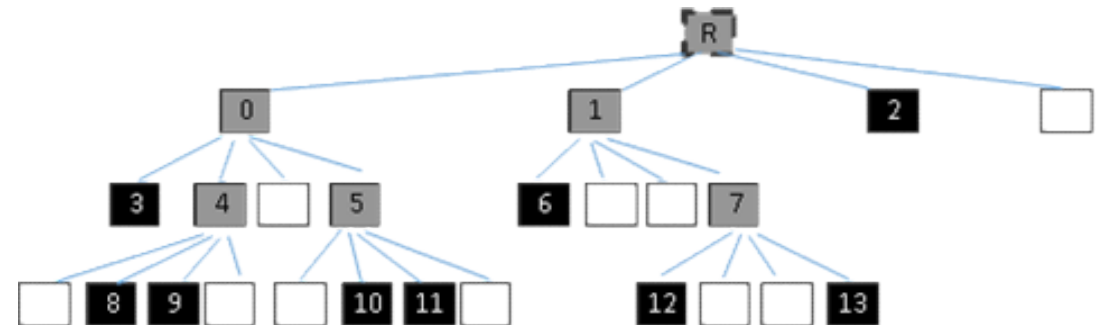
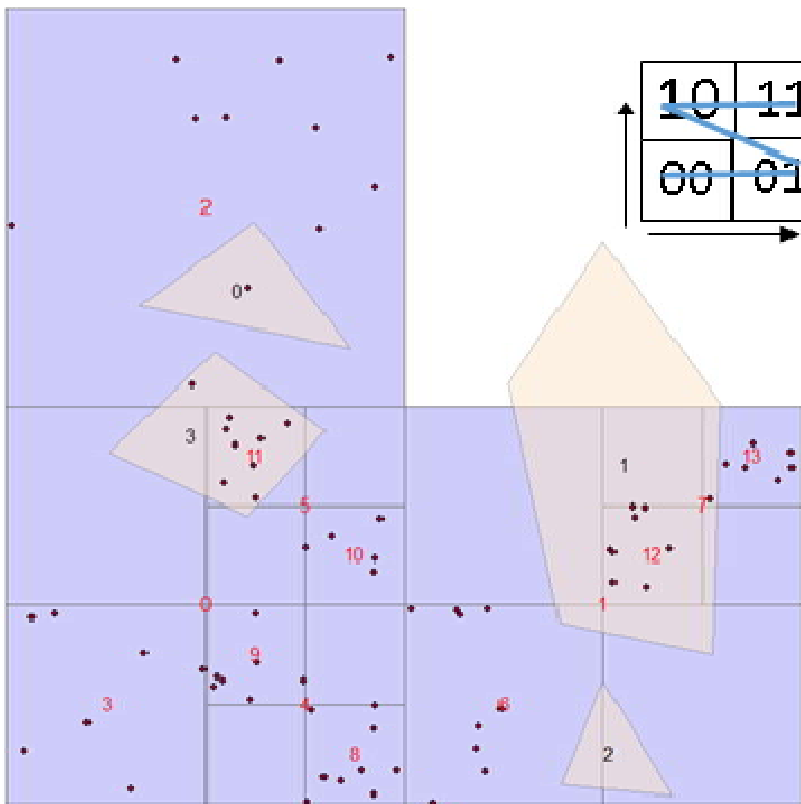
Proposed solution: Apply Indexed Spatial Join Framework

- High-performance indexing on large-number of points
- Using BBoxes to approximate complex polygons
- Pairing point indices (quadrants for quadtree indexing) with polygon bboxes based on simple rectangle intersection test for effective pruning search space
- Performing PIP tests only for points in quadrants that are paired with polygons

Major Challenges

- (Hierarchical) spatial indexing involves irregular data accesses and high data movements
- Point-polygon pairing has many-to-many relationship → output sizes are data dependent

Quadtree Indexing for Large-Scale Point Data

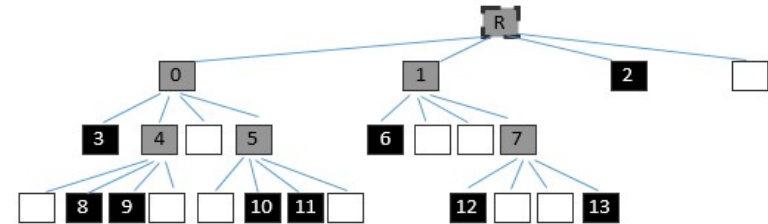


Recursively divides indexing space into four quadrants:

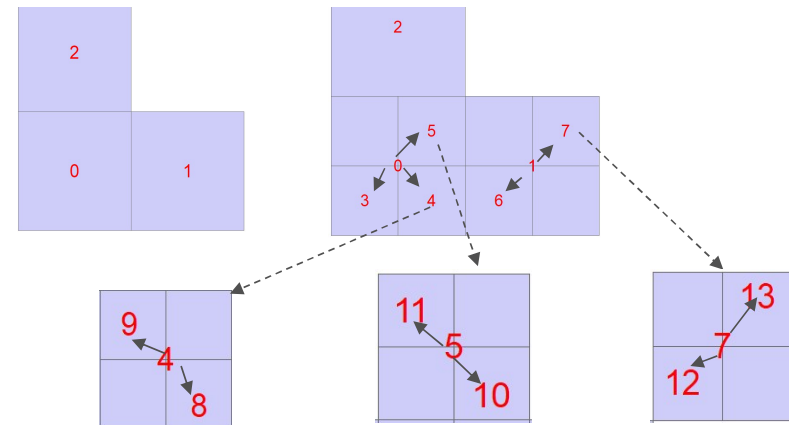
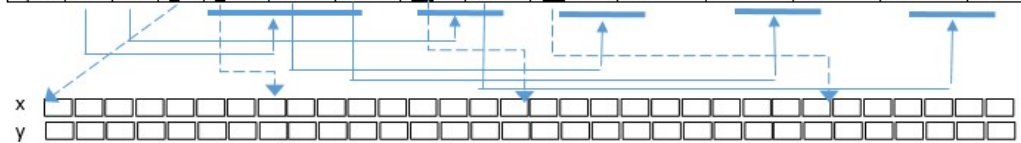
- White node: no points are distributed in the quadrant → **stop**
- Black node: the number of points in the quadrant is below a pre-defined threshold → **stop**
- Gray node: the number of points in the quadrant is above the threshold → **recursive sub-division** until reaching black nodes or the maximum level is reached.
- Reduce searching from linear scanning to **logarithmic** (assuming reasonable balancing)
- Low overheads for implementations, better suitable for in-memory data processing (comparing with R-Trees)

Quadtree Indexing for Large-Scale Point Data

- Novel Structure of Array (SoA) design for GPUs
- Data parallel implementations using parallel primitives
(sort/reduce/transform/gather/scatter...)
- Five arrays to encode resulting quadtree:
 - key (32b): Z-order or Morton code at a level
 - lev (8b): level (<16)
 - sign (1b): needs further division (non-leaf node) or not (leaf node)
 - len (32b): number of child nodes (for non-leaf node) or number of points (for leaf nodes)
 - fpos(32b): first child node position (for non-leaf node) or first point position (for leaf node)
- Level-by-level Breadth-First Search (BFS)
- Access to child nodes using array offsets
 - Node/point position= fpos[p]+i
 - no pointer chasing

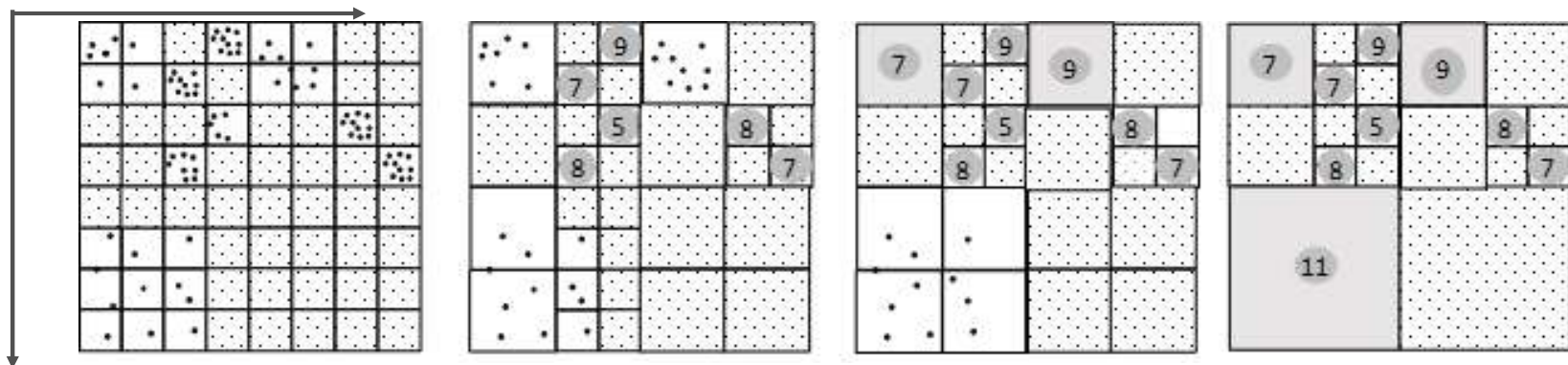


Seq	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Key	00	01	10	0000	0010	0011	0100	0111	001001	001010	001101	001110	011100	011111
Lev	0	0	0	1	1	1	1	1	2	2	2	2	2	2
Sign	1	1	0	0	1	1	0	1	0	0	0	0	0	0
len	3	2	7	9	2	2	7	2	9	5	8	8	7	11
fpos	3	6	0	7	8	10	16	12	23	60	37	45	53	60



Quadtree Indexing for Large-Scale Point Data

- http://www.adms-conf.org/2019-camera-ready/zhang_adms19.pdf
- Compute all non-empty quadrants at the finest level in phase 1
- construct quadtree by removing non-qualified nodes in phase 2
- **Require sorting points only once at the finest level**



- A quadtree node is a leaf node if:
 - $n_k > n_t$ AND at finest level
 - $n_k \leq n_t < n_p \Rightarrow n_k \leq n_t$ AND $n_p > n_t$

- n_t : threshold (# points)
- n_k : #of points at node k
- n_p : #of points at the parent node of node k

Quadtree Indexing for Large-Scale Point Data

Phase 1

- 1 Transform point dataset P to key set l_key using Z-ordering at finest level
- 2 Sort_by_key using l_key as the key and P as the value
- 3 Reduce_by_key to count numbers of points in partitioned quadrants and set $nlen$
- 4 $t_key \leftarrow l_key$
- 5 for $k=0, max_level-1$
- 6 $(t_key, lev) \leftarrow \text{transform}(t_key) \dots (t_key[k] \neq 4)$
- 7 $(pkey, clen) += \text{reduce_by_key}(t_key)$

Phase 2 (next slide):

$(sign, length, fpos) = \text{genValidQuadrants}(pkey, clen, nlen, n_t)$

Inputs: **pkey**: array of Morton codes of quadrants; **clen**: numbers of non-empty sub-quadrants; **nlen**: array of the numbers of points in these quadrants; **n_t**: #of points threshold

Output: **indicator**, **f_pos**

Algorithm **genValidQuadrants**

1 $t_{pos} \leftarrow \text{exclusive_scan}(clen)$

2 $tmap \leftarrow \text{scatter}([0..|clen|], t_{pos})$

3 $tmap \leftarrow \text{inclusive_scan}(tmap, \text{maximum})$

4 $tlen \leftarrow clen$ //tlen is a copy of clen to avoid modification due to remove_if in the next step

5 $\{pkey, clen, nlen, tmap\} \leftarrow \text{remove_if}(\{pkey, tlen, nlen, tmap\}, (nlen, n_t))$

6 $indicator \leftarrow \text{transform}(clen, (n_t))$

7 $nlen \leftarrow \text{replace_if}(nlen, indicator, 0)$

8. (revision) **reorder leaf nodes** based on their z-order codes at the finest level, compute **prefix-sum for ppos** and then **restore** the original order

9 $clen \leftarrow \text{replace_if}(clen, \sim indicator, 0)$

10 $cpos \leftarrow \text{exclusive_scan}(clen)$

11.1 $length \leftarrow \text{transform}(\{nlen, clen\}, indicator)$

11.2 $f_pos \leftarrow \text{transform}(\{ppos, cpos\}, indicator)$

Computing parent node offsets for access in Step 5

Determines leaf node

Remove unqualified nodes if #of points in their parent nodes are less than n_t

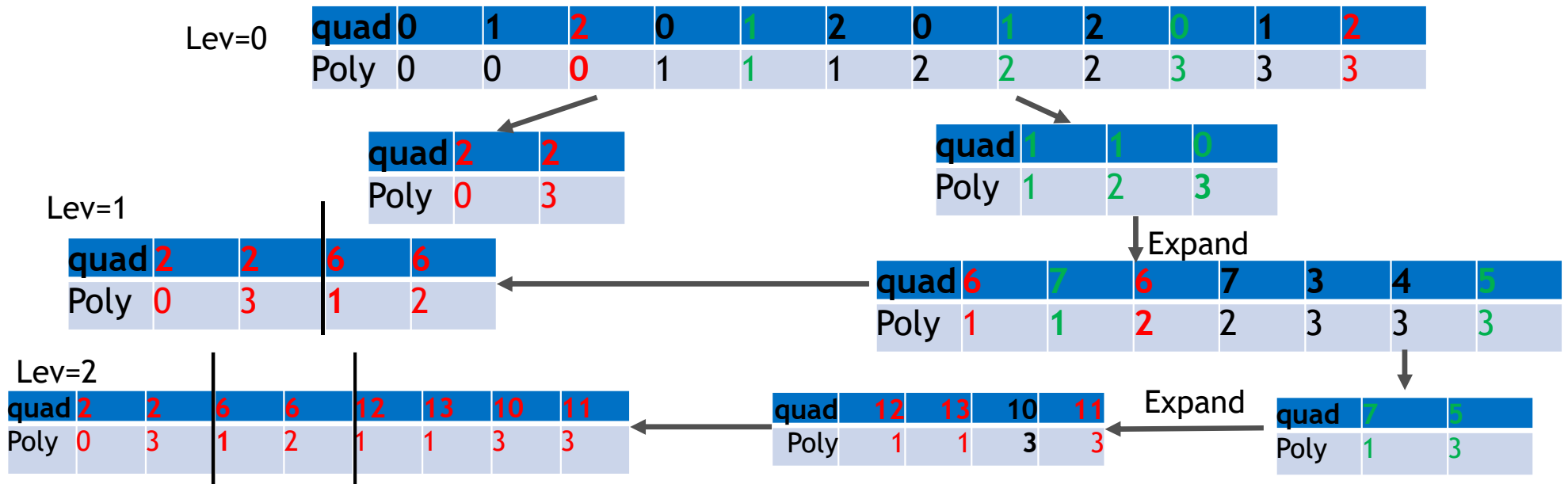
Adjust nlen/clen to prepare for computing pos

Filling f_pos

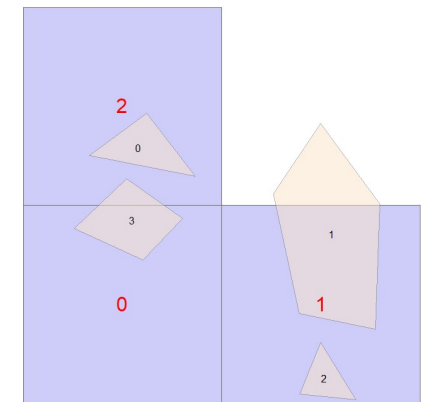
Quadrant and Polygon Intersection Test for Spatial Filtering

- **Basic idea:** If the quadrant that a point falls within does not intersect with the bounding box of a polygon, the point can not be within the polygon.
- The spatial filtering step **pairs point quadrants and polygon bboxes** for later spatial refinement step that actually test whether a point is inside a polygon.
- The paring process starts at the top level of a quadtree and pair quadrant and polygon **level-by-level**.
- All quadrants at a level are compared against all polygon bboxes. Three cases:
 - Quadrant does not intersect with polygon bbox → **discard**
 - Quadrant intersect with polygon bbox and the corresponding quadtree node is a leaf node: → copy the pair to output array, **no further expansion**
 - Quadrant intersect with polygon bbox and the corresponding quadtree node is a non-leaf node → copy the pair to the expansion array, **expand it based on fpos and length**, and get ready for next level iteration

Quadrant and Polygon Intersection Test for Spatial Filtering

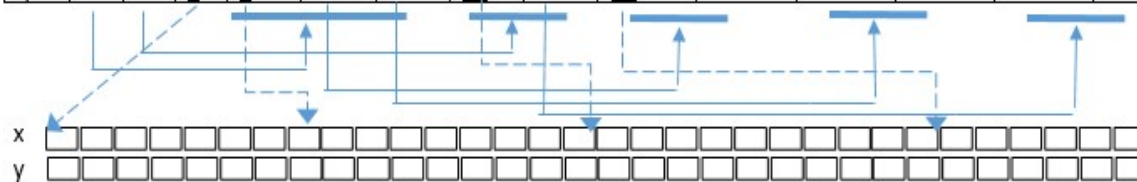


Seq	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Key	00	01	10	0000	0010	0011	0100	0111	001001	001010	001101	001110	011100	011111
Lev	0	0	0	1	1	1	1	1	2	2	2	2	2	2
Sign	1	1	0	0	1	1	0	1	0	0	0	0	0	0
len	3	2	7	9	2	2	7	2	9	5	8	8	7	11
fpos	3	6	0	7	8	10	16	12	23	60	37	45	53	60

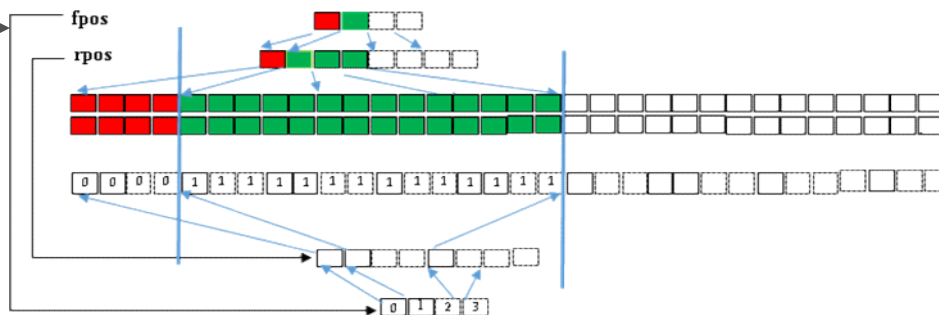


PIP-based Spatial Refinement

Seq	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Key	00	01	10	0000	0010	0011	0100	0111	001001	001010	001101	001110	011100	011111
Lev	0	0	0	1	1	1	1	1	2	2	2	2	2	2
Sign	1	1	0	0	1	1	0	1	0	0	0	0	0	0
len	3	2	<u>7</u>	<u>9</u>	2	2	<u>7</u>	2	<u>9</u>	<u>5</u>	<u>8</u>	<u>8</u>	<u>7</u>	<u>11</u>
fpos	3	6	0	7	8	10	16	12	23	60	37	45	53	60

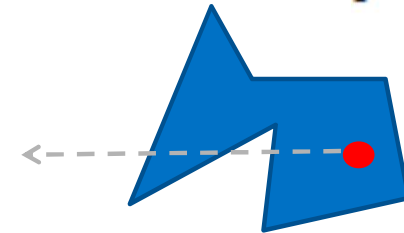


quad	2	2	6	6	12	13	10	11
Poly	0	3	1	2	1	1	3	3



Ray casting algorithm

$$P \propto_{PIP} Q$$



```
bool pnpoly(int npol, float *xp, float *yp, float x, float y)
{
    int i, j, c = 0;
    for (i = 0, j = npol-1; i < npol; j = i++) {
        if (((yp[i] <= y) && (y < yp[j])) ||
            ((yp[j] <= y) && (y < yp[i]))) &&
            (x < (xp[j] - xp[i]) * (y - yp[i]) / (yp[j] - yp[i]) +
             xp[i]))
            c = !c;
    }
    return c;
}
```

https://wrf.ecse.rpi.edu//Research/Short_Notes/pnpoly.html

The even-odd rule works for any closed set of polygons (polygons with multiple rings; polygons with holes...)

PIP-based Spatial Refinement

- Each quadrant-polygon pair is assigned to a thread block
- Each thread processes a point within a thread block
- All threads loop through all vertices of the polygon → coalesced memory access
- # of points in leaf-quadrants can be larger than the threshold (even if the threshold can be limited by the maximum number of threads in a block) → **decompose large quadrants**

Assuming minimum size for non-leaf nodes (threshold) is 512 and #of threads per block is 256

qsz	348	348	233	233	1000	375	432	212
quad	2	2	6	6	12	13	10	11
Poly	0	3	1	2	1	1	3	3

size	256	92	256	92	233	233	256	256	256	216	256	119	256	176	212
offset	0	256	0	256	0	0	0	256	512	784	0	256	0	256	0
quad	2	2	2	2	6	6	12	12	12	12	13	13	10	10	11
Poly	0	0	3	3	1	2	1	1	1	1	3	3	3	3	3

PIP-based Spatial Refinement

- Many-to-many relationship among quadrants (and their points) and polygons
- Output (point-polygon pairs) size is data dependent and can not pre-determined
- **Two-phase strategy:** count # of point-polygon pairs in phase 1 and write the actual output in phase 2 → run PIP twice.

```
bool in_polygon= pip_test (...)
```

```
unsigned mask = __ballot_sync(0xFFFFFFFF, threadIdx.x < num_point);
```

```
uint32_t vote=__ballot_sync(mask, in_polygon);
```

```
if(threadIdx.x%num_threads_per_warp==0)
```

```
    data[threadIdx.x/num_threads_per_warp]=__popc(vote);
```

```
    __syncthreads();
```

```
if(threadIdx.x<max_warps_per_block)
```

```
{ uint32_t num=data[threadIdx.x];
```

```
  for (uint32_t offset = max_warps_per_block/2; offset > 0; offset /= 2)
```

```
    num += __shl_xor_sync(0xFFFFFFFF,num, offset);
```

```
if(threadIdx.x==0)
```

```
    num_hits[blockIdx.x]=num; }
```

Phase1

Combining ballot and popc to
count # of points in polygon in a
complete warp

Warp-level reduction
(for no more than 1024
threads per block)

reduce (sum) on num_hits will give #of output point-polygon pair
scan(prefix-sum) on num_hits will give the offset to output for each block

PIP-based Spatial Refinement

- Pre-processing: sub-quadrant and polygon pairs may have zero point-polygon pairs and need to be removed → `remove_if`
- **Phase 2**

```
....  
if(threadIdx.x < num_threads_per_warp) {  
    uint32_t num = data[threadIdx.x];  
    for (uint8_t i = 1; i <= num_threads_per_warp; i *= 2) {  
        int n = __shfl_up_sync(0xFFFFFFFF, num, i, num_threads_per_warp);  
        if (threadIdx.x >= i) num += n; }  
        sums[threadIdx.x + 1] = num;  
        __syncthreads(); }  
__syncthreads();  
if((threadIdx.x < num_point) && (in_polygon)) {  
    uint16_t num = sums[threadIdx.x / num_threads_per_warp];  
    uint16_t warp_offset = __popc(vote >> (threadIdx.x % num_threads_per_warp)) - 1;  
    uint32_t pos = d_num_hits[blockIdx.x] + num + warp_offset;  
    d_res_poly_idx[pos] = pq_poly_idx[blockIdx.x];  
    d_res_pnt_idx[pos] = qt_fpos[pq_quad_idx[blockIdx.x]] + sub_offset[blockIdx.x] + threadIdx.x; }  
}
```

data stores # of positive PIP test per warp (as in Phase 1)

Warp-level scan (for no more than 1024 threads per block)

Output offset for each point-polygon pair (pos) is the sum of quadrant offset, warp offset and thread ID.

Point idx is the sum of first point position in a quadrant, offset within sub-quadrant and thread ID

PIP-based Spatial Refinement

size	11	11	9	9	8	7	5	8
offset	0	0	0	0	0	0	0	0
quad	2	2	6	6	12	13	10	11
Poly	0	3	1	2	1	1	3	3

Phase1

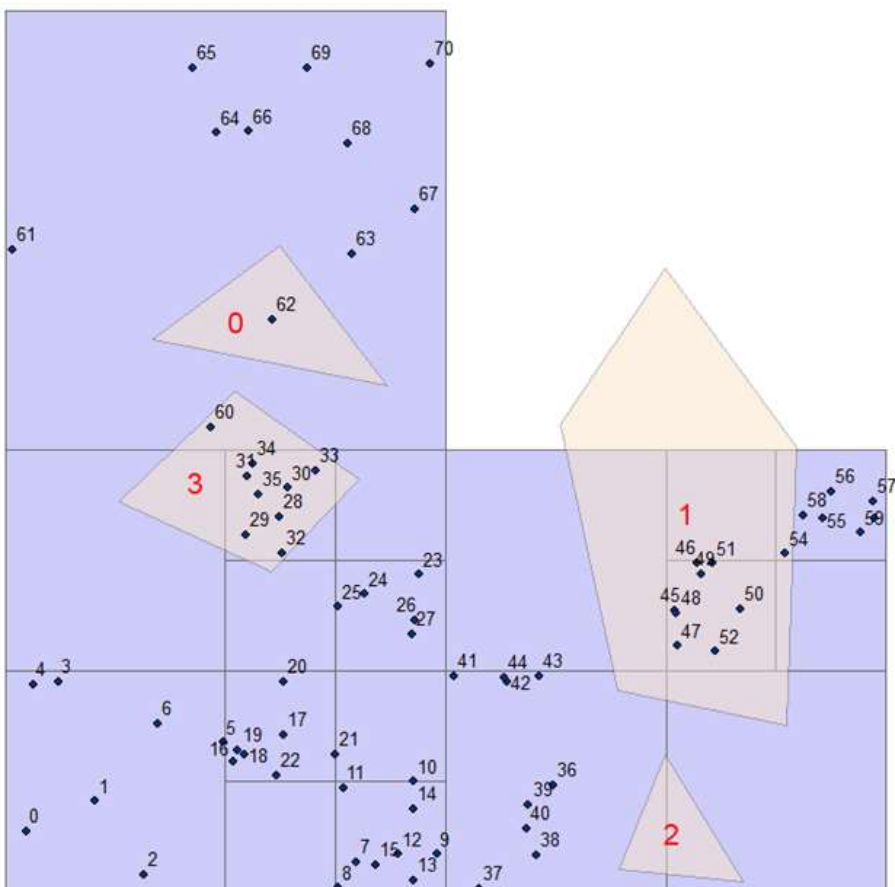
#	1	1	0	0	8	1	0	8
---	---	---	---	---	---	---	---	---

Remove Empty Pairs

1	1	8	1	8
---	---	---	---	---

Prefix-sum

0	1	2	10	11
---	---	---	----	----



Point-idx

Polygon-idx

62	60	52	51	50	49	48	47	46	45	54	35	34	33	32	31	30	29	28
0	3	1	1	1	1	1	1	1	1	1	3	3	3	3	3	3	3	3

Performance on NYC Taxi Trip Data

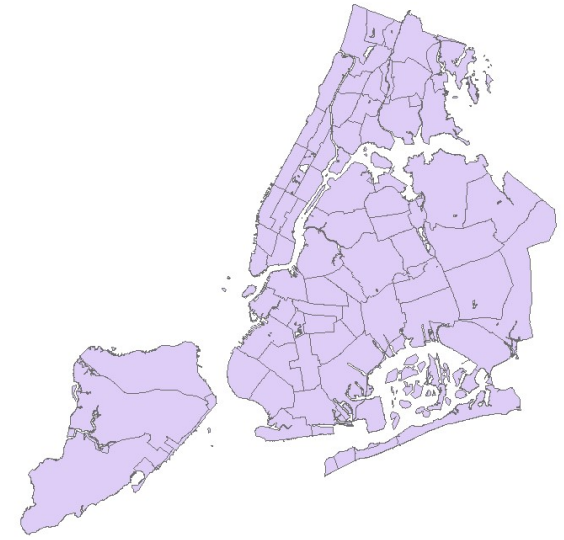
2009 NYC Taxi Tri Pickup Locations

# Mo.	#of points
1	13,887,620
2	27,079,723
3	41,284,081
4	55,383,596
5	69,970,743
6	84,035,490
7	97,553,533
8	111,127,610
9	124,993,700
10	140,444,141
11	154,523,740
12	168,898,952

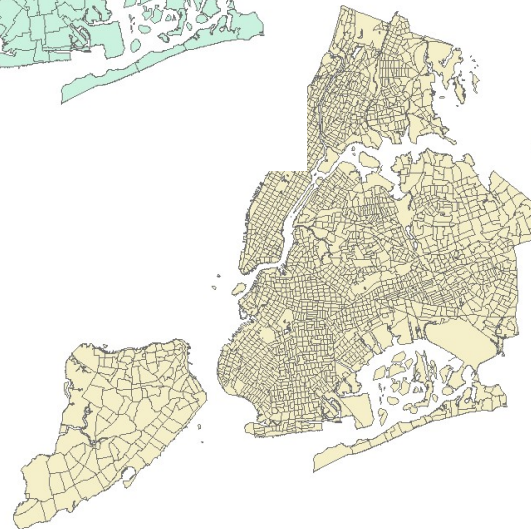
NYC Taxi Zones, 263 polygons,
(354 rings, 98192 vertices)



NYC Community Districts , 71 polygons
(243 rings, 100119 vertices)



NYC Census Tract,
2216 polygons
(2411 rings,
170329 vertices)



Performance on NYC Taxi Trip Data -Titan V 12GB/i7-7800X

	Description	Taxi Zone	CD	CT
0	# of quad-poly pairs	1,623,908	1,496,150	1,860,074
1	Point Quadtree Indexing runtime (ms)	384.6	384.2	385.2
2	Spatial Filtering runtime (ms)	35.2	31.0	36.3
3	Spatial Refinement runtime (ms)	140.5	552.0	277.3
4	GPU End-to-End runtime (ms)	560.9	967.8	699.3
5	GDAL API on CPU runtime (ms) (10k random points - without indexing)	66,890.9	66,959.4	145,975
6	Speedup1= ([5]/10000)/([4]/168,898,952)	2.01E+06	1.17E+06	3.53E+06
7	GDAL API on CPU runtime (ms) (10000 random quad-poly pairs)	71,143.6	420,970	202,320
8	Speedup2=([7]/10000)/([4]/[0])	2.06E+04	6.51E+04	5.38E+04

Performance on NYC Taxi Trip Data -Titan RTX 24 GB/Intel i7-6700K

	Description	Taxi Zone	CD	CT
0	# of quad-poly pairs	1,623,908	1,496,150	1,860,074
1	Point Quadtree Indexing runtime (ms)	297.7	329.5	294.6
2	Spatial Filtering runtime (ms)	52.9	23.0	59.3
3	Spatial Refinement runtime (ms)	985.9	4299.8	2040.8
4	GPU End-to-End runtime (ms)	1347.0	4658.7	2406.3
5	GDAL API on CPU runtime (ms) (10k random points -without indexing)	63,220.3	62,763.5	142,826
6	Speedup1= ([5]/10000)/([4]/168,898,952)	1.90E+06	1.10E+06	3.45E+06
7	GDAL API on CPU runtime (ms) (10000 random quad-poly pairs)	46,758.9	277,164	140,984
8	Speedup2=([7]/10000)/([4]/[0])	1.35E+04	4.28E+04	3.75E+04