Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
B		2116540

Infinite Powers of Drones : A Practical Discrete Optimization Model Helping Fight against Bushfires in Australia

Summary

In recent years, under the influence of global climate change, wildfires have gradually become a potential crisis in human life. From 2019 to 2020, Australia suffered a "Black Summer of death" due to the impact of bushfires. Drones are effective equipment for disaster relief, for example, fires. This paper is mainly based on mathematical models optimize the Victoria's CFA's drone purchase plan, as well as feasible plans to optimize the hovering strategy of drones.

First of all, in the setting of the first question, we are required to create a model to determine the optimal mix of SSA drones and radio repeater drones for the new department proposed by the Victorian National Fire Service. We abstract the total cost as a cost function, and the numbers of both kind of drones are abstracted as 2 parameters to be optimized. Based on the knowledge of **single-objective integer optimization**, we obtained an optimized purchase strategy of the two kinds of drones. For the other two parameters: topography and fire frequency, we also obtained a variety of optimal decisions. Our model successfully minimized the total cost while meeting the requirements of security and communication tasks.

Secondly, we are asked to explain our purchase strategy reasonably, which is an extension of the first question. Due to the randomness of serious wildfires, we assume that the occurrence of extreme fire weather in Australia within ten years is a probability event. We followed the **Poisson distribution** and give the prior probability distribution of bushfire in Victoria within ten years. Based on the **prior probability distribution**, the total cost increment is simulated by software, and the cost increment optimization based on different topography is proposed. The strategy is reasonably explained by our model. Finally, based on the results, we conducted a sensitivity analysis to verify the robustness of the model against changes of the possibility of extreme fire events in the next ten years.

Next, the question asked to determine a model to optimize the position of the hovering VHF/UHF radio repeater drone to deal with fires of different sizes on different terrains. In order to simplify the model, we introduce some concepts, such as grid, area coverage, coverage redundancy and so on. Among them, the introduction of grid is to divide a certain size area, so as to better generate altitude data and UAV positioning. And also, we abstract the drone team carrying the repeater as a **connected network**. By using **Particle Swarm Optimization** algorithm, we have given an optimized solution for the hovering of radio repeater drones. **Monte Carlo method** is used to simulate, and we obtained the optimal hovering position strategy.For this model, we also do sensitivity analysis, generate multiple sets of coverage schemes under the same terrain data and compare their performance. Finally, we conclude that the model has strong stability.

In the end, based on the results of the first two questions and more information about UAVs, fire and terrain learned from the third question, we wrote an annotated Budget Request for FCA which carried a robust conclusion.

Key Words: Single-objective integer optimization Prior probability distribution Graph theory Particle Swarm Optimization Monte Carlo method

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1 Introduction

1.1 Problem Background

Fire is a tough issue for Australia. As we all know, forest fires usually occur during severe drought and continuous high temperature. In order to solve this problem, Australian firefighters began to use SSA to monitor the changing situation of fire as early as a few years ago to assist EOC to fight the fire efficiently. In addition, EOC also uses handheld two-way radios operating and Radio Repeater drones. The range of the former is limited by the transmission power, which has a great relationship with the terrain; the latter can expand the range of radio signals, and as the hub of the front line and EOC.

As we can see from the picture1 , over time, thermal anomalies have become more pronounced and hill fires have become more frequent in southeastern Australia.

Figure 1: South East Australia Fires and Thermal Anomalies From OCT 01 2019 to JAN 07 2020 (Ordered in Month) , 23.5°S 43.21°S, 135°E 153.48°E, Data From Satellite Terra/MODIS [1]

In recent years, the Victorian Fire Service has divided the entire state into nine fire districts in order to combat fires, as shown in Figure 2.

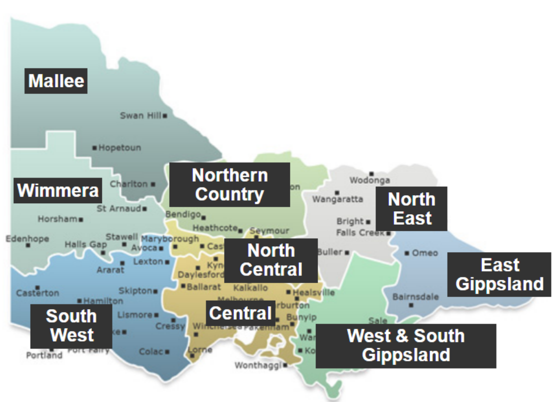


Figure 2: Nine Fire Areas in Victoria

1.2 Restatement of the Problem

Considering the background information and restricted conditions identified in the problem statement, we need to solve the following problems:

- Establish a model to determine the best combination of SSA drones and Radio Repeater drones.
- Based on the changing possibility of extreme fire events in the next ten years, use your model to predict which kind of equipment cost will increase under the condition of constant cost, and how the model can adapt to this situation.
- According to different sizes on different terrains of fires, a model is determined to optimize the position of hovering VHF / UHF radio repeater drones.
- Prepare a reasonable and annotated budget based on the model for government support.

2 Assumptions and Justifications

The following basic assumptions are made to simplify problems.

- The flight process of UAV is not affected by bad weather conditions, wind direction and wind speed at the disaster site.
- As for UAV, the time of its taking-off, landing and the time of attitude adjustment can be ignored.
- The UAV will not experience, resulting in complete failure within the disaster relief time.

3 Notations

The primary notations used in this paper are listed in table 1.

Table 1: Notations

symbols	definition
δ	different spatial locations
R	maximum emission radius
x_i, y_i, z_i	the location coordinates of each nodes
x_j, y_j, z_j	elevation information of each nodes
d(i,j)	the distance of all nodes in the model space
$\mathrm{P_i} \in$ set {FrowardTeams}	maximum transmitting distance of the Forward Teams
$\mathrm{U_j} \in$ set { Drones }	maximum signal receiving distance of the UAVs carrying repeaters
f_{cost}	cost function
a	price of a single flight per UAV
n_1	the number of UAV racks carrying repeaters
n_2	the number of UAV racks carrying SSA
t′	the departure interval of different batches of UAVs
t_{charge}	the charging time of drones
t	the time the drone is on site
t_{max}	maximum flight time of a UAV in the air
x_{max}	maximum range of an aircraft
R	maximum radius of the signal that the repeater can accept
n+1	batches
v	the flight speed of the UAV
k_q	the number of area types owned by the regional city
$\Delta f_{\mathrm{cost}i}$	the incremental cost of a single type of drone
Δmi	area increment .
$f_{\mathrm{cost}i}$	the total cost function
$E(\lambda_j)$	the expectation based on the number of fires
$N^{(r)}$	the number of remaining region cities
$\Delta f'_{\mathrm{cost}\ i}$	the incremental cost of changing
U_i	the UAV coordinates
V_i	the UAV speed

4 Model I :Integer Combination Optimization Model Based on Land Type, Fire Frequency and Disaster Range

First of all, the SSA UAV is used to collect the fire signal. It also needs certain relay equipment, and then transmits the signal from the front line to EOC.

Repeater, is to ensure that in our entire disaster relief process, always can not let the signal interrupt a device. So it needs to have a certain coverage and coverage density.

If we assume that the UAV starts from the EOC and finally returns to the EOC, then we hope that the UAV can stay on site as long as possible, so as to send back as many signals as possible, and then reduce the time spent on the road as much as possible, so as to expand our coverage as much as possible.

The equipment worn by the ground personnel is used to monitor the ground personnel, including the situation of the front line and receiving the instructions issued by EOC, etc. The device also needs repeaters to cover. Once the equipment signal is interrupted, EOC can't monitor the signal of ground personnel, and then ground personnel will be in danger, so the equipment signal can't be interrupted at any time.

So the key of the problem is that we should keep the repeater running normally all the time, and the signal will not be interrupted in the whole field. So the goal of our optimization is how to keep the UAV with repeater flying continuously.

4.1 The Establishment of model I

In our model, the Emergency Operations Center, all drones, and the fire point (there may be more than one, depending on the size of the fire) are deployed in a circular region of radius $r_{\max}$, and all elements are considered as nodes, which are abstracted as mass points with no size and only coordinates.

The UAV nodes flying in the air have access to their own longitude, latitude, and altitude information at any time, and under this topology, the absolute distances of each node are expressed as Euclidean distance in 3D space (i.e., distances under the L_2 norm),

$$d(i,j)=\sqrt{\left(x_i-x_j\right)^2+\left(y_i-y_j\right)^2+\left(z_i-z_j\right)^2}. \tag{4.1}$$

where x_i, y_i, z_i, x_j, y_j, z_j denote the location coordinates, elevation information of each node, and $d(i,j)$ denotes the distance of all nodes in the model space, including Emergency Operations Center $\rightarrow$ UAV, UAV $\rightarrow$ UAV, UAV $\rightarrow$ Fire Spot, and Emergency Operations Center $\rightarrow$ Fire Spot.

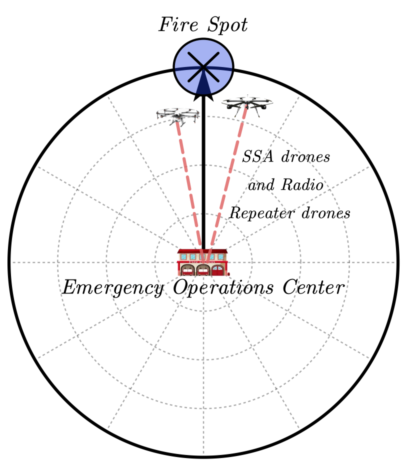


Figure 3: Topological abstraction of the fire rescue scene (in 2-dimensional view)

For any point $C_k(k_x,k_y,k_z)$ in the abstract space, we use Boolean-type variables to determine whether C_k is within the signal search range of the ith repeater drone $U_i(x_i,y_i,y_i)$:

$$\operatorname{Cov}(C_k, U_i) = \begin{cases} 0, & \sqrt{(k_x - x_i)^2 + (k_y - y_i)^2 + (k_z - z_i)^2} > R \\ 1, & \text{otherwise} \end{cases} \tag{4.2}$$

That is, $Cov(C_k, U_i)$ = 1 when the target position is within the signal search range of the ith repeater UAV U_i ; and vice versa $Cov(C_k, U_i)$ = 0.

When considering "Boots-on-the-ground" Forward Teams on the front lines, the radius of each firefighter's VHF/UHF transmitter will change due to different terrain, so we may take the state variable to represent different spatial locations:

$$\delta = \begin{cases} 1, \text{Municipalities} \\ 2, \text{Mountainous} \end{cases} \tag{4.3}$$

We may assume that each firefighter's emitter radiation radius can reach the maximum emission radius, so we have:

$$R = \begin{cases} 2, t = 1 \\ 5, t = 2 \end{cases} \tag{4.4}$$

In the same way as above for any location with a repeater drone, for any frontline firefighter $P_l(l_x, l_y, l_z)$ in abstract space, we use a Boolean type variable to determine if P_l is within the signal search range of the jth repeater drone $U_j(x_j, y_j, y_j)$.

$$\operatorname{Cov}(C_k, U_i) = \begin{cases} 0, & \sqrt{(k_x - x_i)^2 + (k_y - y_i)^2 + (k_z - z_i)^2} > R_{Receive} + R_{Spread} \\ 1, & \text{otherwise} \end{cases} \tag{4.5}$$

That is, $Cov(P_l, U_j)$ = 1 when the firefighter P_l is within the signal search range of the jth repeater drone U_j ; and vice versa $Cov(P_l, U_j)$ = 0.

In the process of fighting a mountain fire, it is essential to keep the signal between the front and the Emergency Operations Center open. The Forward Team is the center of activity for the entire firefighting process, and the Forward Team on the ground has a lot of first-hand information that needs to be transmitted back to the Emergency Operations Center in real time through a smooth network for decision making. The SSA equipment contains information on the physical condition of the forward firefighters, which needs to be monitored throughout the firefighting process to ensure the personal safety of the "Boots-on-the-ground" Forward Teams. For safety reasons, it is assumed that $D_{\text{margin}}(P_i, U_j)$ is the sum of the Euclidean distance $D(P_i, U_j)$ minus the maximum transmitting distance of the Forward Teams $\mathrm{P_i} \in$ set {FrowardTeams} and the maximum signal receiving distance of the UAVs $\mathrm{U_j} \in$ set { Drones } carrying repeaters, when deploying the UAV hovering position. The following constraint exists on the distance between the UAV carrying SSA equipment and repeater equipment and the forward personnel:

$$\begin{aligned} & D_{\text{margin}}(P_i, U_j) = D(P_i, U_j) - (R + R_{\max}) \leq 0, \\ & \exists j \in \text{set}\{\text{ Drones }\}, \forall i \in \text{set}\{\text{ Forward Teams }\} \end{aligned} \tag{4.6}$$

That is, any one forward person P_i has to be under the monitoring range of at least one UAV U_j carrying a repeater, as shown in the figure4 below.

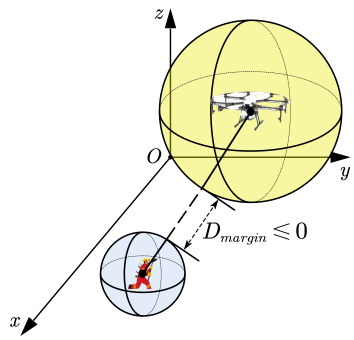


Figure 4: Handheld UHF/VHF transmitters and UAVs carrying repeaters need to keep the signal open at all times

4.1.1 Strategy I :Single-batch Round-trip Program

Here is the cost function we developed for this scenario:

$$\min\ f_{\text{cost}} = a\sum_{i=1}^{2} n_i, \quad i = 1,2 \tag{4.7}$$

Our single-batch round-trip program in these areas would make for a good solution to the safety of residents as well as the economic problems of the government.

4.1.2 Strategy II :Multi-batch Round-trip Program

We model the transmitter carried by the ground personnel into a spherical shape with a radius, and so are the repeaters. The number of aircraft in each voyage depends on the extent of the fire.

We assume that after the fire, the time taken by the first aircraft, the first to carry the repeater, to depart from EOC to the target site is t' , and then the time spent hovering there is t . Then, due to power constraints, the time he takes to return is t' .At the same time that the first batch of aircraft returned, the second batch of aircraft arrived at the hovering position of the first batch of aircraft just after t' . And so on, until the whole batch of $n+1$ aircraft alternated.

To match this multi-batch cycle scheme, we propose the following optimal solution:

$$\min\ f_{\text{cost}} = a\sum_{i=1}^{2} n_i, \quad i = 1,2 \tag{4.8}$$

$$s.t.\begin{cases} 2n_1t' + t_{\text{charge}} = t, \\ 2n_1t' + t \leqslant \mathrm{t}_{\max}, \\ 2n_1t' \cdot v \leqslant x_{\max}, \\ t' \cdot v \leqslant R, \\ n_1 \in R^+, \\ t \geqslant 0, t' \geqslant 0. \end{cases}$$

where, f_{cost} denotes the cost function; a represents the price of a single flight per UAV; n_1 , n_2 denote the number of UAV racks carrying repeaters and the number of UAV racks carrying SSA, respectively; t' is the departure interval of different batches of UAVs; $t_{\mathbf{charge}}$ is the charging time of drones; t is the time the drone is on site; $t_{\max}$ is the maximum flight time of a UAV in the air; $x_{\max}$ is the maximum range of an aircraft; R is the maximum radius of the signal that the repeater can accept; $n+1$ represents batches; v indicates the flight speed of the UAV.

Let us further show from the image5 and 6 the propagation distance of the equipment carried by the personnel on the ground and the reception distance of the repeater.

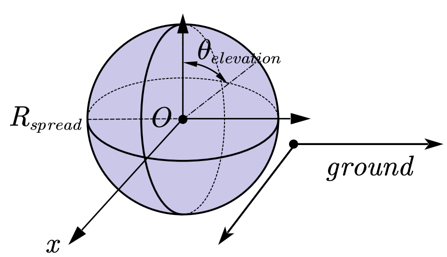


Figure 5: The equipment worn by the ground personnel

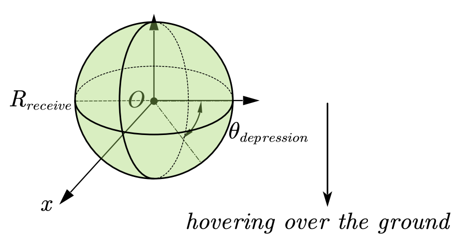


Figure 6: Repeater Drones

In order to make the front signal uninterrupted, the hovering time of the UAV in front is just equal to the sum of the charging time and the time of flying back and forth in the air.

$$2n_1 t' + t_{\text{charge}} = t \tag{4.9}$$

Suppose that the total distance of each aircraft from the air is X.

$$2n_1 t' \cdot v \leqslant x_{\max} \tag{4.10}$$

4.2 Results

- **Urban: Low Frequency of Fires**

  Considering that urban areas are often close to the CFA, EOC often will not be particularly far from the center of the fire location, and the fire range is often not too large, the duration will not be too long, we may wish to assume that the altitude h of our UAV flight satisfies:

  $$300 \leqslant h \leqslant 500 \tag{4.11}$$

  consider the economics as well as to meet the needs of communication tasks, the final only need to purchase two UAVs that $n_1 = n2 = 1$, one for carrying SSA equipment, one for carrying repeaters.

- **Urban: High Frequency of Fires**

  In cities, the difficult points of fire rescue often occur at the level of more high-rise buildings, slow marching of ground troops, and difficulties in signaling front-line teams due to the shielding effect of concrete structures on signals, so the urgency of rescue time and the enhancement of rescue signals are particularly important. Therefore, in cities where fires occur with high frequency, in order to safeguard the lives of people in front, it may still be assumed that the altitude h of our UAV flight satisfies:

  $$300 \leqslant h \leqslant 500 \tag{4.12}$$

  our model combined with the actual fire situation, it is recommended to acquire two UAVs carrying repeaters, which can be set up on both sides of the building near the fire point (two corners of the block) respectively to compensate for the reception of the single UAV situation that exists The blind area problem. In terms of observation range, the dense and densely populated urban buildings and the high-rise buildings themselves are relatively upright, making overhead observation easier, and therefore only two UAVs for carrying SSA equipment are required.

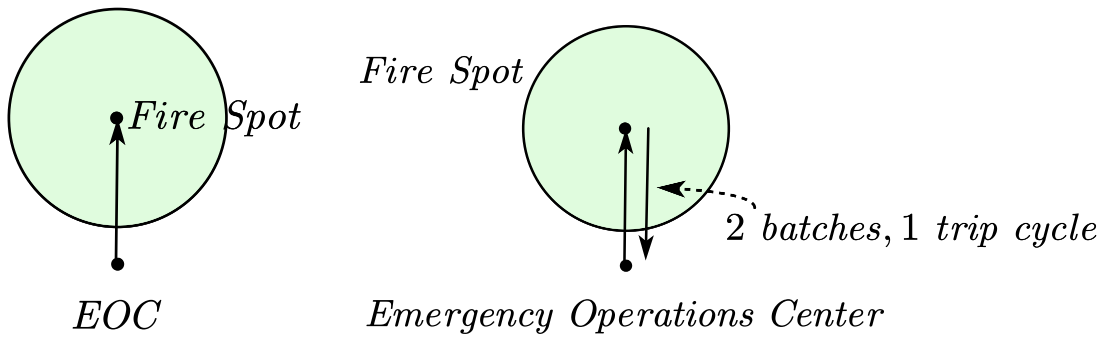


Figure 7: Team Strategy in Urabn: Low(Left) & High(Right) Frequency of Fires

- **Rural: Low Frequency of Fires**

  In rural areas, the fire command center, EOC is often a certain distance from the countryside, but taking into account the small population density of the countryside, sparse housing, when the fire occurs, personnel easy to evacuate, firefighters to reach the disaster area will not take too long, EOC can also carry out timely operations, the acquisition of fewer UAVs can ensure safety. At the same time, the rural terrain is relatively flat, and the handheld radio has a wide range of propagation on flat, unobstructed ground, which facilitates the reception of signals, so there is no huge problem in the signal communication between the front personnel and the EOC in small fires. It may be assumed that the aircraft flight altitude $h = 300$m, so we suggest that only two UAVs need to be acquired, $n_1 = n_2 = 1$, one for carrying SSA equipment, one for carrying a repeater can be, two aircraft flying at the same time.

- **Rural: High Frequency of Fires**

  Since the countryside is far away from the fire command center and EOC, if fires occur frequently, there is a high possibility of causing large casualties and financial losses. Generally speaking, the countryside is large, so the number of UAVs carrying SSA equipment needs to be increased for observation; meanwhile, the signal dispersion caused by the large overfire area needs to be solved by increasing the number of UAVs carrying repeaters. It is still assumed that the flight altitude $h = 300$m, four UAVs need to be purchased, that is, $n_1 = n_2 = 2$, two for carrying SSA equipment, two for carrying repeaters, the UAVs carrying SSA equipment and the UAVs carrying repeaters are carried out in a 2-batch round-trip cycle scheme.

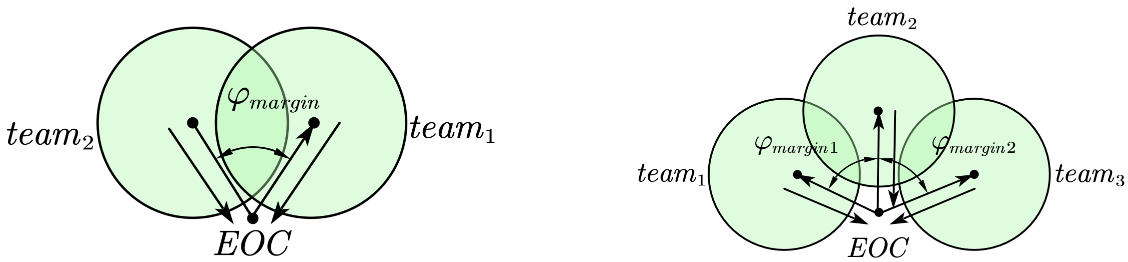


Figure 8: Team Strategy in Rural: Low(Left) & High(Right) Frequency of Fires

- **Mountainous Areas: Low Frequency of Fires**

  In the case of mountainous areas, taking into account the distance between mountainous areas and fire command centers, EOC is often much greater than the distance between the city and the countryside from both, and the mountainous terrain is complex, the front firefighters are often more scattered, but because of the low frequency of fires in mountainous areas usually do not occur large fires, but fires need to be extinguished after the open fire for a longer period of time after a wide range of observation to prevent the recurrence of flames, so assume In mountainous conditions UAV flight altitude $h = 2500$ meters, considering the economy and safety, we propose to purchase a total of 6 UAVs, of which $n_1 = 2, n_2 = 4$, 2 carry repeaters, while equipped with 4 UAVs carrying SSA equipment, take the same batch of 2 UAVs carrying SSA + 1 UAV carrying repeaters to fly with the combination of strategy to increase the search area and maintain the ability to search for a longer period of time.

- **Mountainous Areas: High Frequency of Fires**

  In mountainous areas where fires are frequent, the search area is large, the firefighting time is long, the task of fighting fires is heavy, and the distribution of people ahead is more dispersed and the danger is greatly increased, which greatly increases the difficulty of rescue. For this situation, in order to ensure the safety of the rescue process and meet the communication task needs, assuming that the UAV flight altitude $h = 2500$ meters in mountainous conditions, we take the form of 2 to 3 groups of formations in parallel, distributed participation in fire fighting and rescue operations, where each group of formations contains 2 UAVs carrying repeaters, 4 UAVs carrying SSA equipment, each formation takes 2 batches The circular flight plan, take the same batch of 2 UAVs carrying SSA + 1 UAV carrying repeater accompanying flight combination strategy, so that you can carry out fire fighting tasks in different directions at the same time, at this time we assume that the angle formed within the two groups of formations superior arc is ϕ, with different fire fighting tasksϕ can be adjusted according to the situation, and according to the characteristics of fire-prone mountainous areas with 2-3 Group echelon to adapt to the different fire point needs, so as to expand the source of information.

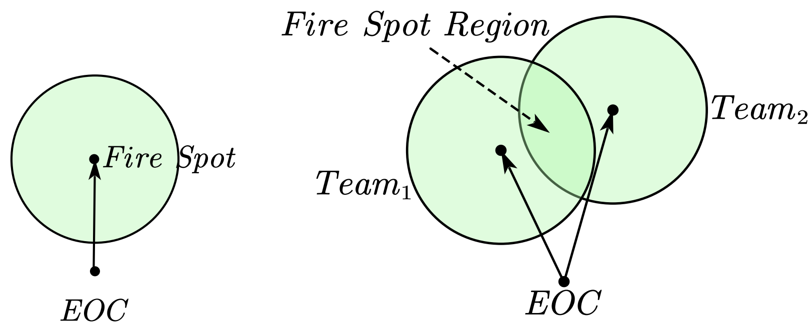


Figure 9: Team Strategy in Mountaionous Areas: Low(Left) & High(Right) Frequency of Fires

- **Conclusion**

  The best mix of 2 kinds of drones and total cost summary sheet is shown as table 2.

Table 2: Best Mix and Total Cost Summary Sheet

AREA	Frequency	UAVs carrying SSA	UAVs carrying Repeter	f_{cost}/AUD
Urban	Low	1	1	20000
	High	2	2	40000
Rural	Low	1	1	20000
	High	2	2	40000
Mountain	Low	4	2	60000
	High	4-6	2-3	60000-90000

5 Model II :Equipment cost prediction model based on a priori probability distribution for large fires

5.1 The Establishment of Model II

Victoria has 48 districts. [4] Of these, Hume and Grippsland have 18 regional and rural council areas, which completely cover the Australian Ranges, so we consider that there are only high frequency areas here. These high frequency areas all include mountainous, urban and rural areas. However, since they do not involve incremental cost functions, we do not consider the exact number of mountain, urban, and rural areas.

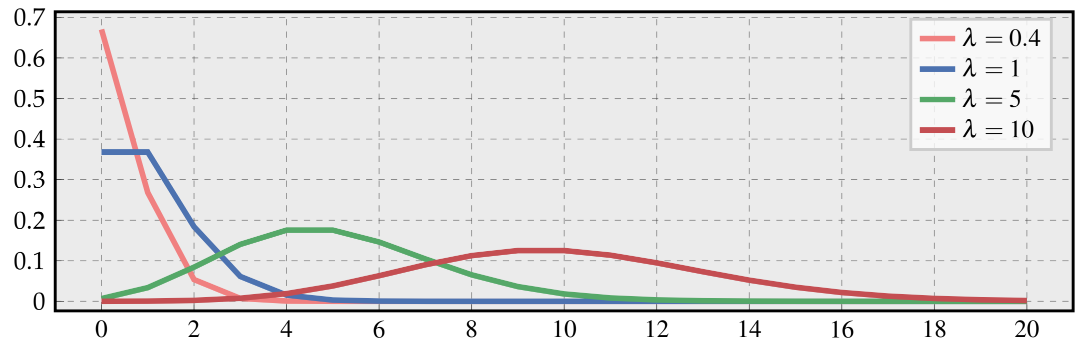
Our model covers a maximum area of $1829.80km^2$, and the remaining 30 Regions can be counted as having 2 urban (low frequency) and 8 rural (low frequency) Regions each, with a total of 9 Regions located at the foot of the mountains, so each of these 9 Regions plus 1 mountain (low frequency). 5 near the foot of the mountains for Loddon and 4 of the Grampians near the foot of the mountains, which are very close to the mountains. To simplify the model, we consider that there is only one low fire frequency mountain area in these 9 Regions.

Since large fires are small probability events that vary with time and obey the conditions of applicability of Poisson distribution [2], we use Poisson distribution for the analysis.

We assume that the number of extreme fires (within one year) obeys a Poisson distribution with λ = 0.4 in the next ten years.

The following graph shows the Poisson distribution for parameters λ of 0.4, 1, 5, and 10.

The Poisson Distribution With Parameter λ



$$\Delta f'_{\text{cost } i} = \begin{cases} 20000, i = 1 \\ 20000, i = 2 \\ 30000, i = 3 \end{cases} \tag{5.1}$$

$$\Delta mi = \begin{cases} 2, & i = 1 \\ 8, & i = 2 \\ 1, & i = 3 \end{cases} \tag{5.2}$$

$$kq = \begin{cases} 2, & 1 \leqslant q \leqslant 21 \\ 3, & 22 \leqslant q \leqslant 30 \end{cases} \tag{5.3}$$

where,kq denotes the number of area types owned by the regional city $(q = 1, 2, 3)$; $\Delta f_{\text{cost } i}$ denotes the incremental cost of a single type of drone; Δmi denotes the number of areas, that is, the amount of drone growth.

Since the probability of extreme fires is small and negligible, the value of $\lambda < 1$.

5.2 Sensitivity Analysis

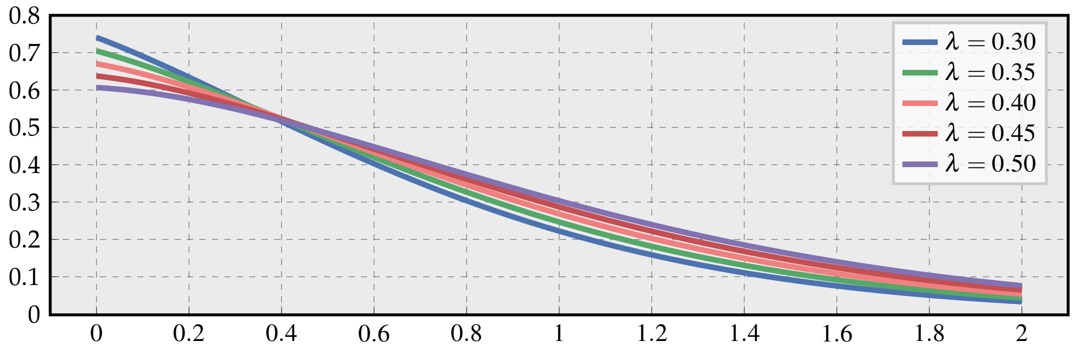
We model the UAS with constant cost to explore how it adapts to the changing likelihood of extreme fire events over the next decade in order to find the optimal solution for increasing equipment costs. And, we used sensitivity analysis in order to allow the reliability of the model to be confirmed. In other words, we vary the lambda parameter of the model (Eq.) and observe the degree of change that causes the output of this model. The purpose of this sensitivity test is to examine the extent to which the cost function fluctuates with the frequency of catastrophes.

Now we take λ to be $0.3, 0.4, 0.5$ respectively.

$$\lambda_j = \begin{cases} 0.3, & j=1 \\ 0.4, & j=3 \\ 0.5, & j=5 \end{cases} \tag{5.4}$$

The following graph shows the Poisson distribution for parameters λ of 0.30, 0.35, 0.40, 0.45, 0.50.

The Poisson Distribution With Parameter λ



$$\Delta f_{\text{cost}i} = E\left(\lambda_j\right) \sum_{q=1}^{N^{(r)}} \left(\sum_{i=1}^{kq} \Delta f'_{\text{cost } i} \cdot \Delta mi \right) \quad j=1,2,3,4,5 \tag{5.5}$$

$N^{(r)} = 30$ is the number of remaining region cities.

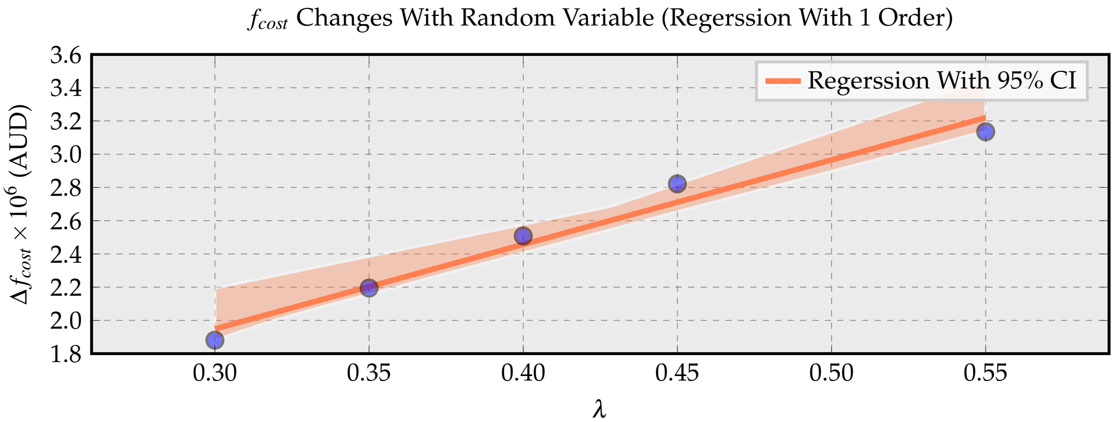
$$\Delta f'_{\text{cost } i} = \begin{cases} 20000, i=1 \\ 20000, i=2 \\ 30000, i=3 \end{cases} \tag{5.6}$$

$$\Delta mi = \begin{cases} 2 & i=1 \\ 8 & i=2 \\ 1 & i=3 \end{cases} \tag{5.7}$$

$$kq = \begin{cases} 2 & 1 \leqslant q \leqslant 21 \\ 3 & 22 \leqslant q \leqslant 30 \end{cases} \tag{5.8}$$

where, $\Delta f_{\text{cost}i}$ is the total cost increment; $E\left(\lambda_j\right)$ is the expectation based on the number of fires (in one year);kq denotes the number of area types owned by the regional city (q=1,2,3); $\Delta f'_{\text{cost } i}$ illustrates the incremental cost of changing from a low frequency of fire to a high frequency of fire for different terrains; Δmi is the description of the area increment corresponding to the low frequency of fire to high frequency of fire for different terrain.

The following graphs represent the different values of f_{cost} in the case of random variations.



5.3 Results

By applying an equipment cost prediction model based on the prior probability distribution of large fires, combined with the properties of the Poisson distribution and the geography of Victoria. The final conclusion of the question is drawn.

Conclusion: Large fires are small probability events that vary over time, and according to our tests, its number of occurrences in a year roughly obeys the Poisson distribution, and its expected number of occurrences is λ, to accommodate the possibility of changing extreme fire events in the next decade, early preparation should be made, i.e., a certain amount of reserve funds should be prepared each year to counteract the occurrence of extreme fire events. By calculating the cost increment, a minimum of 3135,000 reserve funds should be set aside each year, which will be used to purchase SSA equipment and repeaters if an extreme fire event occurs. Therefore the cost of SSA equipment and repeaters will increase.

6 Optimized position of VHF / UHF radio relay UAV for different sizes of fire on different terrains

6.1 Topographic Analysis of Eastern Victoria

The topographic map of Eastern Victoria shows that the elevation ranges from sea level to Mount Bogong, which is 1,986 m above sea level, and is roughly high in the middle and low on both sides.

6.2 Problem Model

Regional Model In order to simplify the problem, we will consider the region is reduced, that is, the specific consideration of different terrain.

We assume that Z is a three-dimensional region, and its x0y plane area is S = 50km ×

50km .The xoy plane is divided into 50km $\times$ 50km grids, each grid is 1km long and 1km wide. The coordinates of each grid are $Sq_k = (k_x, k_y, k_z)$, and k_x, k_y, k_z are the abscissa, ordinate and altitude of the grid, respectively.

Each grid is 1 km long, 1 km wide and the high of altitude.

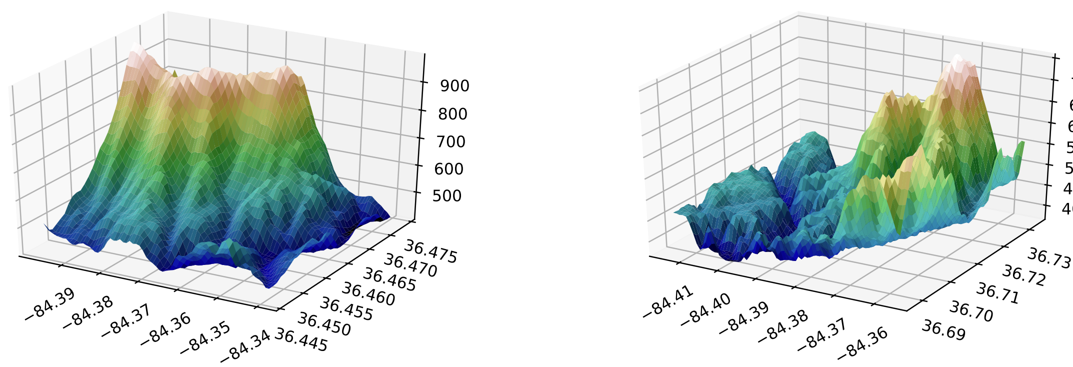


Figure 10: Terrain information simulation

Coverage Model Whether the UAV U_i is marked as cover(Sq_k, U_i) to the target grid .

Lim represents the range of handheld radios.

$$\text{Lim } = \begin{cases} 5km, \text{ Flat terrain} \\ 2km, \text{ otherwise} \end{cases} \tag{6.1}$$

A total of N drones.

The signal acceptance range of UAV is rec = 20 km .

The UAV coordinates are $\mathrm{U_i} = (\ x_i,\ y_i,\ z_i\ ) \quad (\mathrm{i} = 1, 2, 3 \cdots\cdots \mathrm{N})$

$$\begin{cases} 50 \geqslant x_i \geqslant 0(km) \\ 50 \geqslant y_i \geqslant 0(km) \end{cases} \tag{6.2}$$

In order to better receive signals and ensure security, UAV can ' t be lower than $+0.5km$ below it or higher than $+1km$ below it. altitude at $(x_i,\ y_i)$.

$$1 + at(x_i, y_i) \geqslant z_i \geqslant 0.5 + at(x_i, y_i) \tag{6.3}$$

The UAV speed is $V_i = (v_{xi}, v_{yi}, v_{zi})(i = 1, 2, 3 \cdots\cdots N)$. The maximum combined velocity of UAV is 72 km/h , so $v_{xi},\ v_{yi},\ v_{zi} < 72/\sqrt{3}$.

Considering efficiency, let $20 \leqslant v_{xi}, v_{yi}, v_{zi} \leqslant 40(km/h)$

$$cover\,(Sq_k,U_i)=\begin{cases}0, & \sqrt{(k_x-xi)^2+(ky-yi)^2+(kz-zi)^2}>\mathrm{rec}+\mathrm{Lim}_{\leftarrow}\\ 1, & \text{otherwise}\end{cases} \tag{6.4}$$

1 Represents Ui covering target grid;

0 Represents that Ui does not cover the target grid.

Defines the grid set covered by the UAV as Rci, $\mathrm{Sq_k}\in$ Rci , only if cover $(\mathrm{Sq_k},\mathrm{Ui})=1$

Defines the UAV set covering the grid as cRk , Ui $\in$ cRk., only if cover $(\mathrm{Sq_k},\mathrm{Ui})=1$

- Regional coverage

  The total UAV coverage area can be represented by the number of target grids covered.

  The target grid set covered by all UAVs , $\mathrm{Sco}=\cup\mathrm{Rci}(\mathrm{i}=1,2,3\cdots\cdots\mathrm{N})$. Area coverage can be expressed as $\mathrm{Fco}=\sum(\text{ Sco })/S$.

  sum represents the number of elements in a collection, and Fco is proportional to coverage performance.

- Coverage redundancy rate

  Coverage redundancy rate Defines a collection $\mathrm{Fc}=\{\mathrm{Sqk}\mid\sum(\mathrm{cRk})>1\}$ of target grids that are simultaneously covered by multiple UAV nodes Coverage Redundancy $\mathrm{Cr}=\sum(\mathrm{Fc})/\sum(\mathrm{Sco})$.

  Coverage redundancy is proportional to coverage performance with the same regional coverage

6.2.1 Network Connectivity

Communication distance of UAV is com = 20km , Near as an adjacency matrix.

$$Near_{ij}=\begin{cases}1, & \sqrt{(xi-xj)^2+(yi-yj)^2+(zi-zj)^2}<=\mathrm{com}\\ 0, & \text{otherwise}\end{cases} \tag{6.5}$$

If the entire UAV communication graph ALGragh is a connected graph, then according to the criterion of connected graph,

$$MatrixA=\sum_{k=1}^{N-1}Near^k. \tag{6.6}$$

For any element in A, if they are not zero, then ALGragh is a connected graph.

6.2.2 Fitness Function

$$\mathrm{F}=\omega_1*\ \mathrm{Fco}+\omega_2*\mathrm{Cr} \tag{6.7}$$

To ensure coverage performance, regional coverage should be prioritized. Therefore,

$$\omega_1>>\omega_2 \tag{6.8}$$

So, we can take

$$\omega_1 = 0.9, \omega_2 = 0.1 \tag{6.9}$$

6.2.3 Particle swarm optimization algorithm

An evolutionary algorithm, which seeks global optimization by following the optimal value currently searched. Features : easy to implement, high precision, fast convergence.

Through this algorithm, the optimal solution is found from the reasonable UAV coverage scheme.

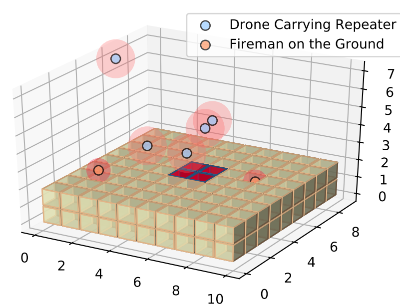


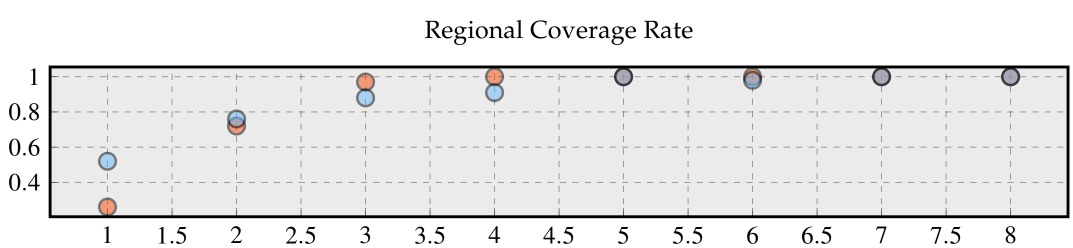
Figure 11: Nine Fire Areas in Victoria

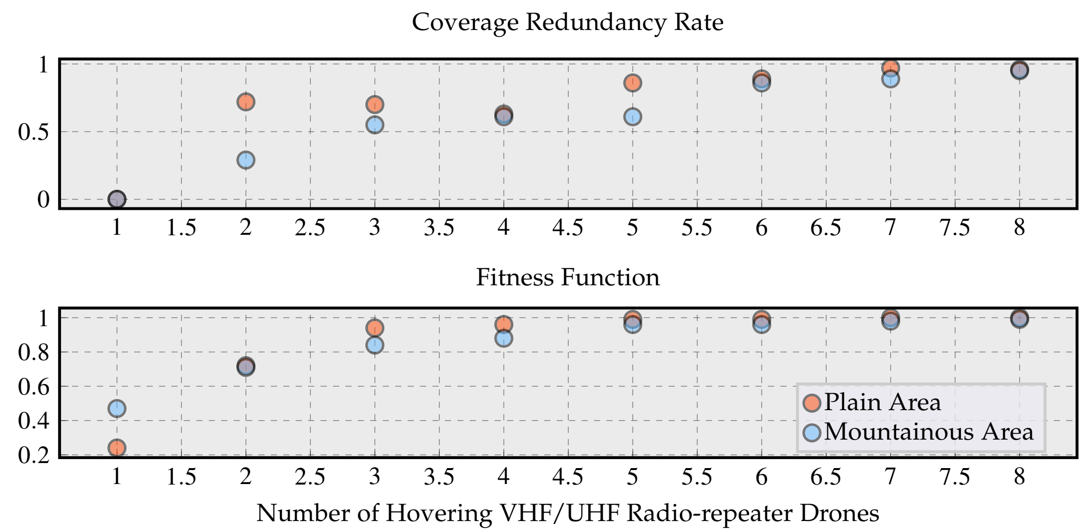
6.3 Model solution

6.3.1 Different terrain

We used the following data for **plains** , i.e. terrain with less elevation fluctuations. For each grid Sqk , k_z ranges from 0.2 km to 0.3 km. Random values are used to generate a terrain with small elevation fluctuations. The optimal coverage scheme corresponding to different number of UAVs is obtained.

The following data are used for **mountain areas** , i.e. terrain with large elevation fluctuations. For each grid Sqk , k_z ranges from 0 km to 1.986 km. Random values are used to generate a terrain with large elevation fluctuation. The optimal coverage scheme corresponding to different number of UAVs is obtained.





According to the figure, the area coverage rate, coverage redundancy rate and fitness degree are compared respectively when the terrain is fixed and the number of UAVs is fixed. It can be concluded that in case of fire, the search and rescue difficulty in plain is less than that in mountain area.

6.3.2 Fire size

- Small fires

  In plain areas, considering that small fires are easy to carry out search and rescue, the fire spread is slow, and the frontline personnel are easy to move. So we get the range of coverage schemes Fco, Cr, F for small fire UAVs in plain areas.

  In mountainous areas, considering the difficulty of search and rescue caused by terrain and the difficulty of frontline personnel movement, it is necessary to improve the coverage rate of UAVs, at the same time, it is necessary to improve the coverage redundancy rate of UAVs, so as to quickly and better understand the information of each area. Moreover, if some UAVs are exhausted, a large coverage redundancy rate can also ensure the efficiency of receiving and sending signals.

- Large fires

  Although the plain is easy to carry out search and rescue, it is difficult for frontline personnel to move because of the rapid spread of large fires. In order to ensure safety and efficiency, it is necessary to improve the coverage rate, coverage redundancy rate and fitness of UAV.

  So we get the range of coverage schemes Fco, Cr, F for large fire UAVs in plain areas.

  In view of the complex terrain and the rapid spread of fire in mountainous areas, it is difficult for frontline personnel to move. In order to ensure safety, it is necessary to improve the regional coverage, coverage redundancy and fitness of UAV.

  So we get the range of coverage schemes Fco, Cr, F for large fire UAVs in mountainous areas.

Table 3: Simulation Results of PSO in Mountainous Area(Showing Part of the Results)

Number of Drones	3	5	7
Particles' Spatial Coordinates	(31.38,31.14,2.44) (10.22,15.94,1.68) (13.68,26.36,1.63)	(36.34,4.99,0.84) (12.67,35.09,0.79) (13.19,28.33,0.93) (4.97,12.36,2.00) (38.54,42.76,2.56)	(33.02,0.08,2.30) (37.19,38.41,0.97) (5.31,42.76,2.62) (10.91,19.46,1.52) (23.36,8.13,2.50) (23.84,15.72,1.19) (49.83,41.50,1.40)
Regional Coverage Rate	0.88	1.00	1.00
Coverage Redundancy Rate	0.55	0.61	0.89
Fitness Function	0.84	0.96	0.98

Table 4: Simulation Results of PSO in Plain(Showing Part of the Results)

Number of Drones	3	5	7
Particles' Spatial Coordinates	(22.35,13.39,1.19) (17.03,28.52,1.20) (33.66,23.08,1.01)	(19.67,15.33,1.26) (34.91,40.63,0.88) (22.43,42.34,1.20) (21.24,27.14,1.10) (46.00,18.98,1.12)	(40.47,15.25,1.11) (25.60,2.63,1.12) (11.70,29.76,0.93) (31.04,36.09,1.25) (12.55,16.93,1.19) (34.29,47.99,1.09) (43.55,33.48,1.05)
Regional Coverage Rate	0.97	1.00	1.00
Coverage Redundancy Rate	0.70	0.86	0.97
Fitness Function	0.94	0.99	1.00

6.4 Sensitivity Analysis

6.4.1 Reason

Due to the nature of the particle swarm algorithm [3] , the results obtained will be different each time even for the same altitude data and the same number of drones.

We explore the best coverage solution for the number of UAVs N=3 to see its solution for the same elevation data each time.

1.Firstly generate a terrain by taking random values for each raster Sq_k (k_z's range is from 0.2km to 0.3km) with the program.

2.Fix the terrain and find the best coverage scheme for 4 groups of UAVs with number N=3.

3.Perform area coverage, coverage redundancy, and adaptation analysis for these groups of coverage schemes.

6.4.2 Conclusion

We conclude by analyzing the coverage area, the coverage redundancy, and the adaptability of these sets of coverage schemes using **Monte Carlo methods**.

1. Although the data of area coverage and adaptation degree of the four groups of coverage schemes are different, the differences of area coverage of each group and the differences of adaptation degree of each group are not significant.

2.Although there is a significant difference between the coverage redundancy rate of one group of coverage schemes and the other three groups, this difference is negligible because the coverage redundancy rate has a small weight in the adaptation function.

3. Based on the first two conclusions, it can be concluded that although the coordinates of the UAVs are different in each group of the obtained UAV coverage scenarios in this case, the coverage rates as well as the adaptation degrees are similar. Therefore, the model has good stability as well as high efficiency to find a satisfactory solution.

7 Model Evaluation

7.1 Strengths

- **1. Our model is robust.** Our model is mainly based on the single-objective integer optimization method, which is a classical optimization model, which is easy for non-professionals to understand, and makes the model easy to improve in other difficult situations. Sensitivity analysis shows the robustness of our model.
- **2. Our model gives appropriate assumptions.** By simulating the bushfire event with a given prior probability, the difficulty of our model is reduced, and it is easy to spread to other research areas.
- **3. Our model is innovative.** The discrete operation optimization problem is solved by Particle Swarm Optimization theory (PSO), which makes the abstract problem concrete and enhances the innovation of solution.

7.2 Solutions to improve

- 1. Considering that the decision space is too complex, many assumptions are made to simplify the model. In the real situation, hypothesis can be appropriately reduced to consider more permutations and combinations, and heuristic optimization algorithms such as genetic algorithm and simulated annealing algorithm can be used in parallel for optimization.

- 2. In real scenes, bushfires are more randomized and difficult to predict, so we can consider solving with different random distributions, and we can use Monte Carlo method to simulate the reality, and we can repeat the test several times to reduce system errors.

8 Conclusion

Through the modeling of the first three problems, the solution of each problem is obtained, and a budget request with notes is written for CFA to submit to the Victorian government.

For the first question, we study the characteristics of UAVs and obtain the optimal procurement strategies of two kinds of UAVs by using the single objective integer programming method. By considering different aspects, the cost function is established and the final solution is obtained.

For the second question, we mainly use the method of prior probability distribution and sensitivity analysis, and finally get how to adapt to the changing possibility of extreme fire events in the next decade and the amount of cost that may increase.

For the third question, we first expand and shrink the area, and then divide it into a certain number of grids with altitude. By defining the area coverage rate, coverage redundancy rate, adaptability function and considering the connectivity of UAV Communication Network, we get the optimal coverage scheme of UAV with different terrain and different sizes of fire events through particle swarm optimization algorithm.

When writing the budget request, we mainly combine the answers of the first two questions with the more in-depth understanding of fire and terrain obtained from the third question to calculate the amount of funds required.

9 Budget Request for Investment in Drones

The Victoria State Government:

During the 2019-2020 fire season, eastern Victoria was severely traumatized by wildfires. Due to the large area of wildfires and the powerful features of SSA drones, we need to use SSA drones to monitor front-line personnel and report data from wearable devices on them. We usually work together with EOC. Repeaters can extend radio range well and Let the frontline communicate smoothly with EOC. Therefore, we plan to use the high-performance WileE-15.2X hybrid drone to assemble SSA functions and repeaters. They will play an important role in firefighting.

In order to ensure the safety of personnel during firefighting and the efficiency, a lot of

SSA drones and Radio Repeater drones need to be prepared. Therefore, we apply to the government for 14,505,000(AUD) to purchase these equipment. The specific reasons are as follows.

1.Current funds

In order to reduce casualties and quickly extinguish wildfires, we propose a new department "Rapid Bushfire Response", and based on factors such as capacity, safety, economy, communication requirements, terrain, size and frequency of fire incidents, we have determined the optimal numbers and mix of SSA drones and Radio Repeater drones to purchase. The main method is to divide each Region in Victoria. For each divided area, determine the number of drones that should be allocated according to the size and frequency of the fire events in the area and its topography. According to the sum of the number of drones that should be allocated in each area, the current funding for purchasing drones is 11,370,000 (AUD).

2. Reserve funds

In order to adapt to the changing possibilities of extreme fire events in the next decade, we should prepare in advance, that is, prepare a certain amount of reserve funds every year to combat the occurrence of extreme fire events. By solving the probability of extreme fire events, it is concluded that at least 3,135,000 (AUD) of reserve funds should be prepared every year.

Looking forward to being approved.

CFA

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