6.5 Chapter Summary

We continued the discussion of neuron modeling that began in chapter 5 by considering models with more complete sets of conductances and techniques for incorporating neuronal morphology. We introduced A-type K^+, transient Ca^{2+}, and Ca^{2+}-dependent K^+ conductances, and noted their effect on neuronal activity. The cable equation and its linearized version were introduced to examine the effects of morphology on membrane potentials. Finally, multi-compartment models were presented and used to discuss propagation of action potentials along unmyelinated and myelinated axons.

6.6 Appendices

A: Gating Functions for Conductance-Based Models

Connor-Stevens Model

The rate functions used for the gating variables n, m, and h of the Connor-Stevens model, in units of 1/ms with V in units of mV, are

$$\begin{aligned}
\alpha_m &= \frac{0.38(V+29.7)}{1-\exp(-0.1(V+29.7))} & \beta_m &= 15.2\exp(-0.0556(V+54.7)) \\
\alpha_h &= 0.266\exp(-0.05(V+48)) & \beta_h &= 3.8/(1+\exp(-0.1(V+18))) \\
\alpha_n &= \frac{0.02(V+45.7)}{1-\exp(-0.1(V+45.7))} & \beta_n &= 0.25\exp(-0.0125(V+55.7))\,.
\end{aligned} \tag{6.33}$$

The A-current is described directly in terms of the asymptotic values and τ functions for its gating variables (with τ_a and τ_b in units of ms and V in units of mV),

$$a_\infty = \left(\frac{0.0761\exp(0.0314(V+94.22))}{1+\exp(0.0346(V+1.17))}\right)^{1/3} \tag{6.34}$$

$$\tau_a = 0.3632 + 1.158/(1+\exp(0.0497(V+55.96))) \tag{6.35}$$

$$b_\infty = \left(\frac{1}{1+\exp(0.0688(V+53.3))}\right)^4 \tag{6.36}$$

and

$$\tau_b = 1.24 + 2.678/(1+\exp(0.0624(V+50)))\,. \tag{6.37}$$

Transient Ca^{2+} Conductance

The gating functions used for the variables M and H in the transient Ca^{2+} conductance model we discussed, with V in units of mV and τ_M and τ_H in ms, are

$$M_\infty = \frac{1}{1+\exp\left(-(V+57)/6.2\right)} \tag{6.38}$$

$$H_\infty = \frac{1}{1+\exp\left((V+81)/4\right)} \tag{6.39}$$

$$\tau_M = 0.612 + \left(\exp\left(-(V+132)/16.7\right) + \exp\left((V+16.8)/18.2\right)\right)^{-1} \tag{6.40}$$

and

$$\tau_H = \begin{cases} \exp\left((V+467)/66.6\right) & \text{if } V < -80\text{ mV} \\ 28+\exp\left(-(V+22)/10.5\right) & \text{if } V \geq -80\text{ mV}. \end{cases} \tag{6.41}$$

Ca^{2+}-dependent K^+ Conductance

The gating functions used for the Ca^{2+}-dependent K^+ conductance we discussed, with V in units of mV and τ_c in ms, are

$$c_\infty = \left(\frac{[\mathrm{Ca}^{2+}]}{[\mathrm{Ca}^{2+}]+3\mu\mathrm{M}}\right)\frac{1}{1+\exp(-(V+28.3)/12.6)} \tag{6.42}$$

and

$$\tau_c = 90.3 - \frac{75.1}{1+\exp(-(V+46)/22.7)}. \tag{6.43}$$

B: Integrating Multi-compartment Models

Multi-compartment models are defined by a coupled set of differential equations (equation 6.29), one for each compartment. There are also gating variables for each compartment, but these involve only the membrane potential (and possibly Ca^{2+} concentration) within that compartment, and integrating their equations can be handled as in the single-compartment case using the approach discussed in appendix B of chapter 5. Integrating the membrane potentials for the different compartments is more complex because they are coupled to each other.

Equation 6.29, for the membrane potential within compartment μ, can be written in the form

$$\frac{dV_\mu}{dt} = A_\mu V_{\mu-1} + B_\mu V_\mu + C_\mu V_{\mu+1} + D_\mu\,, \tag{6.44}$$