



Dynamics of Deffuant Model in Activity-Driven Online Social Network

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Abstract. In many social system the interactions among the individuals are rapidly changing and are characterized with timing. The dynamics of social interaction constantly affects the development of their opinions. However, most of the opinion evolution models characterize interpersonal opinion in static, structural properties of the network such as degree, cluster and distance. In this paper, an Deffuant opinion model based on the activity-driven network is developed to examine how different activity distribution effects the dynamics of opinion evolution. When the activity distribution complies with power-law distribution or random distribution, phase transition transform from polarization to consensus when threshold is 0.6 and 0.4, respectively. In the process of opinion formation the distribution of opinion clusters' scales are complying with power-law distribution. Especially, under the power-law distribution the opinion disparity of the two clusters in polarization state is lower than the others, which means that the burst of the activity helps the individuals converging in opinion clusters in values. Finally we show that the speed to reach stable is influenced by the type of activity distribution. The simulation on power-distribution and random distribution need more time steps to get steady state.

Keywords: Activity-driven network · Temporal network · Opinion dynamic
Deffuant model

1 Introduction

More and more people interact with others by publishing posts or replying, which will facilitate the formation and evolution of public opinions online [1]. In particular, affordable and ubiquitous information and communication technologies (ICT) promote the online interactions changing rapidly over time [2, 3]. The underlying structure of the network and on the temporal activity patterns of humans, deeply influence the formation and the evolution of opinion on social media [4–6]. Consequently the relationships between the time-varying activities and the dynamics of opinion are worthy to deeply explore.

In general models about opinion dynamics consist of the mechanism of interaction and the network of interaction. They can be classified into two classes according to whether the variable that represents the opinion of an agent is discrete or continuous [7]. The voter model [8], the Galam majority rule model [9] and the Sznajd model [10] are

the most common discrete opinion models. Deffuant model [11] and Hegselmann and Krause model [12] are the most typical continuous opinion models. In the above model, the opinion dynamics are always described by stationary states statistics, such as opinion clusters, phase transaction, the difference of opinion, etc. To the Deffuant model, the emphasis are explore the role of social influence [11], which is include but is not limited to threshold d and convergence u . For example, the value of d is proved to be used for control the scale of emerging clusters that will contribute to the phase transition of collective opinion on the complete networks, the square lattices, the random networks, and the scale free networks [13]. The mental capacity, the propaganda, and the fraction of conformists are important effect factors to the Deffuant models [14–16]. In addition there are studies explore the effect of dynamical affinity or the local topology of the network surrounding the individuals in Deffuant model, which show interesting behaviors in regards to the structure of the social networks and their correlation with the opinion formation process or the opinion structures [17].

However most of the Deffuant models are evolved in static networks, which ignore the dynamic of social interaction network emerging from the timing of interacting. But various real-world social interaction systems, such as Twitter, Email messages, blogs broadcast etc., are characterized by processes whose timing and duration are defined on a very short time scale [18, 19]. Recent investigations about online social networks proved that the dynamics of user interactions may be more important than these static relationship networks in determining how information flows and opinion flows shuttle in a social network [20–22]. Specially, the burst nature of human interactions slows down the information spreading and results in the limited scope [23, 24]. Therefore, Maxi San Miguel has carried out exploration based on vote model on temporal network. They focused on the temporal variation in networks' connectivity patterns and the ongoing dynamic processes [25]. Andrzej study the influence of correlation between the activity of an individual and its connectivity on the process of opinion formation on social network [26]. These results represented that human activity plays a role in opinion formation, which is worthwhile to take it into account in the research of opinion model [27].

Motivated by the above analysis, in this paper, we study the opinion dynamics in Deffaunt model specifically devised for a class of time-varying networks, namely activity-driven networks. It is presented for describing the dynamics of a social network with individuals having activity potential and synthesis the interaction dynamics and topology dynamics together [28]. The topological structure of this kind of time-varying networks is measured by the distribution of activity of the nodes. Here we emphasis how activity distribution affect the evolution of opinion. We applied agent-based simulation to compare the effects of three activity distributions on the opinion dynamics of Deffuant model. We consider the normal distribution (corresponding to uniform probability of being active), the random distribution (being active disorder) and the power-law (to reproduce heterogeneous activity patterns) distributions. The main observation is that the phase transition, probability of the scales for opinion clusters, the opinion of each opinion clusters, and the speed to reach stable are influenced by activity distribution, called the structure of the time-varying network.

2 Deffuant Model on Activity-Driven Network

Social media is a typical communication platform, on which the relationships and the interaction among users build new channels for information diffusion and opinion formation. Thus, opinion dynamics is not only depending on online social networks where connections among nodes are long-lasting [29], but also on temporal network driven by individual's instant activity. That means on condition of the individual being active, the opinion interaction has possibility.

Here we illustrate our model with Fig. 1. In online communication platforms like Twitter, there is social network $G(N, E)$. $N = \{1, 2, \dots, n\}$ is the set of nodes and their social relationships are tagged by the set $E = \{e(i, j) | i, j \in N\}$. Each node i has an activity potential $a_i = \text{activity}_i / \sum \text{activity}_i$, which is the probability per unit time to create new tweet or retweet online. activity_i is the number of activities of node i over a period. Thus, a_i are bounded in the interval $[\varepsilon, 1]$ and comply with a given probability distribution $p(a)$. Additionally, node i has an continuous opinion x_i , which is bounded in the interval $[0, 1]$. For two nodes i and j , the opinion difference between them is defined as $\Delta x_{ij} = |x_i(t) - x_j(t)|$.

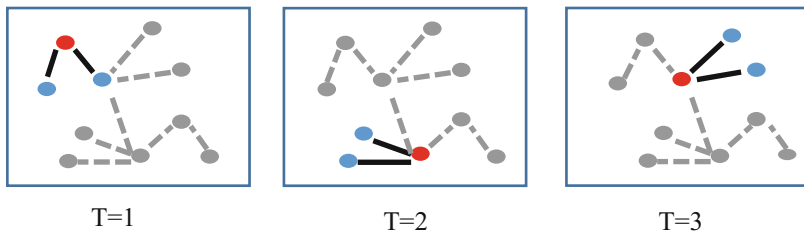


Fig. 1. The colored nodes and edges constructed the activity-driven network based on online social network. Red node is in the active state, who will adjust opinions according to the opinions hold by his neighbors on social network. Grey nodes are in the inactive state, who can't interact with their friends even if they are active. The structure of temporal networks is determined by (a) , where a means the activity potential. For Twitter, $p(a)$ is power-law distribution and the slope value will be between -1.8 and -3 . (Color figure online)

In previous work about Deffuant models, two agents meet and adjust their opinion when their difference of opinion them is smaller than threshold d . While in activity-driven network based on online social network, the opinion interaction will be constrained by both the social network and the activities. Generally at time t , only the nodes, who are in active state and receive the information from neighbors, can change opinion according the interaction rules. Hence, we assume an opinion interaction on activity-driven network according to the following rules:

- At each discrete time step t , there exists an online social network $G(N, E)$.
- With probability a_i node i becomes active. If node i is active, he can take one of the following behaviors randomly. One is creating a new message. The other is

forwarding a message received from node j , where exists an edge between i and j and $e(i, j) \in E$. Also the message is randomly chosen from his information storage. If node i is not active, it will do nothing. The forwarding behavior is recorded as information interaction.

- For two nodes, i and j , if there exists information interactions and $\Delta x_{ij} \leq d$, node i can update opinion as $x_i(t+1) = x_i(t) + \mu[x_j(t) - x_i(t)]$. If the opinion difference Δx_{ij} between them is larger than the tolerance threshold d , that means the opinion difference is out of his considerable range, so he keeps his current opinion.

3 Results

We analyze the effects of individuals' activity on opinion formation of Deffuant model by simulation experimental method. All the simulations based on the online social network $G(N, E)$, which is sampled from the Tencent microblog dataset [30]. $G(N, E)$ included records of 1509 node in two month, consisting of 10396 follower-followee relationships. The topology of it has small-world properties [20], where $\langle L \rangle = 3.25$, $\langle C \rangle = 0.4$, $M = 0.15$ and $\langle k \rangle = 17.7$. The probability distribution of activities of the nodes in the dataset is $p(a) = a^{-2.7}$. At the beginning of each simulation, initially opinions of the nodes were randomly generated across a uniform distribution on $[0, 1]$ and 30 nodes were selected as seeds randomly (0.02% of all nodes) to create new messages and push them to their friends. Then at time t , all the nodes carry on opinion interaction according the rules mentioned in Sect. 2. Besides, we execute the model on the sample network and analysis the opinion evolution at different activity distribution, such as normal distribution $p(a) = \text{Normal}(0.5, 0.5)$, and random distribution with $a \in [0, 1]$. All the simulations carried out 10000 steps and repeated for 100 times.

3.1 The Opinion Phase Transition

The previous studies of Deffaut model showed that threshold values leading to 3 convergence station, consensus, polarization and fragmentation. When the threshold values are lower than 0.3, several opinion clusters can be observed and it is called fragmentation. When the threshold value is 0.3, there exits two opinion clusters and means polarization. When the threshold values are higher than 0.5, there exists only one opinion cluster, named consensus. Figure 2 shows the different results about relationships between the threshold values and the convergence station. One is no matter what the activity distributions are; there always exist some isolated nodes whose opinion cannot belong to any opinion cluster. The second is when the activity distribution is complied with power-law, the consensus will appear at $d = 0.6$. The effect of busty is lessening the level of consensus. In Fig. 2(b), polarization and consensus come up at $d = 0.2$ and 0.4 separately, which shows the uniform probability of being active is speed up the gathering of the opinion.

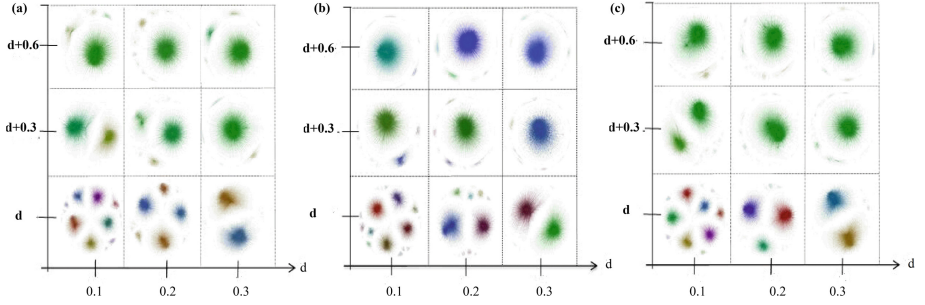


Fig. 2. The results of opinion phase transition with the increase of threshold d increased at different activity distribution. ($\mu = 0.5$, nodes with same color means they have same opinion, the) (a) $p(a) = a^{-2.7}$. Polarization is observed at $d = 0.3$. Consensus is observed at $d = 0.6$. (b) $p(a) = \text{Normal}(0.5, 0.5)$. Polarization is observed at $d = 0.2$. Consensus is observed at $d = 0.5$. (c) $p(a)$ is random distribution $a \in [0, 1]$. Polarization is observed at $d = 0.3$. Consensus is observed at $d = 0.5$.

3.2 The Probability of the Scales of the Opinion Clusters

In this study, we also concern how different activity distributions affect the scales of the opinion clusters. Figure 3 shows the results in details. The scale of opinion cluster is measured by N_{Ci} , where N_{Ci} is the number of nodes in opinion cluster C_i . Most notably, the final macroscopic state is found to be difficult to reach complete consensus by ways of opinion detentions [26]. This is usually presented as the majorities of the individuals holding the same opinion but the rest holding many different opinions [31]. In Fig. 3 it can be seen that the scales of opinion clusters are complying with power-law distribution, except the results of random distribution. It means that the simulation results of the opinion model in this paper reproduce the distribution of the number of opinion clusters on social media by ways of activity driven.

3.3 The Difference of Clusters' Opinion

The difference of clusters' opinion reflects the degree of bias of opinion in the local network. In Fig. 4 we show the proportion of the scales and the opinion values for each opinion clusters at different convergence station and different activity distributions. Table 1 compares the statistics of the opinion and the scale of Top N opinion clusters in details.

In fragmentation state the variance of opinion among the Top4 opinion clusters are 0.28 for power-law distribution, 0.49 for normal distribution, and 0.58 for random distribution. All the scales of Top 4 opinion clusters are around 20% of the number of the whole network. At the same time the opinion of the cluster are well-distributed in $[0, 1]$.

In polarization state the variance of the opinion between the Top2 opinion clusters are 0.23 for power-law distribution, 0.33 for normal distribution and 0.3 for random distribution. Meanwhile, the scales of the two largest opinion clusters in power-law

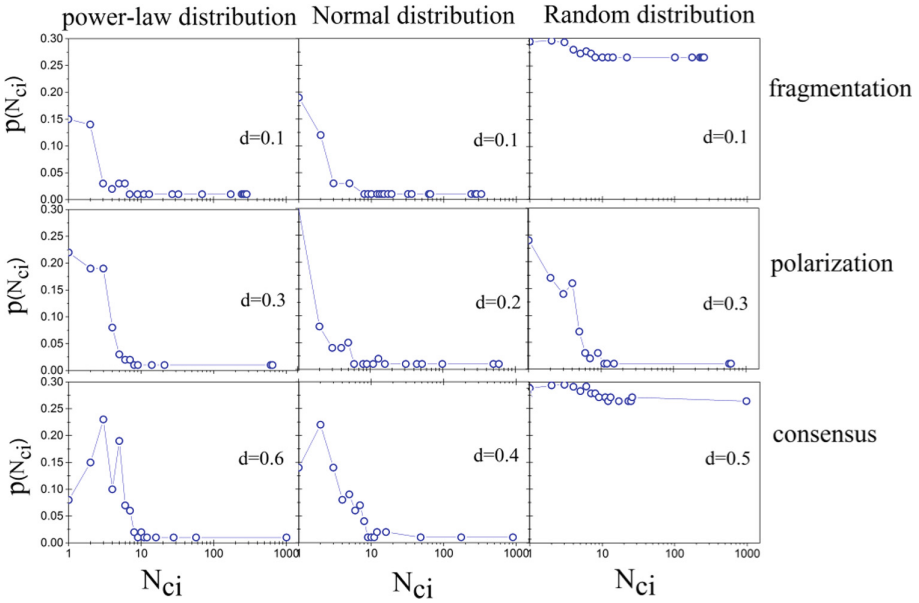


Fig. 3. The probabilities of the scales for opinion clusters at different convergence station and different activity distributions, which are the statistical results of the 100 simulations ($\mu = 0.5$). The left column is for power-law distribution $p(a) = a^{-2.7}$, middle column for normal distribution $p(a) = \text{Normal}(0.5, 0.5)$ and right column for random distribution $a \in [0, 1]$. Top row contains plots for fragmentation state, second row shows the plots for polarization station and third row shows the plots for consensus station.

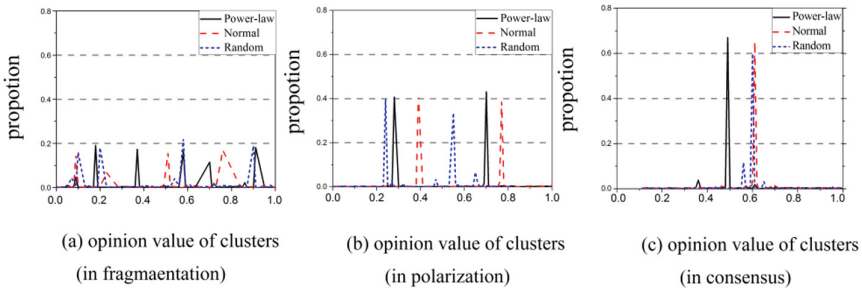


Fig. 4. The proportion of the scales and the opinion of each opinion clusters at different convergence station and different activity distributions, which are the statistical results of the 100 simulations ($\mu = 0.5$). The black line is for power-law distribution $p(a) = a^{-2.7}$. The red line is for normal distribution $p(a) = \text{Normal}(0.5, 0.5)$. The blue line is for random distribution $a \in [0, 1]$. (Color figure online)

Table 1. The proportion and the opinion value of the Top N opinion clusters

State	Power-law distribution	Normal distribution	Random distribution
Fragmentation $d = 0.1$	$N_{C1} = 286$, opinion = 0.19 $N_{C2} = 271$, opinion = 0.37 $N_{C3} = 241$, opinion = 0.71 $N_{C4} = 271$, opinion = 0.92	$N_{C1} = 241$, opinion = 0.05 $N_{C2} = 181$, opinion = 0.12 $N_{C3} = 256$, opinion = 0.51 $N_{C4} = 271$ opinion = 0.77	$N_{C1} = 256$, opinion = 0.06 $N_{C2} = 271$, opinion = 0.21 $N_{C3} = 316$, opinion = 0.59 $N_{C4} = 286$, opinion = 0.92
Polarization $d = 0.2/0.3$	$N_{C1} = 650$, opinion = 0.34 $N_{C2} = 614$, opinion = 0.66	$N_{C1} = 599$, opinion = 0.26 $N_{C2} = 505$, opinion = 0.73	$N_{C1} = 623$, opinion = 0.28 $N_{C2} = 587$, opinion = 0.7
Consensus $d = 0.4/0.5/0.6$	$N_{C1} = 1011$, opinion = 0.5	$N_{C1} = 903$, opinion = 0.52	$N_{C1} = 983$, opinion = 0.51

distribution are bigger than the other two distributions. Although the simulation with normal distribution transferred to polarization state at threshold $d = 0.2$, but the conflict of the two opinion clusters is more violent. In contrast, random activity will reduce the conflict among individuals.

Comparing the results in consensus state, the power-law distribution shows the most conformance. Because the scale of the largest opinion cluster is 1011 and its' opinion value is 0.5. As mentioned in [32], in the process of becoming consensus the system should experience opinion bifurcation, which means small clusters with different opinion values combined into a union gradually. In power-law condition, the difference of the values between opinion clusters is smaller than those of the two distributions. Thus the bursty of the activity helps the individuals converging in opinion clusters. However, the uniform probability of being active, named normal distribution is not benefit to the convergence of the opinion clusters' value.

3.4 The Difference of Nodes' Opinion in the Whole Network

The difference of nodes' opinion in the whole network reflects the overall degree of bias of opinion in the whole network. To measure how individual opinions differ at each step, we here use the standard deviation of the individuals' opinions:

$$s_{opinion}(t) = \sqrt{\frac{1}{N} \sum_{i=1}^N x_i(t) - \overline{X(t)}} \quad (1)$$

Where $\overline{X(t)}$ is the mean value of all individuals' opinion at time t .

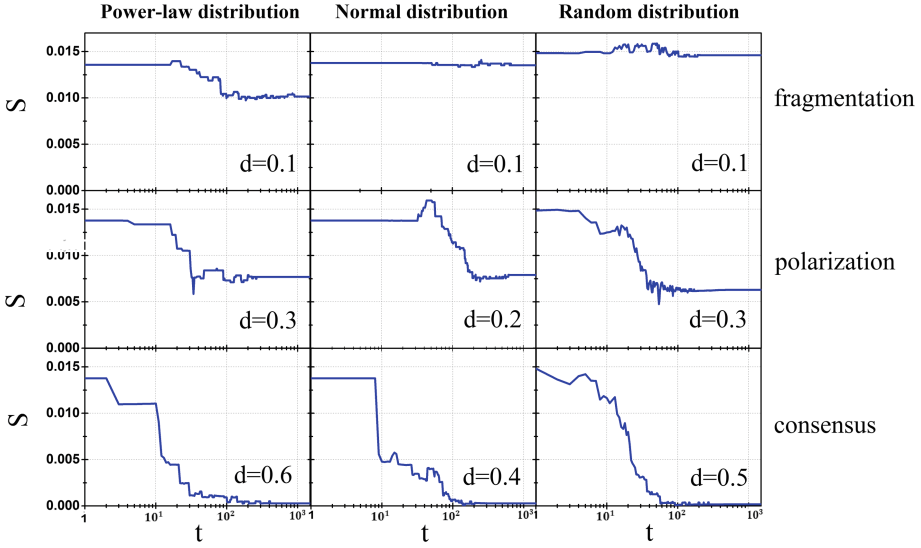


Fig. 5. The difference of opinion in the whole network. $s_{opinion}$ varies with time t for different activity distributions, which is the average results of 100 simulations. The left column is for power-law distribution $p(a) = a^{-2.7}$, middle column for normal distribution $p(a) = \text{Normal}(0.5, 0.5)$ and right column for random distribution $a \in [0, 1]$. Top row contains plots for fragmentation state, second row shows the plots for polarization station and third row shows the plots for consensus station. $s_{opinion}$.

Figure 5 shows $s_{opinion}$ varying with time t for different activity distributions. Overall when the threshold d increased the $s_{opinion}$ decreased. Since the threshold d expressed the tolerance of nodes' to his neighbors, the more the tolerance is, the more the convergence are.

With the increase of time t , the difference of opinion will decrease and reaches a certain value, which presents the individuals' opinion reach a steady. Under the condition of the random distribution, the $s_{opinion}$ achieved stable at $t = 150$. To the power-law distribution and normal distribution, the $s_{opinion}$ achieved stable at $t = 200$, which is similar with the result in [33]. The results can be illustrated as follows. Although the individuals with high activity contact others more frequently, most of their contacts have low activity rate [34]. Because of the individuals having low activity with a small probability, the total number of interacting individuals would be reduced. So in the experiment on power-law distribution, more time is needed to reach stability.

4 Conclusion

Network model plays an important role to many complex systems. Because of the function to describe the rapidly changing interactions among individuals, the activity driven network is gain more and more interest. In this paper, we have studied the

opinion formation of Deffuant model in online social network, named Tengtun Weibo. In our model, we have taken into account the activity of individuals, on which the activity-driven network is generated dynamically. Opinion interaction happens when individuals are active. According to the simulation results, we have found that the activity distribution, namely structure of activity-driven network, affects the opinion dynamic. Because the bursty of activity, the power-law distribution will decrease the speed to get steady. Yet it help individuals' opinion become similar for the smallest variance of opinion among the Top N opinion clusters and $s_{opinion}$. Under the condition of the normal distribution, the simulation transferred to polarization state at threshold $d = 0.2$ and the conflict of the two opinion clusters is more violent. It means that uniform probability of being active will increase the chance to communicate, but the threshold d inhibition the interaction between the two having large opinion gap.

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