

## Mathematics Courses List

### Contexts

#### **I. Analysis and Differential Equations:.....2**

1. Mathematical Analysis A1~A3;
2. Differential Equations I;
3. Real Analysis;
4. Complex Analysis (H);
5. Functional Analysis;
- \*6. Advanced Real Analysis;
- \*7. Differential Equations II (H);
- \*8. Harmonic Analysis;
- \*9. Nonlinear PDEs in Fluid Dynamics;
- \*10. Second Order Elliptic PDEs;
- \*11. Nonlinear Dispersive PDEs;

#### **II. Probability and Stochastic Processes:.....5**

1. Probability Theory;
- \*2. Advanced Probability Theory;
- \*3. Advanced Stochastic Processes;
- \*4. Probability Limiting Theory;
- \*5. Topics in Stochastic Analysis;
- \*6. Martingale Theory and Stochastic Calculus.

#### **III. Geometry and Topology:.....7**

1. Differential Geometry;
2. Topology;
- \*3. Differential Manifolds;
- \*4. Riemann Geometry.

#### **IV. Algebra:.....8**

1. Linear Algebra A1, A2;
2. Introduction to Algebra;
3. Abstract Algebra.

Note: \* means this is a graduate level course

## I. Analysis and Differential Equations:

### 1. Mathematical Analysis A1~A3

**Instructor:** Guangbin Ren (A1), Jihuai Shi (A2, A3); **Grade:** 89, 99, 96/100.

**Textbook:** Gengzhe Chang, Jihuai Shi: *A Course in Mathematical Analysis*.

**Contents: A1: One-Variable Analysis:**

1. Limit of Sequences;
2. Continuity and Differentiation of One-variable Functions;
3. Taylor's Formula;
4. Riemann's Integration and Lebesgue's Theorem, Stirling's Formula;

**A2: Multi-Variable Analysis:**

1. Topology of  $\mathbf{R}^d$ ;
2. Continuity and Differentiation of Multi-variable functions;
3. Inverse and Implicit Mapping Theorem;
4. Multi-variable Integral (Green's Formula, Stokes' Formula, An intro. to Field Theory);

**A3: Series and Generalized Integral:**

1. Convergence of Number Series, Function Series including Power Series;
2. Convergence of Generalized Integral;
3. Fourier Series and Its Convergence;
4. Convergence of Generalized Integral with Parameters.

### 2. Differential Equation I

**Instructor:** Benjin Xuan; **Grade:** 92/100.

**Textbook:** [1] Tongren Ding, Chengzhi Li: *Textbooks of Ordinary Differential Equations*;

[2] Lawrence C. Evans: *Partial Differential Equations*, Ch. 1, 2, 4.1.

**Contents:** 1. Methods of solving to some ODEs;

2. Picard's Iteration: the Existence and the Uniqueness of ODE;
3. Extension of Solutions of ODEs;
4. Solving ODE Systems;
5. Introduction to the Planar Dynamical Systems determined by ODEs;
6. Transport Equation;
7. Laplacian Equation: Fundamental Solutions, Green's Function, Gradient Estimates;
8. Heat Equation: Fourier Transform Method;
9. Classical Solution to Wave Equation;
10. Separation of Variables.

### 3. Real Analysis

**Instructor:** Lifeng Zhao; **Grade:** 97/100.

**Textbook:** [1] Elias M. Stein, R. Shakarchi: *Real Analysis*, Chapter 1, 2, 3, 6.

[2] Minqiang Zhou: *Functions of Real Variables*,  $L^p$  space part.

**Contents:** 1. Lebesgue Measure Theory in  $\mathbf{R}^d$ ;

2. Lebesgue Integration Theory and Convergence Theorems,  $L^p$  space;
3. Differentiation of Functions on  $\mathbf{R}$ : Lebesgue's Density, Functions of Bounded Variation, Absolute and Lipschitz Functions.
4. Abstract Measure Theory: Extension Theorem, Lebesgue-Stieltjes Measure.

**4. Complex Analysis (H)****Instructor: Luo Luo; Grade: 84/100.**Textbook: Jihuai Shi, Taishun Liu: *Functions of Complex Variables*.Contents: 1. Topology of  $\mathbb{C}$ ;

2. Cauchy-Riemann Equations and Holomorphic Functions;
3. Cauchy-Pompeiu Integral Formula;
4. Complex Series: Taylor's and Laurent's Expansion;
5. Meromorphic Functions and Residue Formulae;
6. Holomorphic Extension;
7. An Introduction to Riemann's Mapping Theorem.

**5. Functional Analysis****Instructor: Yi Wang; Grade: 85/100**Textbook: Kung-ching Chang, Yuanqu Lin: *Lecture Notes on Functional Analysis*.

Contents: 1. Metric Space and Contraction Principle;

2. Linear Operators: Riesz Representation Thm, Open Mapping Thm, Hahn-Banach Thm.
3. Weak and Weak-\* Convergence of Linear Operator Series;
4. Spectrum of Compact Linear Operators: Riesz-Schauder Theory.

**6. Advanced Real Analysis****Instructor: Lifeng Zhao; Grade: 97/100**Textbook: [1] Elias M. Stein, R. Shakarchi: *Real Analysis*, Chapter 6;[2] Elias M. Stein, R. Shakarchi: *Functional Analysis*, Chapter 1, 2, 3, 8.

Contents: 1. Abstract Measure Theory: Extension Theorems and Applications, Signed Measure and Radon-Nikodym Theorem;

2.  $L^p$  space and Inequalities;
3. Riesz-Thorin Interpolation, Hilbert Transform, Hardy-Littlewood Maximal Function;
4. Distribution: Basic Calculation, Tempered Distribution, Homogeneous Distribution, Parametrixes of Linear Partial Differential Operators;
5. An Introduction to Oscillatory Integrals.

**7. Differential Equation II (H)****Instructor: Lifeng Zhao; Grade: 90/100**Textbook: Lawrence C. Evans: *Partial Differential Equations*, Chapter 5, 6, 7, 8.1, 8.6, 9.4.Contents: 1. Sobolev Spaces  $W^{k,p}(\mathbf{U})$  and  $H^s(\mathbf{R}^d)$ ;

2. Elliptic Equations: Weak Solutions, Maximum Principle, Fourier Expansion;
3. Parabolic Equ. : Galerkin's Method, Theory of Weak Solutions, Maximum Principle;
4. Hyperbolic Equ. : Galerkin's Method, Existence and Regularity to Weak Solutions;
5. Vanishing Viscosity Method for 1<sup>st</sup> Order Hyperbolic Systems;
6. 3D Cubic Nonlinear Schrodinger's Equation and Its Strichartz Estimates;
7. Conservation Law: Euler-Lagrange Equations, Nother's Theorem, Pohozaev's Identity for  $\Delta u = u|u|^{p-1}$  with zero boundary data.

## 8. Harmonic Analysis

**Instructor: Guangbin Ren; Grade: 95/100**

Textbook: [1] Javier Duoandikoetxea: *Fourier Analysis*, Chapter 1, 2;

[2] Elias M. Stein: *Singular Integrals and Differentiability of Functions*, Chapter 2, 3;

[3] C. Muscalu, W. Schlag: *Classical and Multilinear Harmonic Analysis*, Chapter 8;

[4] M. Ruzhansky: *Pseudo-Differential Operators and Symmetries*, Chapter 6, 7.

[5] M. Wong: *Wavelet Transforms and Localization Operators*, Section 6-9, 17-18.

Contents: 1. Fourier Series and Fourier Transforms, Radon Transforms;

2. Hardy-Littlewood Maximal Operators;

3. Singular Integrals: Calderon-Zygmund Theory;

4. Littlewood-Paley Theory;

5. Representation Theory of Compact Groups(including Peter-Weyl Theorem);

6. Wavelet Transform;

7. Representation Theory of Affined Group and Weyl-Heisenberg Group.

## 9. Nonlinear PDEs in Fluid Dynamics

**Instructor: Lifeng Zhao; Grade: 96/100**

Textbook: [1] Andrew J. Majda, Andrea L. Bertozzi: *Vorticity and Incompressible Flows*;

[2] Changxing Miao, Jiahong Wu, Zhifei Zhang: *Littlewood-Paley Theory and Its Applications to PDEs in Fluid Dynamics*.

[3] J. Bedrossian, N. Masmoudi: *Inviscid Damping and the Asymptotic Stability of Planar Shear Flows in the 2D Euler Equations*, 2015.

Contents: 1. Sobolev Inequalities, Calderon-Zygmund Singular Integral;

2. Littlewood-Paley theory, Besov Spaces and Paraproduct Decomposition;

3. Conservation Laws in 2D, 3D Euler's Equation;

4. Local Existence of the solution to 2D Euler's Equations;

5. Beale-Kato-Majda Criterion and Global Existence of 2D Euler's Equation;

6. Leray-Hopf Weak Solutions;

7. Semi-linear Methods for Navier-Stokes and Fujita-Kato;

8. Strong solution to NSE: Mild solution and weak-strong uniqueness;

9. Spectral stability and Lyapunov stability of Euler's equation;

10. 2D Inviscid Planar Shear Flows and Rayleigh's theorem;

11. Arnold's Nonlinear Stability Theorem for Shear Flows in a Channel;

12. Mixing, Trade regularity for decaying;

13. Zillinger's theorem;

14. *Inviscid Damping and the Asymptotic Stability of Planar Shear Flows in the 2D Euler Equations*, J. Bedrossian, N. Masmoudi, *Publications mathématiques de l'IHÉS*.

## 10. Second Order Elliptic PDEs

**Instructor: Hong Zhang; Grade: 90/100**

Textbook: David Gilbarg, Neil S. Trudinger: *Second Order Elliptic PDEs*, Chapter 1-9.

Contents: 1. Fundamental Solution to Laplacian, Gradient Estimates, Maximum Principle;

2. Newton Potential and Its Holder Estimates;
3. Schauder Estimates;
4. Weak Solution: Sobolev Spaces Theory, Existence and Regularity of Weak Solutions;
5. Boundedness: Di Giorgi-Moser Iteration;
6. Strong Solution:  $L^p$  Theory and Calderon-Zygmund Theorem.

## 11. Nonlinear Dispersive PDEs

**Instructor:** Lifeng Zhao;      **Grade:** Next Semester

Textbook:

- [1] Terence Tao: *Local and global analysis of nonlinear dispersive and wave equations*;
- [2] Christopher D. Sogge: *Lectures on Nonlinear Wave Equations*, 2<sup>nd</sup> edition;
- [3] Jonathan Luk: *Lecture Notes on Nonlinear Wave Equations*;
- [4] Sergiu Klainerman: *Lecture Notes of Analysis in Princeton*, 2011;
- [5] Pierre Germain, Fabio Pusateri, Frederic Rousset: *Asymptotic Stability of solutions for mKdV*, *Advances in Mathematics*, 2016;
- [6] Changxing Miao, Bo Zhang: *Harmonic Analysis Methods in PDEs*.

Contents:

1. Prerequisites: Distribution, Oscillatory Integrals;
2. Derivation of Dispersive PDEs: Hamiltonian, Euler-Lagrange Method, Water wave eq;
3. Linear Wave Equ: Fundamental Solution, Klainerman-Sobolev Inequality, Strichartz Estimates of Wave Equation;
4. Linear Schrodinger Equ: Strichartz Estimates, Christ-Kiselev Lemma, Keel-Tao Endpoint Estimates.
5. Local Well-posedness of NLS in subcritical and critical spaces;
6. Conservation Law: Conservations Laws for NLS, Morawetz Estimates, Global Well-posedness of NLS, Decay and Scattering of NLS and NLW.
7. Asymptotic Stability of mKdV.

## II. Probability and Stochastic Processes:

### 1. Probability Theory

**Instructor:** Dang-zheng Liu;      **Grade:** 88/100

Textbook: G. Grimmett, D. Stirzaker: *Probability and Random Process*, Chapter 1~5, 7.

- Contents:
1. Probability Measure and Random Variable;
  2. Discrete Random Variables;
  3. Continuous Random Variables;
  4. Generating Function Method;
  5. Characteristic Functions;
  6. Convergence of Random Variables;
  7. CLT(iid case, Lindeberg-Feller) and LLN.

### 2. Advanced Probability Theory

**Instructor:** Lijun Bo;      **Grade:** 91/100

Textbook: Kai-lai Chung: *A Course in Probability*.

- Contents: 1. Probability Measure Theory;  
 2. Random Variable and Distribution;  
 3. Expectation and Conditional Expectation (w. r. t. Sigma-Algebra);  
 4. Convergence and Tightness of Random Variable Sequences;  
 5. CLT and LLN.

### 3. Advanced Stochastic Processes

**Instructor: Lijun Bo; Grade: 97/100**

Textbook: I. Karatzas, S. E. Shreve: *Brownian Motion and Stochastic Calculus*, GTM113, Ch 1~3.

- Contents: 1. Basic Definitions about Stochastic Processes;  
 2. Martingale Theory and Its Convergence Theorems;  
 3. Brownian Motion: Wiener Measure, Reflection Principle, Markov Property, Donsker Theorem.

### 4. Probability Limiting Theory

**Instructor: Zhishui Hu; Grade: 93/100**

Textbook: [1] Rick Durrett: *Probability: Theory and Examples*, 4<sup>th</sup> edition, 2010. Ch 2, 3, 5;

[2] Zhengyan Lin, etc: *Basis of Limiting Theory*.

[3] Olav Kallenberg: *Foundations of Modern Probability*, 1997, Ch 4.

- Contents: 1. Weak and Strong Law of Large Numbers;  
 2. Three Series Theorem;  
 3. Law of Iterated Logarithm by Hartman-Witner;  
 4. Central Limit Theorem (iid, Lindberg-Feller) and Karamata Slow-Variation Theorem;  
 5. Stable Law, Poisson Convergence, Infinite Divisible Distribution;  
 6. Discrete Martingale Theory;  
 7. Skorohod's Topology and Probability Theory in Polish Spaces.

### 5. Topics in Stochastic Analysis

**Instructor: Elton P. Hsu, Dang-zheng Liu, Ran Wang; Grade: 97/100**

Textbook: [1] Terence Tao: *Topics in Random Matrix Theory*;

[2] Louis Chen: *An Introduction to Stein's Method*;

[3] Varadhan: *Lecture Notes on Large Deviation Theory*.

Contents: **Part 1: An Introduction to Random Matrix Theory:**

1. Examples of random matrices and problems;
2. Wigner matrices and semicircular law;
3. GOE, GUE and Tracy-Widom distribution.

**Part 2: An Introduction to Stein's Method:**

1. Gaussian Measures;
2. Functional inequalities (Poincare, log-Sobolev, and Beckner)
3. Stein's method and Central Limit Theorem.

**Part 3: An introduction to Large Deviation Theory:**

1. Large Deviation of i.i.d. sequences, Markov Chain, Brownian Motion;
2. Sanov's Theorem and Schilder's Theorem.

## 6. Martingale Theory and Stochastic Integrals

**Instructor: Jianliang Zhai; Grade: 96/100**

Textbook: Elton P. Hsu: *Lecture Notes on Martingale Theory and Stochastic Integrals*.

Contents:

1. Martingale Theory;
2. Brownian Motion: Wiener Measure, Markov Property, Reflection Principle, Quadratic Variation.
3. Stochastic Integration and Ito's Formula;
  - 3.1 Stochastic Integrals w.r.t Brownian Motion;
  - 3.2 Stochastic Integrals w.r.t Continuous Local Martingale;
  - 3.3 Ito's Formula of  $L^2$  semi-martingale;
  - 3.4 Stratonovich's Stochastic Integral.
4. Applications of Ito's Formula;
  - 4.1 Levy's Characterization of Brownian Motion;
  - 4.2 Exponential Martingale and Uniform Integrability;
  - 4.3 Girsanov's Theorem;
  - 4.4 B-D-G inequality;
  - 4.5 Representation Theorem of Martingales;
  - 4.6 Reflective Brownian Motion and Brownian Bridge.
5. Stochastic Differential Equations and Its Applications to Financial Mathematics.

## III. Geometry and Topology:

**1. Differential Geometry Instructor: Xiaowei Xu; Grade: 87/100**

Textbook: Jiagui Peng, Qing Chen: *Differential Geometry*.

- Contents:
1. Local Theory of 2D and 3D Curves;
  2. Local Theory of Surfaces (1<sup>st</sup>, 2<sup>nd</sup> fundamental form, Weingarten Transform, Gauss Curvature);
  3. Frames and the Fundamental Theorem of Theory of Surfaces;
  4. Intrinsic Geometry of Surfaces (Co-variant Differentiation, Geodesic Curve, Gauss-Bonnet Formula, Laplacian of a Surface, Riemann's Metric);
  5. Global Properties of Surfaces (Gauss-Bonnet Formula, Gauss Mapping for Compact Surfaces, Convex Surfaces).

**2. Topology Instructor: Bailin Song; Grade: 80/100**

Textbook: [1] Chengye You: *General Topology*, Peking University;

[2] Allen Hatcher: *Algebraic Topology*.

- Contents:
1. Basic Properties: Topological Spaces, Compactness, Connectness and Path-connectness, Homeomorphisms;
  2. Classification of Closed Surfaces in 3D.
  3. Homotopy and Fundamental Group, Van-Kampen Theorem;
  4. Covering Spaces, Universal Covering Space, Group Actions on Topological Spaces;
  5. Simplex Homology, CW complexes;
  6. Singular Homology: Calculation, Long and Short Exact Sequences;

7. Tracing Graphs Method and Excision Theorem;
8. Degree of Mappings and Cellular Homology.

### 3. Differential Manifolds

**Instructor:** Zuoqin Wang;      **Grade:** 84/100

Textbook: [1] John M. Lee: *An Introduction to Smooth Manifolds*, GTM218, 2<sup>nd</sup> edition, Chapter 1-22.

[2] Loring W. Tu: *An Introduction to Manifolds*.

Contents: 1. Smooth Manifolds and Submanifolds;  
 2. Smooth Mappings and Differentials;  
 3. Vector Bundles, Tangent and Cotangent Bundles, Tensor Bundles;  
 4. Vector Fields and Flows;  
 5. Lie Groups and Their Actions;  
 6. Differential Forms and Integration;  
 7. de Rham Cohomology;  
 8. Riemannian and Symplectic Structures;  
 9. Other Topics (e. g. Chern-Weil).

### 4. Riemann Geometry

**Instructor:** Shiping Liu;      **Grade:** Next Semester

Contexts: TBA.

## IV. Algebra

### 1. Linear Algebra A1~A2

**Instructor:** Guangtian Song;      **Grade:** 87, 90/100

Textbook: Shangzhi Li: *Linear Algebra*.

Contents: 1. Linear Equation System and Matrices;  
 2. Vector Spaces: Rank, Linear Independence;  
 3. Determinants (Including Binet-Cauchy Formula);  
 4. Matrix Theory: Calculation, Rank, Equivalence;  
 5. Eigenvalue Theory: Eigenvector, Eigenspace, Minimal Polynomial;  
 6. Jordan's Form Theory and  $\lambda$ -matrix Theory;  
 7. Congruence of Symmetric Matrices;  
 8. Unitary Matrices.

### 2. Introduction to Algebra

**Instructor:** Yi-huang Shen;      **Grade:** 91/100

Textbook: Yi Ouyang, Yihuang Shen: *An Introduction to Algebra*.

Contents: 1. An Introduction to Group, Ring, Field;  
 2. Elementary Number Theory: Fermat Theorem, Euler's Theorem, Wilson's Theorem;  
 3. Polynomial Rings over  $\mathbf{R}$ ;  
 4. Cyclic Groups;  
 5. Polynomial Rings over Field.



**3. Abstract Algebra****Instructor:** Mao Sheng;      **Grade:** 87/100Textbook: Keqin Feng, Shangzhi Li, Pu Zhang: *An Introduction to Abstract Algebra*.

Contents: 1. Group Theory: Cyclic Group, Abel Group, Group Action, Sylow's Theorem, Free Group, Solvable Group.

2. Ring Theory: Commutative Rings containing unit element.

3. Galois Theory: Field Expansion, Galois Expansion, Galois Theory.