Mathematics Courses List

Contexts

Note: * means this is a graduate level course

I. Analysis and Differential Equations:

1. Mathematical Analysis A1~A3

Instructor: Guangbin Ren (A1), Jihuai Shi (A2, A3); Grade: 89, 99, 96/100.

Textbook: Gengzhe Chang, Jihuai Shi: *A Course in Mathematical Analysis*.

Contents: **A1: One-Variable Analysis:**

1. Limit of Sequences;
2. Continuity and Differentiation of One-variable Functions;
3. Taylor's Formula;
4. Riemann's Integration and Lebesgue's Theorem, Stirling's Formula;

A2: Multi-Variable Analysis:

1. Topology of $\mathbf{R}^d$;
2. Continuity and Differentiation of Multi-variable functions;
3. Inverse and Implicit Mapping Theorem;
4. Multi-variable Integral (Green's Formula, Stokes' Formula, An intro. to Field Theory);

A3: Series and Generalized Integral:

1. Convergence of Number Series, Function Series including Power Series;
2. Convergence of Generalized Integral;
3. Fourier Series and Its Convergence;
4. Convergence of Generalized Integral with Parameters.

2. Differential Equation I

Instructor: Benjin Xuan; Grade: 92/100.

Textbook: [1] Tongren Ding, Chengzhi Li: *Textbooks of Ordinary Differential Equations*;

[2] Lawrence C. Evans: *Partial Differential Equations*, Ch. 1, 2, 4.1.

Contents: 1. Methods of solving to some ODEs;

2. Picard's Iteration: the Existence and the Uniqueness of ODE;
3. Extension of Solutions of ODEs;
4. Solving ODE Systems;
5. Introduction to the Planar Dynamical Systems determined by ODEs;
6. Transport Equation;
7. Laplacian Equation: Fundamental Solutions, Green's Function, Gradient Estimates;
8. Heat Equation: Fourier Transform Method;
9. Classical Solution to Wave Equation;
10. Separation of Variables.

3. Real Analysis

Instructor: Lifeng Zhao; Grade: 97/100.

Textbook: [1] Elias M. Stein, R. Shakarchi: *Real Analysis*, Chapter 1, 2, 3, 6.

[2] Minqiang Zhou: *Functions of Real Variables*, L^p space part.

Contents: 1. Lebesgue Measure Theory in $\mathbf{R}^d$;

2. Lebesgue Integration Theory and Convergence Theorems, L^p space;
3. Differentiation of Functions on $\mathbf{R}$: Lebesgue's Density, Functions of Bounded Variation, Absolute and Lipschitz Functions.
4. Abstract Measure Theory: Extension Theorem, Lebesgue-Stieltjes Measure.

4. Complex Analysis (H)

Instructor: Luo Luo; Grade: 84/100.

Textbook: Jihuai Shi, Taishun Liu: *Functions of Complex Variables*.

Contents: 1. Topology of **C**;

2. Cauchy-Riemann Equations and Holomorphic Functions;
3. Cauchy-Pompeiu Integral Formula;
4. Complex Series: Taylor's and Laurent's Expansion;
5. Memomorphic Functions and Residue Formulae;
6. Holomorphic Extension;
7. An Introduction to Riemann's Mapping Theorem.

5. Functional Analysis

Instructor: Yi Wang; Grade: 85/100

Textbook: Kung-ching Chang, Yuanqu Lin: *Lecture Notes on Functional Analysis*.

Contents: 1. Metric Space and Contraction Principle;

2. Linear Operators: Riesz Representation Thm, Open Mapping Thm, Hahn-Banach Thm.

3. Weak and Weak-* Convergence of Linear Operator Series;

4. Spectrum of Compact Linear Operators: Riesz-Schauder Theory.

6. Advanced Real Analysis

Instructor: Lifeng Zhao; Grade: 97/100

Textbook: [1] Elias M. Stein, R. Shakarchi: *Real Analysis*, Chapter 6;

[2] Elias M. Stein, R. Shakarchi: *Functional Analysis*, Chapter 1, 2, 3, 8.

Contents: 1. Abstract Measure Theory: Extension Theorems and Applications, Signed Measure and Radon-Nikodym Theorem;

2. L^p space and Inequalities;

3. Riesz-Thorin Interpolation, Hilbert Transform, Hardy-Littlewood Maximal Function;

4. Distribution: Basic Calculation, Tempered Distribution, Homogeneous Distribution, Parametrices of Linear Partial Differential Operators;

5. An Introduction to Oscillatory Integrals.

7. Differential Equation II (H)

Instructor: Lifeng Zhao; Grade: 90/100

Textbook: Lawrence C. Evans: *Partial Differential Equations*, Chapter 5, 6, 7, 8.1, 8.6, 9.4.

Contents: 1. Sobolev Spaces $\mathbf{W^{k,p}(U)}$ and $\mathbf{H^s(R^d)}$;

2. Elliptic Equations: Weak Solutions, Maximum Principle, Fourier Expansion;

3. Parabolic Equ. : Galerkin's Method, Theory of Weak Solutions, Maximum Principle;

4. Hyperbolic Equ. : Galerkin's Method, Existence and Regularity to Weak Solutions;

5. Vanishing Viscosity Method for 1st Order Hyperbolic Systems;

6. 3D Cubic Nonlinear Schrodinger's Equation and Its Strichartz Estimates;

7. Conservation Law: Euler-Lagrange Equations, Nother's Theorem, Pohozaev's Identity for $\triangle u=u|u|^{p-1}$ with zero boundary data.

8. Harmonic Analysis

Instructor: Guangbin Ren; Grade: 95/100

Textbook: [1] Javier Duoandikoetxea: *Fourier Analysis*, Chapter 1, 2;

[2] Elias M. Stein: *Singular Integrals and Differentiability of Functions*, Chapter 2, 3;

[3] C. Muscalu, W. Schlag: *Classical and Multilinear Harmonic Analysis*, Chapter 8;

[4] M. Ruzhansky: *Pseudo-Differential Operators and Symmetries,* Chapter 6, 7.

[5] M. Wong: *Wavelet Transforms and Localization Operators*, Section 6-9, 17-18.

Contents: 1. Fourier Series and Fourier Transforms, Radon Transforms;

2. Hardy-Littlewood Maximal Operators;
3. Singular Integrals: Calderon-Zygmund Theory;
4. Littlewood-Paley Theory;
5. Representation Theory of Compact Groups(including Peter-Weyl Theorem);
6. Wavelet Transform;
7. Representation Theory of Affined Group and Weyl-Heisenberg Group.

9. Nonlinear PDEs in Fluid Dynamics

Instructor: Lifeng Zhao; Grade: 96/100

Textbook: [1] Andrew J. Majda, Andrea L. Bertozzi: *Vorticity and Incompressible Flows*;

[2] Changxing Miao, Jiahong Wu, Zhifei Zhang: *Littlewood-Paley Theory and Its Applications to PDEs in Fluid Dyamics*.

[3] **J. Bedrossian, N. Masmoudi:** ***Inviscid Damping and the Asymptotic Stability of Planar Shear Flows in the 2D Euler Equations*, 2015.**

Contents: 1. Sobolev Inequalities, Calderon-Zygmund Singular Integral;

2. Littlewood-Paley theory, Besov Spaces and Paraproduct Decomposition;
3. Conservation Laws in 2D, 3D Euler's Equation;
4. Local Existence of the solution to 2D Euler's Equations;
5. Beale-Kato-Majda Criterion and Global Existence of 2D Euler's Equation;
6. Leray-Hopf Weak Solutions;
7. Semi-linear Methods for Navier-Stokes and Fujita-Kato;
8. Strong solution to NSE: Mild solution and weak-strong uniqueness;
9. Spectral stability and Lyapunov stability of Euler's equation;
10. 2D Inviscid Planar Shear Flows and Rayleigh's theorem;
11. Arnold's Nonlinear Stability Theorem for Shear Flows in a Channel;
12. Mixing, Trade regularity for decaying;
13. Zillinger's theorem;
14. ***Inviscid Damping and the Asymptotic Stability of Planar Shear Flows in the 2D Euler Equations*, J. Bedrossian, N. Masmoudi, *Publications mathématiques de l'IHÉS.***

10. Second Order Elliptic PDEs

Instructor: Hong Zhang; Grade: 90/100

Textbook: David Gilbarg, Neil S. Trudinger: *Second Order Elliptic PDEs,* Chapter 1-9.

Contents: 1. Fundamental Solution to Laplacian, Gradient Estimates, Maximum Principle;

2. Newton Potential and Its Holder Estimates;
3. Schauder Estimates;
4. Weak Solution: Sobolev Spaces Theory, Existence and Regularity of Weak Solutions;
5. Boundedness: Di Giorgi-Moser Iteration;
6. Strong Solution: L^p Theory and Calderon-Zygmund Theorem.

11. Nonlinear Dispersive PDEs
Instructor: Lifeng Zhao; Grade: 97/100
Textbook:
[1] Terence Tao: *Local and global analysis of nonlinear dispersive and wave equations*;
[2] Christopher D. Sogge: *Lectures on Nonlinear Wave Equations, 2nd edition;*
[3] Jonathan Luk: *Lecture Notes on Nonlinear Wave Equations;*
[4] Sergiu Klainerman: *Lecture Notes of Analysis in Princeton, 2011*;
[5] Pierre Germain, Fabio Pusateri, Frederic Rousset: *Asymptotic Stability of solutions for mKdV, Advances in Mathematics, 2016;*
[6] Changxing Miao, Bo Zhang: *Harmonic Analysis Methods in PDEs*.
Contents:
1. Prerequisites: Distribution, Oscillatory Integrals;
2. Derivation of Dispersive PDEs: Hamiltonian, Euler-Lagrange Method, Water wave eq;
3. Linear Wave Equ: Fundamental Solution, Klainerman-Sobolev Inequality, Strichartz Estimates of Wave Equation;
4. Linear Schrodinger Equ: Strichartz Estimates, Christ-Kiselev Lemma, Keel-Tao Endpoint Estimates.
5. Local Well-posedness of NLS in subcritical and critical spaces;
6. Conservation Law: Conservations Laws for NLS, Morawetz Estimates, Global Well-posedness of NLS, Decay and Scattering of NLS and NLW.
7. Asymptotic Stability of mKdV.

II. Probability and Stochastic Processes:

1. Probability Theory
Instructor: Dang-zheng Liu; Grade: 88/100
Textbook: G. Grimmett, D. Stirzaker: *Probability and Random Process*, Chapter 1~5, 7.
Contents: 1. Probability Measure and Random Variable;
2. Discrete Random Variables;
3. Continuous Random Variables;
4. Generating Function Method;
5. Characteristic Functions;
6. Convergence of Random Variables;
7. CLT(iid case, Lindeberg-Feller) and LLN.

2. Advanced Probability Theory
Instructor: Lijun Bo; Grade: 91/100
Textbook: Kai-lai Chung: *A Course in Probability*.

Contents: 1. Probability Measure Theory;
2. Random Variable and Distribution;
3. Expectation and Conditional Expectation (w. r. t. Sigma-Algebra);
4. Convergence and Tightness of Random Variable Sequences;
5. CLT and LLN.

3. Advanced Stochastic Processes
Instructor: Lijun Bo; Grade: 97/100
Textbook: I. Karatzas, S. E. Shreve: *Brownian Motion and Stochastic Calculus*, GTM113, Ch 1~3.
Contents: 1. Basic Definitions about Stochastic Processes;
2. Martingale Theory and Its Convergence Theorems;
3. Brownian Motion: Wiener Measure, Reflection Principle, Markov Property, Donsker Theorem.

4. Probability Limiting Theory
Instructor: Zhishui Hu; Grade: 93/100
Textbook: [1] Rick Durrett: *Probability: Theory and Examples*, 4^{th} edition, 2010. Ch 2, 3, 5;
[2] Zhengyan Lin, etc: Basis of Limiting Theory.
[3] Olav Kallenberg: *Foundations of Modern Probability*, 1997, Ch 4.
Contents: 1. Weak and Strong Law of Large Numbers;
2. Three Series Theorem;
3. Law of Iterated Logarithm by Hartman-Witner;
4. Central Limit Theorem (iid, Lindberg-Feller) and Karamata Slow-Variation Theorem;
5. Stable Law, Poisson Convergence, Infinite Divisible Distribution;
6. Discrete Martingale Theory;
7. Skorohod's Topology and Probability Theory in Polish Spaces.

5. Topics in Stochastic Analysis
Instructor: Elton P. Hsu, Dang-zheng Liu, Ran Wang; Grade: 97/100
Textbook: [1] Terence Tao: *Topics in Random Matrix Theory*;
[2] Louis Chen: *An Introduction to Stein's Method*;
[3] Varadhan: *Lecture Notes on Large Deviation Theory*.
Contents: **Part 1: An Introduction to Random Matrix Theory:**
1. Examples of random matrices and problems;
2. Wigner matrices and semicircular law;
3. GOE, GUE and Tracy-Widom distribution.
Part 2: An Introduction to Stein's Method:
1. Gaussian Measures;
2. Functional inequalities (Poincare, log-Sobolev, and Beckner)
3. Stein's method and Central Limit Theorem.
Part 3: An introduction to Large Deviation Theory:
1. Large Deviation of i.i.d. sequences, Markov Chain, Brownian Motion;
2. Sanov's Theorem and Schilder's Theorem.

6. Martingale Theory and Stochastic Integrals
Instructor: Jianliang Zhai; Grade: 96/100
Textbook: Elton P. Hsu: *Lecture Notes on Martingale Theory and Stochastic Integrals*.
Contents:

1. Martingale Theory;
2. Brownian Motion: Wiener Measure, Markov Property, Reflection Principle, Quadratic Variation.
3. Stochastic Integration and Ito's Formula;
    3.1 Stochastic Integrals w.r.t Brownian Motion;
    3.2 Stochastic Integrals w.r.t Continuous Local Martingale;
    3.3 Ito's Formula of L^2 semi-martingale;
    3.4 Stratonovich's Stochastic Integral.
4. Applications of Ito's Formula;
    4.1 Levy's Characterization of Brownian Motion;
    4.2 Exponential Martingale and Uniform Integrability;
    4.3 Girsanov's Theorem;
    4.4 B-D-G inequality;
    4.5 Representation Theorem of Martingales;
    4.6 Reflective Brownian Motion and Brownian Bridge.
5. Stochastic Differential Equations and Its Applications to Financial Mathematics.

III. Geometry and Topology:

1. Differential Geometry Instructor: Xiaowei Xu; Grade: 87/100
Textbook: Jiagui Peng, Qing Chen: *Differential Geometry*.
Contents: 1. Local Theory of 2D and 3D Curves;

2. Local Theory of Surfaces (1st, 2nd fundamental form, Weingarten Transform, Gauss Curvature);
3. Frames and the Fundamental Theorem of Theory of Surfaces;
4. Intrinsic Geometry of Surfaces (Co-variant Differentiation, Geodesic Curve, Gauss-Bonnet Formula, Laplacian of a Surface, Riemann's Metric);
5. Global Properties of Surfaces (Gauss-Bonnet Formula, Gauss Mapping for Compact Surfaces, Convex Surfaces).

2. Topology Instructor: Bailin Song; Grade: 80/100
Textbook: [1] Chengye You: *General Topology*, Peking University;
[2] Allen Hatcher: *Algebraic Topology*.
Contents: 1. Basic Properties: Toplogical Spaces, Compactness, Connectness and Path-connectness, Homeomorphisms;

2. Classification of Closed Surfaces in 3D.
3. Homotopy and Fundamental Group, Van-Kampen Theorem;
4. Covering Spaces, Universal Covering Space, Group Actions on Topological Spaces;
5. Simplex Homology, CW complexes;
6. Singular Homology: Calculation, Long and Short Exact Sequences;

7. Diagramme Chasing Method and Excision Theorem;
8. Degree of Mappings and Cellular Homology.

3. Differential Manifolds

Instructor: Zuoqin Wang; Grade: 84/100

Textbook: [1] John M. Lee: *An Introduction to Smooth Manifolds*, GTM218, 2nd edition, Chapter 1-22.

[2] Loring W. Tu: *An Introduction to Manifolds*.

Contents: 1. Smooth Manifolds and Submanifolds;
2. Smooth Mappings and Differentials;
3. Vector Bundles, Tangent and Cotangent Bundles, Tensor Bundles;
4. Vector Fields and Flows;
5. Lie Groups and Their Actions;
6. Differential Forms and Integration;
7. de Rham Cohomology;
8. Riemannian and Symplectic Structures;
9. Other Topics (e. g. Chern-Weil).

4. Riemann Geometry

Instructor: Shiping Liu; Grade: TBA

Contexts:

1. Riemann Metric;
2. Geodesics, Exponential Maps, Normal Coordinates, Geodesical Completeness and Hopf-Rinow Theorem;
3. Connections (Affine, Levi-Civita), Paralleism, Covariant Derivatives;
4. Curvature (Riemann Curvature Tensor, Sectional and Ricci Curvature);
5. Index Form, Space Forms, Variational Formulae, Jacobi Fields;
6. Candidates for Synthetic Curvature Conditions;
7. Cartan-Hadamard Thm, Bonnet-Meyer Thm, Synge Thm;
8. Comparison Principle;
9. Laplacian, Hessian, Hodge-Laplace Operator;
10. An Introduction to Discrete Geometry and Applications in Graph Theory.

IV. Algebra

1. Linear Algebra A1~A2

Instructor: Guangtian Song; Grade: 87, 90/100

Textbook: Shangzhi Li: *Linear Algebra*.

Contents: 1. Linear Equation System and Matrices;
2. Vector Spaces: Rank, Linear Independence;
3. Determinants (Including Binet-Cauchy Formula);
4. Matrix Theory: Calculation, Rank, Equivalence;
5. Eigenvalue Theory: Eigenvector, Eigenspace, Minimal Polynomial;
6. Jordan's Form Theory and λ-matrix Theory;
7. Congruence of Symmetric Matrices;

8. Unitary Matrices.

2. Introduction to Algebra

Instructor: Yi-huang Shen; Grade: 91/100

Textbook: Yi Ouyang, Yihuang Shen: *An Introduction to Algebra*.

Contents: 1. An Introduction to Group, Ring, Field;

2. Elementary Number Theory: Fermat Theorem, Euler's Theorem, Wilson's Theorem;
3. Polynomial Rings over **R**;
4. Cylic Groups;
5. Polynomial Rings over Field.

3. Abstract Algebra

Instructor: Mao Sheng; Grade: 87/100

Textbook: Keqin Feng, Shangzhi Li, Pu Zhang: *An Introduction to Abstract Algebra*.

Contents: 1. Group Theory: Cylic Group, Abel Group, Group Action, Sylow's Theorem, Free Group, Solvable Group.

2. Ring Theory: Commutative Rings containing unit element.
3. Galois Theory: Field Expansion, Galois Expansion, Galois Theory.