

2.2.1 $h_{\theta}(x_i) = \theta^T x_i$

$$J(\theta) = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x_i) - y_i)^2 + \lambda \theta^T \theta = \frac{1}{m} \sum_{i=1}^m (\theta^T x_i - y_i)^2 + \lambda \theta^T \theta = \frac{1}{m} (x \theta - y)^T (x \theta - y) + \lambda \theta^T \theta = \frac{1}{m} \|x \theta - y\|_2^2 + \lambda \|\theta\|_2^2$$

2.2.3 $\nabla_{\theta} J(\theta) = \frac{2}{m} x^T (x \theta - y) + 2 \lambda \theta$

2.3.1 - 阶乘展开

$$J(\theta + \eta h) = J(\theta) + (h^T \theta)^T \nabla_{\theta} J(\theta) + \dots$$

$h^T \nabla_{\theta} J(\theta)$ 只有 $h \in \nabla_{\theta} J(\theta)$ 方向相同时取得最大值

2.3.2 $\theta^{(t+1)} = \theta^{(t)} - \eta \nabla_{\theta} J(\theta) \Big|_{\theta=\theta^{(t)}}$

2.3.4 步长为 0.01 时收敛最快，步长为 0.5, 0.1 时会发散

2.4.1 $X_k = \begin{pmatrix} x_{i11} & x_{i12} & \dots & x_{i1d+1} \\ x_{i21} & x_{i22} & \dots & x_{i2d+1} \\ \vdots & & & \\ x_{in1} & x_{in2} & \dots & x_{ind+1} \end{pmatrix} \quad y_k = \begin{pmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{in} \end{pmatrix}$

$$J_{SVD}(\theta) = \frac{1}{n} \sum_{k=1}^n (h_{\theta}(x_{ik}) - y_{ik})^2 + \lambda \theta^T \theta = \frac{1}{n} (x_k \theta - y_k)^T (x_k \theta - y_k) + \lambda \theta^T \theta$$

$$\nabla_{\theta} J_{SVD}(\theta) = \frac{2}{n} X_k^T (X_k \theta - y_k) + 2 \lambda \theta$$

2.4.3 批量增大，损失函数呈收敛趋势

2.4.5 发现在所取数值中，正则化系数越大越准确

3.2 发现 $distance$ 对房价有较大负面影响， $latitude$ 对房价有较大正面影响，当加入将时间运用特征工程加入了模型，且分为年月，可能会影响房屋年份产生关联，同时也会引入过多时间波动，弱化其它指标作用。可以减少时间变量来重新设计模型。

把不同变量都进行特征规范化，但变量间关系不同，可能进行标准化会更好。
(最大最小) (正态)

4.1.4 $h_{\theta}(x) = P(y=1|x; \theta) = \frac{1}{1+e^{-\theta^T x}}$

$$J(\theta) = -y \log(1+e^{-\theta^T x}) + (1-y) \log\left(\frac{1+e^{-\theta^T x}}{e^{-\theta^T x}}\right)$$

$$\nabla_{\theta} J(\theta) = y \frac{-X e^{-\theta^T x}}{1+e^{-\theta^T x}} - (1-y) \frac{-X e^{-\theta^T x}}{e^{-\theta^T x}} + (1-y) \frac{X e^{-\theta^T x}}{1+e^{-\theta^T x}} = -X e^{-\theta^T} h_{\theta}(x) + (1-y) x = -x(e^{\theta^T} + 1) h_{\theta}(x) + x h_{\theta}(x) + (1-y) x = (h_{\theta}(x) - y) x$$

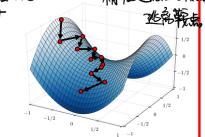
4.1.5. 发现手动得出的结果与调用算法得到的结果一样

4.2.2 选取了花萼长度与花瓣宽度

4.2.3 引入基函数决策面变弯曲拟合效果变好

Stochastic Gradient Descent (SGD)

- In each iteration t ($\leq T$):
 - 隨机采样 $\text{d}x \text{d}y$
 - Randomly sample a **minibatch** of $m \ll n$ points $\{(x_i, y_i)\}_{i=1}^m$
 - Set $J^t(\mathbf{w}^t) = \frac{1}{m} \sum_{i=1}^m \ell(\mathbf{w}^t; x_i, y_i)$
 - Compute gradient on minibatch: $\Delta^t = \nabla_{\mathbf{w}} J^t(\mathbf{w}^t)$
 - Update parameters with learning rate η : $\mathbf{w}^{t+1} = \mathbf{w}^t - \eta \Delta^t$



- For linear regression: $\mathbf{w}^{t+1} \leftarrow \mathbf{w}^t - 2\eta \mathbf{X}_m^T (\mathbf{X}_m \mathbf{w}^t - \mathbf{y})$

沿着负梯度

- Gradient shows direction that function varies fastest.

Same dimension with parameter \mathbf{w} 's dimension	$\mathbf{g} = \nabla_{\mathbf{w}} J(\mathbf{w})$ 梯度向量
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$$g_j = \nabla_{w_j} J(\mathbf{w})$$



- First-order Taylor approximation of the objective function:

$$J(\mathbf{w}) = J(\mathbf{w}_0) + (\mathbf{w} - \mathbf{w}_0)^T \mathbf{g} + \dots$$

梯度近似

- Go along gradient for a step with a small rate η :

$$J(\mathbf{w} - \eta \mathbf{g}) \approx J(\mathbf{w}) - \eta \mathbf{g}^T \mathbf{g}$$

- Reach a point with **smaller loss**.

$$-\eta \mathbf{g}^T \mathbf{g} \leq 0$$

- Repeat this step, and we get the **Gradient Descent (GD)** algorithm.

- Compute the gradient and set it to zero

$$\begin{aligned} \nabla_{\mathbf{w}} \hat{\epsilon}(\mathbf{w}) &= 2\mathbf{X}^T(\mathbf{X}\mathbf{w} - \mathbf{y}) + 2\lambda\mathbf{w} = 0 \\ &\Rightarrow (\mathbf{X}^T\mathbf{X} + \lambda\mathbf{I})\mathbf{w} = \mathbf{X}^T\mathbf{y} \\ &\Rightarrow \mathbf{w} = (\mathbf{X}^T\mathbf{X} + \lambda\mathbf{I})^{-1}\mathbf{X}^T\mathbf{y} \end{aligned}$$

有逆-解
不带解

- Or using Gradient Descent (GD): **Always invertible** for $\lambda > 0$

$$\mathbf{w}^{t+1} \leftarrow \mathbf{w}^t - \eta \nabla_{\mathbf{w}} \hat{\epsilon}(\mathbf{w})|_{\mathbf{w}=\mathbf{w}^t}$$

$d \times d$ -dim identity matrix