# Identification and Estimation of Market Size in Discrete Choice Demand Models

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**Abstract**

Within the framework of Berry (1994) and Berry, Levinsohn, and Pakes (1995), I prove that market size can be point identified along with all demand parameters in a random coefficients logit (BLP) model. I require no additional data beyond what is needed to estimate standard BLP models. Identification comes from exogenous variation that shifts quantities and the nonlinearity of the demand system. I illustrate the method using a merger simulation in the ready-to-eat cereal market, and find that assuming a market size larger than the true estimated size would underestimate merger price increases and has non-trivial implications for welfare.

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# 1 Introduction

Aggregate demand models of differentiated products are crucial for analyzing market power and firm competition in a wide range of industries. The most widely adopted estimation approach developed by Berry (1994) and Berry, Levinsohn, and Pakes (1995) (hereafter referred to as BLP) involves using observed aggregate market shares. Constructing market shares requires researchers to observe the size of the market. Market size consists of all observed sales (the inside goods) plus all potential purchases (the outside goods or no-purchase). Potential purchases are generally unobservable and are therefore a source of possible mismeasurement of market size.[1]

Many empirical results are sensitive to market size (see section 1.1 and Supplement B for details and examples). Yet how to choose market size in demand models has received limited formal attention in the literature. Table 1 shows that, over the past six years, around 30 articles published in the top 5 journals used a parametric BLP demand model. Of these, more than 80% made ad-hoc assumptions on market size or the outside option, and only 5 out of 24 studies performed a robustness check on these assumptions. A few researchers have commented on this problem,[2] but provide little guidance on what to do about unobserved or mismeasured market size.

Table 1: Empirical BLP Studies in Top 5 Journals from 2018 to 2024

| | Number of Empirical BLP Studies in Top 5 Journals | Studies with Ad-hoc Market Size Assumptions | Studies Conducting Market Size Robustness Tests |
|---|---|---|---|
| Count | 29 | 24 | 5 |

Note: The top 5 journals refer to the American Economic Review, Econometrica, the Journal of Political Economy, the Quarterly Journal of Economics, and the Review of Economic Studies.

A common empirical choice is to assume the market size equals the population of the

1. For instance, when estimating airline demand, a market is typically defined as an origin-destination pair of cities. This raises questions about how to determine the number of potential flyers – whether it comprises only those currently traveling by other means, individuals who might opt for travel with lower prices, or the entire population of end-point cities, some of whom may never travel to the destination.

2. For example, Berry (1994) says that "issues that might be examined include questions of how to estimate market size when this is not directly observed".

market times a constant.[3] For example, in the demand for soft drinks, this constant represents the maximum amount an individual can potentially consume, which is not observed or estimated in general but chosen ad hoc based on institutional background or consumer behavior. It is important to note that this constant is not a free normalization as it affects the estimates of preferences and counterfactual simulations.

This paper shows how to correct for the unknown market size in random coefficients BLP and other related demand models. I focus mainly on a class of market size specifications in which market size is modeled as a polynomial function of observed market-level variables, and I provide sufficient conditions under which market size and all demand parameters of the BLP model are point identified. For example, in an airline demand model, market size can be a function of the population in the origin city, population in the destination city, city characteristics like being a hub or not, and a vector of unknown parameters that are identified and estimated along with the rest of the BLP model. A special case of this framework is when market size is a constant multiple of the observed population, in which case the constant can be identified and estimated jointly with the remaining model parameters. I then extend the analysis to allow for the possibility of random variation in market size, so that it is not exactly determined by a function of observables. Specifically, I consider a model of the form $s(M_t; \gamma) + v_t$, where $s$ is a function of market-level observables $M_t$, known up to a finite-dimensional parameter vector $\gamma$, and $v_t$ is an unobserved error term.

Identification exploits two important features: exogenous variation that shifts quantities across products and markets, and the nonlinearity of the demand model. It does not rely on other information such as micro-moments or additional data beyond those typically used in standard BLP. That is, the results in my paper use the same BLP parametric assumptions and data as in the 29 empirical applications of Table 1. A key insight is that any exogenous changes in product characteristics affect the quantities of inside goods, and the responses of quantities to this variation depends on the true size of the market. Why does this variation

3. Well-known examples include Nevo (2001), Petrin (2002), Rysman (2004), Berto Villas-Boas (2007), Berry and Jia (2010), Ho, Ho, and Mortimer (2012), and Ghose, Ipeirotis, and Li (2012). Of the 29 papers in Table 1, half explicitly assume market size be proportional to an observed measure.

have extra identifying power for parameters beyond ordinary demand coefficients? In section 3, I show that the log of product shares in the plain multinomial logit model can be written as a function linear in product characteristics but nonlinear in market size parameters, making identification possible.

More formally, identification is based on conditional moment restrictions and full rank conditions. By explicitly computing the associated Jacobian matrix, I provide low-level assumptions on instruments that serve to jointly identify the market size and demand parameters (including random coefficients). These conditions clarify the role of instruments and the sources of exogenous variation that are informative for identifying market size. In the special case where market size takes the form of $\gamma M_t$, the required instruments are *exogenous quantity shifters*, which will be defined later in the paper; importantly, these instruments are neither $M_t$ nor functions of $M_t$. In the more general polynomial specification of market size, the required instruments include *interactions* between quantity shifters and market-level covariates $M_t$.

In practice, researchers often argue that nested logit models or the inclusion of market fixed effects alleviate concerns about unknown market size.[4] However, as I will show, such practices do not fully eliminate biases. I demonstrate how the method proposed in this paper is related to, yet distinct from, these commonly used model specifications. In the case of a nested logit model with the outside option as a standalone nest, I show that the nesting parameter governing substitution between inside goods and the outside option is separately identified from the market size parameter, meaning the two drive substitution differently (see section 3.4).

For market fixed effects, a plain logit model with market fixed effects identifies demand parameters, but market size remains unidentified if it is the object of interest. More importantly, in section 4.5, I show that market fixed effects remove bias only in plain or nested logit models, not in the random coefficients logit model, and the latter is the main focus

4. Market fixed effects are primarily intended to capture unobserved market-level attributes that might otherwise violate conditional independence assumptions. The method proposed in this paper is intended to complement, rather than replace, the use of market fixed effects.

of this paper. In principle, market size is separately identified from fixed effects – in other words, the market size model is still identifiable in a random coefficients model specification that includes market fixed effects.[5] Specifically, the instruments that identify market size typically have product-market variation, and are therefore not perfectly collinear with the market fixed effects.

In addition to providing these identification results, I also (a) derive the bias caused by mismeasured market size; (b) establish a test based on linear regression to detect instrument relevance; (c) show identification in models where market size is an unknown function of observed variables; (d) provide stronger conditions that permit point identification (up to scale) of market size, even when the demand model is not known or nonparametric (e.g., in Berry and Haile (2014)'s nonparametric BLP framework), which allows for testing market size specifications without estimating the demand model; (e) show that a special case of nonparametric estimation of random coefficients is equivalent to estimating the market size, but it requires imposing particular assumptions on the distribution of random coefficients.

Based on these identification results, I apply the proposed method to a merger simulation in the ready-to-eat cereal industry using the constructed data from Nevo (2000a). In the merger analysis, I use both the proposed method and the standard BLP to estimate demand, while assuming a Bertrand competition among firms. Using the estimated market size that equals 0.78 times the population size, the predicted post-merger price effects are more than twice as large as those obtained under an assumption of 2 servings per day, leading to a larger reduction in consumer surplus. The results highlight the sensitivity of demand estimates to market size assumptions: the estimated price coefficient is $-35.82$ under a market size of 0.78 servings, compared with $-28.9$ under 2 servings, while the estimated standard deviation of the random price coefficient is 0.68 versus 0.25. The estimated outside diversion ratios are 23% and 60%, respectively. In Supplement J, I present a second merger analysis for carbonated soft drink firms using NielsenIQ scanner data.

5. In remark 3, I cover the case in which market size is treated as a free, market-specific parameter, and I discuss the conditions on instruments that achieve separate identification of these parameters and market fixed effects.

In the Monte Carlo simulations, I examine what parameters are most sensitive to errors in market size measurements. Furthermore, I find that adding random coefficients on an intercept or prices does not fix the bias with mismeasured market size. I also show that my proposed approach performs well, particularly when the true share of the outside option is not extremely large, and so my method will generally be useful in applications.

The proposed method in this paper is transparent and simple to implement. It requires estimating only a few extra nonlinear parameters in a Generalized Method of Moments (GMM) context, along with the standard BLP estimation. Researchers may have tried to estimate market size, but the lack of identification theorem and the unsatisfactory empirical performance with the estimator have hindered the widespread adoption of market size estimation in applied work. This paper provides conditions under which the market size is identified, shows valid sources of data variation that achieves identification, and discusses under what market structures identification may be poor. We hope that researchers who find their outcomes sensitive to market size assumptions can flexibly incorporate our model as an extra specification in their analysis. Moreover, whenever the market size itself is important to practitioners or regulators, this method can serve as a means to infer the size of the market. Note, that although the solution is simple, it goes beyond merely adding a regressor or market fixed effects.

### 1.1 Why Market Size Matters

One argument for not correcting the market size issue is the belief that random coefficients or a nesting parameter can partially account for the bias. Indeed, calculations such as own-price elasticities, may exhibit less sensitivity when the model includes random coefficients, as seen in Rysman (2004), Iizuka (2007), and Duch-Brown et al. (2017). However, my simulations and empirical study reveal that random coefficients do not fully eliminate biases. Biases are particularly pronounced in certain calculations, such as outside good elasticities, outside good diversion ratios, choice probabilities, and aggregate price elasticities, even with a random coefficient on price or the intercept. Conlon and Mortimer (2021) (Table 4) also find that outside diversion ratios and aggregate elasticities are sensitive to market size in both the BLP automobile application and Nevo's cereal application.

Moreover, as market shares and their derivatives enter the supply-side pricing first order condition, researchers aiming to recover marginal costs and markups from the pricing condition may end up with biased estimates. This bias can propagate through the structural model and substantially affect results for empirical questions, particularly those related to the outside option share, such as the willingness-to-pay for a new good (see discussion in Conlon and Mortimer 2021), tax or subsidy policies (dependent on aggregate elasticities), and merger analysis[6]. Table 2 summarizes a list of merger studies using different logit-based demand models with various specifications, with and without market fixed effects, highlighting calculations and counterfactual estimates that are sensitive to market size assumptions. See Supplement B for additional examples and discussion in other empirical settings.

Table 2: Sensitivity Analysis to Market Size Assumptions in Horizontal Merger Studies

| Article | Demand Model | Market FE | Sensitivity Analysis<br>Market size affects: |
|---|---|---|---|
| Ivaldi and Verboven (2005) | Nested Logit | Yes | Aggregate price elasticities |
| Weinberg and Hosken (2013) | Plain Logit | No | Simulated merger price changes |
| Bokhari and Mariuzzo (2018) | BLP | Yes | Cross-price elasticities and simulated price changes |
| Wollmann (2018) | BLP | No | Total output changes and compensating variation |

Furthermore, the size of a market by itself could be a piece of critical and useful information for firms and regulators.[7] This suggests that obtaining a consistent estimate of the true market size is important in itself, in addition to its use in removing model estimate biases.

6. *Market size* and *market definition* are different concepts, with the latter more commonly used in traditional merger analysis. Screening tools like the Herfindahl-Hirschman Index depends on market definition, while market size issue becomes relevant in structural demand models for merger simulations. Merger simulations as a new screening tool, introduced in the 2010 Horizontal Merger Guidelines, predict lost competition and counterfactual price effects. It includes an outside option in modeling consumer choices, whereas market definitions exclude it, considering only products in the competitive set. In other words, *market size* used in demand models captures relevant consumers or purchase opportunities, while *market definition* focuses on relevant substitute products.

7. For example, firms predict product quantities on the basis of potential market size. The Comments of the Department of Justice (DOJ) on Joint Application Of American Airlines Et Al. state that "To model the benefits of an alliance ... Given a fixed market size, passengers are assigned based on relative attractiveness of different airline offerings."

The next section is a literature review. In section 3, I start with a multinomial logit demand model to provide simple identification results. In section 4, I show general identification for the random coefficients logit model. Section 5 provides extensions. Section 6 assesses the performance of the method using Monte Carlo simulations. Section 7 presents an empirical application, and section 8 concludes. Proofs of main theorems are in the Appendix.

# 2 Literature Review

Some researchers realize the issue and explicitly incorporate market size estimation into demand models. Bresnahan and Reiss (1991) and Greenstein (1996) both specify market size as a linear function of market characteristics, though theirs is a vertical model rather than BLP. Berry (1990) and Berry, Carnall, and Spiller (2006) estimate a scaling factor similar to this paper, however, they do not formally proves identification, and they do not allow for market size being a more general function of multiple measures. Chu, Leslie, and Sorensen (2011) and Byrne et al. (2022) utilize observed cost data and the supply side first order condition to obtain identification of market size. Sweeting, Roberts, and Gedge (2020) and Li et al. (2022) estimate a generalized gravity equation and define market size as proportional to the expected total passengers predicted from the gravity equation but leave the choice of the proportionality factor to the researcher. Hortaçsu, Oery, and Williams (2022) estimate a Poisson arrival process and use the arrival rate as a proxy measure of market size. Their method applies to settings with individual choice data, whereas I focus on aggregate data. Huang and Rojas (2013) consider a plain logit model, where they use market fixed effects to eliminate bias in the first stage and recover market size by a minimum distance estimator in the second stage.

The closest study to ours is Huang and Rojas (2014), which provides theoretically-founded methods to deal with the market size problem in a random coefficients logit setting, by approximating the unobserved market size as a linear function of market characteristics (Chamberlain's device). They employ the control function method to handle price endogeneity as in Petrin and Train (2010). By doing so, the unobserved market size becomes an additive term outside of the nonlinear part of the demand function. In contrast, ours is built on the standard BLP framework, where market size enters the moment restrictions in a nonlinear manner. Huang and Rojas (2014)'s method largely relies on the linear additivity and thus can not extend directly to the BLP framework.[8] Their primary focus is on removing bias, while this paper also aims to identify and estimate the market size.

Two other papers have looked at issues arising in constructing market shares. Gandhi, Lu, and Shi (2023) handle the problem of zeros in market share data. Berry, Linton, and Pakes (2004) take into account sampling errors in estimating shares from a sample of consumers. While both papers deal with errors in aggregate market shares, the present paper tackles a different problem, inherent to the model itself rather than features of the data sample. The goal of this paper is to address the more fundamental problem of the unobserved share of the outside option and that all shares will be inconsistent in the limit. Unlike sampling errors that diminish as the sample size increases, the errors I address persist and do not vanish.

More recently, theoretical literature on the identification and estimation of random coefficients aggregate demand model has been growing. Berry and Haile (2014) and Gandhi and Houde (2019) emphasize that identification of BLP demand models requires instruments for not only endogenous prices but also endogenous market shares. Other studies that discuss the role of instruments in BLP models include Reynaert and Verboven (2014), and Conlon and Gortmaker (2020). I contribute to this literature by providing low-level conditions on instruments for identification of random coefficients in the standard BLP model, both with and without identifying market size.

Recent work generalizes the parametric demand models to more flexible nonparametric, nonseparable demand systems. Nonparametric identification of aggregate demand models is studied by Berry and Haile (2014), Gandhi and Houde (2019), Lu, Shi, and Tao (2021), and Dunker, Hoderlein, and Kaido (2022), among others. This paper also provides conditions

8. Petrin and Train (2010)'s control function approach is an alternative to the BLP approach in dealing with the price endogeneity; which method to use will be application-specific. This discussion is outside the scope of the present paper.

for identification of market size in nonparametric specified demand models.

# 3 Simple Identification in Multinomial Logit Demand Model

I begin with a simple special case of our general results, by showing identification of market size in plain logit and nested logit demand models without random coefficients. Throughout this section, I assume exogenous prices to simplify the exposition. The results in this section are not as general as the main identification theorem, but they provide demonstration of how market size is identified from aggregate data and can be applied in empirical contexts with substitution patterns that can reasonably be characterized by a parsimonious demand model.

## 3.1 Demand Model

Suppose that we observe $T$ independent markets. A market $t$ can refer to a single region in a single time period. Let $\mathcal{J}_t = (1, \cdots, J_t)$ be the set of differentiated products in market $t$, referred to as inside goods. Let $j = 0$ denote the outside option. As in Berry (1994), I assume the indirect utility of consumer $i$ for product $j$ in market $t$ is characterized by a linear index structure

$$U_{ijt} = X_{jt}'\beta + \xi_{jt} + \varepsilon_{ijt},$$

which depends on a vector of observed product characteristics $X_{jt} \in \mathbb{R}^L$, unobserved characteristics $\xi_{jt}$, and idiosyncratic tastes of consumers $\varepsilon_{ijt}$. Consumer tastes are assumed to be independently and identically distributed across consumers and products, with extreme value type I distribution. Let the average utility index of product $j$ at market $t$ be denoted as $\delta_{jt} = X_{jt}'\beta + \xi_{jt}$, with the mean utility for the outside option being normalized as $\delta_{0t} = 0$.

Let $\pi_{jt}$ denote the true conditional probability of choosing product $j$ in market $t$. Each consumer chooses the product that gives rise to the highest utility. The probability of choosing good $j$ is obtained by integrating out over the distribution of consumer tastes $\varepsilon_{ijt}$. Given the functional form and parametric assumptions, the true choice probability takes an analytic form:

$$\pi_{jt} = \frac{exp(\delta_{jt})}{1 + \sum_{k=1}^{J_t} exp(\delta_{kt})} \quad \forall j \in \mathcal{J}_t, \quad \text{and } \pi_{0t} = \frac{1}{1 + \sum_{k=1}^{J_t} exp(\delta_{kt})}.$$

In a plain logit context, the nonlinear demand system can be inverted to solve for $\delta_{jt}$ as a function of choice probabilities, yielding

$$\ln(\pi_{jt}/\pi_{0t}) = X_{jt}'\beta + \xi_{jt} \quad \forall j \in \mathcal{J}_t. \tag{1}$$

If the value of $\pi_{jt}$ and $\pi_{0t}$ were observed, parameters $\beta$ can be consistently estimated by regressing $\ln(\pi_{jt}/\pi_{0t})$ on $X_{jt}$. GMM estimators can be constructed based on mean independence conditions $E(\xi_{jt} \mid X_{jt}) = 0$. In the case with endogenous product characteristics, one can use excluded instruments along with exogenous characteristics to address endogeneity. The conditions imposed so far are standard assumptions from Berry (1994) and the empirical IO literature, which are sufficient to identify the demand parameters $\beta$ when the market size is correctly measured and therefore $\pi_{jt}$ and $\pi_{0t}$ are observed without errors.

In Supplement C, I formally derive the bias in the estimated value of $\beta$ under the assumption that market size equals $\gamma M_t$ for an observable $M_t$, and mismeasurement corresponds to misspecifying the value of $\gamma$. For instance, when the market size used in estimation is larger than the true size, the estimated price coefficient will be biased downward (in absolute value), resulting in an underestimation of price sensitivity.

## 3.2 Market Size Model

In this subsection I provide modeling assumptions for unobserved $\pi_{jt}$ and $\pi_{0t}$. These assumptions allow us to characterize the connection between unobserved probabilities and measures of market size. I then combine these assumptions with the demand system to obtain a new model which I will later prove identification.

Define $r^*_{jt}$ by

$$r^*_{jt} = \frac{\pi_{jt}}{\sum_{k=1}^{J_t} \pi_{kt}} \quad \forall j \in \mathcal{J}_t, \tag{2}$$

which is the true conditional choice probability of choosing product $j$, conditional on purchasing any inside goods. Using equations (1) and (2), we have

$$\ln\left(r^*_{jt}\right) = \ln\left(\frac{\pi_{0t}}{1-\pi_{0t}}\right) + X'_{jt}\beta + \xi_{jt} \quad \forall j \in \mathcal{J}_t. \tag{3}$$

Let $Q_{jt}$ be the observed sales of good $j$ in market $t$, and let $Q^{total}_t = \sum_{j=1}^{J_t} Q_{jt}$ denote the total observed sales of all goods. We observe $r_{jt}$,[9] where $r_{jt} = Q_{jt}/Q^{total}_t$ represents the fraction of total purchases spent on good $j$ in market $t$, and therefore does not depend on the outside option or the size of the total market. I call these $r_{jt}$ *relative shares*, and assume $r_{jt} = r^*_{jt}$. In Supplement D, I relax this assumption and allow the true $r^*_{jt}$ to be unobservable, introducing sampling errors or measurement errors in $r_{jt}$.

The issue with not observing market size is not observing $\pi_{0t}$ in equation (3). If the total market size were directly observed, $\pi_{0t}$ can be calculated from the observed $Q^{total}_t$ and the market size. However, observing only the relative shares $r_{jt}$ for all $J_t$ goods does not provide sufficient information to determine $\pi_{0t}$. Therefore, we need to specify a model for the unobserved outside share. Compared to equation (1), the model of equation (3) offers the advantage that only the first term on the right side depends on the outside share, and thus it is easier and more natural to impose assumptions on this additively separable term.

Let $M_t$ be some observed population or quantity measure of market $t$ that we believe is related to the true market size.[10] Assume that the true market size is $\gamma M_t$, and hence

9. In most empirical contexts, we might directly observe $Q_{jt}$. For example, the number of passengers on flights by airline $j$ in city pair $t$, or servings of cereals of brand $j$ sold in city $t$. From these observed $Q_{jt}$ we can calculate $r_{jt}$ and $Q^{total}_t$. In other applications, $r_{jt}$ and $Q^{total}_t$ might come from separate sources. For instance, $r_{jt}$ could be the fraction of a set of sampled consumers who buy product $j$ in time period $t$, and $Q^{total}_t$ could be separate estimates of total sales in time $t$.

10. For instance, if a market is defined to be a city, $M_t$ could be the population size (e.g. Nevo 2001; Berto Villas-Boas 2007; Rysman 2004; Ho, Ho, and Mortimer 2012; and Ghose, Ipeirotis, and Li 2012). Alternatively, $M_t$ could be a prediction of total product sales or the number of passengers on a flight (e.g. Sweeting, Roberts, and Gedge 2020; Li et al. 2022; and Backus, Conlon, and Sinkinson 2021).

$1 - \pi_{0t} = Q_t^{total}/\gamma M_t$. Let $W_t = M_t/Q_t^{total}$ denote *observed market to sales.* Given the assumption on $\pi_{0t}$, we have

$$\ln\left(\frac{\pi_{0t}}{1-\pi_{0t}}\right) = \ln\left(\gamma W_t - 1\right) \tag{4}$$

for some unknown constant $\gamma$. In Supplement D, I relax equation (4) by introducing a random error term $v_t$, so that this relationship is approximate rather than exact. In section 4, I further generalize the model by allowing $\pi_{0t}$ to depend on multiple $\gamma$'s.[11]

Putting the above equations and assumptions together we get the estimating equation

$$\ln\left(r_{jt}\right) = \ln\left(\gamma W_t - 1\right) + X'_{jt}\beta + \xi_{jt} \quad \forall j \in \mathcal{J}_t. \tag{5}$$

Unknown parameters in model (5) include the market size parameter $\gamma$ and demand coefficients $\beta$. Note, that $\beta$ can be identified from a market fixed effects regression without identifying $\gamma$. In Supplement E, I present the formal identification of a model with market fixed effects $\kappa_t$. However, as I show in section 4.5, market fixed effects remove bias only in the simple logit or nested logit models, not in the more general random coefficients logit model, and the latter is the main focus of this paper.

Before moving forward, it is important to clarify the concept of market size and highlight some caveats. Interpreting market size is inherently challenging because the definition of the market and the outside good have been less clear in aggregate logit demand models. We follow the statement in Gandhi and Nevo (2021): a market $t$ is implicitly defined by consumers who face the same prices, product attributes, and demand shocks. Market size is a measure of this set of consumers. Given the logit form, one way to interpret market size is (as is in standard BLP papers), for example: "Imagine a scenario where soft drinks were free, available in endless varieties, and universally healthy—how many people would opt to buy them?" In our model, we assume that the aforementioned concept of market

11. An alternative approach to relaxing this modeling assumption is to consider $\gamma$ as a function of observed market-level covariates that affect preferences. I leave this possibility for future research.

size is measurable by some observable variable $M_t$ in the real world (see Supplement H for a micro economic foundation motivating the model). Consequently, whether $\gamma M_t$ serves a good proxy for the true market size becomes a specification issue. In other words, the interpretation of market size would also depend on the choice of variable(s) $M_t$.

Another caveat is that in a counterfactual scenario, market size may change, which becomes problematic if this change is not fully captured by $M_t$ and if the parameter $\gamma$ also changes. For example, factors omitted in the utility function such as advertising or recent experiences could lead people who previously did not consider buying a product (even if it were free) to start doing so. In such cases, our model is likely unable to capture this change in the counterfactual.

## 3.3 Identification

**Assumption 1.** $E\left(\xi_{jt} \mid \tilde{Z}_t, X_{1t}, \ldots, X_{J_t t}\right) = 0$, *where* $\tilde{Z}_t$ *represents instruments for* $W_t$. $W_t$ *is continuously distributed. The number of markets* $T \to \infty$.

Assumption 1 assumes that the additive error $\xi_{jt}$ is mean independent of product characteristics and some instrument $\tilde{Z}_t$, and that the regressor have a continuous distribution. Note that the nonlinear variable $W_t$ in equation (5) is endogenous since it is a function of quantities. The instrument $\tilde{Z}_t$ can take the form of a vector or a scalar. For the sake of convenience, Theorem 1 employs a scalar $\tilde{Z}_t$. The large $T$ assumption is necessary as the theorem is based on a conditional expectation conditioning on $\tilde{Z}_t$, and the derivatives of the conditional expectation. These derivatives would be estimated using nonparametric regression techniques such as kernel regression or local polynomials (Li and Racine 2007).

**Theorem 1.** *Given Assumption 1 and equation (5), let* $\Gamma$ *be the set of all possible values of* $\gamma$, *if*

1. *function* $f(c, z, x)$ *is twice differentiable in* $(c, z)$ *for every* $x \in supp(X_{jt})$, *where*

$$f(c, z, x) = E\left(\ln(r_{jt}) - \ln(cW_t - 1) \mid \tilde{Z}_t = z, X_{jt} = x\right),$$

*2. and* $\partial E\left(-\frac{W_t}{cW_t-1} \mid \tilde{Z}_t = z, X_{jt} = x\right)/\partial z > 0$ *or* $< 0$ *for all* $c \in \Gamma$,[12]

*then* $\gamma$ *and* $\beta$ *are identified.*

The proof of Theorem 1, provided in the Appendix, works by showing that there exists $z$ and $x$ such that $g(c, z, x) = 0$ has a unique solution $c$, where $g(c, z, x) = \partial f(c, z, x)/\partial z$.[13] Generally, the second condition in Theorem 1 is a nonlinear analog of the traditional relevance restriction required in the classical linear IV model, requiring $W_t$ to vary with $\tilde{Z}_t$ in a certain way. One key insight is that the instrument works by shifting the quantity rather than $M_t$. To see this, note that scaling $M_t$ proportionately scales $Q_t^{total}$, since by definition $Q_t^{total} = \sum_{j=1}^{J} \pi_{jt}\gamma M_t$. Recall that $W_t = M_t/Q_t^{total}$, which can be rewritten as $1/(\gamma \sum_{j=1}^{J} \pi_{jt})$. Fundamentally, to satisfy the relevance condition, valid $\tilde{Z}_t$ should shift the choice probabilities and thus we refer to them as *exogenous quantity shifters*, which will be discussed in detail in section 4.2.2.

Estimation of the model of equation (5) based on Theorem 1 is straightforward. It could be done by a standard GMM estimation or nonlinear two-stage least squares estimation using $\tilde{Z}_t$ as instruments.

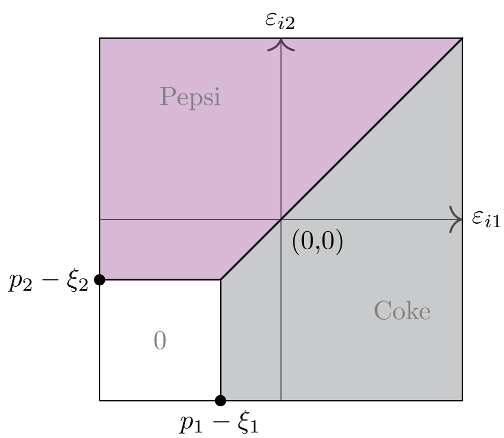
### 3.3.1 Visual Intuition

Figure 1 offers intuition for the identification result. In a simplified model where $\delta_{jt} = -p_{jt} + \xi_{jt}$, with two goods ($j = 1$ Coke and $j = 2$ Pepsi), the space of $\varepsilon_{ij}$ is partitioned into three regions, each corresponding to the choice of $j = 0, 1, 2$ (Berry and Haile 2014 and Thompson 1989). The measure of consumers in each region, i.e. integral of $\varepsilon$ over the region, reflects choice probabilities. For example, $Pr(j = 1 \mid p, \xi) = Pr(\varepsilon_{i1} > p_1 - \xi_1; \varepsilon_{i1} > \varepsilon_{i2} + (p_1 - \xi_1) - (p_2 - \xi_2))$.

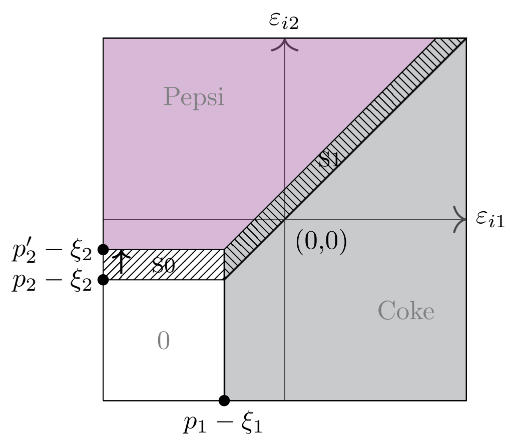
12. For a binary instrument $Q$, we can replace the derivative $\partial E\left(\frac{W_t}{cW_t-1} \mid \tilde{Z}_t = z, X_{jt} = x\right)/\partial z$ with $E\left(\frac{W_t}{cW_t-1} \mid \tilde{Z}_t = 1\right) - E\left(\frac{W_t}{cW_t-1} \mid \tilde{Z}_t = 0\right)$.

13. The identification result in Theorem 1 is one way to achieve identification. Alternatively, in the absence of external instruments $\tilde{Z}_t$, identification can be achieved using nonlinear functions of $X_{jt}$ (e.g. $X_{jt}^2$) as instruments. Accordingly, condition 2 in Theorem 1 can be replaced by requiring that the second derivative with respect to $x$ be either strictly positive or negative.

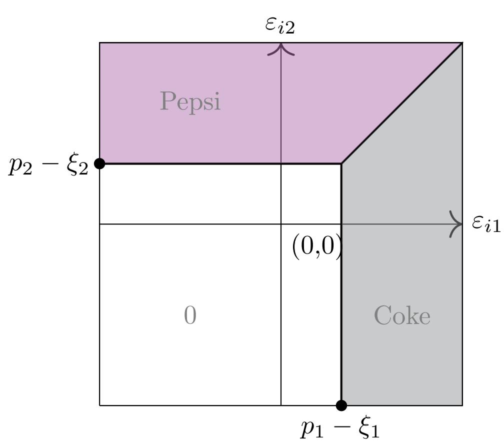
Panels (a) and (b) of Figure 1 depict a dgp where the true $\pi_{0t}$ is small. Panels (c) and (d) show similar graphs but with large true $\pi_{0t}$. Suppose there is an exogenous positive tax shock to good 2, which increases its price. The resulting changes in choice probabilities $\pi_{0t}$ and $\pi_{1t}$ are captured by the shaded boundaries $S_0$ and $S_1$. In panel (b), the price increase prompts more consumers to switch to good 1, while in panel (d), the same price change leads to more consumers switching to the outside option. The relative diversion to the outside option compared to good 1, which is known, relies on the original sizes of each region, which is unknown, and this relationship provides identification of the underlying market size.



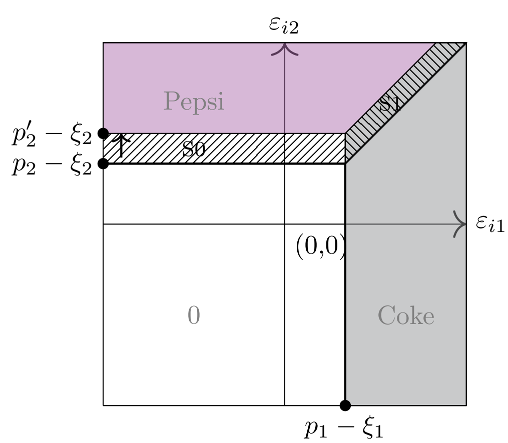

(a) small $\pi_{0t}$




(b) small $\pi_{0t}$, when $p_2 \uparrow$ by $\Delta p$




(c) big $\pi_{0t}$




(d) big $\pi_{0t}$, when $p_2 \uparrow$ by $\Delta p$

Figure 1: Intuition for Identification in Multinomial Logit Demand Model

In this example, the tax shock to good 2 acts as the instrument. It shifts the demand for good 1 and the outside good. Since the degree of substitution to the outside good depends on the true market shares, relative changes in quantities of inside versus outside goods can be exploited to recover the true market size. Note that $M_t$ or functions of $M_t$ cannot serve as instruments: any changes in $M_t$ do not affect these choice probabilities, so the regions shown in panels (a) and (c) remain unchanged and do not generate the variation observed in panels (b) and (d).

### 3.4 The Nested Logit Demand Model

In Supplement F, I establish formal identification of market size in a nested logit demand model. Here I briefly summarize the intuition. Consider the case where all goods are divided up into two nests, one with the outside good as the only choice and the other containing all inside goods. Using our notation, the estimating equation is a nonlinear function of the market size parameter $\gamma$ and the nesting parameter $\rho$, $\ln(r_{jt}) = \frac{1}{1-\rho}\ln(\gamma W_t - 1) + X'_{jt}\frac{\beta}{1-\rho} + \frac{\xi_{jt}}{1-\rho}$. The total derivative with respect to these two parameters has independent variation.

We leverage instruments that shift $W_t$ and the nonlinearity of the estimating equation to separately identify $\gamma$ and $\rho$. When $\tilde{Z}_t$ is a scalar random variable, we can use $\tilde{Z}_t$ and any nonlinear function of $\tilde{Z}_t$ as instruments to estimate $\gamma$ and $\rho$. Nonlinear functions of $\tilde{Z}_t$ (e.g. $\sqrt{\tilde{Z}_t}$ or $\tilde{Z}_t^2$) will have additional explanatory power to separately identify $\gamma$ and $\rho$.[14]

## 4 General Identification in Random Coefficients Logit Demand Model

This section generalizes previous results to the random coefficients demand model. I start by introducing the notation and model assumptions, followed by presenting sufficient conditions for model identification and suggesting potential instruments. The main results assume a

14. In practice, separately identifying the two parameters can be challenging without strong instrument. Observe that if $\gamma W_t - 1$ were close to zero or if the logarithm were not in the equation, $\rho$ tends to be not identified. We can also see this from a first order Taylor expansion around $W_t = \overline{W}$ (White 1980), where $\overline{W}$ is the mean of $W_t$. The coefficient of the Taylor series depends on both $\gamma$ and $\rho$. This result partly confirms the commonly held intuition that a nest structure can mitigate biases caused by unknown market size.

polynomial specification for market size in market-level covariates and are then extended to settings where market size is not fully characterized by observables. A test for instrument relevance is included in Supplement G. Next, I provide a numerical illustration to offer intuition for separately identifying market size and random coefficients. Additionally, I derive results for market fixed effects and demonstrate that market size remains identified even after conditioning on market-level dummies.

## 4.1 Demand Model and Market Size

The utility of consumer $i$ for product $j$ in market $t$ is now given by

$$U_{ijt} \quad = \quad X_{jt}'\beta_i + \xi_{jt} + \varepsilon_{ijt}, \tag{6}$$

where $\beta_i = (\beta_{i1}, \cdots, \beta_{iL})$. The individual-specific taste parameter for the $l$-th characteristics can be decomposed into a mean level term $\beta_l$ and a deviation from the mean $\sigma_l \nu_{il}$: $\beta_{il} = \beta_l + \sigma_l \nu_{il}$, with $\nu_i \sim f_\nu(\nu)$. $\nu_{il}$ captures consumer characteristics, which could be either observed individual characteristics or unobserved characteristics. When estimating demand models with aggregate data, observed individual characteristics are typically unavailable. Therefore, $\nu_{il}$ in the current analysis is assumed to be unobserved characteristics with a known distribution $f_\nu$. Extending the model to include observed consumer characteristics would be straightforward if individual-level data were available.

Let $\delta_{jt}$ denote the mean utility $X_{jt}'\beta + \xi_{jt}$. Combining equations we have $U_{ijt} = \delta_{jt} + \sum_l \sigma_l x_{jtl}^{(2)} \nu_{il} + \varepsilon_{ijt}$, where $X_{jt}^{(2)} = (x_{jt1}^{(2)}, \cdots, x_{jt\tilde{L}}^{(2)})$ is a $\tilde{L} \times 1$ subvector of $X_{jt}$ that has random coefficients and is the nonlinear components of the indirect utility function. Note that we can include market-level dummies in the mean utility, and $X_{jt}^{(2)}$ may contain a constant term. Therefore, the results in this section do not exclude the possibility of market fixed effects or a random coefficient on the intercept.

After integrating out over the logit error $\varepsilon_{ijt}$, the true aggregate choice probability is

$$\pi_{jt}\left(\delta_t, X_t^{(2)}; \sigma\right) = \int \frac{exp(\delta_{jt} + \sum_l \sigma_l x_{jtl}^{(2)} \nu_l)}{1 + \sum_{k=1}^{J_t} exp(\delta_{kt} + \sum_l \sigma_l x_{ktl}^{(2)} \nu_l)} f_\nu(\nu) d\nu, \tag{7}$$

where the arguments in the choice probability function are mean utilities $\delta_t = (\delta_{1t}, \cdots, \delta_{J_t t})$, nonlinear attributes $X_t^{(2)} = (X_{1t}^{(2)}, \cdots, X_{J_t t}^{(2)})$ and taste parameters $\sigma = (\sigma_1, \cdots, \sigma_{\tilde{L}})$. The choice probability is written as a function of $\delta_t$, $X_t^{(2)}$ and $\sigma$ in order to highlight its dependence on the mean utilities, nonlinear attributes, and parameters of the model. I suppress the dependence of the choice probability function on $\nu$ for brevity. The mean utility of outside good is normalized to $\delta_{0t} = 0$.

Let $M_t = (M_{1t}, \cdots, M_{Kt})$ be a vector of measures of the market size, and $\gamma = (\gamma_1, \gamma_2)$, $\gamma_1 = (\gamma_{11}, \cdots, \gamma_{K1})$ and $\gamma_2 = (\gamma_{12}, \cdots, \gamma_{K2})$ are two vectors of market size parameters. Recall that $Q_{jt}$ is the observed sales of each good and $Q_t^{total}$ the total sales of all inside goods. Assumption 2 incorporates a general model of market size into the demand system. In section 4.3, I relax the specification to allow for error in how the market size model (the function of $M_t$) matches the actual market size.

**Assumption 2.** $\Gamma = \{(\gamma_1, \gamma_2) \mid \sum_k \gamma_{k1} M_{kt}^{\gamma_{k2}} > Q_t^{total}$ *for all* $t \in 1 \cdots T\}$ *is the set of all possible values of* $\gamma$. *The implicit system of demand equations in a given market* $t$ *is given by*

$$\frac{Q_t}{\sum_{k=1}^{K} \gamma_{k1} M_{kt}^{\gamma_{k2}}} = \pi_t\left(\delta_t, X_t^{(2)}; \sigma\right), \tag{8}$$

*where* $Q_t = (Q_{1t}, \cdots, Q_{J_t t})$ *and* $\pi_t(\cdot) = (\pi_{1t}(\cdot), \cdots, \pi_{J_t t}(\cdot))$ *represent vectors of observed quantities and choice probability functions.*

The market size formula $\sum \gamma_{k1} M_{kt}^{\gamma_{k2}}$ has several appealing features. Taking the airline market as an example, suppose $M_{1t}$ is the population of city A (a small market) and $M_{2t}$ is the population of city B (a big market). The true size of a market defined by these two end-point cities could be $M_{1t}^2 + 3M_{2t}^2$. First, this formula allows for different coefficients for each term. For instance, city B might have a larger coefficient due to being a major transportation hub. Second, it accommodates nonlinearity in $M_t$. In the airline example, larger metropolitan areas are more likely to have alternative transportation options, such as high-speed rail or highways in multiple directions. In addition, this polynomial function nests the specification $\gamma_{11} + \gamma_{21} M_t$ by letting $\gamma_{12} = 0$ and $\gamma_{22} = 1$. So, not only the slope but also the "level" is identified. In section 5, we generalize the functional form $\sum \gamma_{k1} M_{kt}^{\gamma_{k2}}$ to any known function $s(M_t; \gamma)$ described by the vector $\gamma$.

## 4.2 Identification

In a standard BLP model, the link between the choice probability $\pi_{jt}(\delta_t, X_t^{(2)}; \sigma)$ predicted by the model and the observed market shares is crucial. The key to identification and estimation in a standard BLP model is to recover the mean utility $\delta_t$ as a function of the observed variables and parameters, by the *inversion* of the demand equation system. This paper builds on the same form of demand inversion while replacing observed market shares with the unobserved ones.

The identification argument can be summarized into two parts: First, I show that for any given parameters $(\gamma, \sigma)$ and data $(Q_t, M_t, X_{jt})$, the implicit system of equations (8) has a unique solution $\delta_t$ for each market. Lemma 1, adapted from Berry (1994) and Berry, Levinsohn, and Pakes (1995), establishes the existence and uniqueness of demand inversion (see also Berry and Haile (2014) for demand inversion in nonparametric models). Second, once we have a unique sequence of *inverse demand function* $\delta_{jt}(Q_t, M_t, X_t^{(2)}; \gamma, \sigma)$, we can construct a corresponding sequence of *residual function* $\xi_{jt}(Q_t, M_t, X_t; \gamma, \sigma, \beta)$, which will be defined later. Identification is then based on conditional moment restrictions, and we will require the moment conditions to local identify the true-parameter values.

**Lemma 1.** *Let equations (7) and Assumption (2) hold. Define the function* $g_t : \mathbb{R}^{J_t} \to \mathbb{R}^{J_t}$, *as* $g_t(\delta_t) = \delta_t + \ln(Q_t) - \ln(\sum_{k=1}^{K} \gamma_{k1} M_{kt}^{\gamma_{k2}}) - \ln(\pi_t(\delta_t, X_t^{(2)}; \sigma))$. *For any* $\gamma \in \Gamma$ *and* $\sigma \in \Sigma \subset \mathbb{R}^{\tilde{L}}$ *and any given* $(Q_t, M_t, X_t^{(2)})$, *there is a unique fixed point* $\delta_t(Q_t, M_t, X_t^{(2)}; \gamma, \sigma)$ *to the function* $g_t$ *in* $\mathbb{R}^{J_t}$.

This result is based on the contraction mapping argument in Berry, Levinsohn, and Pakes (1995). All conditions in the contraction mapping theorem continue to hold in our specific structure with the extra vector $\gamma$. The parameter space of $\gamma$ ensures that the total quantity is bounded above by the true market size and is necessary for the existence of a fixed point. Therefore, the function $g(\delta)$ is a contraction mapping.

Let $\theta = (\gamma, \sigma, \beta) \in \Theta$ be the full vector of model parameters of dimension $dim(\theta)$. Lemma 1 shows that there is a unique fixed point $\delta_t$ to the function $g_t(\delta_t)$. Given the inverse demand function $\delta_{jt}(Q_t, M_t, X_t^{(2)}; \gamma, \sigma)$, I define the residual function as

$$\xi_{jt}\left(Q_t, M_t, X_t; \theta\right) = \delta_{jt}\left(Q_t, M_t, X_t^{(2)}; \gamma, \sigma\right) - X_{jt}'\beta. \tag{9}$$

The uniqueness of $\delta_{jt}(Q_t, M_t, X_t^{(2)}; \gamma, \sigma)$ implies a unique sequence of $\xi_{jt}(Q_t, M_t, X_t; \theta)$. Following Berry, Levinsohn, and Pakes (1995), Berry and Haile (2014), and Gandhi and Houde (2019), I assume that the unobserved structural error term is mean independent of a set of exogenous instruments $Z_t$, based off which we can construct unconditional moment conditions.[15]

**Assumption 3.** *Let $Z_t = (Z_{1t}, \cdots, Z_{Jt})$. The unobserved product-specific quality is mean independent of a vector of instruments $Z_t$, so $E\left(\xi_{jt}(Q_t, M_t, X_t; \theta_0) \mid Z_t\right) = 0$.*

*Define $h_{jt}(\theta) = \xi_{jt}(Q_t, M_t, X_t; \theta)\phi_{jt}(Z_t)$, where $\phi_{jt}(Z_t)$ is a $m \times 1$ vector function of the instruments with $m \geq dim(\theta)$. Then the conditional moment restriction implies $E\left(h_{jt}(\theta_0)\right) = 0$.*

The instrument vector $Z_t$ typically includes a subvector of $X_t$ that contains exogenous characteristics and excluded price instruments such as cost shifters. The assumption posits that the structural error is mean independent not only of the exogenous covariates of product $j$ but also of all other products. I discuss the choice of instruments in detail in section 4.2.2.

15. The choice of unconditional moment restrictions affects whether the selected moments preserve global identification from the conditional moments in nonlinear models (Domínguez and Lobato 2004). In this paper, I derive identification results directly for unconditional moments and focus on local identification, thereby abstracting from the issue considered in Domínguez and Lobato (2004).

**Definition 1.** *$\theta_0$ is locally identified if and only if there exists an open neighborhood of $\theta_0$ in which the equations $E\left(h_{jt}(\theta)\right) = 0$ have a unique solution at $\theta = \theta_0$. In other words,*

$$E\left(h_{jt}(\tilde{\theta})\right) = 0 \iff \tilde{\theta} = \theta_0, \tag{10}$$

*for $\tilde{\theta}$ in an open neighborhood of $\theta_0$.*

I formally define *local identification* in Definition 1. Assumption 4 in Berry and Haile (2014) and equation (5) in Gandhi and Houde (2019) impose high-level identification assumptions similar to (10). Theorem 5.1.1 in Hsiao (1983) (in line with Fisher 1966 and Rothenberg 1971) provides sufficient rank conditions for the identification assumption stated above to hold locally, which I summarize in Lemma 2.[16]

**Lemma 2** (Theorem 5.1.1 in Hsiao 1983)**.** *If $\theta_0$ is a regular point,*[17] *a necessary and sufficient condition that $\theta_0$ be a locally isolated solution is that the $m \times dim(\theta)$ Jacobian matrix formed by taking partial derivatives of $E\left(h_{jt}(\theta)\right)$ with respect to $\theta$, $\nabla_\theta E\left(h_{jt}(\theta)\right)$ has rank $dim(\theta)$ at $\theta_0$.*

Using Lemma 2, I can now establish an identification theorem for the random coefficients demand model with an unobserved market size.[18]

**Theorem 2.** *Assume that $E\|h_{jt}(\theta)\| < \infty$ for all $\theta \in \Theta$ and is continuously differentiable with respect to $\theta$ with probability 1, and $\nabla_\theta h_{jt}(\theta)$ is dominated by a random variable $G_{jt}$*

16. Because of the nonlinear structure of BLP, identification results based on rank conditions hold only locally. Primitive conditions that guarantee global identification in such settings are generally restrictive; accordingly, I focus on local identification and treat global uniqueness of the population moment root as an additional high-level assumption when needed. Rothenberg (1971) provides sufficient conditions for global identification, which require that the symmetric part of the Jacobian be positive semidefinite and that its determinant be strictly positive for all $\theta \in \Theta$. Unfortunately, these conditions are difficult to be verified explicitly in our setting.

17. $\theta_0$ is a regular point if and only if for all $\theta$ in some small neighborhood of $\theta_0$, the Jacobian matrices have constant rank.

18. The application of full rank conditions for achieving local identification is seen in various studies, including McConnell and Phipps (1987), Iskrev (2010), Qu and Tkachenko (2012), Milunovich and Yang (2013), and Gospodinov and Ng (2015).

*($\|h_{jt}(\theta)\| \leq G_{jt}$ for all $\theta$) such that $E(G_{jt}) < \infty$. Under Assumptions 2 and 3, if the rank of*

$$E\left[\phi_{jt}(Z_t)\frac{\partial \delta_{jt}(Q_t, M_t, X_t^{(2)}; \gamma, \sigma)}{\partial \gamma'} \quad \phi_{jt}(Z_t)\frac{\partial \delta_{jt}(Q_t, M_t, X_t^{(2)}; \gamma, \sigma)}{\partial \sigma'} \quad -\phi_{jt}(Z_t)X_{jt}'\right]$$

*is $dim(\theta)$ at $\theta_0$, then $\theta$ is locally identified.*

The identification proof follows directly from Lemma 2 and the rank condition that the Jacobian matrix has full column rank. Standard BLP models require a rank condition similar to the one stated in Theorem 2, but not the same because it does not have the extra $\gamma$ rows and columns in the Jacobian matrix. These moments depend on the inverse demand function, which lacks a closed-form expression, making it challenging to directly verify full column rank. However, I show that the full rank condition is generally satisfied due to the high nonlinearity of the demand system. The rank condition is testable using the test of the null of underidentification proposed by Wright (2003).

### 4.2.1 Sufficient Conditions for Identification

This section provides the theoretical foundation for the discussion of instruments in the next section. I replace the high-level rank condition with some low-level conditions on instruments. The identification theorem imposes an assumption regarding the rank of the Jacobian matrix. To verify this condition and to clarify the role of instruments, I explicitly compute the derivatives of $h_{jt}(\theta)$. The Jacobian matrix encompasses four sets of derivatives: derivatives with respect to $\gamma_1$, $\gamma_2$, $\sigma$ and $\beta$, respectively. By utilizing the implicit function theorem for a system of equations (Sydsæter et al. 2008) and applying the Cramer's rule, the first two sets of derivatives can be explicitly computed as

$$
\begin{aligned}
J_1 &= \frac{\partial h_{jt}(\theta)}{\partial \gamma_{k1}} = \begin{vmatrix} \frac{\partial \pi_{1t}}{\partial \delta_{1t}} & \cdots & \frac{\partial \pi_{1t}}{\partial \delta_{Jt}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \pi_{Jt}}{\partial \delta_{1t}} & \cdots & \frac{\partial \pi_{Jt}}{\partial \delta_{Jt}} \end{vmatrix}^{-1} \underbrace{\begin{vmatrix} \frac{\partial \pi_{1t}}{\partial \delta_{1t}} & \cdots & -\pi_{1t} & \cdots & \frac{\partial \pi_{1t}}{\partial \delta_{Jt}} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{\partial \pi_{Jt}}{\partial \delta_{1t}} & \cdots & -\pi_{Jt} & \cdots & \frac{\partial \pi_{Jt}}{\partial \delta_{Jt}} \end{vmatrix}}_{(\pi_{1t},\cdots,\pi_{Jt})' \text{ is in the } j\text{-th column}} \frac{M_{kt}^{\gamma_{k2}}}{\sum_k \gamma_{k1} M_{kt}^{\gamma_{k2}}} \phi_{jt}(Z_t), \\
&= \Psi_{jt}\left(\delta_t, X_t^{(2)}; \sigma\right) \frac{M_{kt}^{\gamma_{k2}}}{\sum_k \gamma_{k1} M_{kt}^{\gamma_{k2}}} \phi_{jt}(Z_t), \qquad (11)
\end{aligned}
$$

and

$$
J_2 = \frac{\partial h_{jt}(\theta)}{\partial \gamma_{k2}} = \Psi_{jt}\left(\delta_t, X_t^{(2)}; \sigma\right) \frac{\gamma_{k1} \ln(M_{kt}) M_{kt}^{\gamma_{k2}}}{\sum_k \gamma_{k1} M_{kt}^{\gamma_{k2}}} \phi_{jt}(Z_t) \qquad (12)
$$

where $J_1$ and $J_2$ are $m \times 1$ vectors, and $\Psi_{jt}(\delta_t, X_t^{(2)}; \sigma)$ denotes the product of the first two matrix determinants in equation (11). I emphasize its dependence on $\delta_t$ and $X_t^{(2)}$ because the partial derivatives of $\pi_{jt}$ with respect to $\delta_{jt}$ are functions of mean utilities and characteristics of all products. I provide the calculation of these partial derivatives in Supplement K. The Jacobian determinant of $(\pi_{1t}, \cdots, \pi_{Jt})'$ with respect to $(\delta_{1t}, \cdots, \delta_{Jt})$ is different from zero, so the condition of implicit function theorem is satisfied.[19]

**Remark 1.** *The Jacobian matrices show that identification fails in cases where $\gamma_{k1} = 0$ for some $k$ (so that the corresponding $\gamma_{k2}$ is not identified), or when $M_t$ is independent of $\phi_{jt}(Z_t)$ and all other components in the demand model. In the latter case, $E\left(\frac{\partial h_{jt}}{\partial \gamma_{k1}}\right) = cE\left(\frac{\partial h_{jt}}{\partial \gamma_{k2}}\right)$, for some non-zero constant $c$. This makes it impossible to separately identify $\gamma_{k1}$ and $\gamma_{k2}$, neither could we distinguish $\gamma_{k1}$ and $\gamma_{j1}$ for $j \neq k$.*

19. When market size takes a general form $s(M_t;\gamma)$, the column of Jacobian matrix corresponding to the first element of $\gamma$ is given by $J_1 = \Psi_{jt}\left(\delta_t, X_t^{(2)}; \sigma\right) \frac{\partial s(M_t;\gamma)}{\partial \gamma_1} \frac{1}{s(M_t;\gamma)} \phi_{jt}(Z_t)$.

The third group of derivatives is

$$
\begin{aligned}
J_3 &= \frac{\partial h_{jt}(\theta)}{\partial \sigma_l} = \begin{vmatrix} \frac{\partial \pi_{1t}}{\partial \delta_{1t}} & \cdots & \frac{\partial \pi_{1t}}{\partial \delta_{Jt}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \pi_{Jt}}{\partial \delta_{1t}} & \cdots & \frac{\partial \pi_{Jt}}{\partial \delta_{Jt}} \end{vmatrix}^{-1} \underbrace{\begin{vmatrix} \frac{\partial \pi_{1t}}{\partial \delta_{1t}} & \cdots & -\frac{\partial \pi_{1t}}{\partial \sigma_l} & \cdots & \frac{\partial \pi_{1t}}{\partial \delta_{Jt}} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{\partial \pi_{Jt}}{\partial \delta_{1t}} & \cdots & -\frac{\partial \pi_{Jt}}{\partial \sigma_l} & \cdots & \frac{\partial \pi_{Jt}}{\partial \delta_{Jt}} \end{vmatrix}}_{(-\partial \pi_{1t}/\partial \sigma_l, \cdots, -\partial \pi_{Jt}/\partial \sigma_l)' \text{ is in the } j\text{-th column}} \phi_{jt}(Z_t) \\
&= \Phi_{jtl}(\delta_t, X_t^{(2)}; \sigma)\phi_{jt}(Z_t), \qquad (13)
\end{aligned}
$$

where I let the product of the two determinants of $J_3$ be denoted as $\Phi_{jtl}(\delta_t, X_t^{(2)}; \sigma)$. Comparing $J_3$ with $J_1$ (or $J_2$), the first determinant term of $\Phi_{jtl}$ and $\Psi_{jt}$ are identical. The difference lies in the $j$-th column of the second determinant term, which is $\left(-\frac{\partial \pi_{1t}}{\partial \sigma_l}, \cdots, -\frac{\partial \pi_{Jt}}{\partial \sigma_l}\right)'$ for $J_3$, and $(-\pi_{1t}, \cdots, -\pi_{Jt})'$ for $J_1$ and $J_2$. Observe that the derivative $\frac{\partial \pi_{jt}}{\partial \sigma_l}$ and $\pi_{jt}$ are not perfectly collinear in general,[20] implying that $\Psi_{jt}$ is not perfect multicollinear with $\Phi_{jtl}$. The column vectors of the Jacobian matrix are therefore linearly independent as long as we have a sufficient number of instruments that are correlated with $\Psi_{jt}$ and $\Phi_{jtl}$, respectively. Proposition 1 formalizes this argument.

**Proposition 1.** *Let $\bar{\gamma} \in \mathbb{R}$ be a known constant. Suppose $K = 1$ and $\Gamma = \{\gamma_1 \mid \gamma_1 M_t^{\bar{\gamma}} > Q_t^{total}$ for all $t \in 1 \ldots T\} \times \{\bar{\gamma}\}$. Let $\phi_{jt}^{(1)}(Z_t)$ , $\phi_{jt}^{(2)}(Z_t)$ and $\phi_{jt}^{(3)}(Z_t)$ be subvectors of $\phi_{jt}(Z_t)$. The rank condition for identification given in Theorem 2 is satisfied if $E(\phi_{jt}^{(1)}(Z_t)X_{jt}')$ is non-singular, the support of $\phi_{jt}(Z_t)$ does not lie in a proper linear subspace of $\mathbb{R}^{dim(\theta)}$, the joint support of $\partial \pi_{jt}(\delta_t, X_t^{(2)}; \sigma_0)/\partial \sigma_l$ and $\pi_{jt}(\delta_t, X_t^{(2)}; \sigma_0)$ does not lie in a proper linear subspace of $\mathbb{R}^2$ for all $l$, and there are instruments that satisfy*

$$
Cov\left(\Psi_{jt}\left(\delta_t, X_t^{(2)}; \sigma_0\right), \phi_{jt}^{(2)}(Z_t)\right) \neq 0, \qquad (14)
$$

20. Specifically, for the $j$-th column of the above matrices, we have

$$
\pi_{jt}\left(\delta_t, X_t^{(2)}; \sigma\right) = \int \pi_{jti}\left(\delta_t, X_t^{(2)}; \sigma\right) f_\nu(\nu) d\nu \quad \text{for } J_1 \text{ (or } J_2\text{), and}
$$

$$
\frac{\partial \pi_{jt}\left(\delta_t, X_t^{(2)}; \sigma\right)}{\partial \sigma_l} = \int \pi_{jti}\left(\delta_t, X_t^{(2)}; \sigma\right)\left(x_{jtl}^{(2)} - \sum_{k=1}^{J} x_{ktl}^{(2)} \pi_{kti}\left(\delta_t, X_t^{(2)}; \sigma\right)\right) \nu_l f_\nu(\nu) d\nu \quad \text{for } J_3.
$$

*and*

$$Cov\left(\Phi_{jt}\left(\delta_t, X_t^{(2)}; \sigma_0\right), \phi_{jt}^{(3)}(Z_t)\right) \neq 0, \tag{15}$$

*where $\phi_{jt}^{(2)}(Z_t)$ is of dimension one, and $\phi_{jt}^{(3)}(Z_t)$ has the same dimension as $\sigma$.*

**Remark 2.** *As implied by the Jacobian matrix, there are two cases where identification becomes poor. The first occurs when the number of products $J$ in a market tends to infinity. In this case, $\Psi_{jt}$ and $\Phi_{jt}$ are close to identical relative to data variability. The second case arises when the choice probabilities for all inside goods approach zero – that is, when the choice probability of the outside option approaches one – causing the Jacobian column corresponding to $\gamma$ to be close to zero. I illustrate this poor identification case in an Monte Carlo experiment.*

### 4.2.2 Instruments

Here, I discuss intuition from (14) and (15) for guiding the choice of instruments. I begin by considering candidates of $Z_t$. Valid excluded instruments are random variables that satisfy the conditional moment restrictions and the relevance condition.

Note that $\Psi_{jt}$ and $\Phi_{jt}$ are functions of the choice probabilities $\pi_{jt}$ of all products and their derivatives. Exogenous variables that satisfy the relevance condition are those that either enter the choice probability function directly *or* affect it indirectly through prices. Such excluded observables could be exogenous characteristics of rival products, since the choice probability depends on the characteristics of all products. They may also be exogenous measures of market structure, such as the number of products $J$, since the denominator of the choice probability involves summation over $J$. In addition, $J$ directly affects the determinant of the matrices in $\Psi_{jt}$ and $\Phi_{jt}$ through their dimensions.

Other exogenous variables that do not show up in the choice probability function directly but shift choice probabilities by generating exogenous variation in prices also satisfy the relevance condition. Examples include cost shifters, Hausman instruments (prices of the same products in other markets), or a tax change. Traditionally, such price shifters serve as instrumental variables for identifying price coefficients (see Berry and Haile 2021 for a detailed discussion). A perhaps surprising but sensible insight is that exogenous price shifters of own and *rival products* are valid instruments for $\gamma$ and $\sigma$ as well, since choice probabilities are functions of prices of all products. For example, a tax on product $j$ has an impact on the demand for all other products in equilibrium. In this sense, price shifters not only serve as price instruments but also have identifying power for nonlinear parameters.

Collectively, all of the instruments discussed above can be broadly referred to as *exogenous quantity shifters*.

Based on the candidate exogenous variables $Z_t$, one can choose appropriate functions $\phi_{jt}(Z_t)$ to construct unconditional moments. We start with Chamberlain (1987)'s optimal IV to motivate the choice of $\phi_{jt}$. Let $\Omega(Z_t) = E\left(\xi(\theta_0)\xi(\theta_0)' \mid Z_t\right)$. Then, by Chamberlain (1987), the efficient choice is given by $\phi^*_{jt}(Z_t) = E\left(\frac{\partial \xi(\theta_0)}{\partial \theta} \mid Z_t\right)\Omega(Z_t)^{-1}$. Note that, in our context, $E\left(\frac{\partial \xi_{jt}(\theta_0)}{\partial \gamma} \mid Z_t\right) \propto E(\Psi_{jt} \mid Z_t)$ and $E\left(\frac{\partial \xi_{jt}(\theta_0)}{\partial \sigma} \mid Z_t\right) \propto E(\Phi_{jt} \mid Z_t)$.[21] Since $\Psi_{jt}$ and $\Phi_{jt}$ are complicated nonlinear functions, polynomials in $Z_t$ could be used to approximate the optimal instruments. Standard BLP instruments and the instruments proposed by Gandhi and Houde (2019) are both functions of product characteristics of own and competing products, differing mainly in their choice of basis functions.

As mentioned previously, separating $\gamma$ from $\sigma$ relies on $\Psi_{jt}$ and $\Phi_{jt}$ varying independently, and such independent variation is generated from unobserved preference heterogeneity. First, it is evident from the term $\frac{\partial \pi_{jt}}{\partial \delta_{jt}}$ in (11) and (13) that the demand responses to changes in mean utility provide identifying information for both $\gamma$ and $\sigma$. The key question, then, is what additional variation distinguishes the two parameters. As shown in (13), where $\frac{\partial \pi_{jt}}{\partial \sigma}$ shows up, and from its formula given in footnote 20, one source that helps pin down $\sigma$ is variation in nonlinear covariates $X^{(2)}_{jt}$. Specifically, the demand for products with higher (or lower) than average values of $X^{(2)}_{jt}$ typically responds more strongly to changes in the distribution of random coefficients. In addition, the interaction between $\pi_{jti}$ and $\nu_i$ appears in

21. Since the focus of this paper is on identification rather than the properties of estimators, we therefore refer readers to Reynaert and Verboven (2014) and Conlon and Gortmaker (2020) for approaches to approximating the optimal instruments.

$\frac{\partial \pi_{jt}}{\partial \sigma}$, capturing how individual-specific choice probabilities covary with draws of the random component $\nu_i$. As a result, cross-consumer variation in $\nu_i$ provides another source that makes identification of $\sigma$ possible. By contrast, as seen in (11), $\pi_{jt}$ shows up in place of $\frac{\partial \pi_{jt}}{\partial \sigma}$. Consequently, the sources of variation discussed above that are informative for identifying $\sigma$ would not provide identifying information for $\gamma$, which allows the two parameters to be separately identified.

More generally, when $K > 1$ and $\gamma$ is a vector, equations (11) and (12) imply that we need at least some exogenous observables $Z_t$ be correlated with $M_t$ and $\Psi_{jt}$. Accordingly, instruments can be constructed by interacting $M_t$ with the exogenous quantity shifters, or, when $M_t$ itself is endogenous, with other variables that shift $M_t$ exogenously. For example, if $M_t$ is population, such variables may include highway expansions in a city. Note that quantity shifters typically vary by both $j$ and $t$, and are therefore not perfectly collinear with the market fixed effects, making it possible to identify $\gamma$ in a model with fixed effects.

In section 4.4, a numerical example is provided to further illustrate why it is possible to distinguish between $\sigma$ and $\gamma$. More formally, one may apply the approach proposed by Andrews, Gentzkow, and Shapiro (2017) to evaluate the local sensitivity of parameter estimates to IV moments.

Weak IV issues are a common concern in highly nonlinear models like BLP (see Armstrong 2016 and Gandhi and Houde 2019). Our theoretical conditions on identifiability do not rule out weak or poor identification in practice. In Supplement G, I adapt an approach from Gandhi and Houde (2019) to test the relevance of the instruments for identifying the nonlinear parameters. Jointly identifying multiple nonlinear parameters can lead to imprecise estimates when strong instruments are not available, especially for empirical work that attempts to accommodate rich specifications with both market fixed effects and random coefficients on the intercept. Nonetheless, the identification analysis in this paper could be useful in facilitating the development of improved methods for addressing these estimation challenges.

**Remark 3.** *If we treat market size as a free, market-specific parameter, identification can be* *achieved under additional conditions on the instruments. I focus on the practically relevant case where we have market fixed effects in utility.*[22] *In order to describe how these parameters are separately identified, it is useful to introduce additional notation. Let* $\tau_t \equiv Q_t^{total} + Q_{0t}$ *denote the unknown market size parameter, and thus the demand equation in market* $t$ *is given by* $Q_{jt}/\tau_t = \pi_{jt}\left(\delta_t, X_t^{(2)}; \sigma\right)$. *Recall that* $\kappa_t$ *are market fixed effects entering utility. The market-specific parameter* $\tau_t$ *scales all products in market* $t$ *proportionally, while* $\kappa_t$ *affects choice probabilities through the mean utility* $\delta_t$. *The Jacobian matrix therefore contains extra columns for* $\tau = (\tau_1, \ldots, \tau_T)$ *and* $\kappa = (\kappa_2, \ldots, \kappa_T)$. *We can show that, for market* $k$,

$$\frac{\partial h_{jt}(\theta)}{\partial \tau_k} = \Psi_{jt}(\delta_t, X_t^{(2)}; \sigma)\frac{1}{\tau_t}\mathbb{1}\{t = k\}\phi_{jt}(Z_t),$$

*and*

$$\frac{\partial h_{jt}(\theta)}{\partial \kappa_k} = -\mathbb{1}\{t = k\}\phi_{jt}(Z_t),$$

*where* $\Psi_{jt}(\delta_t, X_t^{(2)}; \sigma)$ *is defined as in equation (11), and* $\mathbb{1}\{t = k\}$ *denotes market dummy variables. The derivative with respect to* $\kappa_t$ *indicates that identifying* $\kappa$ *requires a subvector of* $\phi_{jt}(Z_t)$ *that is correlated with* $\mathbb{1}\{t = k\}$, *whereas the derivative with respect to* $\tau_k$ *implies that identifying* $\tau$ *requires correlation with* $\Psi_{jt}\mathbb{1}\{t = k\}$. *The former is constant across products and does not depend on substitution patterns, while the latter is proportional to* $\Psi_{jt}$, *which depends on how shares respond to changes in mean utility and therefore varies across* $j$. *If products exhibit heterogeneous substitution patterns (i.e. when the distribution of random coefficient is non-degenerate), the vector of derivatives with respect to* $\tau_k$ *is generally not perfectly collinear with the vector of derivatives with respect to* $\kappa_k$.

*In practice, market dummies serve as their own instruments for identifying* $\kappa$. *To separately identify* $\tau$ *from* $\kappa$, *additional instruments are required, which can be formed by in-*

22. One may allow for a random coefficient on the constant term in utility. This parameter can be separately identified using an argument similar to that in footnote 20. The column of the Jacobian matrix corresponding to $\sigma_0$ is $\partial h_{jt}(\theta)/\partial\sigma_0 = \Phi_{jt0}(\delta_t, X_t^{(2)}; \sigma)\phi_{jt}(Z_t)$, with the $j$-th column of the second determinant term in $\Phi_{jt0}$ given by $\partial\pi_{jt}/\partial\sigma_0 = \int \pi_{jti}\pi_{0ti}\nu_0 f(\nu)d\nu$, while the relevant column in $\Psi_{jt}$ is given by $\int \pi_{jti} f(\nu)d\nu$. Separating the effect of $\sigma_0$ from $\tau$ relies on unobserved heterogeneity entering $\pi_{0ti}$ and the extra $\nu_0$ term in the integral.

*teracting exogenous quantity shifters with market dummies. Intuitively, when the size of the market varies, the response of $\Psi_{jt}$ to demand shifters differs across products. When interacted with the market dummy, these instruments generate moment variation that reflects how a common scale change (i.e. change in $\tau_t$) propagates differently across products depending on substitution, conditional on the market.*

*Several caveats are worth noting. First, identifying both market fixed effects and market-specific size parameters requires sufficient within-market cross-product variation. As a result, the approach requires a large set of instruments, growing proportionally with the number of markets. Second, the result relies on parametric structure in demand: as shown in the next section, since $\tau_t$ acts like a multiplicative factor, it is not identified when the function $\pi_{jt}$ is unknown. Finally, without random coefficients, $\Psi_{jt}$ collapses and looses cross-product variation, making $\tau$ and $\kappa$ indistinguishable. We chose our parsimonious specification in equation (8) for empirical tractability.*

### 4.3 Unobserved Errors in Market Size

It is possible that market size cannot be perfectly determined using covariates. In this section, we allow for an error term $v_t$ in how the model implied market size matches the true market size.[23] For example, in the airline demand model, even if the populations of the two cities were observed, the size of the airline market might still not be perfectly known. Here $v_t$ is interpreted as a random shock to the true size that is not captured by market-level covariates, and therefore is independent of other variables.

Consider the true market size given by

$$s(M_t; \gamma) + v_t,$$

where the function $s(M_t; \gamma)$ is known and $v_t$ represents market-specific unobserved error or other independent variation in the true market size that is not captured by observed

23. I thank one anonymous referee for this useful suggestion.

covariates.[24] We assume that $v_t$ is independent of the model components $(Q_t, M_t, X_t)$, and has a distribution $F(v;\lambda)$ and density function $f(v;\lambda)$ that is known up to a finite parameter vector $\lambda$.[25]

Define the residual function as

$$\xi_{jt}\left(Q_t, M_t, X_t, v_t; \theta\right) = \delta_{jt}\left(Q_t, M_t, X_t^{(2)}, v_t; \gamma, \sigma\right) - X_{jt}'\beta, \tag{16}$$

where $\delta_{jt}\left(Q_t, M_t, X_t^{(2)}, v_t; \gamma, \sigma\right)$ is the inverse demand function obtained from the demand system (8), with the true market size replaced by $s(M_t;\gamma)+v_t$, for any given $Q_t$, $M_t$, $X_t^{(2)}$ and $v_t$. Since $v_t$ is unobserved, it needs to be integrated out. The mean independence condition $E(\xi_{jt} \mid Z_t) = 0$ implies that $E(E(\xi_{jt} \mid Z_t, Q_t, M_t, X_t) \mid Z_t) = E(\xi_{jt} \mid Z_t) = 0$. That is, the conditional moment restriction is given by

$$\begin{aligned} 0 &= E\left(E\left(\xi_{jt} \mid Z_t, Q_t, M_t, X_t\right) \mid Z_t\right) \\ &= E\left(\int \delta_{jt}\left(Q_t, M_t, X_t^{(2)}, v; \gamma, \sigma\right) dF_{v|Z,N,M,X}(v;\lambda) - X_{jt}'\beta \mid Z_t\right) \\ &= E\left(\int \delta_{jt}\left(Q_t, M_t, X_t^{(2)}, v; \gamma, \sigma\right) dF_v(v;\lambda) - X_{jt}'\beta \mid Z_t\right), \end{aligned}$$

where the last equality follows from the independence assumption.

An extension to Theorem 2 is the following:

**Corollary 1.** *Suppose that the system of demand equations in a market $t$ is given by $\frac{Q_t}{s(M_t;\gamma)+v_t} = \pi_t\left(\delta_t, X_t^{(2)}; \sigma\right)$. Let $\theta = (\gamma, \lambda, \sigma, \beta) \in \Gamma \times \mathbb{R}^{dim(\lambda)} \times \mathbb{R}^{\tilde{L}} \times \mathbb{R}^{L}$, where $\Gamma = \{\gamma \mid s(M_t;\gamma) + v_t > Q_t^{total} \text{ for all } t \in 1, \ldots, T\}$ and $F(v;\lambda)$ is a proper distribution of $v_t$.*

24. Note that our strategy differs from a two-step procedure used in some empirical studies, in which researchers first estimate a separate regression to predict total demand, denoted by $\mathcal{M}_t$, and then scale it using an ad-hoc assumed factor. Such a two-step approach is equivalent to assuming $\mathcal{M}_t = s(M_t;\gamma) + v_t$ for some observed variable $M_t$, and then dealing with $E(\mathcal{M}_t \mid M_t)$. In that setting, $\mathcal{M}_t$ is typically taken to be, e.g., the observed total number of passengers in previous periods. However, this does not address the fundamental issue that the potential market size is unobserved. In our framework, $\mathcal{M}_t$ would represent the potential market size, and is therefore unobserved. We cannot estimate $\gamma$ by regressing $\mathcal{M}_t$ on $M_t$ and then plugging in $s(M_t;\hat{\gamma})$.

25. If the density function of $v_t$ were unknown, identifying the distribution of $v_t$ is likely possible by making distributional assumption on $\xi_{jt}$.

*Let $h_{jt}(\theta) \equiv E(\xi_{jt} \mid Z_t, Q_t, M_t, X_t)\phi_{jt}(Z_t)$. Assume that $E\|h_{jt}(\theta)\| < \infty$ for all $\theta \in \Theta$ and is continuously differentiable in $\theta$ with probability 1, and $\nabla_\theta h_{jt}(\theta)$ is dominated by a random variable $G_{jt}$ ($\|h_{jt}(\theta)\| \leq G_{jt}$ for all $\theta$) such that $E(G_{jt}) < \infty$. Assume further that the density $f(v;\lambda)$ is continuously differentiable in $\lambda$ for almost all $u$, and that the derivative $\nabla_\lambda f(v;\lambda)$ is bounded by an integrable function. Assume that $E(\xi_{jt} \mid Z_t) = 0$ and $v_t \perp (Z_t, Q_t, M_t, X_t)$, and that the matrix*

$$E\left[\phi_{jt}(Z_t)\begin{pmatrix}\dfrac{\partial \int \delta_{jt}(Q_t, M_t, X_t^{(2)}, v; \gamma, \sigma) dF_v(v;\lambda)}{\partial \begin{pmatrix}\gamma' & \lambda' & \sigma'\end{pmatrix}} & -X_{jt}'\end{pmatrix}\right]$$

*has full column rank at $\theta_0$. Then, $\theta$ is locally identified.*

With a more general specification of $s(M_t;\gamma)$ and the additional unobservable $v_t$, sufficient rank condition for identification requires extending Proposition 1. This extension builds on similar ideas but involves more cumbersome notation. We therefore focus on providing intuition for why identification is generally achievable. For example, when $s(M_t;\gamma) = \gamma M_t$, the column corresponding to $\gamma$ simplifies to $\Psi_{jt} \int -\frac{M_t}{\gamma M_t + v} dF(v;\lambda)$, while the column for $\lambda$ is $\int \delta_{jt}(Q_t, M_t, X_t^{(2)}, v; \gamma, \sigma) \frac{\partial f(v;\lambda)}{\partial \lambda} dv$. The latter can be interpreted as the expectation of the inverse demand function weighted by the score, where the expectation is taken with respect to the distribution of $v_t$. These columns are generally linearly independent due to the non-linear interaction between the observable $M_t$ and the integration over the density $f(v;\lambda)$, provided $Z_t$ contains instruments that shift $M_t$ and $\Psi_{jt}$ independently of the shocks $v_t$.

To estimate the parameters, one can nest simulations within the GMM algorithm. For a given value of the parameter vector $\lambda$, the integral over the inverse demand function can be numerically calculated by drawing values of $v$ from its known distribution and averaging the results to approximate the expectation.[26]

26. With the inclusion of an error term, one might be concerned that for some $t$, an extreme draw of $v_t$ could lead to $s(M_t;\gamma) + v_t < Q_t^{total}$, and thus a negative outside good share. This possibility is ruled out by restrictions on the parameter space of $\gamma$. In estimation, if one plug in a value of $\gamma \notin \Gamma$, the necessary condition for a contraction mapping would be violated. In this case, one can show that no fixed point exists and thus the inverse of the demand system cannot be obtained.

## 4.4 A Numerical Illustration

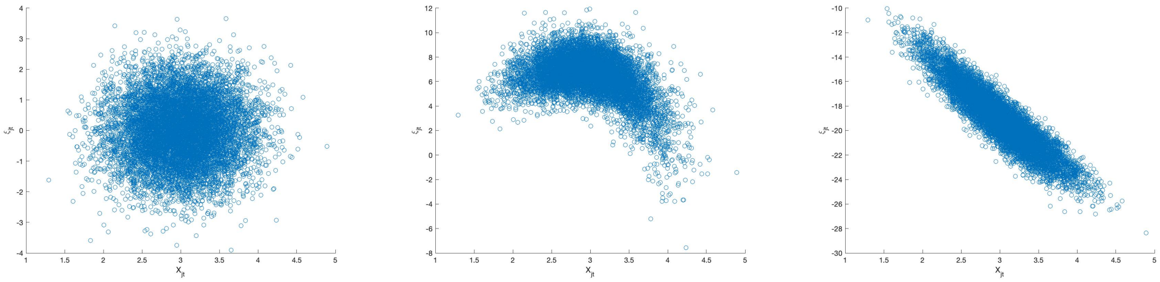
In this section, I provide a brief numerical example to visually illustrate the intuition for separately identifying market size and random coefficients.

Consider a model that has one random coefficient $\sigma$. The utility to consumer $i$ for product $j$ in market $t$ is $U_{ijt} = \sigma\nu_i X_{jt} + \xi_{jt} + \varepsilon_{ijt}$, and the market size is parameterized by a single scalar $\gamma$. Equation (8) can be written as $\frac{Q_{jt}}{\gamma M_t} = \int \frac{\exp(\xi_{jt}+\sigma\nu_i X_{jt})}{1+\sum_{k=1}^{J}(\xi_{kt}+\sigma\nu_i X_{kt})} f_\nu(\nu)d\nu$.

If we do not impose the conditional moment restrictions as in Assumption 3, $\gamma$ is not point identified. To see this, recognize that for a given wrong value $\tilde{\gamma}$, one can construct a corresponding wrong $\tilde{\xi}_{jt}$ that fits the observed data equally well by letting $\tilde{\xi}_{jt}$ be given by $\frac{Q_{jt}}{\tilde{\gamma} M_t} = \int \frac{\exp(\tilde{\xi}_{jt}+\sigma\nu_i X_{jt})}{1+\sum_{k=1}^{J}(\tilde{\xi}_{kt}+\sigma\nu_i X_{kt})} f_\nu(\nu)d\nu$. Put differently, for any value of $\tilde{\gamma}$, the implied $\tilde{\xi}_{jt}$ will adjust to set the predicted choice probabilities equal to the observed shares $Q_{jt}/\tilde{\gamma}M_t$.

Following a similar idea in Gandhi and Nevo (2021), in Figure 2, I visually illustrate the variation that distinguishes $\gamma$ and $\sigma$.

Figure 2 plots $X_{jt}$ against the implied residual function $\xi_{jt}(\sigma, \gamma)$ for different values of $(\sigma, \gamma)$. As depicted in Figure 2(a), there is no correlation between $\xi$ and the $X$ at the true parameter values. Figure 2(b) shows that when $\sigma$ is different from the truth, it exhibits a hump-shaped correlation and Figure 2(c) shows that when $\gamma$ is different from the truth, there is a linear correlation. For the wrong $\sigma$ or $\gamma$ to fit the data, $\xi$ would have to be correlated with the instruments. Therefore once we assume that $\xi$ is mean independent of $X$, we are shutting down this channel (as in Gandhi and Nevo 2021). Only at the true parameter values can we match the market shares. Furthermore, the graphs with wrong $\sigma$ or wrong $\gamma$ have different shapes, which provide information to distinguish these two parameters.



(a) $\sigma = \sigma^0, \gamma = \gamma^0$ (b) $\sigma > \sigma^0, \gamma = \gamma^0$ (c) $\sigma = \sigma^0, \gamma > \gamma^0$

Figure 2: Intuition for Identification in Random Coefficients Logit

Notes: The figure shows a scatter plot of $\xi_{jt}$ and the characteristics $X_{jt}$ under three scenarios. (a) $\sigma = \sigma^0 = 5, \gamma = \gamma^0 = 1$, (b) $\sigma = 15, \gamma = \gamma^0 = 1$, and (c) $\sigma = \sigma^0 = 5, \gamma = 4$.

I conducted two additional exercises. For brevity, I omit the plot and summarize the patterns here. The first introduces a random coefficient on the intercept: $U_{ijt} = \sigma_0 \nu_{0i} + \sigma_1 \nu_{1i} X_{jt} + \xi_{jt} + \varepsilon_{ijt}$. When $\sigma_0$ differs from the truth, there is no correlation between $\xi_{jt}$ and $X_{jt}$, but there is correlation between $\xi_{jt}$ and $\sum_{k \neq j} X_{kt}$, which distinguishes itself from $\sigma_1$ and $\gamma$. The second exercise incorporates market fixed effects in the utility function. For different values of $(\sigma, \gamma)$, market fixed effects are estimated and the resulting residuals are plotted against $X_{jt}$. The shape of the correlation plots, after conditioning on fixed effects, resembles those in Figure 2, with the only difference being a shift in the location of $\xi_{jt}$ to center at 0. This exercise highlights that market fixed effects and $\gamma$ are separately identified, a point I formally demonstrate next.

## 4.5 Market Fixed Effects

The conditions provided in previous sections do not preclude $X_{jt}$ from containing market-level dummies, and thus, in principle, both market fixed effects and market size are identified. In Supplement E I show that in a plain logit model, by including market fixed effects in the regression, one can obtain consistent estimators of $\beta$ without observing or estimating the true market size. In this section, I show why the same argument cannot be made in the random coefficients case.

By Assumption 3, we have $E\left[\left(\delta_{jt}(Q_{jt}, M_t, X_t^{(2)}; \gamma_0, \sigma_0) - X'_{jt}\beta_0\right)\phi_{jt}(Z_t)\right] = 0$. We can rewrite the moment condition as

$$E\left[\left(\delta_{jt}\left(Q_{jt}, M_t, X_t^{(2)}; \tilde{\gamma}, \sigma_0\right) - X_{jt}'\beta_0 + \delta_{jt}\left(Q_{jt}, M_t, X_t^{(2)}; \gamma_0, \sigma_0\right)\right.\right.$$
$$\left.\left. -\delta_{jt}\left(Q_{jt}, M_t, X_t^{(2)}; \tilde{\gamma}, \sigma_0\right)\right)\phi_{jt}(Z_t)\right] = 0, \quad (17)$$

where $\tilde{\gamma} \in \Gamma$ can be any value in the parameter space of $\gamma$. Suppose one assumes the market size coefficient is $\tilde{\gamma}$ and estimates the model following the standard BLP procedure, then the probability limit of the empirical moment used in estimation is $E\left[\left(\delta_{jt}(Q_{jt}, M_t, X_t^{(2)}; \tilde{\gamma}, \sigma_0) - X_{jt}'\beta_0\right)\phi_{jt}(Z_t)\right]$. Now we explore the possibility of consistently estimating the parameters $\beta$ and $\sigma$ by adding market-level fixed effects as in the plain logit case (Supplement E). The question then arises as to whether the term showing up in equation (17), $\delta_{jt}(Q_{jt}, M_t, X_t^{(2)}; \gamma_0, \sigma_0) - \delta_{jt}(Q_{jt}, M_t, X_t^{(2)}; \tilde{\gamma}, \sigma_0)$, is invariant across products in a given market. If yes, then this gap can be captured by a product-invariant parameter $\kappa_t$, and the true moment condition (17) would be $E\left[\left(\delta_{jt}\left(Q_{jt}, M_t, X_t^{(2)}; \tilde{\gamma}, \sigma_0\right) - X_{jt}'\beta_0 - \kappa_t\right)\phi_{jt}(Z_t)\right] = 0$, from which we can consistently estimate $\sigma$ and $\beta$ by including market-level dummies, regardless of the value of $\tilde{\gamma}$. In other words, the choice of $\tilde{\gamma}$ would be a free normalization.

I verify this by deriving the changes in $\delta_{jt}$ resulting from changes in $\gamma$. First consider the plain logit model, where $\delta_{jt}$ has an analytic form. For a scalar $\gamma$, the derivative of $\delta_{jt}$ with respect to $\gamma$ is $-\frac{1}{\gamma} - \frac{\sum_k (Q_{kt}/M_t)}{\gamma^2 - \gamma\sum_k (Q_{kt}/M_t)}$, which depends only on $t$, implying that the variation in $\delta_{jt}$ as $\gamma$ changes is not product specific and thus $\delta_{jt}(Q_{jt}, M_t, X_t^{(2)}; \gamma_0) - \delta_{jt}(Q_{jt}, M_t, X_t^{(2)}; \tilde{\gamma})$ can be captured by $\kappa_t$.

Now consider the random coefficients logit. Suppose $J = 2$, we have

$$\frac{\partial \delta_{1t}\left(Q_{jt}, M_t, X_t^{(2)}; \gamma, \sigma\right)}{\partial \gamma} = \begin{vmatrix} \frac{\partial \pi_{1t}}{\partial \delta_{1t}} & \frac{\partial \pi_{1t}}{\partial \delta_{2t}} \\ \frac{\partial \pi_{2t}}{\partial \delta_{1t}} & \frac{\partial \pi_{2t}}{\partial \delta_{2t}} \end{vmatrix}^{-1} \begin{vmatrix} \frac{\pi_{1t}}{\gamma} & \frac{\partial \pi_{1t}}{\partial \delta_{2t}} \\ \frac{\pi_{2t}}{\gamma} & \frac{\partial \pi_{2t}}{\partial \delta_{2t}} \end{vmatrix},$$

$$\text{and } \frac{\partial \delta_{2t}\left(Q_{jt}, M_t, X_t^{(2)}; \gamma, \sigma\right)}{\partial \gamma} = \begin{vmatrix} \frac{\partial \pi_{1t}}{\partial \delta_{1t}} & \frac{\partial \pi_{1t}}{\partial \delta_{2t}} \\ \frac{\partial \pi_{2t}}{\partial \delta_{1t}} & \frac{\partial \pi_{2t}}{\partial \delta_{2t}} \end{vmatrix}^{-1} \begin{vmatrix} \frac{\partial \pi_{1t}}{\partial \delta_{1t}} & \frac{\pi_{1t}}{\gamma} \\ \frac{\partial \pi_{2t}}{\partial \delta_{1t}} & \frac{\pi_{2t}}{\gamma} \end{vmatrix},$$

respectively. The denominators are identical for $j = 1, 2$. When $j = 1$, the determinant in the numerator is $\frac{1}{\gamma}\left(\int \pi_{1ti} f_\nu(\nu) d\nu\right)\left(\int \pi_{2ti}(1-\pi_{2ti}) f_\nu(\nu) d\nu\right) + \frac{1}{\gamma}\left(\int \pi_{2ti} f_\nu(\nu) d\nu\right)\left(\int \pi_{1ti}\pi_{2ti} f_\nu(\nu) d\nu\right)$. Similarly, when $j = 2$, the determinant in the numerator is $\frac{1}{\gamma}\left(\int \pi_{2ti} f_\nu(\nu) d\nu\right)\left(\int \pi_{1ti}(1-\pi_{1ti}) f_\nu(\nu) d\nu\right) + \frac{1}{\gamma}\left(\int \pi_{1ti} f_\nu(\nu) d\nu\right)\left(\int \pi_{1ti}\pi_{2ti} f_\nu(\nu) d\nu\right)$. The two are equivalent only when $\nu$ has a degenerate distribution and the individual choice probabilities are identical. Overall, $\delta_{jt}(Q_{jt}, M_t, X_t^{(2)}; \gamma_0, \sigma_0) - \delta_{jt}(Q_{jt}, M_t, X_t^{(2)}; \tilde{\gamma}, \sigma_0)$ would have a $j$ subscript and cannot be captured market fixed effects.

In practice, in applications where the number of markets is much larger than the number of products, including a full set of market fixed effects sometimes results in noisier estimates of random coefficients (see section 7 for illustrations). Due to loss of power and numerical instability, as shown in Table 5, the inclusion of market fixed effects might even make estimates more sensitive to market size assumptions.

# 5 Extensions

## 5.1 Nonparametric Identification of Market Size

Under stronger conditions, the parametric model of market size considered in prior sections can be extended to a more general specification where the true market size is an unknown function of the vector of measures $M_t \in \mathbb{R}^K$. For the moment, I consider the plain logit setting and replace the model of true market size with $s(M_t)$, where $s(\cdot)$ is an unknown function. Under this assumption, the estimating equation becomes

$$\ln(r_{jt}) = \ln\left(\frac{s(M_t)}{Q_t^{total}} - 1\right) + X'_{jt}\beta + \xi_{jt},$$

which is a partially linear regression with an endogenous nonparametric part studied by Ai and Chen (2003) (see also Newey and Powell 2003 and Chen and Pouzo 2009; see Robinson 1988 for an exogenous nonparametric part). Implicitly, I allow market size measures to be endogenous in the sense that $E(M_t\xi_{jt}) \neq 0$. Identification of $\beta$ and $s(\cdot)$ can be achieved by imposing assumptions similar to those in Ai and Chen (2003). I summarize it in the following theorem.

**Theorem 3.** *Let* $Y_t = (Q_t^{total}, M_t)$, $Z_{jt} = (X_{jt}, \tilde{Z}_t)$, *and* $dim(\tilde{Z}_t) = dim(Y_t) = K+1$. *Suppose the following hold: (i) The joint distribution of* $(Y_t, Z_{jt})$ *is observable; (ii)* $E(\xi_{jt} \mid Z_{jt}) = 0$; *(iii) The conditional distribution of* $Y_t$ *given* $Z_{jt}$ *is complete; (iv)* $E\left(\ln\left(\frac{s(M_t)}{Q_t^{total}} - 1\right) \mid Z_{jt}\right) \notin$ *linear span*$(X_{jt})$, *and* $E\left(X_{jt}X_{jt}'\right)$ *is non-singular. Then* $\beta$ *and* $s(\cdot)$ *are identified.*

The proof follows from Newey and Powell (2003) and Proposition 3.1 in Ai and Chen (2003), relying on the completeness of the conditional distribution[27]. Condition (i) rules out that $M_t$ be an unknown measurable function of $Q_t^{total}$, so that $s(M_t)$ is recoverable from $\ln\left(\frac{s(M_t)}{Q_t^{total}} - 1\right)$. Ai and Chen (2003) propose a sieve minimum distance estimator to estimate $\beta$ and $s(\cdot)$. By restricting the unknown function to a space of smooth functions such as the Hölder space, the function can approximate be approximated using a wide range of sieve basis.

## 5.2 Identification With a Nonparametric Demand Model

The identification and estimation in sections 3 and 4 are based on parametric demand models with logit error terms and a known distribution of the random variable $\nu$. However, in some applications, these distribution assumptions on individual tastes may appear to be arbitrary and relatively strong. Thus, I generalize the results to a fully nonparametric model of BLP in the spirit of Berry and Haile (2014) to accommodate less restrictive consumer preferences. The demand system is as equation (8), but with an unknown function $\pi_t(\cdot)$ replacing the regular logit formula and an unknown function $s(\cdot)$ being the true market size, yielding

$$\frac{Q_{jt}}{s(M_t)} = \pi_j\left(\delta_t, X_t^{(2)}\right), \quad j = 1, \cdots, J. \tag{18}$$

The following results show that under a stronger exogeneity condition, the market size function $s(\cdot)$ can be identified up to scale, without even knowing the whole demand model.

27. See Lehmann and Romano (2005) for the concept of statistical completeness. Andrews (2017) provides examples of distributions that are complete.

**Theorem 4.** *Assume that $M_t$ is continuously distributed, and is independent of $(\xi_t, X_t)$. Assume that $s(m)$ is differentiable in $m$. Then $s(m) = e^{\int g(m)}\tilde{c}$ is identified up to a constant $\tilde{c}$, where $g(m) = \partial E(\ln(Q_{jt}) \mid m)/\partial m$.*

The proof is in the Appendix. Market size can only be identified up to scale because the unknown function $\pi_t(\cdot)$ can absorb any scaling constant. If one imposes an additional scale normalization on the market size model, nonparametric identification of the demand model follows under conditions in Berry and Haile (2014).

# 6 Monte Carlo Simulations

In this section, I use a Monte Carlo experiment to assess the performance of our proposed method. Simulation results suggest that our estimator works well when the true outside good share is not too large. In addition to the experiment presented in this section, two additional sets of Monte Carlo experiments are reported in Supplement I. The first evaluates whether random coefficients on an intercept or attributes remove bias induced by incorrect market size assumptions. The second explores how sensitive parameter estimates and elasticities are to market size assumptions in a random coefficients logit model.

The data generating process for the simulated datasets follows closely that in Armstrong (2016). Prices are endogenously generated from a demand and supply model, where firms compete a la Bertrand in the market. More details on the model specification and the data generating process are provided in Supplement I.

In the baseline design (Design 1), two-thirds of markets have 20 products per market and the rest of markets have 60 products in the market. The true values of demand parameters are $\beta = (2, -2, 2)$. In Design 2, I use the same parameter values as in Design 1 but assume that all markets have 20 products, which leads to less variation in market structure. In Design 3, I use the same market structure as Design 1, but set $\beta = (2, -3, 2)$. This particular choice of parameters generates a larger true outside share $\pi_{0t}$. Across $1,000$ simulated samples, the average $\pi_{0t}$ is 0.55 in Design 1, while 0.9 in Design 3. Designs 3 is intentionally constructed to mimic a setting in which identification is expected to be poor, and therefore serves as a cautionary case.

I assume that the true market size is given by $\gamma M_t$. Recall that with a scalar $\gamma$, exogenous quantity shifters serve as instruments for the nonlinear parameters. I include BLP-type instruments or Gandhi and Houde differentiation instruments, as well as functions of excluded cost instruments.

Tables 3 and 4 report results from each design. In addition to the mean, the standard deviation, and the median, we also report the 25% quantile (LQ), the 75% quantile (UQ), the root mean squared error (RMSE), the mean absolute error (MAE), and the median absolute error (MDAE).

Table 3 shows results for the baseline design. The primary parameter of interest, $\gamma$, tends to be estimated precisely, with the RMSE being 0.2. Estimates of $\beta$ and $\sigma$ are mostly close to the true parameter values, and the RMSEs are small. Only the estimate of the constant term coefficient $\beta_0$ is somewhat variable, having a larger RMSE of 0.9. Although not reported in the main tables, we have estimated the same specification replacing BLP-type instruments with Gandhi and Houde differentiation instruments. The resulting estimates are qualitatively similar overall but somewhat more precise with smaller RMSEs.

In Panel A of Table 4, due to reduced variation in market structure, estimates from design 2 are generally noisier than those in design 1, with most RMSEs in the range of 0.7 to 1.3. The median of estimates remains close to the true values. Although $\gamma$ and demand parameters are less precisely estimated in design 2, our proposed estimation is still more preferable to making wrong assumptions of the market size. As shown in the table, the mean of $\gamma$ estimates is 1.447, which is closer to the true value than any $\tilde{\gamma} > 1.5$. Panel B provides results for design 3. $\gamma$, $\sigma$ and $\beta_p$ appear to be difficult to be precisely estimated, with large standard deviations.

The relatively poor performance in design 3 is not a failure of the method but instead due to the Jacobian matrix being close to singular. This case falls exactly into the situation noted in remark 2. Intuitively, when the share of the outside option is too large, the variation in the market shares of inside goods is compressed. As discussed in section 4, both $\gamma$ and $\sigma$ rely on exogenous quantity shifters for identification, and in principle the same set of instruments has identifying power for both. However, a large outside share poses greater challenges for identifying $\gamma$ than in a standard BLP model that estimates only $\sigma$. For $\sigma$, the function $\Phi_{jt}$ in (13) depends only on the *derivatives* of $\pi_{jt}$ for $j = 1, \ldots J$. That is, even when all $\pi_{jt}$ are close to zero, their derivatives may be significantly different from zero and have sufficient variation. By contrast, for $\gamma$, the function $\Psi_{jt}$ involves a column of $\pi_{jt}$. When the outside option share approaches one, $\pi_{jt}$ is pushed close to zero for all $j$ and thus $\Psi_{jt}$ is also close to zero.[28]

In sum, our proposed estimator performs well, particularly in cases where the true outside good share is not too large and has enough variation across markets.

Table 3: Estimating $\gamma$ in the Random Coefficients Logit Model, Design 1

| | TRUE | MEAN | SD | LQ | MED | UQ | RMSE | MAE | MDAE |
|---|---|---|---|---|---|---|---|---|---|
| $\gamma$ | 1 | 1.032 | 0.211 | 0.861 | 1.004 | 1.195 | 0.213 | 0.178 | 0.173 |
| $\sigma$ | 1 | 0.969 | 0.226 | 0.805 | 1.019 | 1.16 | 0.228 | 0.19 | 0.169 |
| $\beta_0$ | 2 | 1.655 | 0.924 | 1.146 | 1.842 | 2.296 | 0.985 | 0.704 | 0.517 |
| $\beta_p$ | -2 | -1.956 | 0.358 | -2.26 | -2.036 | -1.686 | 0.361 | 0.303 | 0.273 |
| $\beta_2$ | 2 | 1.989 | 0.059 | 1.95 | 1.994 | 2.026 | 0.06 | 0.047 | 0.038 |

Notes: The table report summary statistics of the demand parameters. The GMM estimates are based on $1,000$ generated data sets of sample size $T = 100$ and varied $J$. The true model is a random coefficients logit model with a random coefficient for price. Parameters $\beta$, $\sigma$ and $\gamma$ are estimated from IV-GMM estimations using excluded cost shifters and BLP instruments. Design 1: $\beta = (2, -2, 2)$, varied number of products per market.

28. The use of optimal instruments or alternative choices of the instrument function $\phi_{jt}$ that better approximate the optimal instruments might be a potential avenue for improving performance.

Table 4: Estimating $\gamma$ in the Random Coefficients Logit Model, Alternative Designs

| | TRUE | MEAN | SD | LQ | MED | UQ | RMSE | MAE | MDAE |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Panel A: Design 2 | | | | | |
| $\gamma$ | 1 | 1.447 | 1.188 | 0.887 | 1.006 | 1.711 | 1.269 | 0.607 | 0.222 |
| $\sigma$ | 1 | 1.169 | 0.712 | 0.913 | 1.034 | 1.291 | 0.732 | 0.312 | 0.156 |
| $\beta_0$ | 2 | 1.744 | 0.835 | 1.285 | 1.771 | 2.287 | 0.873 | 0.663 | 0.511 |
| $\beta_p$ | -2 | -2.273 | 1.109 | -2.483 | -2.052 | -1.863 | 1.142 | 0.502 | 0.255 |
| $\beta_2$ | 2 | 1.991 | 0.077 | 1.936 | 1.994 | 2.044 | 0.078 | 0.062 | 0.052 |
| | | | | Panel B: Design 3 | | | | | |
| $\gamma$ | 1 | 2.234 | 2.143 | 0.67 | 1.011 | 3.452 | 2.472 | 1.574 | 0.457 |
| $\sigma$ | 1 | 2.518 | 5.15 | 0.795 | 0.994 | 2.223 | 5.367 | 1.743 | 0.287 |
| $\beta_0$ | 2 | 1.844 | 1.511 | 1.309 | 1.835 | 2.305 | 1.518 | 0.659 | 0.511 |
| $\beta_p$ | -3 | -5.351 | 7.901 | -4.938 | -2.988 | -2.665 | 8.24 | 2.731 | 0.537 |
| $\beta_2$ | 2 | 1.989 | 0.119 | 1.958 | 1.994 | 2.028 | 0.12 | 0.046 | 0.034 |

Notes: The table report summary statistics of the demand parameters. The GMM estimates are based on $1,000$ generated data sets of sample size $T = 100$. The true model is a random coefficients logit model with a random coefficient for price. Parameters $\beta$, $\sigma$ and $\gamma$ are estimated from IV-GMM estimations using excluded cost shifters and BLP instruments. Design 2: $\beta = (2, -2, 2)$, fixed number of products per market. Design 3: $\beta = (2, -3, 2)$, varied number of products per market.

# 7 Merger Analysis: Ready-to-Eat Cereal Market

In this section, I apply the proposed method to a hypothetical merger in the cereal market using constructed data from Nevo (2000a). This exercise is for demonstrating the results one can obtain with and without correctly capturing market size. In Supplement J, I have a second merger analysis in the Carbonated Soft Drink market showing the performance of the proposed method in different empirical contexts using NielsonIQ data.

Market size plays a crucial role in merger simulations that are based on aggregate discrete choice demand models.[29] The analysis of unilateral effects hinges on whether an increase in the price of one product will lead consumers to choose an alternative in the market; also important is whether the consumer will divert to an outside option. Throughout this section, I assume that firms are under a static Nash-Bertrand pricing game. The empirical specification of the supply side follows Nevo (2000b) and Weinberg and Hosken (2013).

29. The merger analysis in this paper is based on an implicit assumption that specifying demand using a discrete choice model is correct. In practice, if there are relatively few products – so that too many parameters is not a concern – an alternative is to estimate demand in the product space (i.e. a continuous demand model). Bokhari and Mariuzzo (2018) compare the performance of these two types of models in the context of mergers.

Market shares (or market sizes) used in estimation not only affect estimates of marginal effects ($\beta's$) but also enter firms' first-order conditions for pricing. Thus, assumptions about market size can influence estimates of firms' markup and consumer surplus.

Suppose there are two firms each producing a single product. The upward pricing pressure (UPP) of good 1 depends on the substitution between good 1 and good 2, as well as the markup of good 2. The size of the outside market matters for a firm's optimization problem and, therefore, has a substantial effect on the estimated markup. More generally, in mergers involving multiple firms and products, the strategic complements result in all market participants increasing their prices, which in turn generates substitution to the outside option.

## Data and Demand Specification

The data in Nevo (2000a) is simulated from a model of demand and supply, and consists of 24 brands of the ready-to-eat cereal products for 94 markets. Nevo's specification contains a price variable and brand fixed effects. Variables that enter the nonlinear part of the model are the constant, price, sugar content and a mushy dummy. For each market 20 i.i.d. simulation draws are provided for each of the nonlinear variables. In addition to the unobserved tastes, $\nu_i$, demographics are drawn from the current population survey (CPS) for 20 individuals in each market. It allows for interactions between demographics such as income and the child dummy with price, sugar content and the mushy dummy, capturing heterogeneity on the tastes for product characteristics across demographic groups. Nevo (2000a) uses as instruments the prices of the brand in other cities, variables that serve as proxies for the marginal costs, distribution costs and so on. As discussed in section 4.2.2, these constitute an adequate set of instruments with identifying power for the price coefficient and the nonlinear parameters.

A market is defined as a city-quarter pair and the market size is the total potential number of servings. Nevo assumes the potential consumption is one serving of cereal per day. Using notations in this paper, the assumed market potential is therefore $1 \cdot M_t$, where

$M_t$ is the population in city $t$ in this case.

The baseline specification replicates that in Nevo (2000a). I calculate the estimated own- and cross-price elasticities and diversion ratios, which are the mean of all entries of the elasticity/diversion ratio matrix over the 94 markets. On the basis of the baseline estimation, I consider a hypothetical merger analysis between two multi-products firms. Post-merger equilibrium prices are solved from the Bertrand first order condition. Consumer surplus calculations are provided to show the impacts of the hypothetical merger. Next, I consider an alternative choice of potential market size. I rescale the market shares for all inside goods by a factor of 1/2, which is equivalent to taking the potential market size to be double as large as in the baseline case. I re-simulate the merger using the rescaled market shares. Finally, I assume the true market size is $\gamma$ servings per person per day, estimate $\gamma$ and repeat the merger simulation.[30]

## Results

Table 5 reports the demand coefficients and the implied mean elasticities and diversion ratios. The baseline estimation replicates the results in Nevo (2000a). Interestingly, doubling the market size has little impact on the estimates of demand coefficients $\beta$ and $\sigma$. The baseline estimation has a price coefficient of $-32$ and the rescaled of $-28.9$. However, translating it to elasticities and diversion ratios, we see a substantial increment in the diversion to outside option. In particular, the average outside-good diversion increase from 37.5% to 60.2%. These estimates imply that, if one assumed a larger market size, more consumers would switch to outside good rather than alternative substitutes upon an increase in price of inside goods. The second and third columns include a full set of market fixed effects. Estimates are much less precise than the ones without fixed effects and more sensitive to variations in the

30. While the identification analysis allows market size to be a general function of market observables, the empirical application adopts a linear specification as a benchmark commonly used in the literature. This choice facilitates direct comparison of estimates across studies. The empirical application is intended as an illustration of the proposed identification strategy rather than as an exhaustive empirical analysis.

potential market size.[31] The last column presents the estimated $\gamma$ and demand parameters (including a random coefficient on the constant term) using two-step GMM estimation. $\hat{\gamma} = 0.78$ means that the true market size is a potential daily consumption of 0.78 servings per person. The implied market size is smaller than the baseline case, leading to a lower true diversion ratio. Given $\gamma$ estimate being 0.78, we can calculate the outside share is about 40%.

In order to quantify the overall effect of uncertainty in market size on merger analysis, we look at the impact on both the simulated prices and consumer surplus. Figure 3 plots the distribution of percentage price changes pre- and post-merger, where the three curves plot the baseline case, rescaled case and the case for our estimate of $\gamma$. Predicted price increase is the smallest when we assume $\gamma = 2$. When the potential market size is two times the baseline case, prices of the merging brands respond relatively less to the merger, with a median increase of 5.4%. While in the baseline case, the median price increase is 10.7% for the merging brands. Under the true estimated market size $\hat{\gamma} = 0.78$, the predicted price increase is larger than assuming $\gamma = 1$. This is consistent with our intuition: when there are less people substitute to outside good, the merging firms will have a greater increase in market power.

Next we consider the implications of our estimates for the consumer surplus change after the merger.[32] As expected, we predict a larger decrease in consumer surplus when the price increase is high. Overall, different market sizes affect how much we predict a merger harms consumer welfare.

31. Based on 2256 observations, the inclusion of market fixed effects – which are not in Nevo's original model – introduces 93 more parameters, which can lead to noisier estimates. In this situation, the given data variation and available instruments may not support precise joint estimation of the large number of fixed effects and market size.

32. The consumer surplus is the expected value of the highest utility one can get measured in dollar values. It is calculated by $CS = \sum_{i=1}^{NS} w_{it} CS_{it}$, where the consumer surplus for individual $i$ is

$$CS = \ln\left(1 + \sum_{j \in J_t} \exp V_{ijt}\right) \Big/ \left(-\frac{\partial V_{i1t}}{\partial p_{1t}}\right), \text{ and } V_{ijt} \equiv U_{ijt} - \varepsilon_{ijt}.$$

Table 5: Parameter Estimates for the Cereal Demand

| | Baseline ($M_t$) | Rescaled ($2M_t$) | Baseline ($M_t$) | Rescaled ($2M_t$) | Estimate $\gamma$ |
|---|---|---|---|---|---|
| $\beta_{price}$ | -32.00 | -28.90 | -37.19 | -43.30 | -35.82 |
| | (2.30) | (3.29) | (8.17) | (15.28) | (7.06) |
| $\sigma_{cons}$ | 0.38 | 0.25 | -0.03 | 14.59 | 0.68 |
| | (0.12) | (0.16) | (1.28) | (1.83) | (0.33) |
| $\sigma_{price}$ | 1.80 | 3.31 | 9.76 | -25.70 | 2.13 |
| | (0.92) | (0.97) | (11.21) | (10.65) | (1.74) |
| $\sigma_{sugar}$ | 0.00 | 0.02 | -0.09 | -0.09 | -0.03 |
| | (0.01) | (0.01) | (0.04) | (0.04) | (0.03) |
| $\sigma_{mushy}$ | 0.09 | 0.03 | 0.32 | -0.07 | 0.17 |
| | (0.19) | (0.19) | (0.45) | (0.40) | (0.27) |
| $\sigma_{cons \times inc}$ | 3.10 | 3.22 | 0.26 | 81.68 | 4.12 |
| | (1.05) | (0.88) | (0.99) | (76.06) | (1.80) |
| $\sigma_{cons \times age}$ | 1.20 | 0.70 | 0.56 | -4.11 | 2.12 |
| | (1.05) | (0.68) | (3.95) | (12.99) | (1.76) |
| $\sigma_{price \times inc}$ | 4.19 | -2.94 | -4.83 | -56.15 | 8.98 |
| | (4.64) | (5.16) | (7.09) | (26.64) | (152.36) |
| $\sigma_{price \times child}$ | 11.75 | 10.87 | 2.61 | -297.70 | 14.50 |
| | (5.20) | (4.75) | (30.32) | (104.20) | (7.52) |
| $\sigma_{sugar \times inc}$ | -0.19 | -0.14 | -0.13 | -0.15 | -0.30 |
| | (0.04) | (0.03) | (0.04) | (0.11) | (0.08) |
| $\sigma_{sugar \times age}$ | 0.03 | 0.03 | 0.14 | -0.12 | 0.02 |
| | (0.03) | (0.03) | (0.10) | (0.08) | (0.04) |
| $\sigma_{mushy \times inc}$ | 1.50 | 1.40 | 3.40 | 2.05 | 1.53 |
| | (0.65) | (0.47) | (1.28) | (0.97) | (0.90) |
| $\sigma_{mushy \times age}$ | -1.54 | -1.25 | -3.38 | -2.17 | -1.92 |
| | (1.11) | (0.68) | (2.58) | (0.90) | (1.68) |
| $\gamma$ | | | | | 0.78 |
| | | | | | (0.06) |
| Market fixed effects | No | No | Yes | Yes | No |
| Own-elasticity | -3.70 | -3.68 | -3.83 | -3.21 | -3.80 |
| Cross-elasticity | 0.10 | 0.06 | 0.09 | 0.13 | 0.12 |
| Outside-good diversion | 0.38 | 0.60 | 0.42 | 0.06 | 0.23 |

Notes: The first column is the baseline estimation where market potential is 1 serving per person per day. The second column is the rescaled estimation where the market potential is 2 servings per person per day. The third and fourth columns replicate the first two but including market fixed effects. In the last column we estimate the market size parameter $\gamma$. Elasticities and the diversion ratio are averaged across markets and products.

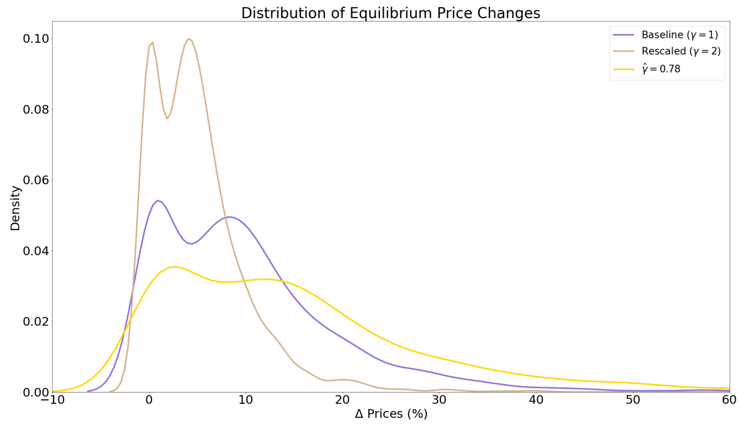



Figure 3: Equilibrium Price Changes

Notes: The figure shows changes in equilibrium prices after a merger between firms 1 and 2.

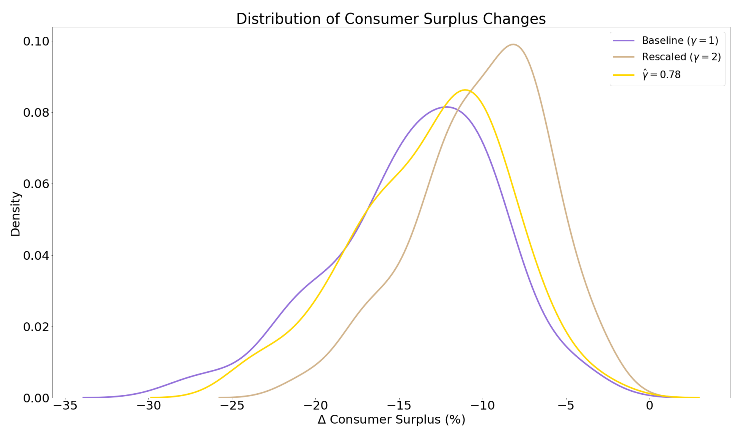



Figure 4: Consumer Surplus Changes

Notes: The figure shows changes in equilibrium prices after a merger between firms 1 and 2.

# 8 Conclusions

This paper shows that market size is point identified in aggregate discrete choice demand models. My identification results use the same parametric assumptions that are commonly imposed in practice, like those in my survey of papers published in top 5 journals (see Table 1). Point identification relies on observed substitution patterns induced by exogenous variation in product characteristics and the nonlinearity of the demand model. The required data are conventional market-level data used in standard BLP estimation. I illustrate the results using Monte Carlo simulations and provide an empirical application to merger analysis using Nevo (2000a)'s cereal data. Our application shows that correctly measuring market size is economically important. For instance, I find that assuming a market size larger than the true size leads to a non-negligible downward bias in the estimated merger price increase, which could affect the conclusions of the merger evaluation. Apart from the merger application, my results would also have important implications for social welfare, markup calculations, tax and subsidy policies, and the entry of new firms. It could also potentially be adapted as a test for defining relevant product markets, which I leave for future exploration.

Potential areas for future theoretical research include deriving conditions for strong identification and instrument selection, extending the model to micro-BLP which uses individual choice data, and allowing for dependence among logit errors to make the results applicable to panel data settings as in Khan, Ouyang, and Tamer (2021).

In the current application, I consider a scalar $\gamma$. A possible extension would be to allow $\gamma$ to vary based on market characteristics, such as demographic composition and the number of retail stores. It would also be useful to test my model in an industry where the true market size is known, such as the pharmaceutical market, where researchers generally know the number of patients, which can be considered as the potential market size. Another possibility for further work is generalizing the model to empirical contexts where inside good quantity is mismeasured or unknown, such as the consumption of informal goods or services (Pissarides and Weber 1989).

# Appendix: Proofs

*Proof of Theorem 1.* By the mean independence condition given in Assumption 1, we have $E\left(\ln(r_{jt}) \mid \tilde{Z}_t = z, X_{jt} = x\right) = E\left(\ln(\gamma W_t - 1) \mid \tilde{Z}_t = z, X_{jt} = x\right) - x'\beta$. Taking derivative with respect to $z$ yields $0 = \partial E\left(\ln(r_{jt}) - \ln(\gamma W_t - 1) \mid \tilde{Z}_t = z, X_{jt} = x\right)/\partial z$.

Let $\Gamma$ be the set of all possible values of $\gamma$. For any given constant $c \in \Gamma$, define the function $g(c,z,x) = \partial E\left(\ln(r_{jt}) - \ln(cW_t - 1) \mid \tilde{Z}_t = z, X_{jt} = x\right)/\partial z$.

We observe $r_{jt}$, $W_t$, $\tilde{Z}_t$ and $X_{jt}$. For any constant $c$, observed $z$ and $x$, we can therefore nonparametrically identify $g(c,z,x)$. In order to show point identification, we need to verify that there exists at most one value of $c \in \Gamma$ such that $g(c,z,x) = 0$ for all observed $z \in \mathrm{Supp}(\tilde{Z}_t)$ and $x \in \mathrm{Supp}(X_{jt})$. Taking the derivative of $g(c,z,x)$ with respect to $c$, we have

$$\frac{\partial^2 E\left(\ln(r_{jt}) - \ln(cW_t - 1) \mid \tilde{Z}_t = z, X_{jt} = x\right)}{\partial c \partial z} = \frac{\partial E\left(-\frac{W_t}{cW_t - 1} \mid \tilde{Z}_t = z, X_{jt} = x\right)}{\partial z}.$$

The identification then follows from the assumption that there exists $(z,x)$ on the support of $(\tilde{Z}_t, X_{jt})$ such that $\partial E\left(-\frac{W_t}{cW_t-1} \mid \tilde{Z}_t = z, X_{jt} = x\right)/\partial z$ is strictly positive or strictly negative for all $c \in \Gamma$.

Given $\gamma$, the model becomes equivalent to a standard multinomial choice model, and therefore $\beta$ is identified the same way. □

*Proof of Theorem 2.* Assuming that $\nabla_\theta h_{jt}(\theta)$ is dominated by some Lebesgue integrable function and applying the dominated convergence theorem, we can take the derivative inside the expectation and have $\nabla_\theta E(h_{jt}(\theta)) = E(\nabla_\theta h_{jt}(\theta))$. The Jacobian matrix is

$$\begin{aligned} E(\nabla_\theta h_{jt}(\theta)) &= E\left[\frac{\partial h_{jt}(\theta)}{\partial \gamma'} \quad \frac{\partial h_{jt}(\theta)}{\partial \sigma'} \quad \frac{\partial h_{jt}(\theta)}{\partial \beta'}\right] \\ &= E\left[\phi_{jt}(Z_t)\frac{\partial \delta_{jt}(Q_t, M_t, X_t^{(2)}; \gamma, \sigma)}{\partial \gamma'} \quad \phi_{jt}(Z_t)\frac{\partial \delta_{jt}(Q_t, M_t, X_t^{(2)}; \gamma, \sigma)}{\partial \sigma'} \quad \phi_{jt}(Z_t)X'_{jt}\right] \end{aligned}$$

Recall that $h_{jt}(\theta) = (\delta_{jt}(Q_t, M_t, X_t^{(2)}; \gamma, \sigma) - X'_{jt}\beta)\phi_{jt}(Z_t)$. The first derivative of the above matrix is an $m \times 2K$ vector. $\partial \pi_{jt}(\delta_t; \sigma)/\partial \sigma'$ is a $1 \times L$ row vector, so the second derivative of the above matrix is an $m \times L$ matrix. Similarly, the dimension of the last derivative is $m \times L$. The identification proof follows directly from Lemma 2 and the rank condition that the Jacobian matrix has rank $K$. □

*Proof of Proposition 1.* To ease notation in the proof, we drop the subscript $j$ and $t$ and suppress the dependence of $\Phi$ and $\Psi$ on $(\delta_t, X_t^{(2)}; \sigma)$, and the dependence of $\phi$ on $Z$. We make a simplifying assumption w.l.o.g.: Suppose $X$ are exogenous and thus can serve as its own instruments, i.e. $\phi^{(1)} = X$. When $K = 1$ and $\gamma_2$ is known, the Jacobian matrix reduces to

$$\begin{pmatrix} E\left(\begin{pmatrix}\phi^{(2)}\\ \phi^{(3)}\end{pmatrix}\begin{pmatrix}\frac{1}{\gamma_1}\Psi\\ \Phi\end{pmatrix}'\right) & E\left(\begin{pmatrix}\phi^{(2)}\\ \phi^{(3)}\end{pmatrix}X'\right) \\ E\left(X\begin{pmatrix}\frac{1}{\gamma_1}\Psi\\ \Phi\end{pmatrix}'\right) & E\left(XX'\right) \end{pmatrix},$$

and recall that

$$A = E\left(\begin{pmatrix}\phi^{(2)}\\ \phi^{(3)}\end{pmatrix}\begin{pmatrix}\frac{1}{\gamma_1}\Psi\\ \Phi\end{pmatrix}'\right) \quad B = E\left(\begin{pmatrix}\phi^{(2)}\\ \phi^{(3)}\end{pmatrix}X'\right)$$

$$C = E\left(X\begin{pmatrix}\frac{1}{\gamma_1}\Psi\\ \Phi\end{pmatrix}'\right) \quad D = E\left(XX'\right),$$

Let $X = (1, \tilde{X}')'$. Denote $\Omega = (E(\tilde{X}\tilde{X}') - E(\tilde{X})E(\tilde{X}'))^{-1}$, then we have

$$D^{-1} = \begin{pmatrix} 1 + E(\tilde{X}')\Omega E(\tilde{X}) & -E(\tilde{X}')\Omega \\ -\Omega E(\tilde{X}) & \Omega \end{pmatrix},$$

and

$$A - BD^{-1}C = \frac{1}{\gamma_1}\left(Cov\left(\begin{pmatrix}\phi^{(2)}\\ \phi^{(3)}\end{pmatrix}, (\Psi, \Phi)\right) - Cov\left(\begin{pmatrix}\phi^{(2)}\\ \phi^{(3)}\end{pmatrix}, \tilde{X}'\right)\Omega Cov\left(\tilde{X}, (\Psi, \Phi)\right)\right)$$

For the Jacobian matrix to have full rank, we make a technical assumption that $det(A - BD^{-1}C) \neq 0$. This assumption is generically satisfied when

$$Cov\left(\begin{pmatrix}\phi^{(2)}\\ \phi^{(3)}\end{pmatrix}, (\Psi, \Phi)\right)$$

has full rank. Note that given the regularity assumptions in the Proposition, when the above matrix has full rank, $det(A - BD^{-1}C) = 0$ only at a set of measure zero. $\square$

*Proof of Theorem 3.* Following Ai and Chen (2003), under conditions (i)-(iv), one can identify $\beta$ and $\ln\left(\frac{s(M_t)}{Q_t^{total}} - 1\right)$. Let $h(q,m) = \ln\left(\frac{s(m)}{q} - 1\right)$. The function $h(q,m)$ is identified. Take conditional expectation on both sides, we have

$$\begin{aligned} g(m) \equiv E\left(h(Q_t^{total}, M_t) \mid M_t = m\right) &= E\left(\ln\left(\frac{s(m)}{Q_t^{total}} - 1\right) \middle| M_t = m\right) \\ &= \int \ln\left(\frac{s(m)}{q} - 1\right) dF_{Q|M}(q \mid m) \end{aligned}$$

$g(m)$ is identified and $F_{Q|M}$ is identified by condition (i). Identification of $s(m)$ is achieved by showing the existence of a unique solution to the above equation. Denote $c_m = s(m)$. Note that $\ln\left(\frac{c_m}{q} - 1\right)$ is strictly increasing in $c_m$ for all $c_m > q$. When $F_{Q|M}$ is well-defined and integrable on the support of $Q \mid M$, $\int \ln\left(\frac{c_m}{q} - 1\right) dF_{Q|M}(q \mid m)$ is strictly increasing in $c_m$. Thus, for each $m$, there exists a unique $s(m)$ satisfying the equation. $\square$

*Proof of Theorem 4.* Assuming $M_t \perp (\xi_t,\ X_t)$, we take log and conditional expectation on both sides: $E\left(\ln(Q_{jt}) \mid M_t\right) = \ln(s(M_t)) + E\left(\ln\left(\pi_j(\delta_t, X_t^{(2)})\right)\right)$.

Take derivative w.r.t. $m$, we have $\frac{\partial E(\ln(Q_{jt})|M_t=m)}{\partial m} = \frac{\partial \ln(s(M_t))}{\partial m} \equiv g(m)$, from which $g(m)$ is identified. Then $\ln(s(M_t)) = \int g(m) + c$ is identified up to location. Thus, $s(m) = e^{\int g(m)}\tilde{c}$ is identified up to scale.

$\square$

# References


Ai, C., and X. Chen. 2003. "Efficient estimation of models with conditional moment restrictions containing unknown functions." *Econometrica* 71 (6): 1795–1843.

Andrews, D. W. 2017. "Examples of L2-complete and boundedly-complete distributions." *Journal of econometrics* 199 (2): 213–220.

Andrews, I., M. Gentzkow, and J. M. Shapiro. 2017. "Measuring the sensitivity of parameter estimates to estimation moments." *The Quarterly Journal of Economics* 132 (4): 1553–1592.

Armstrong, T. B. 2016. "Large market asymptotics for differentiated product demand estimators with economic models of supply." *Econometrica* 84 (5): 1961–1980.

Backus, M., C. Conlon, and M. Sinkinson. 2021. *Common ownership and competition in the ready-to-eat cereal industry.* Technical report. National Bureau of Economic Research.

Berry, S., O. B. Linton, and A. Pakes. 2004. "Limit theorems for estimating the parameters of differentiated product demand systems." *The Review of Economic Studies* 71 (3): 613–654.

Berry, S. 1990. "Airport presence as product differentiation." *The American Economic Review* 80 (2): 394–399.

———. 1994. "Estimating discrete-choice models of product differentiation." *The RAND Journal of Economics,* 242–262.

Berry, S., M. Carnall, and P. T. Spiller. 2006. "Airline hubs: costs, markups and the implications of customer heterogeneity." *Competition policy and antitrust.*

Berry, S., and P. A. Haile. 2014. "Identification in differentiated products markets using market level data." *Econometrica* 82 (5): 1749–1797.

———. 2021. "Foundations of demand estimation." In *Handbook of industrial organization,* 4:1–62. 1. Elsevier.

Berry, S., and P. Jia. 2010. "Tracing the woes: An empirical analysis of the airline industry." *American Economic Journal: Microeconomics* 2 (3): 1–43.

Berry, S., J. Levinsohn, and A. Pakes. 1995. "Automobile prices in market equilibrium." *Econometrica: Journal of the Econometric Society,* 841–890.

Berto Villas-Boas, S. 2007. "Vertical relationships between manufacturers and retailers: Inference with limited data." *The Review of Economic Studies* 74 (2): 625–652.

Bokhari, F. A., and F. Mariuzzo. 2018. "Demand estimation and merger simulations for drugs: Logits v. AIDS." *International Journal of Industrial Organization* 61:653–685.

Bresnahan, T. F., and P. C. Reiss. 1991. "Entry and competition in concentrated markets." *Journal of political economy* 99 (5): 977–1009.

Byrne, D. P., S. Imai, N. Jain, and V. Sarafidis. 2022. "Instrument-free identification and estimation of differentiated products models using cost data." *Journal of Econometrics* 228 (2): 278–301.

Chamberlain, G. 1987. "Asymptotic efficiency in estimation with conditional moment restrictions." *Journal of econometrics* 34 (3): 305–334.

Chen, X., and D. Pouzo. 2009. "Efficient estimation of semiparametric conditional moment models with possibly nonsmooth residuals." *Journal of Econometrics* 152 (1): 46–60.

Chu, C. S., P. Leslie, and A. Sorensen. 2011. "Bundle-size pricing as an approximation to mixed bundling." *The American Economic Review,* 263–303.

Conlon, C., and J. Gortmaker. 2020. "Best practices for differentiated products demand estimation with pyblp." *The RAND Journal of Economics* 51 (4): 1108–1161.

Conlon, C., and J. H. Mortimer. 2021. "Empirical properties of diversion ratios." *The RAND Journal of Economics* 52 (4): 693–726.

Domínguez, M. A., and I. N. Lobato. 2004. "Consistent estimation of models defined by conditional moment restrictions." *Econometrica* 72 (5): 1601–1615.

Duch-Brown, N., L. Grzybowski, A. Romahn, and F. Verboven. 2017. "The impact of online sales on consumers and firms. Evidence from consumer electronics." *International Journal of Industrial Organization* 52:30–62.

Dunker, F., S. Hoderlein, and H. Kaido. 2022. "Nonparametric identification of random coefficients in endogenous and heterogeneous aggregate demand models." *arXiv preprint arXiv:2201.06140.*

Fisher, F. M. 1966. *The identification problem in econometrics.* McGraw-Hill.

Gandhi, A., and J.-F. Houde. 2019. *Measuring substitution patterns in differentiated products industries.* Technical report. National Bureau of Economic Research.

Gandhi, A., Z. Lu, and X. Shi. 2023. "Estimating demand for differentiated products with zeroes in market share data." *Quantitative Economics* 14 (2): 381–418.

Gandhi, A., and A. Nevo. 2021. *Empirical Models of Demand and Supply in Differentiated Products Industries.* Technical report. National Bureau of Economic Research.

Ghose, A., P. G. Ipeirotis, and B. Li. 2012. "Designing ranking systems for hotels on travel search engines by mining user-generated and crowdsourced content." *Marketing Science* 31 (3): 493–520.

Gospodinov, N., and S. Ng. 2015. "Minimum distance estimation of possibly noninvertible moving average models." *Journal of Business & Economic Statistics* 33 (3): 403–417.

Greenstein, S. M. 1996. "From superminis to supercomputers: Estimating surplus in the computing market." In *The economics of new goods,* 329–372. University of Chicago Press.

Ho, K., J. Ho, and J. H. Mortimer. 2012. "The use of full-line forcing contracts in the video rental industry." *American Economic Review* 102 (2): 686–719.

Hortaçsu, A., A. Oery, and K. R. Williams. 2022. *Dynamic price competition: Theory and evidence from airline markets.* Technical report. National Bureau of Economic Research.

Hsiao, C. 1983. "Identification." *Handbook of econometrics* 1:223–283.

Huang, D., and C. Rojas. 2013. "The Outside Good Bias in Logit Models of Demand with Aggregate Data." *Economics Bulletin* 33 (1): 198–206.

———. 2014. "Eliminating the outside good bias in logit models of demand with aggregate data." *Review of Marketing Science* 12 (1): 1–36.

Iizuka, T. 2007. "Experts' agency problems: evidence from the prescription drug market in Japan." *The Rand journal of economics* 38 (3): 844–862.

Iskrev, N. 2010. "Local identification in DSGE models." *Journal of Monetary Economics* 57 (2): 189–202.

Ivaldi, M., and F. Verboven. 2005. "Quantifying the effects from horizontal mergers in European competition policy." *International Journal of Industrial Organization* 23 (9-10): 669–691.

Khan, S., F. Ouyang, and E. Tamer. 2021. "Inference on semiparametric multinomial response models." *Quantitative Economics* 12 (3): 743–777.

Lehmann, E. L., and J. P. Romano. 2005. *Testing statistical hypotheses.* Vol. 3. Springer.

Li, Q., and J. S. Racine. 2007. *Nonparametric econometrics: theory and practice.* Princeton University Press.

Li, S., J. Mazur, Y. Park, J. Roberts, A. Sweeting, and J. Zhang. 2022. "Repositioning and market power after airline mergers." *The RAND Journal of Economics.*

Lu, Z., X. Shi, and J. Tao. 2021. "Semi-nonparametric estimation of random coefficient logit model for aggregate demand." *Available at SSRN 3503560.*

McConnell, K. E., and T. T. Phipps. 1987. "Identification of preference parameters in hedonic models: Consumer demands with nonlinear budgets." *Journal of Urban Economics* 22 (1): 35–52.

Milunovich, G., and M. Yang. 2013. "On identifying structural VAR models via ARCH effects." *Journal of Time Series Econometrics* 5 (2): 117–131.

Nevo, A. 2000a. "A practitioner's guide to estimation of random-coefficients logit models of demand." *Journal of economics & management strategy* 9 (4): 513–548.

———. 2000b. "Mergers with differentiated products: The case of the ready-to-eat cereal industry." *The RAND Journal of Economics,* 395–421.

———. 2001. "Measuring market power in the ready-to-eat cereal industry." *Econometrica* 69 (2): 307–342.

Newey, W. K., and J. L. Powell. 2003. "Instrumental variable estimation of nonparametric models." *Econometrica* 71 (5): 1565–1578.

Petrin, A. 2002. "Quantifying the benefits of new products: The case of the minivan." *Journal of political Economy* 110 (4): 705–729.

Petrin, A., and K. Train. 2010. "A control function approach to endogeneity in consumer choice models." *Journal of marketing research* 47 (1): 3–13.

Pissarides, C. A., and G. Weber. 1989. "An expenditure-based estimate of Britain's black economy." *Journal of public economics* 39 (1): 17–32.

Qu, Z., and D. Tkachenko. 2012. "Identification and frequency domain quasi-maximum likelihood estimation of linearized dynamic stochastic general equilibrium models." *Quantitative Economics* 3 (1): 95–132.

Reynaert, M., and F. Verboven. 2014. "Improving the performance of random coefficients demand models: The role of optimal instruments." *Journal of Econometrics* 179 (1): 83–98.

Robinson, P. M. 1988. "Root-N-consistent semiparametric regression." *Econometrica: Journal of the Econometric Society,* 931–954.

Rothenberg, T. J. 1971. "Identification in parametric models." *Econometrica: Journal of the Econometric Society,* 577–591.

Rysman, M. 2004. "Competition between networks: A study of the market for yellow pages." *The Review of Economic Studies* 71 (2): 483–512.

Sweeting, A., J. W. Roberts, and C. Gedge. 2020. "A model of dynamic limit pricing with an application to the airline industry." *Journal of Political Economy* 128 (3): 1148–1193.

Sydsæter, K., P. Hammond, A. Seierstad, and A. Strom. 2008. *Further mathematics for economic analysis.* Pearson education.

Thompson, T. S. 1989. "Identification of semiparametric discrete choice models."

Weinberg, M. C., and D. Hosken. 2013. "Evidence on the accuracy of merger simulations." *Review of Economics and Statistics* 95 (5): 1584–1600.

White, H. 1980. "Using least squares to approximate unknown regression functions." *International economic review,* 149–170.

Wollmann, T. G. 2018. "Trucks without bailouts: Equilibrium product characteristics for commercial vehicles." *American Economic Review* 108 (6): 1364–1406.

Wright, J. H. 2003. "Detecting lack of identification in GMM." *Econometric theory,* 322–330.

# Identification and Estimation of Market Size in Discrete Choice Demand Models - Online Supplemental Appendix

# A Additional Proofs

Lemma 3 is the contraction mapping theorem in the appendix from Berry, Levinsohn, and Pakes (1995).

**Lemma 3.** *Consider the metric space* $(\mathbb{R}^J, d)$ *with* $d(x,y) = \|x - y\|$. *Let* $g : \mathbb{R}^J \to \mathbb{R}^J$ *have the properties:*

*(1)* $\forall \delta \in \mathbb{R}^J$, $f(\delta)$ *is continuously differentiable, with,* $\forall k$ *and* $j$,

$$\frac{\partial g_k(\delta)}{\partial \delta_j} \geq 0,$$

*and*

$$\sum_{j=1}^{J} \frac{\partial g_k(\delta)}{\partial \delta_j} < 1.$$

*(2)* $\min_j \inf_\delta g_j(\delta) = \underline{\delta} > -\infty$.*(There is a lower bound to* $g_j(\delta)$*, denoted* $\underline{\delta}$*)*

*(3) There is a value* $\overline{\delta}$*, with the property that if for any* $j$, $\delta_j \geq \overline{\delta}$*, then for some* $k$, $g_k(\delta) < \delta_k$.

*Then, there is a unique fixed point* $\delta^*$ *to* $g$ *in* $\mathbb{R}^J$.

*Proof of Lemma 1.* The implicit system of equations is solved for each market, therefore we drop the $t$ subscript in the proof to simplify the notation. We show the lemma for a scalar $\gamma$. The generalized result can be obtained similarly by replacing $\ln(s_j/\gamma)$ with $\ln(Q_j / \sum \gamma_1 M^{\gamma_2})$. Let $s_j = Q_j/M$ and $s_0 = 1 - \sum_j Q_j/M$. Now we show that the function $g(\delta) = \delta + \ln(s) - \ln(\gamma) - \ln(\pi(\delta;\sigma))$ satisfies the three conditions in Lemma 3.

(1) The function $g(\delta)$ is continuously differentiable by the differentiability of the predicted choice probability function $\pi(\delta;\sigma)$.
First we want to show that

$$\frac{\partial g_j(\delta)}{\partial \delta_j} = 1 - \frac{1}{\pi_j(\delta;\sigma)} \frac{\partial \pi_j(\delta;\sigma)}{\partial \delta_j} \geq 0$$

Take the derivative of $\pi_j(\delta;\sigma)$ with respect to $\delta_j$, we have

$$
\begin{aligned}
& \frac{\partial \pi_j(\delta;\sigma)}{\partial \delta_j} \\
= & \int \frac{exp(\delta_j + \sum_l \sigma_l x_{jl}\nu_l)\left(1 + \sum_{k=1}^{J_t} exp(\delta_k + \sum_l \sigma_l x_{kl}\nu_l)\right)}{\left(1 + \sum_{k=1}^{J_t} exp(\delta_k + \sum_l \sigma_l x_{kl}\nu_l)\right)^2} \\
& - \frac{\left(exp(\delta_j + \sum_l \sigma_l x_{jl}\nu_l)\right)^2}{\left(1 + \sum_{k=1}^{J_t} exp(\delta_k + \sum_l \sigma_l x_{kl}\nu_l)\right)^2} f_\nu(\nu)d\nu \\
= & \int \frac{exp(\delta_j + \sum_l \sigma_l x_{jl}\nu_l)}{1 + \sum_{k=1}^{J_t} exp(\delta_k + \sum_l \sigma_l x_{kl}\nu_l)} - \left(\frac{exp(\delta_j + \sum_l \sigma_l x_{jl}\nu_l)}{1 + \sum_{k=1}^{J_t} exp(\delta_k + \sum_l \sigma_l x_{kl}\nu_l)}\right)^2 f_\nu(\nu)d\nu \\
= & \; \pi_j(\delta;\sigma) - \int \left(\frac{exp(\delta_j + \sum_l \sigma_l x_{jl}\nu_l)}{1 + \sum_{k=1}^{J_t} exp(\delta_k + \sum_l \sigma_l x_{kl}\nu_l)}\right)^2 f_\nu(\nu)d\nu
\end{aligned}
$$

Then we can rewrite the derivative of function $g_j(\delta)$ as

$$
\begin{aligned}
\frac{\partial g_j(\delta)}{\partial \delta_j} &= 1 - \frac{1}{\pi_j(\delta;\sigma)}\frac{\partial \pi_j(\delta;\sigma)}{\partial \delta_j} \\
&= \frac{1}{\pi_j(\delta;\sigma)} \int \left(\frac{exp(\delta_j + \sum_l \sigma_l x_{jl}\nu_l)}{1 + \sum_{k=1}^{J_t} exp(\delta_k + \sum_l \sigma_l x_{kl}\nu_l)}\right)^2 f_\nu(\nu)d\nu,
\end{aligned}
$$

which is non-negative because $\pi_j(\delta;\sigma)$ is strictly positive, and the integrand of the second term is continuous and strictly positive, hence the integral over any closed integral is strictly positive, so the same must hold over the entire real line.

Take the derivative of $\pi(\delta;\sigma)$ with respect to $\delta_j$, we have

$$
\frac{\partial \pi_k(\delta;\sigma)}{\partial \delta_j} = -\int \frac{exp(\delta_k + \sum_l \sigma_l x_{kl}\nu_l)exp(\delta_j + \sum_l \sigma_l x_{jl}\nu_l)}{\left(1 + \sum_{k=1}^{J_t} exp(\delta_k + \sum_l \sigma_l x_{kl}\nu_l)\right)^2} f_\nu(\nu)d\nu.
$$

Therefore the derivative of $g_k(\delta)$ with respect to $\delta_j$ is

$$
\begin{aligned}
\frac{\partial g_k(\delta)}{\partial \delta_j} &= -\frac{1}{\pi_k(\delta;\sigma)}\frac{\partial \pi_k(\delta;\sigma)}{\partial \delta_j} \\
&= \frac{1}{\pi_k(\delta;\sigma)} \int \frac{exp(\delta_k + \sum_l \sigma_l x_{jl}\nu_l)exp(\delta_j + \sum_l \sigma_l x_{jl}\nu_l)}{\left(1 + \sum_{k=1}^{J_t} exp(\delta_k + \sum_l \sigma_l x_{kl}\nu_l)\right)^2} f_\nu(\nu)d\nu,
\end{aligned}
$$

which is non-negative because $\pi_k(\delta;\sigma)$ and the integrand of the second term are strictly positive.

To show the condition $\sum_{j=1}^{J} \partial g_k(\delta)/\partial\delta_j < 1$, note that increasing all the $\delta_j$ including $\delta_0$ simultaneously will not change the market shares, implying that $\sum_{j=0}^{J} \partial\pi_k(\delta;\sigma)/\partial\delta_j = 0$. Then

$$\sum_{j=1}^{J} \frac{\partial\pi_k(\delta;\sigma)}{\partial\delta_j} = -\frac{\partial\pi_k(\delta;\sigma)}{\partial\delta_0} > 0$$

We can therefore establish the condition that the derivatives of $g_k$ sums to less than one

$$\sum_{j=1}^{J} \frac{\partial g_k(\delta)}{\partial\delta_j} = 1 - \frac{1}{\pi_k(\delta;\sigma)} \sum_{j=1}^{J} \frac{\partial\pi_k(\delta;\sigma)}{\partial\delta_j} < 1.$$

(2) Rewrite $g_j(\delta)$ as

$$\begin{aligned} g_j(\delta) &= \ln(s_j) - \ln(\gamma) - \ln\left(D_j(\delta)\right), \\ \text{where } D_j(\delta) &= \int \frac{exp(\sum_l \sigma_l x_{jl}\nu_l)}{1 + \sum_{k=1}^{J_t} exp(\delta_k + \sum_l \sigma_l x_{kl}\nu_l)} f_\nu(\nu)d\nu. \end{aligned}$$

A lower bound of $g_j$ can be obtained by letting all of $\delta_k$ go to $-\infty$, then $D_j(\delta) \to \int exp(\sum_l \sigma_l x_{jl}\nu_l) f_\nu(\nu)d\nu$. So the lower bound on $g_j(\delta)$ is

$$\underline{\delta} \equiv \ln(s_j) - \ln(\gamma) - \ln\left(\int exp(\sum_l \sigma_l x_{jl}\nu_l) f_\nu(\nu)d\nu\right)$$

(3) The proof of this part follows Berry (1994). He shows condition (3) of Lemma 3 is satisfied by first showing that if for any product $j$, $\delta_j \geq \overline{\delta}$, then there is at least one element $k$ with $\pi_k(\delta;\sigma) > s_k/\gamma$.
To construct a $\overline{\delta}$ that satisfies the above requirement, first set all of $\delta_k$ (other than good $j$ and outside good) to $-\infty$. Define $\overline{\delta}_j$ to be the value of $\delta_j$ that makes $\pi_0(\delta;\sigma) = 1 - (1-s_0)/\gamma$. Then define $\overline{\delta} = \max_j \overline{\delta}_j$.
Now if there is any element of $\delta$ with $\delta_j > \overline{\delta}$, then $\pi_0(\delta;\sigma) < 1 - (1-s_0)/\gamma$. It then follows from $\sum_{j=0}^{J} \pi_j(\delta;\sigma) = 1$ that $\sum_{j=1}^{J} \pi_j(\delta;\sigma) > \sum_{j=1}^{J} s_j/\gamma$. Thus there is at least one good $k$ with $\pi_k(\delta;\sigma) > s_k/\gamma$, which implies $g_k(\delta) < \delta_k$:

$$\begin{aligned} & \pi_k(\delta;\sigma) > \frac{s_k}{\gamma} \\ \iff & \ln\left(\pi_k(\delta;\sigma)\right) > \ln(s_k) - \ln(\gamma) \\ \iff & \ln(s_k) - \ln(\gamma) - \ln\left(\pi_k(\delta;\sigma)\right) < 0 \\ \iff & g_k(\delta) = \delta_k + \ln(s_k) - \ln(\gamma) - \ln\left(\pi_k(\delta;\sigma)\right) < \delta_k \end{aligned}$$

□

*Proof of Theorem 5.* By Assumption 4, the conditional mean function is

$$E\left(\ln(r_{jt}) \mid X_{jt} = x\right) = \kappa_t + x'\beta \quad \forall t \in (1, \cdots, T).$$

If $X_{jt}$ is continuous, then $\partial E\left(\ln\left(r_{jt}\right) \mid X_{jt}=x\right)/\partial x=\beta$. If $X_{jt}$ is discrete, then $E\left(\ln\left(r_{jt}\right) \mid X_{jt}=x_1\right) - E\left(\ln\left(r_{jt}\right) \mid X_{jt}=x_2\right)=(x_1-x_2)'\beta$. $\beta$ is therefore identified given that the support of $X_{jt}$ does not lie in a proper linear subspace of $\mathbb{R}^{\dim(X)}$ for $t=1,\cdots,T$ and $X_{it}$ does not contain a constant.

Now that we have shown $\beta$ is identified, the conditional mean function becomes

$$E\left(\ln\left(r_{jt}\right) \mid X_{jt}=x\right)-x'\beta=\kappa_t \quad \forall t\in(1,\cdots,T).$$

The left hand side is identified, and each of the $T$ equations pins down a unique $\kappa_t$. Therefore $(\kappa_1,\cdots,\kappa_T)$ are identified. □

*Proof of Theorem 6.* By the mean independence condition given in Assumption 1, we have

$$E\left(\ln\left(r_{jt}\right) \mid \tilde{Z}_t=z, X_{jt}=x\right)=\frac{1}{1-\sigma}E\left(\ln\left(\gamma W_t-1\right) \mid \tilde{Z}_t=z, X_{jt}=x\right)-x'\frac{\beta}{1-\sigma}.$$

Taking first-order derivative with respect to $z$ yields

$$\frac{\partial E\left(\ln\left(r_{jt}\right) \mid \tilde{Z}_t=z, X_{jt}=x\right)}{\partial z}=\frac{1}{1-\sigma}\frac{\partial E\left(\ln\left(\gamma W_t-1\right) \mid \tilde{Z}_t=z, X_{jt}=x\right)}{\partial z}. \tag{19}$$

Taking second-order derivative with respect to $z$ yields

$$\frac{\partial^2 E\left(\ln\left(r_{jt}\right) \mid \tilde{Z}_t=z, X_{jt}=x\right)}{\partial z^2}=\frac{1}{1-\sigma}\frac{\partial^2 E\left(\ln\left(\gamma W_t-1\right) \mid \tilde{Z}_t=z, X_{jt}=x\right)}{\partial z^2}. \tag{20}$$

Define functions

$$g(z,x)=\frac{\partial E\left(\ln\left(r_{jt}\right) \mid \tilde{Z}_t=z, X_{jt}=x\right)}{\partial z},$$

and

$$h(\gamma,z,x)=\frac{\partial E\left(\ln\left(\gamma W_t-1\right) \mid \tilde{Z}_t=z, X_{jt}=x\right)}{\partial z}.$$

Dividing equation (20) by (19) yields

$$\frac{\partial g(z,x)}{\partial z}\frac{1}{g(z,x)}=\frac{\partial h(\gamma,z,x)}{\partial z}\frac{1}{h(\gamma,z,x)}$$

Let $\Gamma$ be the set of all possible values of $\gamma$. For any given $c\in\Gamma$, define function

$$f(c,z,x)=\frac{\partial h(c,z,x)}{\partial z}\frac{1}{h(c,z,x)}-\frac{\partial g(z,x)}{\partial z}\frac{1}{g(z,x)}.$$

We observe $r_{jt}, W_t, \tilde{Z}_t$ and $X_{jt}$. For any constant $c$ and observed $z$ and $x$, we can therefore nonparametrically identify $f(c,z,x)$. In order to show point identification of $\gamma$, we need to verify that there exists at most one value of $c\in\Gamma$ such that $f(c_q,z,x)=0$ for all observed $z\in\mathrm{Supp}(\tilde{Z}_t)$ and $x\in\mathrm{Supp}(X_{jt})$. Taking the derivative of $f(c,z,x)$ with respect to $c$, we have

$$\frac{\partial f(c,z,x)}{\partial c} = \frac{\partial^2(h(c,z,x))}{\partial z \partial c}\frac{1}{h(c,z,x)} - \frac{\partial h(c,z,x)}{\partial z}\frac{h(c,z,x)}{\partial c}\frac{1}{h(c,z,x)^2}$$
$$= \frac{1}{h(c,z,x)}\frac{\partial^2 E\left(\frac{W_t}{cW_t - 1} \mid \tilde{Z}_t = z, X_{jt} = x\right)}{\partial z^2} -$$
$$\frac{1}{h(c,z,x)^2}\frac{\partial^2 E\left(\ln(cW_t - 1) \mid \tilde{Z}_t = z, X_{jt} = x\right)}{\partial z^2}\frac{\partial E\left(\frac{W_t}{cW_t - 1} \mid \tilde{Z}_t = z, X_{jt} = x\right)}{\partial z}.$$

The identification of $\gamma$ then follows from the assumption that there exists $(z,x)$ on the support of $(\tilde{Z}_t, X_{jt})$ such that $\frac{\partial f(c,z,x)}{\partial c}$ is strictly positive or strictly negative for all $c \in \Gamma$.

Given a unique $\gamma$, and the assumption that $\frac{h(\gamma,z,x)}{g(z,x)} \neq 0$, we can solve for $\sigma$ explicitly as

$$\sigma = 1 - \frac{h(\gamma,z,x)}{g(z,x)}.$$

Given $\gamma$ and $\sigma$, the model reduces to a standard multinomial logit model, and $\beta/(1-\sigma)$ is identified in a linear regression model. Given $\beta/(1-\sigma)$ and $\sigma$, we can solve for $\beta$. □

# B Robustness Tests in the Literature

A full list of all 29 papers in Table 1 is available upon requested. Here, I provide a brief overview of studies published in top 5 journals that have conducted robustness checks on the impact of market size definitions in demand models. For example, in Aguiar and Waldfogel (2018), while their primary measure of interest, the change in consumer surplus ratio ($\Delta CS$ Ratio), remains stable regardless of market size definitions, the absolute value of ($\Delta CS$) is sensitive. When the number of potential consumer is rescaled from 24 times to 6 times the number of internet users, $\Delta CS$ falls from 11.8 to 5.07, though the ratio between the $\Delta CS$ of two different counterfactual scenarios remains unchanged. Similarly, Wollmann (2018) studies mergers with endogenous repositioning and finds that total output and compensating variation are sensitive to market size assumptions. In the main text, total output decreases by 1.3%, with compensating variation ranging between \$22M and \$28M. However, increasing the market size by 7% results in a 0.2% increase in output and compensating variation turning negative, ranging from -\$4M to -\$15M. In Bourreau, Sun, and Verboven (2021), increasing the potential market size by 50% significantly alters both point estimates and confidence intervals, For example, the estimate of the random coefficient on price changes from -3.9 to -2.4, and for forfait bloque from 37.7 to 75.4. Standard errors also shift notably, from 0.6 to 1.8, and 5.5 to 21.5, respectively. Li (2018) tests sensitivity in the car market by doubling market size. The estimated impacts on sales and key parameters such as the price coefficient, as well as counterfactual outcomes remain relatively stable, with changes of at most 4%. However, it is important to note that doubling the total market size only shifts the implied outside option share from 97% to 99% in this context. Lastly, Egan, MacKay, and Yang (2022) examine the sensitivity of their results by scaling the outside option share by a factor of 5. Certain estimates like the expected return in the year of 2009, show notable changes, dropping from 30% to 10%.

These studies together demonstrate that while some results may be robust to market size changes, certain calculations and counterfactual results can still be quite sensitive.

# C Bias Caused by Mismeasured Market Size

I show that the usual approach that estimates demand based on equation (1) with a mismeasured market size will lead to biased estimates of $\beta$. To see this, suppose the true model is given by equation (5) with true value of $\gamma \neq 1$. Without loss of generality, let $s_{jt} = Q_{jt}/M_t$ and $s_{0t} = (M_t - Q_t^{total})/M_t$ denote the mismeasured market shares calculated based on the incorrect assumption that market size is $\tilde{\gamma} M_t$, with $\tilde{\gamma} = 1$. Define $\mu_{jt}$ to be the difference between the true choice probabilities $\ln(\pi_{jt}/\pi_{0t})$ and the mismeasured market shares $\ln(s_{jt}/s_{0t})$, so it gives the model that relates observed market shares to covariates and errors

$$\ln\left(\frac{s_{jt}}{s_{0t}}\right) = X'_{jt}\beta + \xi_{jt} + \mu_{jt},$$

with

$$\begin{aligned}
\mu_{jt} &= \ln\left(\frac{s_{jt}}{s_{0t}}\right) - \ln\left(\frac{\pi_{jt}}{\pi_{0t}}\right) \\
&= \ln\left(\frac{\gamma W_t - 1}{W_t - 1}\right) \\
&= \ln\left(1 \Big/ \left(\frac{1}{\gamma} + \left(\frac{1}{\gamma} - 1\right)\frac{1-\pi_{0t}}{\pi_{0t}}\right)\right)
\end{aligned}$$

by construction. The first equality is by the definition of $\mu_{jt}$. The second equality follows from the definition of mismeasured market shares and equations (1) and (5). The third equality follows from equation (4). It is not reasonable to believe that $\pi_{0t}$ would be independent of $X_{jt}$ because by the model, $\pi_{0t}$ depends on the characteristics of all goods. One possible technique to fix the problem is using a standard 2SLS regression or GMM with appropriate instruments. In this case, a valid instrument should be correlated with the demand covariates $X_{jt}$, and in the meanwhile, uncorrelated with $\pi_{0t}$, which again is a function of $X_{jt}$. In general, it is unlikely to construct an instrument that satisfies both restrictions.

Using the relationship provided above, we can predict the direction of the bias: Suppose that the observed market size is larger than the true size (i.e. $\gamma < 1$), the model predicts that the price of good $j$ will be positively correlated with $\mu_{jt}$, and negatively correlated with its own market share. Therefore, the estimate of the price coefficient will be biased downward (in absolute value), implying an underestimated price sensitivity.

# D Extension of the Simple Logit Case

$r_{jt}$ and $r^*_{jt}$ are defined as in section 3. Now we assume

$$\ln(r_{jt}) = \ln\left(r^*_{jt}\right) + e_{jt}.$$

Here, $e_{jt}$ is the error in $\ln(r_{jt})$ that we will later assume to have mean zero. It can include sampling errors, measurement errors, or aggregate unobserved heterogeneity in individual utility.

Then we assume that the mismeasurement in $W_t$ relative to $\pi_{0t}$ takes the form

$$\ln\left(\frac{\pi_{0t}}{1-\pi_{0t}}\right) = \ln(\gamma W_t - 1) + v_t$$

for some constant $\gamma$ and some random mean zero noise $v_t$. I add the error term $v_t$ to account for this relationship being approximate rather than exact. With the additional $v_t$, $1-\pi_{0t}$ would approximately equal $1/(\gamma W_t)$, and therefore $\ln(\pi_{0t}/(1-\pi_{0t}))$ would approximately equal $\ln(\gamma W_t - 1)$.

Putting the above equations and assumptions together we get the estimating equation

$$\ln(r_{jt}) = \ln(\gamma W_t - 1) + X'_{jt}\beta + u_{jt} \quad \forall j \in \mathcal{J}_t$$

where

$$u_{jt} = \xi_{jt} + e_{jt} + v_t.$$

To achieve identification as in section 3, we only need to modify the mean independence assumption such that $E(u_{jt} \mid Z_t, X_{1t}, \ldots, X_{J_t t}) = 0$, where everything else is defined as in section 3.

# E Market Fixed Effects Approach for Simple Logit

Returning to equation (5), observe that the term with the unknown $\pi_{0t}$ shows up additively, and it varies by market, not by product. I could allow for separate intercepts for each market to capture the unknown $\pi_{0t}$. The inclusion of the market level intercepts allows for unobserved aggregate market effects of the kind introduced by the presence of outside goods. Let $(\kappa_1, \cdots, \kappa_T)$ denote the aggregate market-varying and product-invariant parameters, then we can rewrite the model of equation (5) as

$$\ln(r_{jt}) \quad = \quad \kappa_t + X'_{jt}\beta + u_{jt} \quad \text{for each } t = 1, \cdots, T.$$

**Assumption 4.** *$E(u_{jt} \mid X_{jt}) = 0$ for all $t \in (1, \cdots, T)$. The support of $X_{jt}$ does not lie in a proper linear subspace of $\mathbb{R}^L$. The number of products $J \to \infty$.*

The conditional mean in Assumption 4 takes expectation across all products $j$ for a fixed market $t$. Assumption 4 first assumes all $X_{jt}$ are exogenous characteristics. Prices are taken to be exogenous throughout the context of the plain logit model for expositional purposes. Assumption 4 also imposes no multicollinearity requirements on $X_{jt}$.

**Theorem 5.** *Let Assumption 4 hold. Let $\beta^0$ be the coefficient on the constant. Normalize $\beta^0 = 0$. Then $(\kappa_1, \cdots, \kappa_T, \beta)$ are identified.*

The proof is in Supplement A. Theorem 5 indicates that all parameters are identified except for the constant. This result has straightforward and important implications for how one can deal with the unobserved market size. In particular, when we observe data from a single market ($T = 1$), estimating $\kappa_t$ resembles estimating the constant term. The desirable thing is that it would only bias the estimate of the constant in the consumer's indirect utility function and does not affect estimates of elasticities. For $T \geq 2$, when there are repeated measures of the same market/region over multiple time periods, or when we have cross-sectional data from more than one market/region, including market or time dummies in the model ensures consistent estimation of all parameters but the constant.

However, this method comes with some costs. First, it incurs efficiency loss because the data variation across markets is not exploited. Moreover, coefficients of market-level regressors will not be identified, so we cannot estimate marginal effects of any market characteristics. The biggest limitation is that this method relies on the functional form of the model specification. It works only in the plain logit model as a special case and cannot be generalized to the random coefficients demand model (see section 4.5).

# F Identification of Market Size in Nested Logit Model

Following the nested logit framework in McFadden (1977) and Cardell (1997), we assume the utility of consumer $i$ for product $j$ belonging to group $g$ is

$$U_{ijt} = \delta_{jt} + \zeta_{igt} + (1-\rho)\varepsilon_{ijt},$$

where $\delta_{jt} = X'_{jt}\beta + \xi_{jt}$ and $\varepsilon_{ijt}$ is independently and identically distributed with extreme value type I distribution as before. The unobserved group specific taste $\zeta_{igt}$ follows a distribution such that $\zeta_{igt} + (1-\rho)\varepsilon_{ijt}$ is also distributed extreme value. $\rho$ measures the correlation of unobserved utility among products in group $g$. A larger value of $\rho$ indicates greater correlation within nest. When $\rho = 0$, the within group correlation of unobserved utility is zero, and the nested logit model degenerates to the plain multinomial logit model.

Berry (1994) shows that demand parameters $\beta$ and $\rho$ can be consistently estimated from a linear regression similar to the logit equation (1), with an additional regressor $\ln(\pi_{j|gt})$,

$$\ln(\pi_{jt}/\pi_{0t}) = X'_{jt}\beta + \rho \ln(\pi_{j|gt}) + \xi_{jt}, \tag{21}$$

where $\pi_{j|gt}$ is the conditional choice probability of product $j$ given that a product in group $g$ is chosen.

Consider the case where all goods are divided up into two nests, with the outside good as the only choice in group $g = 0$ and all inside goods belonging to group $g = 1$. In this case, $\pi_{j|gt} = r^*_{jt}$ for $j \neq 0$, where $r^*_{jt}$ is defined in section 3. Then we can rewrite (21) as

$$\ln\left(r^*_{jt}\right) = \frac{1}{1-\rho}\ln\left(\frac{\pi_{0t}}{1-\pi_{0t}}\right) + X'_{jt}\frac{\beta}{1-\rho} + \frac{\xi_{jt}}{1-\rho}.$$

Following the same exposition of the market size model as in section 3, we assume equation (4) hold. Combining above equations and assumptions we get the estimating equation for the nested logit model

$$\ln(r_{jt}) = \frac{1}{1-\rho}\ln(\gamma W_t - 1) + X'_{jt}\frac{\beta}{1-\rho} + \frac{\xi_{jt}}{1-\rho}. \tag{22}$$

**Theorem 6.** *Given Assumption 1 and equation (22), let $\Gamma$ be the set of all possible values of $\gamma$, if*

1. *all relevant first and second order derivatives exist,*
2. *$\partial f(c,z,x)/\partial c > 0$ or $< 0$ for all $c \in \Gamma$, where*

$$f(c,z,x) = \frac{\partial h(c,z,x)}{\partial z}\frac{1}{h(c,z,x)} - \frac{\partial g(z,x)}{\partial z}\frac{1}{g(z,x)},$$
$$g(z,x) = \frac{\partial E\left(\ln(r_{jt}) \mid \tilde{Z}_t = z, X_{jt} = x\right)}{\partial z},$$
$$h(c,z,x) = \frac{\partial E\left(\ln(cW_t - 1) \mid \tilde{Z}_t = z, X_{jt} = x\right)}{\partial z},$$

3. *and $h(c,z,x) \neq 0$ for all $c \in \Gamma$.*

*Then $\gamma$, $\beta$ and $\rho$ are identified.*

The proof of theorem 6 works by showing that there exists $z$ and $x$ such that $f(c,z,x) = 0$ has a unique solution of $c$. In practice, if $\tilde{Z}_t$ is a scalar random variable, we can use $\tilde{Z}_t$ and any nonlinear function of $\tilde{Z}_t$ as instruments to estimate $\gamma$ and $\rho$. Nonlinear functions of $\tilde{Z}_t$ (e.g. $\sqrt{\tilde{Z}_t}$ or $\tilde{Z}_t^2$) will have additional explanatory power to separately identify $\gamma$ and $\rho$. A Monte Carlo simulation for the nested logit model is available upon request.

One might be concerned that the identification result of theorem 6 relies on the functional form assumption we made in equation (4). There might exist some different functional form assumption of market size which would make $\gamma$ and $\rho$ unidentified. For example, the model would be unidentified by letting the true market size be $(\exp(\gamma\tilde{W}_t)+1)Q_t^{total}$, for some variable $\tilde{W}_t$. In this case, equation (22) reduces to $\ln(r_{jt}) = 1/(1-\rho)\gamma\tilde{W}_t + X'_{jt}\beta/(1-\rho) + \xi_{jt}$. However, a market size model of this form is odd and lack of economic meaning.

# G Testing for Relevance of Instruments

Gandhi and Houde (2019) show that the relevance of instruments in BLP models can be tested by estimating a plain logit regression on product characteristics and instruments, with the coefficients determining the strength of these instruments. I re-define the parameters and show that the same test of instrument relevance can be applied in the setting of this paper, for both the random coefficients and the market size parameter.

Gandhi and Houde (2019) use $\lambda$ to denote the vector of parameters that determine the joint distribution of the random coefficients. Here I follow this notation and extend it to include the market size parameters. Specifically, let $\lambda_\sigma = \sigma$, $\lambda_{\gamma_1} = \gamma_1 - 1$ and $\lambda_{\gamma_2} = \gamma_2$, and $\lambda = (\lambda_\sigma, \lambda_{\gamma_1}, \lambda_{\gamma_2})$ be the full vector of nonlinear parameters in the model. By absorbing $\lambda_\gamma$ into the conditioning parameter vector, we rewrite equation (9) as

$$\xi_{jt}\left(Q_t, M_t, X_t; \theta\right) = \delta_{jt}\left(Q_t, M_t, X_t^{(2)}; \lambda\right) - X_{jt}'\beta. \tag{23}$$

Equation (23) encompasses equation (9) and is similar to equation (4) in Gandhi and Houde (2019). Here I have $(Q_t, M_t)$ instead of the observed market shares $s_t$ in their function.

The endogenous problem arises for $\lambda_\sigma$ and $\lambda_\gamma$ because the inverse demand function depends on quantities $Q_t$ (or market shares) of all products, and these endogenous quantities interact nonlinearly with $\lambda_\sigma$ and $\lambda_\gamma$ in the inverse demand function. Therefore, we need instrumental variables for quantities (or market shares) of products to identify $\lambda_\sigma$ and $\lambda_\gamma$. This is the nonlinear simultaneous equations model that has been previously studied by Jorgenson and Laffont (1974) and Amemiya (1974). Unlike in linear models, where the strength of instruments can be assessed by linear regression of endogenous variables on excluded instruments, for nonlinear models, how to detect weak instruments is not obvious.

I use the method as in Gandhi and Houde (2019) to test the relevance of instruments for identifying $\lambda_\sigma$ and $\lambda_\gamma$, which I summarize here. By equation (7) in Gandhi and Houde (2019), the reduced form of the inverse demand function $E\left(\delta_{jt}(Q_t, M_t, X_t^{(2)}; \lambda) \mid Z_t\right)$ can be approximated by a linear projection onto functions of instruments:

$$E\left(\delta_{jt}\left(Q_t, M_t, X_t^{(2)}; \lambda\right) \mid Z_t\right) \approx \phi_j(Z_t)'\alpha.$$

Definition 1 in Gandhi and Houde (2019) provides a practical method referred to as "IIA-test" to detect the strength of the instruments by evaluating the inverse demand function at $\lambda = 0$ (suppose the true parameters are $\lambda_0 \neq 0$). Evaluating the inverse demand function at $\lambda_\sigma = \lambda_{\gamma_1} = \lambda_{\gamma_2} = 0$, we have

$$\begin{aligned} E\left(\delta_{jt}\left(Q_t, M_t, X_t^{(2)}; \lambda = 0\right) \mid Z_t\right) &= E\left(\ln\left(\frac{Q_{jt}}{M_t - \sum_{j=1}^{J_t} Q_{jt}}\right) \mid Z_t\right) \\ &\approx X_{jt}'\alpha_1 + \alpha_p \hat{P}_{jt} + \phi_j^{-X}(Z_t)'\alpha_2, \end{aligned}$$

where $\hat{P}_{jt}$ is the projection of prices on $X_t$ and price instruments, and $\phi_j^{-X}(Z_t)$ is a subvector of instruments excluding $X_{jt}$. Note that $\hat{P}_{jt}$ is constructed based on exogenous variables and thus satisfied the mean independence restriction of Assumption 3. The regression relates the observed product quantities to product characteristics and functions of instruments. The null hypothesis of the test is that the model exhibits IIA preference and market shares calculated by $Q_{jt}/M_t$ are not mismeasured. One can reject the null hypothesis when the parameter vector $\alpha_2$ in the reduced form regression is different from zero. On the other hand, when $\alpha_2$ is close to zero, it indicates that the instruments are weak.

# H A Peculiar Case of Nonparametric Random Coefficients

In this section I show that identifying and estimating market size in the form of $\gamma M_t$ can be equivalent to nonparametric identification and estimation of a peculiar form of random coefficients. On one hand, this provides a structural interpretation of the $\gamma M_t$ specification. On the other hand, it explains why flexible random coefficients can partly address the market size issue. More specifically, consider a model with indirect utility given by equation (6) and $\beta_i \sim F(\beta)$ follows an unknown distribution. Identifying and estimating $F(\beta)$ can be done nonparametrically. Following the approach of Fox, Kim, and Yang (2016) (Example 1 in their paper), using a sieve space approximation to the distribution of random coefficients, we can write

$$\pi_{jt}(\delta_{jt};\sigma) = \sum_{r=1}^{R} \sigma_r \frac{\exp\left(\delta_{jt} + \sum_l \eta_l^r x_{jtl}\right)}{1 + \sum_{k=1}^{J_t} \exp(\delta_{kt} + \sum_l \eta_l^r x_{ktl})} \tag{24}$$

with restrictions

$$\sum_{r=1}^{R} \sigma_r = 1 \text{ and } 0 \leq \sigma_r \leq 1,$$

where $\eta_l = (\eta_l^1 \cdots \eta_l^R)$ is a fixed grid of values chosen by researchers. Parameters to be estimated are the weights $\sigma = (\sigma_1 \cdots \sigma_R)$.

Consider a special case where there are only two types of consumers ($R = 2$), and we aim to identify the probability mass of each type of consumer. Suppose, without loss of generality, that only the constant term has a random coefficient. Let $\eta_1 = -\infty$ and $\eta_2 = 0$ (any values other than 0 would be absorbed into the constant term of $\delta$). The model reduces to $\pi_{jt}(\delta_{jt};\sigma) = \sigma_2 \frac{\exp(\delta_{jt})}{1+\sum_{k=1}^{J_t}\exp(\delta_{kt})}$. Note that $\sigma_2$ plays the same role as the scalar $\gamma$ discussed in section 3 for the simple logit model when $\gamma < 1$. This result can be extended to $R > 2$. If an element of $\eta$ is $-\infty$, it implies that certain consumers will never purchase any inside goods under any circumstances. These consumers should not be considered *potential* consumers and should be excluded from the measure of market size[1]. In general, the most flexible model of this kind can be approximated by a distribution with a probability mass at negative infinity.

Nonparametric random coefficients can address the unknown market size issue if the distribution follows the specified form. Identification of random coefficients distribution of this particular type (one that has a probability mass point at negative infinity) would require strong assumptions. In the literature on nonparametric identification of random coefficients for aggregate demand, Berry and Haile (2014) and Dunker, Hoderlein, and Kaido (2022) prove identification of random coefficients without any restriction on the distribution (i.e., allow for infinite absolute moments). However, both require full/large support of product characteristics or prices (e.g., Assumption 3.3(i) in Dunker, Hoderlein, and Kaido 2022).

1. A limitation of our market size model is that, for example, when $\gamma = 0.5$ (or equivalently, $\sigma_2 = 0.5$), we cannot differentiate between half of the population never buying any inside goods and the rest buying one unit per person on average, versus the entire population buying half a unit per person on average.

# I Additional Monte Carlo Results

The data generating process for the simulation datasets follows closely that in Armstrong (2016), but we only consider small $J$ environments to avoid the weak instruments problem Armstrong raised. Prices are endogenously generated from a demand and supply model, where firms compete a la Bertrand in the market. In the baseline design of the Monte Carlo study, the number of products varies across markets. 2/3 of markets have 20 products per market, and the remaining 1/3 of markets have 60 products in the market. Each firm has 2 products. Other choices of number of products per firm do not significantly alter the results. I consider a relatively small sample size of $T = 100$. I use $R = 1000$ replications of each design.

Consumer utility is given by the random coefficients model described in section 4

$$U_{ijt} = \beta_0 + (\beta_p + \sigma\nu_i)P_{jt} + \beta_1 X_{1,jt} + \xi_{jt} + \varepsilon_{ijt}, \tag{25}$$

where $\nu_i$ is generated from a standard normal distribution. Firm marginal cost is $MC_{jt} = \alpha_0 + \alpha_1 X_{1,jt} + \alpha_2 X_{S,jt} + \eta_{jt}$. $\xi_{jt}$ and $\eta_{jt}$ are generated from a mean-zero bivariate normal distribution with standard deviations $\sigma_\xi = \sigma_\eta = 0.8$ and covariance $\sigma_{\xi\eta} = 0.2$. $X_{1,jt}$ and the excluded cost shifter $X_{S,jt}$ are drawn from a uniform $(0, 1)$ distribution and independent of each other. All random variables are independent across products $j$ and markets $t$.

The true values of cost parameters are $(\alpha_0, \alpha_1, \alpha_2) = (2, 1, 1)$. Demand coefficients and the random coefficient take different values depending on designs.

I compute the true choice probabilities $\pi_{jt}$ in accordance with equation (7). By equations (4), we can compute $Q_{jt}/M_t = \gamma\pi_{jt}$, where the true value is $\gamma = 1$ throughout the Monte Carlo exercise. In the estimation, one assumes a possibly wrong $\tilde{\gamma}$ and uses the mismeasured $s_{jt} \equiv Q_{jt}/\tilde{\gamma}M_t$ as the observed market shares.

The instruments we use in the GMM estimation in all experiments are

$$Z_{jt} = (1, X_{1,jt}, \sum_{k=1}^{J_t} X_{1,kt}, \sum_{k\in\mathcal{J}_f} X_{1,kt}, X_{S,jt}, X_{S,jt}^2),$$

where product $j$ is produced by firm $f$ and $\mathcal{J}_f$ is the set of all products produced by firm $f$. I include BLP-type instruments or Gandhi and Houde differentiation instruments as well as functions of excluded cost instruments. The optimization algorithm we use for the GMM estimation is the gradient-based quasi-Newton algorithm (fminunc in MATLAB).

## I.1 Random Coefficients on Constant Term and Price

The first simulation is designed to assess whether and to what extent random coefficients removes the biases induced from the wrong market size. I generate data from a plain logit model ($\sigma = 0$ in the model of equation (25)). It is widely believed that random coefficients partly take over the role of $\gamma$ and can fix issues caused by unobserved market size. To see if this is true, for each of the 1,000 simulated datasets, we consider three values of $\tilde{\gamma}$ ($\tilde{\gamma} = 1, 2, 4$) and estimate both the correctly specified plain logit model and the random coefficients model with a random coefficient on the constant term and price, respectively. I assume that the true demand coefficients are $\beta = (2, -1, 2)$.

Tables I.1 to I.3 report results from estimating the plain logit model and the more flexible random coefficients models. Each table shows results for three different assumed market size $\tilde{\gamma}$. I report estimates of $\beta, \sigma$, and nonlinear functions of demand parameters, including the own- and cross-price elasticities, and diversion ratios averaged across products for the first market. Reported summary statistics of each parameter estimate across simulations are the mean (MEAN), the standard deviation (SD), and the median (MED).

In Table I.1, comparing to estimates for the specification with correctly measured market size ($\tilde{\gamma} = 1$) in the first three columns, the means of $\beta$'s change monotonically as we increase the assumed market size, and their standard deviations change as well. The implied elasticities and diversion ratios are all sensitive to the assumed market size. When we quadruple the assumed market size, the mean of the own-price elasticity increases from $-5.99$ to $-4.17$, the cross-price elasticity decreases from 0.077 to 0.028, the individual diversion ratio falls by half and the diversion to the outside good rises from around 17% to 79%.

Table I.2 shows the results for estimating the random coefficients model with a random coefficient on the constant term. Although the incorrectly assumed market size results in biased estimates of $\beta$'s, the own-price elasticities and individual diversion ratios of $\tilde{\gamma} = 2, 4$ are comparable to the ones of $\tilde{\gamma} = 1$. The cross-price elasticities of the model with incorrectly assumed market size are also closer to those of $\tilde{\gamma} = 1$, relative to the plain logit model in Table I.1 (decreases from 0.078 to 0.069 versus from 0.077 to 0.028). In contrast, the biases in the outside good elasticity and outside good diversion ratio remain large. When we quadruple the assumed market size, the mean of outside good diversion ratio rises from roughly 17% to 27% and the outside-good price elasticity decreases from 0.077 to 0.007.

In Table I.3, we estimate the model with a random coefficient on price. Including the random coefficient improves especially the estimates of own- and cross-price elasticities as well as individual diversion ratios, similar to those in Table I.2.

Although not shown in the table, we also experimented with different numbers of products per market. The design where the number of products varies across markets generally yields larger biases than the design where the number of products is fixed.

Finally, in Table I.4, we report the estimates from our proposed method of equation (5). Results are based on the IV-GMM estimation that uses cost shifters and sum of characteristics as instruments for both price and the observed market to sales variable $W_t$ defined in section 3. Estimates of $\beta$ and $\gamma$ are very close to the true values, with small standard deviations. The implied elasticities and diversion ratios are quite comparable to the estimates of the logit model with correctly assumed market size shown in the first three columns of Table I.1.

To summarize, we find that including a random coefficient on either term accounts for the incorrectly assumed $\tilde{\gamma}$, so that the biases in certain calculations are relatively small. This finding is consistent with the intuition that $\sigma$ partly corrects for the mismeasured market size. However, biases in other substitution patterns such as cross-price elasticities, outside-good elasticities and diversion ratios are not fully removed.

Table I.1: Monte Carlo Results: Plain Logit, True $\gamma = 1$

| | | $\tilde{\gamma} = 1$ | | | $\tilde{\gamma} = 2$ | | | $\tilde{\gamma} = 4$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | TRUE | MEAN | SD | MED | MEAN | SD | MED | MEAN | SD | MED |
| $\beta_0$ | 2 | 1.99 | 0.318 | 2.006 | -1.205 | 0.534 | -1.192 | -2.401 | 0.594 | -2.379 |
| $\beta_p$ | -1 | -0.998 | 0.056 | -1.002 | -0.731 | 0.094 | -0.732 | -0.688 | 0.105 | -0.691 |
| $\beta_1$ | 2 | 1.998 | 0.076 | 2 | 1.725 | 0.105 | 1.724 | 1.681 | 0.114 | 1.681 |
| Own-Elasticity | | -5.994 | 0.354 | -6.006 | -4.415 | 0.584 | -4.418 | -4.17 | 0.649 | -4.181 |
| Cross-Elasticity | | 0.077 | 0.005 | 0.077 | 0.028 | 0.004 | 0.028 | 0.013 | 0.002 | 0.013 |
| Outside-Good Elasticity | | 0.077 | 0.005 | 0.077 | 0.028 | 0.004 | 0.028 | 0.013 | 0.002 | 0.013 |
| Diversion Ratio | | 0.014 | 0 | 0.014 | 0.007 | 0 | 0.007 | 0.003 | 0 | 0.003 |
| Outside-Good Diversion | | 0.167 | 0.027 | 0.166 | 0.587 | 0.013 | 0.586 | 0.794 | 0.007 | 0.794 |

Notes: The table reports the empirical mean (MEAN), the standard deviation (SD), and the median (MED) of the demand parameters, the implied price elasticities and diversion ratios for the first market. The GMM estimates are based on $1,000$ generated data sets of sample size $T = 100$ and varied $J$. The true model is a plain logit model, with $\gamma = 1$. Parameters are estimated from the plain logit model assuming $\tilde{\gamma} = 1, 2, 4$.

Table I.2: Monte Carlo Results: Random Coefficient on Constant Term, True $\gamma = 1$

| | | $\tilde{\gamma} = 1$ | | | $\tilde{\gamma} = 2$ | | | $\tilde{\gamma} = 4$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | TRUE | MEAN | SD | MED | MEAN | SD | MED | MEAN | SD | MED |
| $\sigma$ | 0 | 0.037 | 0.273 | 0 | 3.998 | 0.168 | 3.992 | 5.116 | 0.172 | 5.11 |
| $\beta_0$ | 2 | 2.039 | 0.343 | 2.05 | 0.86 | 0.333 | 0.862 | -1.806 | 0.321 | -1.79 |
| $\beta_p$ | -1 | -1.003 | 0.057 | -1.005 | -1.001 | 0.058 | -1.003 | -1.001 | 0.058 | -1.003 |
| $\beta_1$ | 2 | 2.003 | 0.076 | 2.005 | 2.004 | 0.078 | 2.005 | 2.004 | 0.078 | 2.005 |
| Own-Elasticity | | -6.022 | 0.357 | -6.031 | -6.018 | 0.364 | -6.029 | -6.02 | 0.365 | -6.03 |
| Cross-Elasticity | | 0.078 | 0.005 | 0.078 | 0.069 | 0.005 | 0.069 | 0.068 | 0.005 | 0.068 |
| Outside-Good Elasticity | | 0.077 | 0.005 | 0.077 | 0.017 | 0.001 | 0.017 | 0.007 | 0 | 0.007 |
| Diversion Ratio | | 0.014 | 0 | 0.014 | 0.013 | 0 | 0.013 | 0.012 | 0 | 0.012 |
| Outside-Good Diversion | | 0.166 | 0.027 | 0.165 | 0.255 | 0.01 | 0.255 | 0.271 | 0.009 | 0.271 |

Notes: The table reports the empirical mean (MEAN), the standard deviation (SD), and the median (MED) of the demand parameters, the implied price elasticities and diversion ratios for the first market. The GMM estimates are based on $1,000$ generated data sets of sample size $T = 100$ and varied $J$. The true model is a plain logit model, with $\gamma = 1$. Parameters are estimated from a random coefficients model with the random coefficient on the constant term, assuming $\tilde{\gamma} = 1, 2, 4$.

Table I.3: Monte Carlo Results: Random Coefficient on Price, True $\gamma = 1$

| | | $\tilde{\gamma} = 1$ | | | $\tilde{\gamma} = 2$ | | | $\tilde{\gamma} = 4$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | TRUE | MEAN | SD | MED | MEAN | SD | MED | MEAN | SD | MED |
| $\sigma$ | 0 | 0.013 | 0.064 | 0 | 0.712 | 0.057 | 0.712 | 0.92 | 0.044 | 0.919 |
| $\beta_0$ | 2 | 2.063 | 0.534 | 2.057 | 2.946 | 0.417 | 2.951 | 2.879 | 0.408 | 2.88 |
| $\beta_p$ | -1 | -1.005 | 0.074 | -1.006 | -1.39 | 0.071 | -1.389 | -1.86 | 0.084 | -1.858 |
| $\beta_1$ | 2 | 2.006 | 0.09 | 2.006 | 2.013 | 0.08 | 2.013 | 2.013 | 0.08 | 2.014 |
| Own-Elasticity | | -6.034 | 0.434 | -6.031 | -6.005 | 0.402 | -6.013 | -6.026 | 0.403 | -6.032 |
| Cross-Elasticity | | 0.078 | 0.007 | 0.078 | 0.065 | 0.006 | 0.065 | 0.063 | 0.005 | 0.063 |
| Outside-Good Elasticity | | 0.078 | 0.005 | 0.078 | 0.025 | 0.002 | 0.025 | 0.01 | 0.001 | 0.01 |
| Diversion Ratio | | 0.014 | 0 | 0.014 | 0.012 | 0 | 0.012 | 0.011 | 0 | 0.011 |
| Outside-Good Diversion | | 0.167 | 0.027 | 0.165 | 0.308 | 0.019 | 0.308 | 0.329 | 0.02 | 0.329 |

Notes: The table reports the empirical mean (MEAN), the standard deviation (SD), and the median (MED) of the demand parameters, the implied price elasticities and diversion ratios for the first market. The GMM estimates are based on $1,000$ generated data sets of sample size $T = 100$ and varied $J$. The true model is a plain logit model, with $\gamma = 1$. Parameters are estimated from a random coefficients model with the random coefficient on price, assuming $\tilde{\gamma} = 1, 2, 4$.

Table I.4: Monte Carlo Results: Estimating $\gamma$ in the Plain Logit Model

| | TRUE | MEAN | SD | MED |
|---|---|---|---|---|
| $\gamma$ | 1 | 1.001 | 0.011 | 1.001 |
| $\beta_0$ | 2 | 1.99 | 0.341 | 1.993 |
| $\beta_p$ | -1 | -0.999 | 0.058 | -1 |
| $\beta_1$ | 2 | 1.999 | 0.077 | 2 |
| Own-Elasticity | | -5.996 | 0.362 | -6.004 |
| Cross-Elasticity | | 0.077 | 0.005 | 0.077 |
| Outside-Good Elasticity | | 0.077 | 0.005 | 0.077 |
| Diversion Ratio | | 0.014 | 0 | 0.014 |
| Outside-Good Diversion | | 0.168 | 0.028 | 0.167 |

Notes: The table reports the empirical mean (MEAN), the standard deviation (SD), and the median (MED) of the demand parameters, the implied price elasticities and diversion ratios for the first market. The GMM estimates are based on $1,000$ generated data sets of sample size $T = 100$ and varied $J$. The true model is a plain logit model. Parameters $\beta$ and $\gamma$ are estimated from IV-GMM estimations using excluded cost shifters and BLP instruments.

## I.2 Sensitivity to Market Size Assumption

The second experiment complements the first experiment. I now generate data from a random coefficients model, with a random coefficient for the price. More specifically, we assume that $\beta = (2, -2, 2)$, and $\sigma = 1$. For each of the $1,000$ simulated datasets, we estimate the random coefficients model and consider four values of $\tilde{\gamma}$ ($\tilde{\gamma} = 1, 2, 4, 8$). This experiment is designed to assess how parameter estimates and the implied substitution patterns vary with market size assumptions in a random coefficients logit model.

Table I.5 shows results of demand estimates and the implied statistics. Some general tendencies stand out. First, consumer heterogeneity ($\sigma$) and disutility for price ($\beta_p$) tend to be overestimated as $\tilde{\gamma}$ increases. The direction of biases in $\beta_0$ is ambiguous. Second, the implied elasticities and diversion ratios give similar results as those in Table I.3. The outside-good elasticities and the outside-good diversion ratios are most sensitive to the choice of $\tilde{\gamma}$. The cross-price elasticities are also affected, but not as sensitive as the former two calculations. However, biases in elasticities and diversion ratios tend not to be monotonic in $\tilde{\gamma}$. For instance, $\tilde{\gamma} = 2$ leads to an upward bias of the diversion to outside good (from around 17% to 20%), but $\tilde{\gamma} = 4$ gives a modest downward bias of the outside-good diversion (from 17% to 16%). The extreme case, which imposes $\tilde{\gamma} = 8$, results in a much larger bias (from 17% to 25%). Hence, imposing different assumptions of the market size is not a simple rescaling of the calculations. This again confirms that random coefficients logit models do not correct for all biases induced by wrong market size assumptions.

Table I.5: Sensitivity to Market Size Assumptions in Random Coefficients Logit, True $\gamma = 1$

| | | $\tilde{\gamma} = 1$ | | | $\tilde{\gamma} = 2$ | | | $\tilde{\gamma} = 4$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | TRUE | MEAN | SD | MED | MEAN | SD | MED | MEAN | SD | MED |
| $\sigma$ | 1 | 1 | 0.034 | 0.999 | 1.413 | 0.036 | 1.413 | 2.646 | 0.173 | 2.629 |
| $\beta_0$ | 2 | 2.012 | 0.447 | 1.999 | 1.431 | 0.396 | 1.418 | 2.164 | 0.604 | 2.143 |
| $\beta_p$ | -2 | -2.001 | 0.068 | -2 | -2.68 | 0.069 | -2.681 | -4.604 | 0.273 | -4.577 |
| $\beta_1$ | 2 | 1.998 | 0.054 | 2.001 | 1.984 | 0.055 | 1.987 | 2 | 0.055 | 2.001 |
| Own-Elasticity | | -7.095 | 0.328 | -7.079 | -6.922 | 0.334 | -6.913 | -7.025 | 0.392 | -6.986 |
| Cross-Elasticity | | 0.077 | 0.005 | 0.076 | 0.071 | 0.004 | 0.071 | 0.075 | 0.005 | 0.074 |
| Outside-Good Elasticity | | 0.029 | 0.003 | 0.029 | 0.011 | 0.001 | 0.011 | 0.004 | 0 | 0.004 |
| Diversion Ratio | | 0.014 | 0 | 0.014 | 0.014 | 0 | 0.014 | 0.014 | 0.001 | 0.014 |
| Outside-Good Diversion | | 0.175 | 0.025 | 0.176 | 0.201 | 0.022 | 0.201 | 0.167 | 0.033 | 0.168 |
| | | $\tilde{\gamma} = 8$ | | | | | | | | |
| $\sigma$ | 1 | 2.427 | 0.048 | 2.426 | | | | | | |
| $\beta_0$ | 2 | -1.252 | 0.416 | -1.247 | | | | | | |
| $\beta_p$ | -2 | -4.307 | 0.091 | -4.306 | | | | | | |
| $\beta_1$ | 2 | 1.91 | 0.066 | 1.909 | | | | | | |
| Own-Elasticity | | -5.84 | 0.445 | -5.826 | | | | | | |
| Cross-Elasticity | | 0.052 | 0.004 | 0.052 | | | | | | |
| Outside-Good Elasticity | | 0.002 | 0 | 0.002 | | | | | | |
| Diversion Ratio | | 0.013 | 0 | 0.013 | | | | | | |
| Outside-Good Diversion | | 0.247 | 0.023 | 0.246 | | | | | | |

Notes: The table reports the empirical mean (MEAN), the standard deviation (SD), and the median (MED) of the demand parameters, the implied price elasticities and diversion ratios for the first market. The GMM estimates are based on $1,000$ generated data sets of sample size $T = 100$ and varied $J$. The true model is a random coefficients logit model with a random coefficient for price, with $\gamma = 1$. Parameters are estimated from the random coefficients model, assuming $\tilde{\gamma} = 1, 2, 4, 8$.

# J Additional Empirical Application: The Soda Market

## J.1 Carbonated Soft Drink (CSD) Market

The soft drink market has received significant attention in the literature, primarily driven by health and regulatory concerns. The conventional discrete choice model remains a widely used approach in modeling consumer purchasing behavior in this field of research.

The soft drink market is suitable for this study due to three key factors. First, the existing literature lacks a consensus on how to define market size. Second, this industry is one where we generally believe the outside option is not too large. Our simulation findings suggest that the proposed method achieves stronger identification in cases where the true choice probability of the outside option is not excessively large. Third, the occurrence of several horizontal mergers in the soft drink industry in recent years. For example, in 2018, the Coca-Cola Company acquired Costa Coffee and PepsiCo acquired SodaStream.

## J.2 Data

I use a panel of weekly scanner data from NielsenIQ for our analysis. The NielsenIQ scanner data provides comprehensive information on prices, sales, and product attributes, including package size, flavor, and nutritional contents. The dataset covers 202 designated market areas (DMAs) in the US and spans 52 weeks, encompassing the period from January 2019 to December 2019. I aggregate the dataset from the retailer level to the market level. Consistent with the literature, I define a market as a combination of a specific DMA and week, resulting in a total of 10504 DMA-week markets[2].

In addition to the NielsenIQ data, I augment the dataset with input price information, which serves as excluded price instruments. This includes raw sugar prices from the US Department of Agriculture, Economic Research Service; local wage from the U.S. Bureau of Labor Statistics; as well as electricity and fuel prices from the US Department of Energy, Energy Information Administration.

Following Eizenberg and Salvo (2015), I aggregate flavors and products in different sized packages into 15 brand-groups, denoted as $j = 1, \cdots, 15$ (e.g., Coca-Cola Cherry 12-oz and Coca-Cola Original 16.9-oz are treated as the same brand). Following Dubé (2005), I consider diet and regular drinks as separate brands due to their distinct target demographics and separate advertising and promotion strategies within the industry. These brand categories include 11 brands owned by the three leading companies. The 12th and 13th brand categories represent aggregate private label (PL) brands for regular and diet drinks, respectively. To account for numerous niche brands (each with a volume share below 1 percent), I aggregate them into the 14th and 15th brand categories for regular and diet drinks, respectively. By doing so, I implicitly assume that product differentiation among these small brands is not of importance in the context of our study. I limit the sample to soft drinks sold in package types that have substantial sales, specifically including the 12-pack of 12-oz cans, 67.6-oz bottle, 6-pack of 16.9-oz bottles, 20-oz bottle, and 8-pack of 12-oz cans. These five package sizes dominate in terms of volume sales compared to other package types.

## J.3 Demand Model

As in section 4, the indirect utility of consumer $i$ in market $t$ from consuming brand $j$ is given by $U_{ijt} = \delta_{jt} + \sigma\nu_i P_{jt} + \varepsilon_{ijt}$. $\delta_{jt}$ denotes a market-specific, individual-invariant mean utility

2. I drop markets with extremely large or small sales relative to their respective populations, leaving us with $9,658$ markets.

from brand $j$: $\delta_{jt} = X'_{jt}\beta + \alpha P_{jt} + \xi_{jt}$. $X_{jt}$ includes in-store presence, brand fixed effects, seasonal effects and region fixed effects. In-store presence is measured by the proportion of stores within a market that carry a particular brand. Brand fixed effects capture the time invariant unobserved product characteristics, while seasonal effects capture temporal demand fluctuations. $P_{jt}$ represents the price of brand $j$, and $\xi_{jt}$ denotes demand shocks specific to a brand-market combination, observable to consumers but unobservable to the econometrician. The second term $\sigma\nu_i P_{jt}$ introduces consumer heterogeneity. $\nu_i$ follows a standard normal distribution. $\varepsilon_{ijt}$ follow the Extreme Value Type I distribution and are iid across consumers, brands, and markets.

One issue is the potential endogeneity of in-store presence due to correlation with unobservables $\xi_{jt}$, if local assortments cater to local demand (Quan and Williams (2018)). I address this potential endogeneity concern by flexibly controlling for brand-, quarter- and region-specific fixed effects. With a rich set of fixed effects included, the unobservables that remain are brand-region specific demand shocks that vary by time. I assume retailers or firms lack full information on consumer preferences in the sense that they do not observe these demand shocks when making product assortment decisions. It is worth noting that in-store presence has been used as an exogenous covariate in previous studies such as Eizenberg and Salvo (2015). Similarly, in the airline industry, *carrier presence* is often considered as an exogenous attribute. The economic interpretation of in-store presence in the present context aligns closely with carrier presence in the airline market. Just as carrier presence may raise concerns of endogeneity, it has typically been addressed through via fixed effects.[3][4]

Table J.1 presents summary statistics for prices and in-store presence in the dataset, demonstrating their sufficient variation. Prices and in-store presence are averaged across all UPCs within each brand, weighted by the volume sales of UPCs. The last three columns of Table J.1 show the percentage of variance explained by brand, DMA, and month dummy variables. The results indicate that a majority of the variation in prices and in-store presence is attributed to differences between brands. After accounting for this brand-level variation, the remaining variation is primarily driven by disparities across geographic areas.

3. Ackerberg and Rysman (2005) deal with what they call "product crowding" effect by including retail presence in the estimating equation, where the the number of retail stores carrying a product is parametrically specified as a function of number of products $J$. We acknowledge that $J$ may affect the differences in the assortments or in-store presence. However, in my application, this may be less of a concern due to minimal or no variation in $J$ across markets but significant variation in the in-store presence.

4. If stores make assortment decisions after the realization of all demand shocks (as assumed in Ciliberto, Murry, and Tamer 2021), fixed effects may not fully address the endogeneity of in-store presence. As an alternative, though not explored in this paper, one can use exogenous changes in soda taxes as instruments.

Table J.1: Prices and In-store Presence of Brands in Sample

| | Mean | Median | Std | Min | Max | Brand Variation | DMA Variation | Month Variation |
|---|---|---|---|---|---|---|---|---|
| Prices ($ per 12 oz.) | 0.40 | 0.39 | 0.12 | 0.11 | 2.75 | 39.73% | 39.50% | 0.50% |
| In-store Presence | 0.50 | 0.51 | 0.22 | 0.01 | 1.00 | 75.12% | 13.44% | 0.06% |

Notes: Variance contribution of brands, DMAs and months is the R-squared value added by each variable when it is added to the regression of price (or in-store presence) on the other two independent variables. In-store presence: the proportion of stores with the given brand in stock.

## J.4 Market Size Definition

I define one serving of soft drink as 12 ounces. In calculating the market share of the outside good, Eizenberg and Salvo (2015) assume a potential weekly consumption of 6 liters (approximately 17 servings) per household. Similarly, Zheng, Huang, and Ross (2019) use as $\gamma$ the documented average per capita consumption of non-alcoholic beverages, including CSDs, water, juice, tea and sports drinks. The average consumption is around 30 ounces per person per day, equivalent to 17.5 servings per week. Other studies, such as Liu, Lopez, and Zhu (2014) and Lopez, Liu, and Zhu (2015), also utilize per capita consumption of non-alcoholic beverages as a proxy for market size. The specific proportional factor varies depending on the inclusion of different beverages as outside options. For example, Liu, Lopez, and Zhu (2014) include milk consumption, while Zheng, Huang, and Ross (2019) do not. The per capita weekly consumption of non-alcoholic beverages in Liu, Lopez, and Zhu (2014) reaches as high as 32 servings, nearly double the amount used in Zheng, Huang, and Ross (2019).

The market size assumptions can be expressed in our notation as $\gamma M_t$, where $M_t$ represents the total population in a DMA area. We focus on population scaling, as this is the standard choice in the aforementioned soda literature we are comparing to.[5] Throughout this section, all comparisons will be made with regard to assuming $\gamma = 17$ servings. Specifically, I estimate $\gamma$ along with other demand parameters and calculate elasticities and diversion ratios. I then simulate the merger using two potential market sizes: one assumes a market size that equals 17 times the population size, and the other assumes $\hat{\gamma}$ times the population size.

## J.5 Instruments

To address the likely correlation of the demand errors $\xi_{jt}$ with prices, as well as identify the random coefficients and market size parameters, I employ three sets of instruments. The first two sets are standard excluded instruments suggested by Berry and Haile (2014) and

5. While the identification analysis allows market size to be a general function of market observables, the empirical application adopts a linear specification as a benchmark commonly used in the literature. This choice facilitates direct comparison of estimates across studies. The empirical application is intended as an illustration of the proposed identification strategy rather than as an exhaustive empirical analysis.

have been widely used in empirical studies (e.g. Eizenberg and Salvo 2015; Petrin and Train 2010; and Nevo 2001).

The first set of price instruments belongs to the *Hausman-type* instrument, proposed by Hausman, Leonard, and Zona (1994). Specifically, the instrument for the price of brand $j$ in a given DMA is the average price of this brand in other DMAs belonging to the same Census Region. These instruments provide variation across brands and DMAs, and are valid due to the correlation of prices across geographic regions through a common cost structure.[6]

The second class of price instruments consists of cost shifters. Specifically, I use input prices such as electricity prices, fuel prices and local wages. These cost shifters are excluded from the demand equation but affect prices through the supply side.

The third set of instruments serves to identify random coefficients and market size parameters, which should satisfy conditions in Proposition 1. Here I use the traditional BLP type instruments. Specifically, they involve sums over exogenous characteristics of brands produced by the same company and sums over rival brands. I construct this class of instruments based on in-store presence and fitted values of prices. The instrument constructed based on the retail presence variable helps identify market size. Consider a thought experiment that aligns with graphical intuition in section 3.3.1: Suppose Pepsi's retail presence increases due to exogenous supply-side factors uncorrelated with demand shocks $\xi_{jt}$, such as the opening of a new distribution hub, leading more stores to carry Pepsi. This shift in the retail environment would cause consumers to shift from other drinks to Pepsi. If Coke's sales decline significantly while overall soda demand remains largely unchanged, it suggests that switching occurs primarily at the intensive margin rather than the extensive margin, indicating a small outside good share. The fitted values of prices are obtained by regressing prices on $X_{jt}$ and excluded price instrument. The projection of prices on exogenous variables would be mean independent of the unobservables $\xi_{jt}$. This exogenous variation in price facilitates the identification of the parameters associated with heterogeneity in price sensitivity. As a robustness check, I also use the differentiation instruments proposed by Gandhi and Houde (2019).

## J.6 Results

Table J.2 reports five sets of demand model estimates. The first two columns correspond to plain logit and random coefficients logit models, where $\gamma$ is estimated along with other demand parameters. Columns 3 to 5 are standard BLP estimates assuming $\gamma = 17$. Column 3 replicates the specification of column 2, while column 4 introduces an additional random coefficient on the constant term to capture unobserved preferences for the outside option. In column 5, DMA-week specific fixed effects are included.

The strength of instruments, measured by the F-statistic of an IIA-test (as discussed in Supplement G), is 2819 with a p-value of 0.00, rejecting the null hypothesis of weak instruments.

The estimated values of $\gamma$ are 12.478 and 11.767 for the plain logit and random coefficients logit models, respectively. The standard errors are relatively small, suggesting that

6. The Hausman-type instruments could be problematic if demand unobservables are correlated across markets (e.g., launching a national campaign). To lessen this concern, I control for DMA-specific, brand-level in-store presence, which partially absorbs common demand shocks.

empirically the instruments provide sufficient variation. These estimates are lower than the range assumed in the literature (between 17.5 and 32), suggesting that a market size defined based on per capita consumption of all non-alcoholic beverages may be too large. It implies that not all beverage categories should be considered as outside alternatives to soda[7].

**Profiled GMM Objective Function.** To verify that the estimated $\gamma$ achieves global minimum for the random coefficients logit model, I plot the GMM objective function while keeping $\gamma$ fixed over a grid of values and re-optimizing the remaining parameters with the weighting matrix fixed. There are no multiple minima within the specified interval. However, the function is not steep around the minimum, which could pose challenges for numerical optimization. Stronger instruments may help improve parameter identification and numerical optimization.

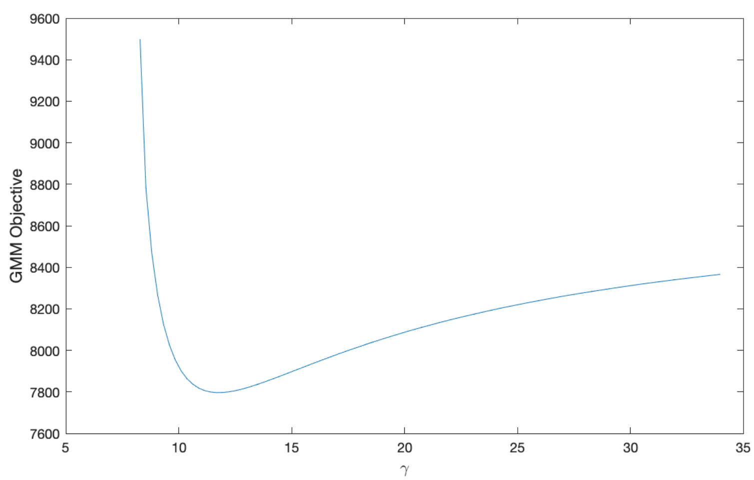



Figure J.1: Profiled GMM Objective

Notes: The figure shows the profiled GMM objective. $\gamma$ is fixed while the remaining parameters are re-optimized.

In columns 1 and 2 of Table J.2, the estimated price sensitivities are $-8.748$ and $-9.86$. The estimate of random coefficient parameter $\sigma$ in column 2 is 1.952 and is statistically significant, indicating a rejection of the plain logit model. Column 3, assuming $\gamma = 17$, exhibits higher price sensitivity ($-13.033$) and a larger standard deviation (4.395) in the preference for price. This aligns with what one would expect when assuming a larger potential market size. Column 4, which includes a second random coefficient on the constant term, produces estimates comparable to column 3. The estimate of $\sigma$ for the constant term is small in magnitude $-0.09$ and statistically insignificant. In the last column, with market fixed effects, the estimate of price sensitivity is much lower. Precisely estimating $\sigma$ becomes challenging, with extremely large standard errors, which is expected due to the inclusion of near $10,000$ dummy variables in the GMM estimation. Therefore, there is limited exogenous variation to identify the random coefficient. Note that although market fixed effects are

7. In 2019, the soft drink consumption per person per week in the US is approximately 107 ounces, or 8.9 servings. See: https://www.ibisworld.com/us/bed/per-capita-soft-drink-consumption/1786/. This reassures that our estimated value of potential consumption, which amounts to 12 servings, is reasonable.

not included in the specification used to estimate $\gamma$ (column 2) due to the large number of markets (near $10,000$), in principle, $\gamma$ is still identifiable in a specification that includes market fixed effects.

Table J.2: Baseline Demand Estimation Results

| | Estimate $\gamma$ | | Assume $\gamma = 17$ servings | | |
|---|---|---|---|---|---|
| | Plain Logit | RC Logit | RC Logit | RC Logit with two RC's | RC Logit with Market FE |
| Means $\beta$ | | | | | |
| Price | -8.748 | -9.860 | -13.033 | -12.793 | -5.245 |
| | (0.084) | (0.222) | (0.289) | (0.434) | (0.311) |
| In-store Presence | 3.281 | 3.311 | 3.309 | 3.314 | 5.061 |
| | (0.022) | (0.022) | (0.023) | (0.024) | (0.019) |
| Standard Deviations $\sigma$ | | | | | |
| Price | | 1.952 | 4.395 | 4.257 | 0.007 |
| | | (0.211) | (0.155) | (0.247) | (53.834) |
| Constant | | | | -0.090 | |
| | | | | (1.189) | |
| Market Size Parameter | | | | | |
| $\gamma$ | 12.478 | 11.767 | | | |
| | (0.263) | (0.210) | | | |
| Product Fixed Effects | Yes | Yes | Yes | Yes | Yes |
| Seasonal Effects | Yes | Yes | Yes | Yes | No |
| Region Fixed Effects | Yes | Yes | Yes | Yes | No |
| DMA-Week (Market) Fixed Effects | No | No | No | No | Yes |

Notes: This table reports demand model estimates. Columns 1 and 2 correspond to plain logit and random coefficients logit models, and $\gamma$ is to be estimated. Columns 3 to 5 are standard BLP estimates assuming $\gamma = 17$. Column 3 replicates the specification of column 2. Column 4 introduces an additional random coefficient on the constant term and column 5 includes market fixed effects. Standard errors in parentheses. Constant terms are omitted due to collinearity with product fixed effects.

Table J.3 provides estimated own-price elasticities and outside-good diversion ratios. Column 1 reports the elasticities based on our estimate of $\hat{\gamma} = 12$. The own-price elasticities range from $-3.651$ to $-1.887$, which is consistent with previous literature[8]. Note that PLs have lower own-price elasticities compared to other brands. This can be attributed to PLs being composite brands consisting of numerous niche products. The demand for an entire category are expected be less elastic than for each individual product.

The remaining columns in Table J.3 are based on estimates from columns 3 to 5 of Table J.2. Assuming $\gamma = 17$ when the true value is $\gamma = 12$, the biases in own-price elasticities are small. However, the biases in outside diversion ratios are more substantial, with a difference of 9 percentage points for PLs and approximately 3 to 4 percentage points for other brands, indicating even less substitutions across brands. Including a second random coefficient on the constant term yields results similar to those in column 2. This is mainly due to the fact that the estimated $\sigma$ for the constant term is not significantly different from zero.

8. For example, the estimated own-price elasticities in Dubé (2005) are in the range of $-3$ to $-6$. Lopez, Liu, and Zhu (2015) report elasticities between $-1$ and $-2$. The magnitude of elasticities varies with the aggregation level of product.

The inclusion of market fixed effects leads to slightly lower own-price elasticities and higher outside diversion ratios. Although the results with market fixed effects are comparable to our estimates, the standard error of the random coefficient estimate is so large that we can not conclude any statistically significant results. The key takeaway from Table J.3 is that none of the commonly employed solutions produce elasticities and diversion ratios close to those obtained using our estimated market size.

Table J.3: Demand Elasticities and Diversion Ratios

| | RC Logit with $\hat{\gamma} = 12$ | RC Logit Assuming $\gamma = 17$ | RC Logit with two RC's Assuming $\gamma = 17$ | RC Logit with Market FE Assuming $\gamma = 17$ |
|---|---|---|---|---|
| Own-Price Elasticities | | | | |
| Product 1 | -3.398 | -3.362 | -3.351 | -2.097 |
| Product 2 | -3.597 | -3.493 | -3.482 | -2.224 |
| Product 3 | -3.651 | -3.528 | -3.518 | -2.262 |
| Private Label R | -1.887 | -2.181 | -2.151 | -1.000 |
| Outside-Good Diversion Ratios | | | | |
| Product 1 | 62.8% | 66.0% | 66.5% | 78.5% |
| Product 2 | 60.3% | 63.0% | 63.5% | 77.2% |
| Product 3 | 59.8% | 62.4% | 62.9% | 77.0% |
| Private Label R | 68.4% | 77.7% | 77.7% | 76.9% |

Notes: This table reports estimates of elasticities and diversion ratio. Columns 1 is based on a random coefficients logit model with estimated $\gamma$. Columns 2 to 4 assume $\gamma = 17$. Column 2 replicates the specification of column 1. Column 3 introduces an additional random coefficient on the constant term and column 4 includes market fixed effects. To save space, only top-3 regular drink products are reported in the table. R represents regular.

**Aggregate Price Elasticity.** I provide additional results for the soft drink application. First, we calculate the price elasticity of aggregate demand, which is the percentage change in total sales for soft drinks when the prices of all soft drinks increase. Note that we can link aggregate demand directly to the outside share, by recognizing that without an outside option defined in the model, the aggregate market demand is perfectly inelastic. More formally, in a simple logit model, the price elasticity of aggregate demand can be calculated by $\alpha\pi_0\hat{p}$, where $\alpha$ is the price coefficient and $\hat{p}$ the average price.

This aggregate elasticity can be thought of as the market-level response to a proportional tax imposed on all products. It is economically important, for example, when policymakers aim to evaluate the effectiveness and targeting of soda taxes.

Figure J.2 illustrates the estimated aggregate elasticities of demand in each market when $\gamma = 17$ and 12, respectively. With a larger market size, the aggregate elasticity falls (in absolute value). The direction of bias is same as those found in Conlon and Mortimer (2021). Moreover, it not only changes the mean level but also the overall distribution across markets. This finding confirms that market size definition is relevant for questions that affect all products in a market.

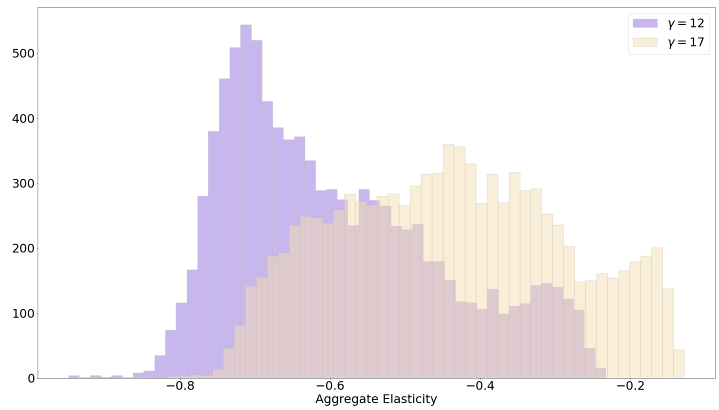



Figure J.2: Distribution of Aggregate Elasticities across Markets
Notes: The figure shows the aggregate elasticities of demand across markets for $\gamma = 12$ and 17.

Finally, I simulate a merger between the largest manufacturer and private label manufacturers.[9] The merger simulation abstracts away from cost reduction, or changes in the model of competition (e.g. coordination between other firms). Table J.4 shows the percentage change in prices for the merging products. In column 1, the estimates (approximately 2.22% to 8.41% price increases) are reasonably comparable to those of Dubé (2005), who estimated the price effect after a simulated merger between two leading manufacturers. The merger simulations predict larger price increases for the PLs than products of the leading manufacturer. This results from the relatively lower own-price elasticities of PLs, and is consistent with previous findings on higher pricing margins for PLs.

In columns 2 and 3, which assume $\gamma = 17$, the price effects of the merger for brands owned by the merging parties tend to be underestimated. The bias is the most pronounced for PLs. Simulated price increases are approximately 8 percent when the market size parameter is estimated to be 12, while assuming $\gamma = 17$ yields a price increase of 5.5 percent, biased by 31%. For brands from the leading manufacturer, the simulated price effects are relatively lower with the assumed $\gamma = 12$, although I acknowledge that the differences are not economically significant. In the last column, the estimate is relatively closer to our estimates but is imprecisely estimated with large standard errors.

In summary, both the diversion ratios and merger simulations generated by different market sizes vary and may lead to different policy evaluations. As the potential market size increases, the simulated price changes display a monotonic decrease.

9. I used the PyBLP package (Conlon and Gortmaker 2020) to conduct the merger simulation.

Table J.4: Simulated Percentage Price Effects for Merging Firms' Brands

| | RC Logit with $\hat{\gamma} = 12$ | RC Logit Assuming $\gamma = 17$ | RC Logit with two RC's Assuming $\gamma = 17$ | RC Logit with Market FE Assuming $\gamma = 17$ |
|---|---|---|---|---|
| Manufacturer A Products | 2.33 | 1.65 | 1.65 | 2.80 |
| | 2.37 | 1.66 | 1.67 | 2.85 |
| | 2.22 | 1.58 | 1.58 | 2.66 |
| | 2.49 | 1.73 | 1.73 | 3.01 |
| Private Label R | 8.41 | 5.64 | 5.66 | 10.14 |
| Private Label DT | 8.21 | 5.56 | 5.57 | 9.83 |

Notes: This table reports the percentage price change after a simulated merger between Manufacturer A and private label manufacturers. Columns 1 is based on a random coefficients logit model with estimated $\gamma$. Columns 2 to 4 assume $\gamma = 17$. Column 2 replicates the specification of column 1. Column 3 introduces an additional random coefficient on the constant term and column 4 includes market fixed effects. To save space, only merging firms' brands are reported in the table. R represents regular. DT stands for diet.

# K Additional Derivations

## Partial Derivatives of $\pi_{jt}$

The partial derivatives of $\pi_{jt}$ with respect to $\delta_{jt}$ and $\delta_{kt}$ are functions of mean utilities and characteristics of all products:

$$\frac{\partial \pi_{jt}}{\partial \delta_{jt}} = \int \pi_{jti}\left(\delta_t, X_t^{(2)}; \sigma\right)\left(1 - \pi_{jti}\left(\delta_t, X_t^{(2)}; \sigma\right)\right) f_\nu(\nu) d\nu, \quad \frac{\partial \pi_{jt}}{\partial \delta_{kt}} = -\int \pi_{jti}\left(\delta_t, X_t^{(2)}; \sigma\right) \pi_{kti}\left(\delta_t, X_t^{(2)}; \sigma\right.$$

where

$$\pi_{jti}\left(\delta_t, X_t^{(2)}; \sigma\right) = \frac{exp\left(\delta_{jt} + \sum_l \sigma_l x_{jtl}^{(2)} \nu_{il}\right)}{1 + \sum_{k=1}^{J_t} exp\left(\delta_{kt} + \sum_l \sigma_l x_{ktl}^{(2)} \nu_{il}\right)}.$$

The partial derivatives of $\pi_{jt}$ with respect to $\sigma_l$ is

$$\frac{\partial \pi_{jt}\left(\delta_t, X_t^{(2)}; \sigma\right)}{\partial \sigma_l} = \int \pi_{jti}\left(\delta_t, X_t^{(2)}; \sigma\right)\left(x_{jtl}^{(2)} - \sum_{k=1}^{J} x_{ktl}^{(2)} \pi_{kti}\left(\delta_t, X_t^{(2)}; \sigma\right)\right) \nu_{il} f_\nu(\nu) d\nu$$

# References


Ackerberg, D. A., and M. Rysman. 2005. "Unobserved product differentiation in discrete-choice models: estimating price elasticities and welfare effects." *RAND Journal of Economics* 36 (4): 771–789.

Aguiar, L., and J. Waldfogel. 2018. "Quality predictability and the welfare benefits from new products: Evidence from the digitization of recorded music." *Journal of Political Economy* 126 (2): 492–524.

Amemiya, T. 1974. “The nonlinear two-stage least-squares estimator.” *Journal of econometrics* 2 (2): 105–110.

Armstrong, T. B. 2016. “Large market asymptotics for differentiated product demand estimators with economic models of supply.” *Econometrica* 84 (5): 1961–1980.

Berry, S. 1994. “Estimating discrete-choice models of product differentiation.” *The RAND Journal of Economics,* 242–262.

Berry, S., and P. A. Haile. 2014. “Identification in differentiated products markets using market level data.” *Econometrica* 82 (5): 1749–1797.

Berry, S., J. Levinsohn, and A. Pakes. 1995. “Automobile prices in market equilibrium.” *Econometrica: Journal of the Econometric Society,* 841–890.

Bourreau, M., Y. Sun, and F. Verboven. 2021. “Market entry, fighting brands, and tacit collusion: Evidence from the French mobile telecommunications market.” *American Economic Review* 111 (11): 3459–3499.

Cardell, N. S. 1997. “Variance components structures for the extreme-value and logistic distributions with application to models of heterogeneity.” *Econometric Theory* 13 (2): 185–213.

Ciliberto, F., C. Murry, and E. Tamer. 2021. “Market structure and competition in airline markets.” *Journal of Political Economy* 129 (11): 2995–3038.

Conlon, C., and J. Gortmaker. 2020. “Best practices for differentiated products demand estimation with pyblp.” *The RAND Journal of Economics* 51 (4): 1108–1161.

Conlon, C., and J. H. Mortimer. 2021. “Empirical properties of diversion ratios.” *The RAND Journal of Economics* 52 (4): 693–726.

Dubé, J.-P. 2005. “Product differentiation and mergers in the carbonated soft drink industry.” *Journal of Economics & Management Strategy* 14 (4): 879–904.

Dunker, F., S. Hoderlein, and H. Kaido. 2022. “Nonparametric identification of random coefficients in endogenous and heterogeneous aggregate demand models.” *arXiv preprint arXiv:2201.06140.*

Egan, M., A. MacKay, and H. Yang. 2022. “Recovering investor expectations from demand for index funds.” *The Review of Economic Studies* 89 (5): 2559–2599.

Eizenberg, A., and A. Salvo. 2015. “The rise of fringe competitors in the wake of an emerging middle class: An empirical analysis.” *American Economic Journal: Applied Economics* 7 (3): 85–122.

Fox, J. T., K. il Kim, and C. Yang. 2016. “A simple nonparametric approach to estimating the distribution of random coefficients in structural models.” *Journal of Econometrics* 195 (2): 236–254.

Gandhi, A., and J.-F. Houde. 2019. *Measuring substitution patterns in differentiated products industries.* Technical report. National Bureau of Economic Research.

Hausman, J., G. Leonard, and J. D. Zona. 1994. “Competitive analysis with differenciated products.” *Annales d'Economie et de Statistique,* 159–180.

Jorgenson, D. W., and J.-J. Laffont. 1974. “Efficient estimation of nonlinear simultaneous equations with additive disturbances.” In *Annals of Economic and Social Measurement, Volume 3, number 4,* 615–640. NBER.

Li, S. 2018. "Better lucky than rich? Welfare analysis of automobile licence allocations in Beijing and Shanghai." *The Review of Economic Studies* 85 (4): 2389–2428.

Liu, Y., R. A. Lopez, and C. Zhu. 2014. "The impact of four alternative policies to decrease soda consumption." *Agricultural and Resource Economics Review* 43 (1): 53–68.

Lopez, R. A., Y. Liu, and C. Zhu. 2015. "TV advertising spillovers and demand for private labels: the case of carbonated soft drinks." *Applied Economics* 47 (25): 2563–2576.

McFadden, D. 1977. "Modelling the choice of residential location."

Nevo, A. 2001. "Measuring market power in the ready-to-eat cereal industry." *Econometrica* 69 (2): 307–342.

Petrin, A., and K. Train. 2010. "A control function approach to endogeneity in consumer choice models." *Journal of marketing research* 47 (1): 3–13.

Quan, T. W., and K. R. Williams. 2018. "Product variety, across-market demand heterogeneity, and the value of online retail." *The RAND Journal of Economics* 49 (4): 877–913.

Wollmann, T. G. 2018. "Trucks without bailouts: Equilibrium product characteristics for commercial vehicles." *American Economic Review* 108 (6): 1364–1406.

Zheng, H., L. Huang, and W. Ross. 2019. "Reducing obesity by taxing soft drinks: tax salience and firms' strategic responses." *Journal of Public Policy & Marketing* 38 (3): 297–315.