Carnegie Mellon University

Database Systems

Bloom Filters, Tries, Skip Lists, Inverted Indexes, Vector Indexes

15-445/645 FALL 2024 » PROF. ANDY PAVLO

ADMINISTRIVIA

Project #1 is due Sunday Sept 29th @ 11:59pm
→ Recitation + Profiling Tutorial: **@160**
→ **Extra Office Hours** on Saturday Sept 28th @ 3:00-5:00pm
→ Location: GHC 5207
→ https://ohq.eberly.cmu.edu

Homework #3 is due Sunday Sept 6th @ 11:59pm

Mid-term Exam on Wednesday Oct 9th @ 2:00pm
→ In-class in this room.
→ More info next week.

INDEXES VS. FILTERS

An **index** data structure of a subset of a table's attributes that are organized and/or sorted to the location of specific tuples using those attributes.
→ Example: B+Tree

A **filter** is a data structure that answers set membership queries; it tells you whether a key (likely) exists in a set but not where it is located.
→ Example: Bloom Filter

TODAY'S AGENDA

Bloom Filters

Skip Lists

Tries / Radix Trees

Inverted Indexes

Vector Indexes

DB Flash Talk: TiDB

BLOOM FILTERS

Probabilistic data structure (bitmap) that answers set membership queries.

可能存在假阳性：布隆过滤器计算某个元素存在集合中，但是该元素实际不存在.

→ False negatives will never occur.
→ False positives can sometimes occur.
→ See Bloom Filter Calculator.

k: 哈希函数的个数
m: 数组空间的总位数

Insert(x):

→ Use *k* hash functions to set bits in the filter to 1.

Lookup(x):

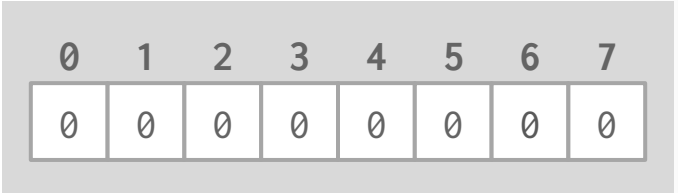
→ Check whether the bits are 1 for each hash function.

普通布隆过滤器不支持删除操作.

BLOOM FILTERS

```
Insert 'RZA'
```

Bloom Filter

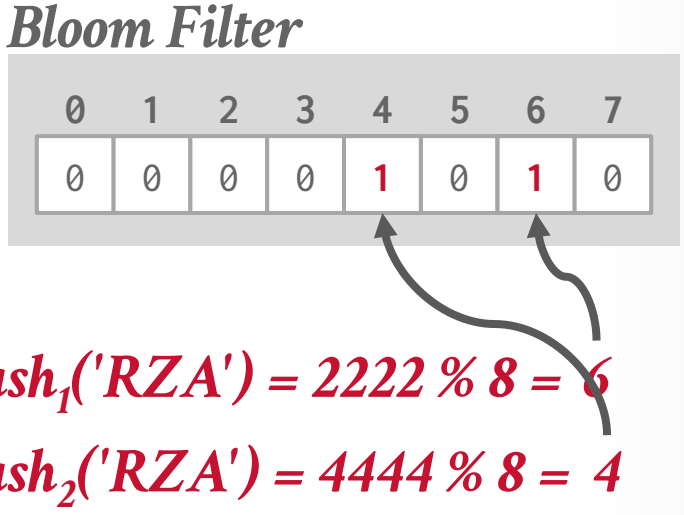


$hash_1('RZA') = 2222 \% 8 = 6$

$hash_2('RZA') = 4444 \% 8 = 4$

BLOOM FILTERS

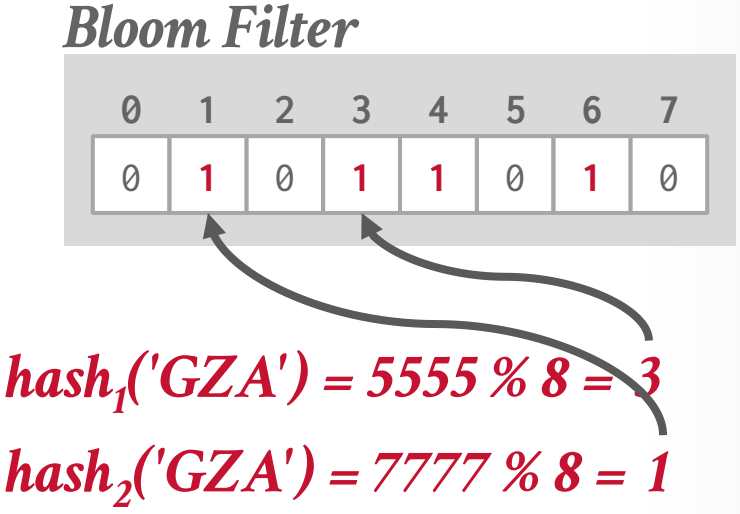
```
Insert 'RZA'
```



BLOOM FILTERS

Insert 'RZA'

Insert 'GZA'

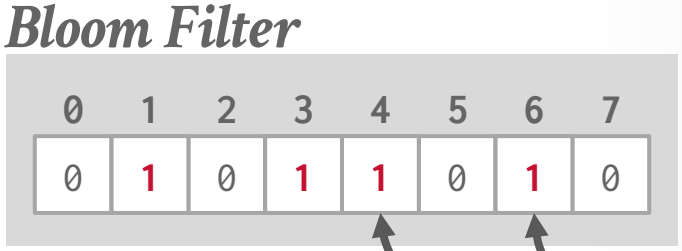


BLOOM FILTERS

Insert 'RZA'

Insert 'GZA'

Lookup 'RZA' → *TRUE*



$hash_1('RZA') = 2222 \% 8 = 6$

$hash_2('RZA') = 4444 \% 8 = 4$

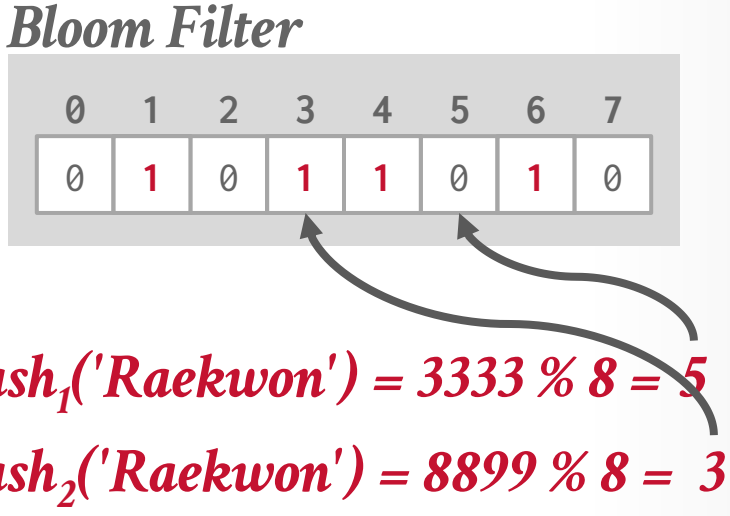
BLOOM FILTERS

Insert 'RZA'

Insert 'GZA'

Lookup 'RZA' → *TRUE*

Lookup 'Raekwon' → *FALSE*



BLOOM FILTERS

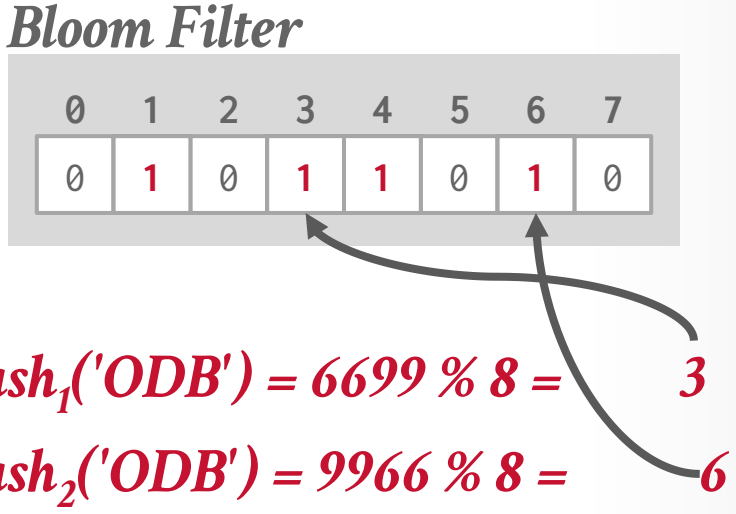
Insert 'RZA'

Insert 'GZA'

Lookup 'RZA' → *TRUE*

Lookup 'Raekwon' → *FALSE*

Lookup 'ODB' → *TRUE*



OTHER FILTERS

Counting Bloom Filter

m: 计数器的个数
k: 哈希函数的个数

→ Supports dynamically adding and removing keys.
→ Uses integers instead of bits to count the number of occurrences of a key in a set.

当插入一个元素时，对应位置的计数值+1.

Carnegie Mellon University

Cuckoo Filter

→ Also supports dynamically adding and removing keys.
→ Uses a Cuckoo Hash Table but stores fingerprints instead of full keys.

更小的计算量.

Carnegie Mellon University

Succinct Range Filter (SuRF)

必须预先获取所有键.

→ Immutable compact trie that supports approximate exact matches and range filtering.

在 SST 文件写入时, 可以构建此过滤器一次写入所有键的更新.

OTHER F

Counting Bloom Filter

→ Supports dynamically adding
→ Uses integers instead of bits t
occurrences of a key in a set.

Carnegie Mellon University

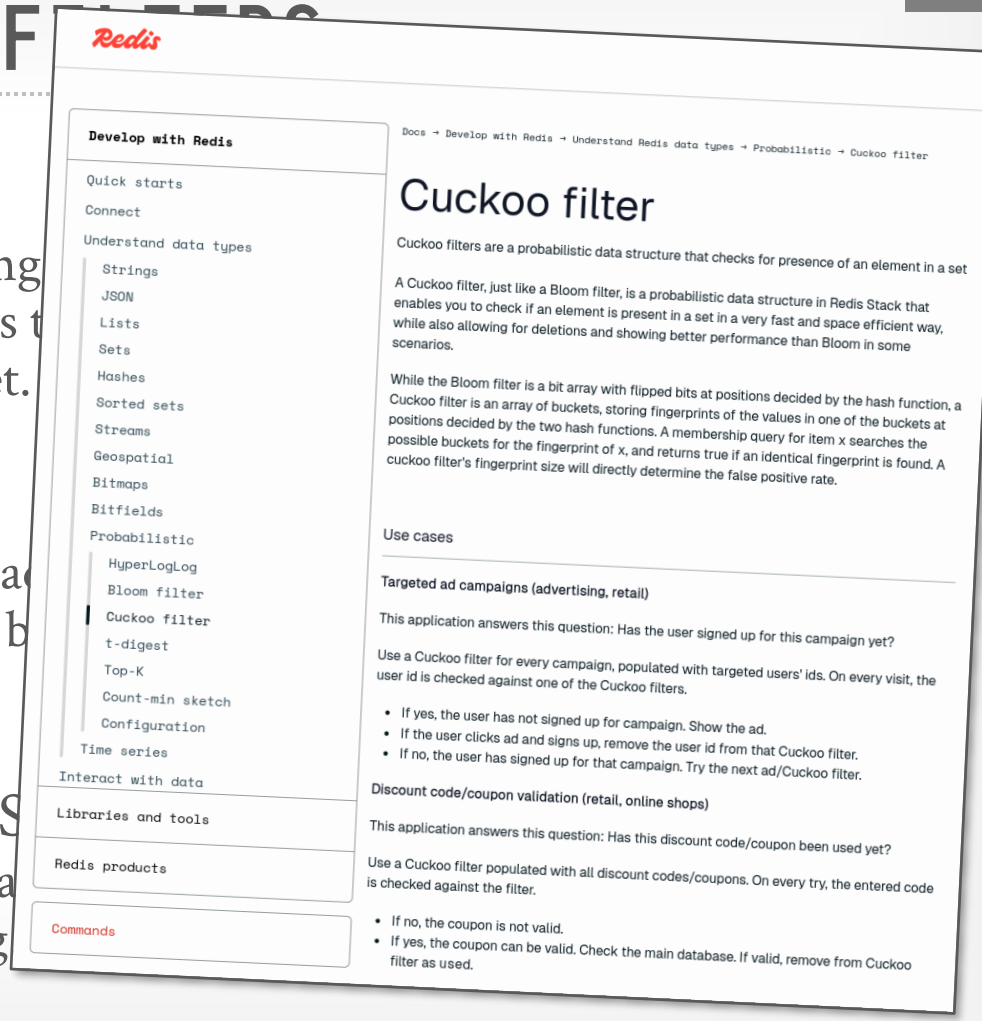
Cuckoo Filter

→ Also supports dynamically a
→ Uses a Cuckoo Hash Table b
of full keys.

Carnegie Mellon University

Succinct Range Filter (S

→ Immutable compact trie tha
matches and range filtering

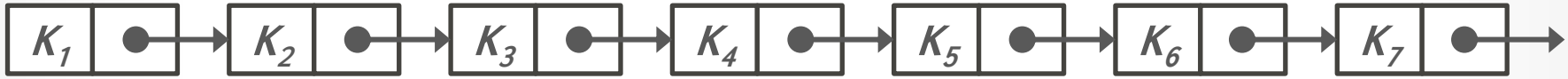


OBSERVATION

The easiest way to implement a dynamic order-preserving index is to use a sorted linked list.

All operations have to linear search.

→ Average Cost: **O(n)**

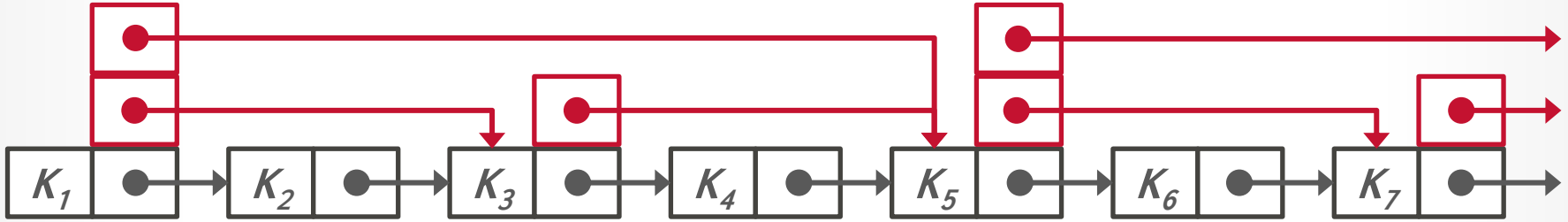


OBSERVATION

The easiest way to implement a dynamic order-preserving index is to use a sorted linked list.

All operations have to linear search.

→ Average Cost: **O(n)**



SKIP LISTS 概率数据结构

在链表的基础上构建了多层索引以跳跃下方链表不同的位置.

Multiple levels of linked lists with extra pointers to skip over entries.

→ 1st level is a sorted list of all keys.
→ 2nd level links every other key
→ 3rd level links every fourth key
→ Each level has ½ the keys of one below it

Maintains keys in sorted order without requiring global rebalancing.

→ Approximate **O(log n)** search times.

Mostly for in-memory data structures.

→ Example: LSM MemTable

Skip Lists: A Probabilistic Alternative to Balanced Trees

Skip lists are a data structure that can be used in place of balanced trees. Skip lists use probabilistic balancing rather than strictly enforced balancing and as a result the algorithms for insertion and deletion in skip lists are much simpler and significantly faster than equivalent algorithms for balanced trees.

William Pugh

Binary trees can be used for representing abstract data types such as dictionaries and ordered lists. They work well when the elements are inserted in a random order. Some sequences of operations, such as inserting the elements in order, produce degenerate data structures that give very poor performance. If it were possible to randomly permute the list of items to be inserted, trees would work well with high probability for any input sequence. In most cases queries must be answered on-line, so randomly permuting the input is impractical. *Balanced* tree algorithms re-arrange the tree as operations are performed to maintain certain balance conditions and assure good performance.

Skip lists are a probabilistic alternative to balanced trees. Skip lists are balanced by consulting a random number generator. Although skip lists have bad worst-case performance, no input sequence consistently produces the worst-case performance (much like quicksort when the pivot element is chosen randomly). It is very unlikely a skip list data structure will be significantly unbalanced (e.g., for a dictionary of more than 250 elements, the chance that a search will take more than 3 times the expected time is less than one in a million). Skip lists have balance properties similar to that of search trees built by random insertions, yet do not require insertions to be random.

Balancing a data structure probabilistically is easier than explicitly maintaining the balance. For many applications, skip lists are a more natural representation than trees, also leading to simpler algorithms. The simplicity of skip list algorithms makes them easier to implement and provides significant constant factor speed improvements over balanced tree and self-adjusting tree algorithms. Skip lists are also very space efficient. They can easily be configured to require an average of $1\frac{1}{3}$ pointers per element (or even less) and do not require balance or priority information to be stored with each node.

SKIP LISTS

We might need to examine every node of the list when searching a linked list (*Figure 1a*). If the list is stored in sorted order and every other node of the list also has a pointer to the node two ahead it in the list (*Figure 1b*), we have to examine no more than $\lceil n/2 \rceil + 1$ nodes (where n is the length of the list).

Also giving every fourth node a pointer four ahead (*Figure 1c*) requires that no more than $\lceil n/4 \rceil + 2$ nodes be examined. If every $(2^i)^{th}$ node has a pointer 2^i nodes ahead (*Figure 1d*), the number of nodes that must be examined can be reduced to $\lceil \log_2 n \rceil$ while only doubling the number of pointers. This data structure could be used for fast searching, but insertion and deletion would be impractical.

A node that has k forward pointers is called a *level* k node. If every $(2^i)^{th}$ node has a pointer 2^i nodes ahead, then levels of nodes are distributed in a simple pattern: 50% are level 1, 25% are level 2, 12.5% are level 3 and so on. What would happen if the levels of nodes were chosen randomly, but in the same proportions (e.g., as in Figure 1e)? A node's i^{th} forward pointer, instead of pointing 2^{i-1} nodes ahead, points to the next node of level i or higher. Insertions or deletions would require only local modifications; the level of a node, chosen randomly when the node is inserted, need never change. Some arrangements of levels would give poor execution times, but we will see that such arrangements are rare. Because these data structures are linked lists with extra pointers that skip over intermediate nodes, I named them *skip lists*.

SKIP LIST ALGORITHMS

This section gives algorithms to search for, insert and delete elements in a dictionary or symbol table. The *Search* operation returns the contents of the value associated with the desired key or *failure* if the key is not present. The *Insert* operation associates a specified key with a new value (inserting the key if it had not already been present). The *Delete* operation deletes the specified key. It is easy to support additional operations such as "find the minimum key" or "find the next key".

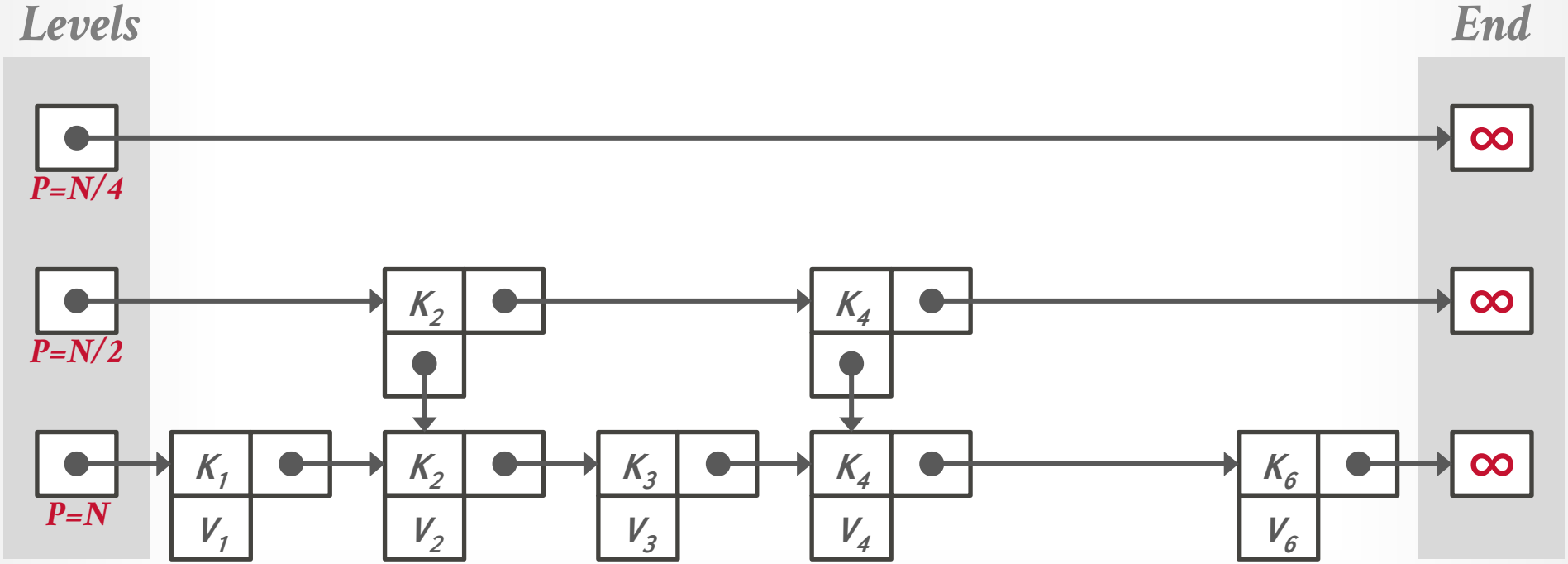
Each element is represented by a node, the level of which is chosen randomly when the node is inserted without regard for the number of elements in the data structure. A *level* i node has i *forward* pointers, indexed 1 through i. We do not need to store the level of a node in the node. Levels are capped at some appropriate constant *MaxLevel*. The *level* of a list is the maximum level currently in the list (or 1 if the list is empty). The *header* of a list has forward pointers at levels one through *MaxLevel*. The forward pointers of the header at levels higher than the current maximum level of the list point to NIL.

SingleStore WIREDTIGER RocksDB Couchbase

SKIP LISTS: INSERT

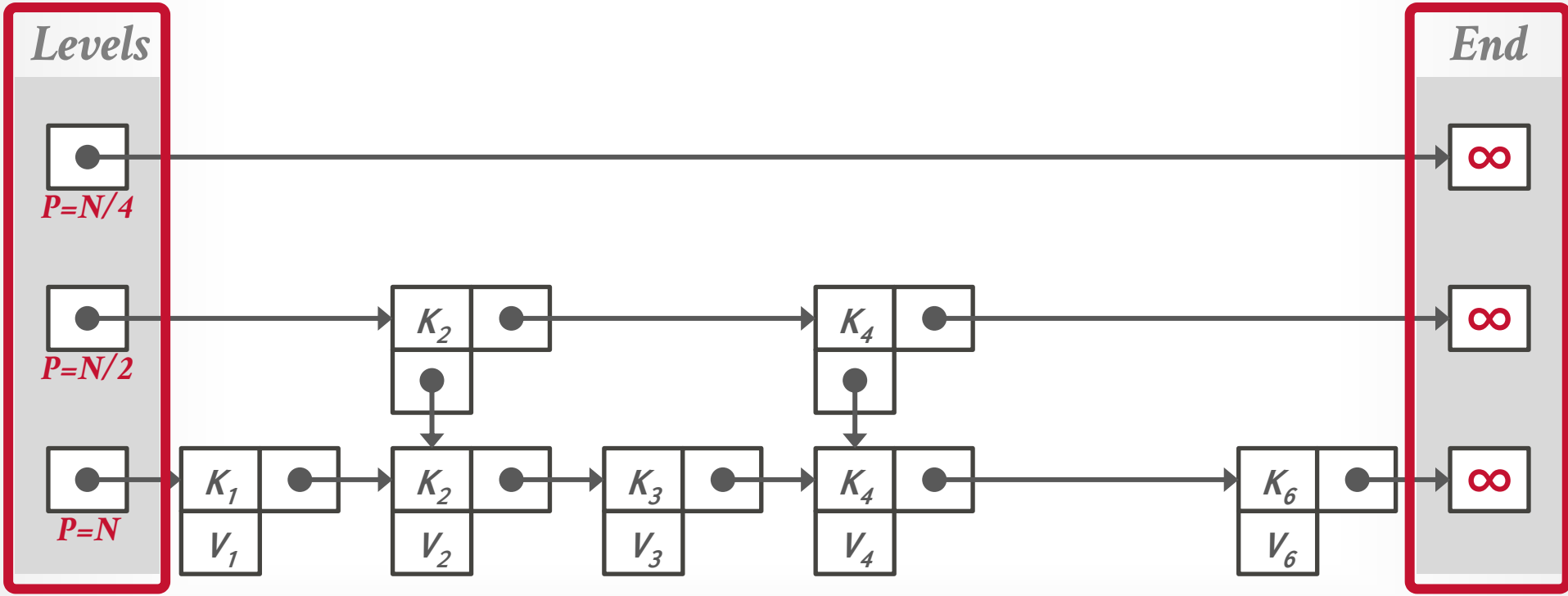
Insert K_5

进入不同层级数据结构的入口



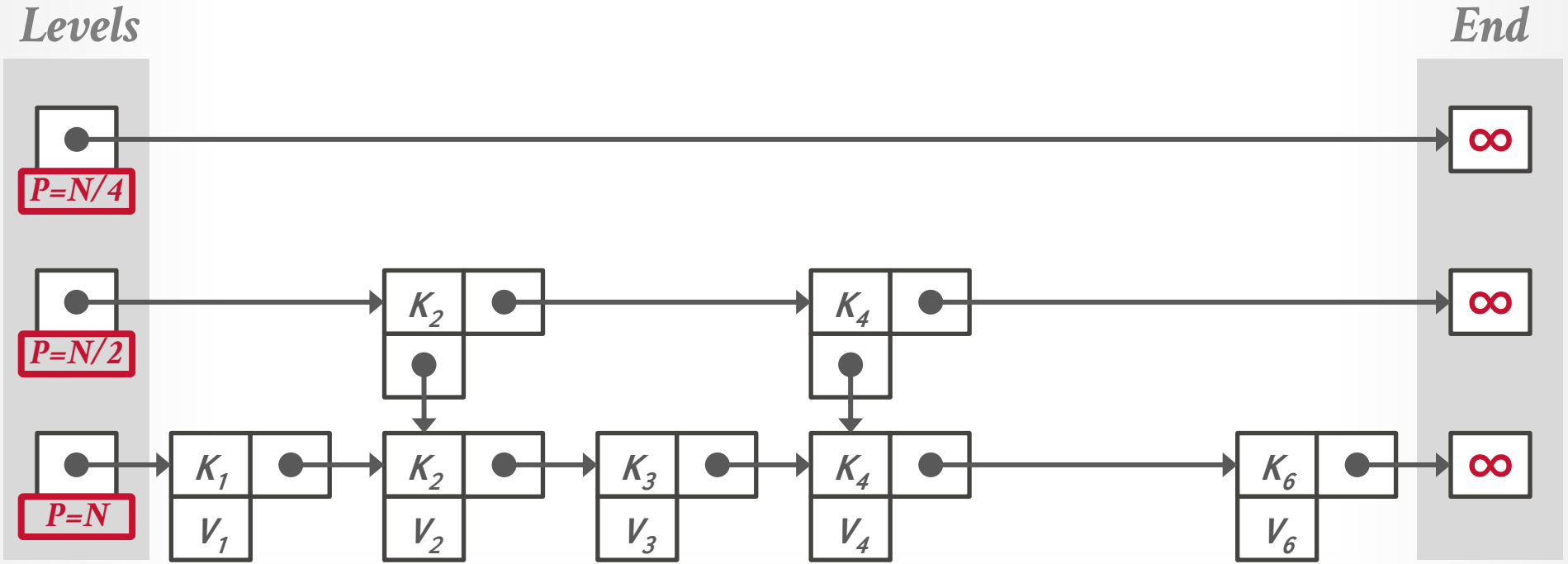
SKIP LISTS: INSERT

Insert K_5



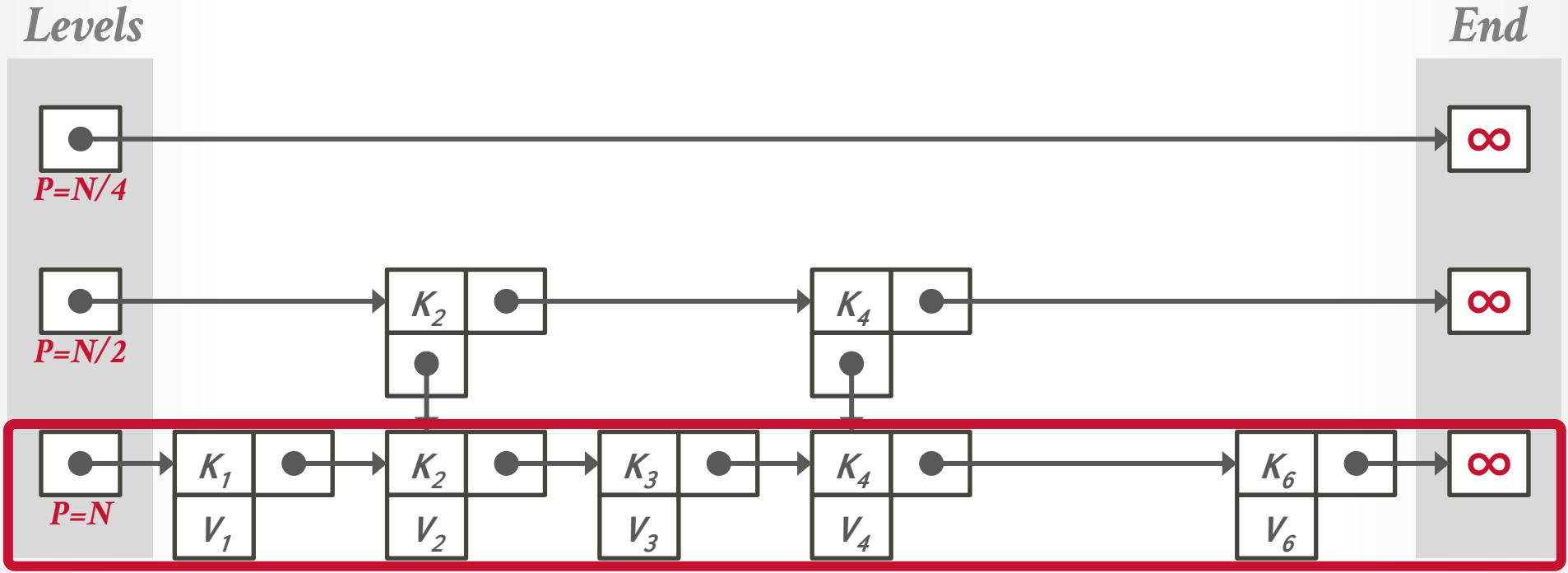
SKIP LISTS: INSERT

Insert K_5



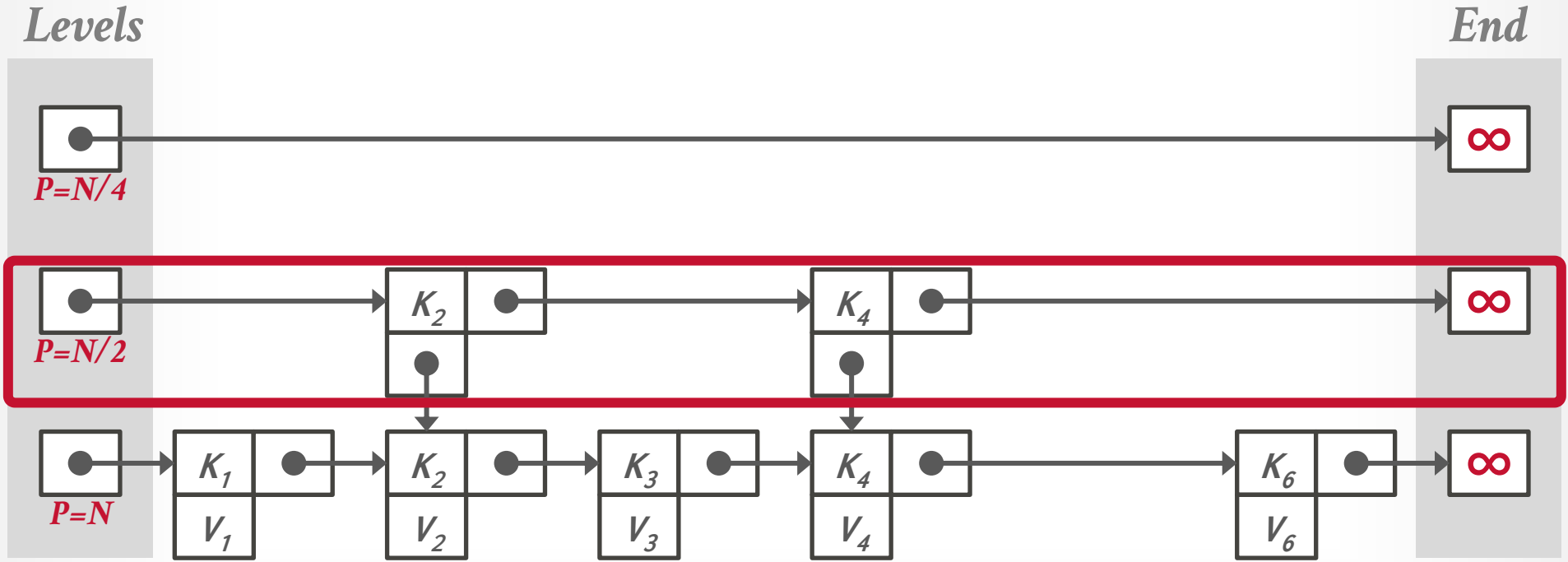
SKIP LISTS: INSERT

Insert K_5



SKIP LISTS: INSERT

Insert K_5

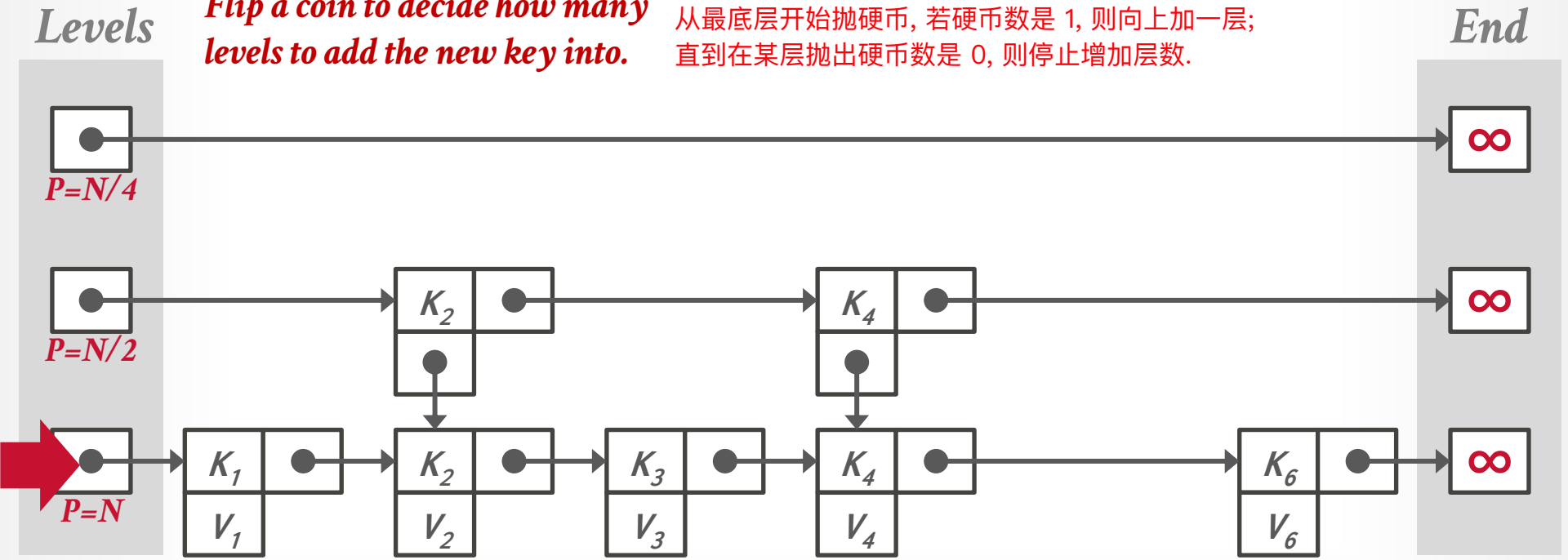


SKIP LISTS: INSERT

Insert K_5

Flip a coin to decide how many levels to add the new key into.

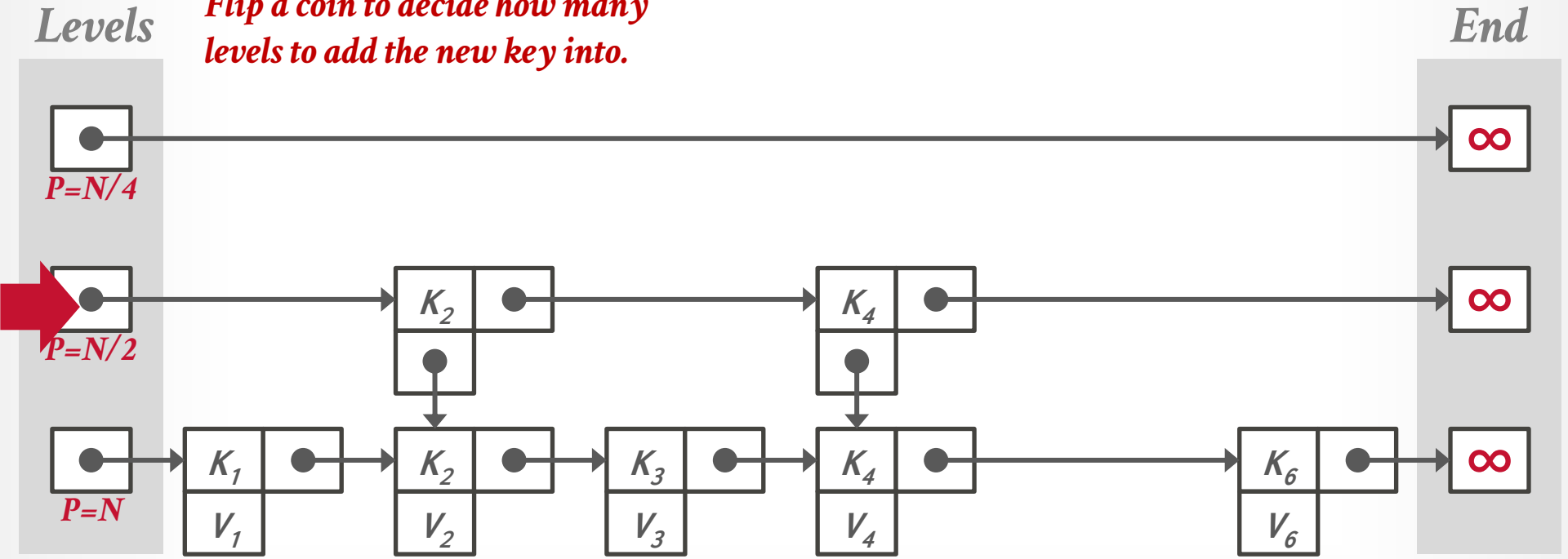
从最底层开始抛硬币, 若硬币数是 1, 则向上加一层;
直到在某层抛出硬币数是 0, 则停止增加层数.



SKIP LISTS: INSERT

Insert K_5

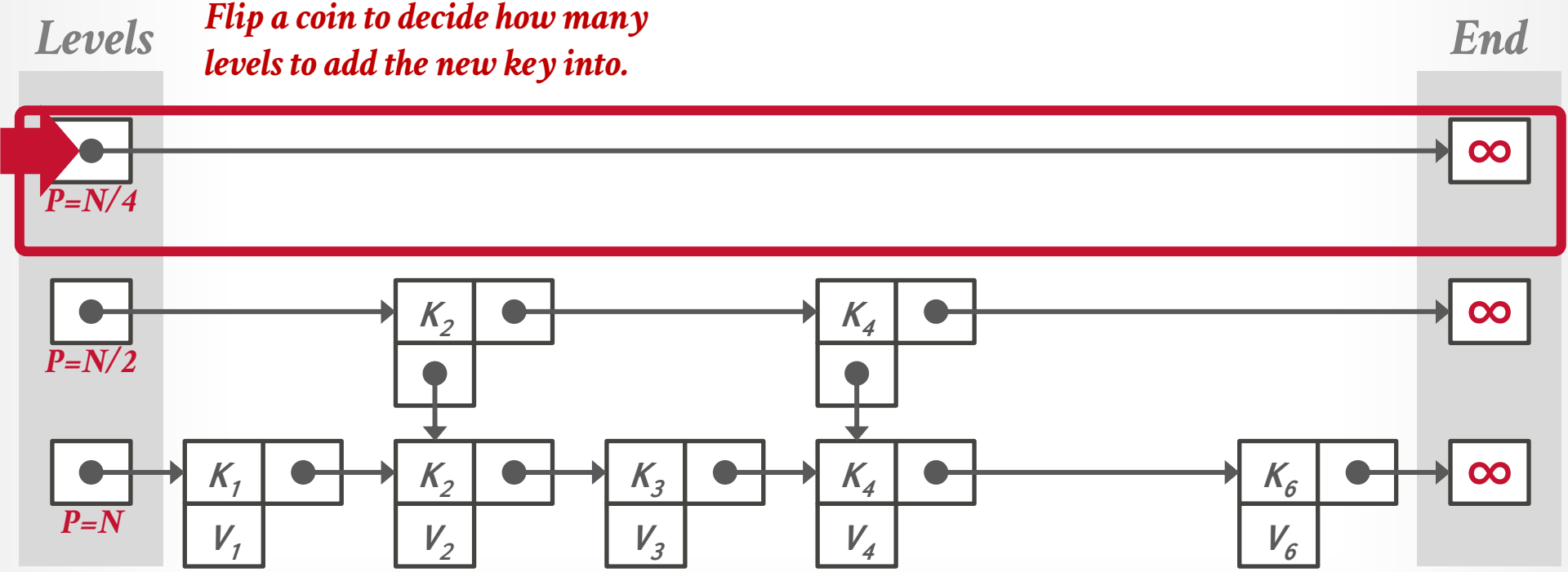
Flip a coin to decide how many levels to add the new key into.



SKIP LISTS: INSERT

Insert K_5

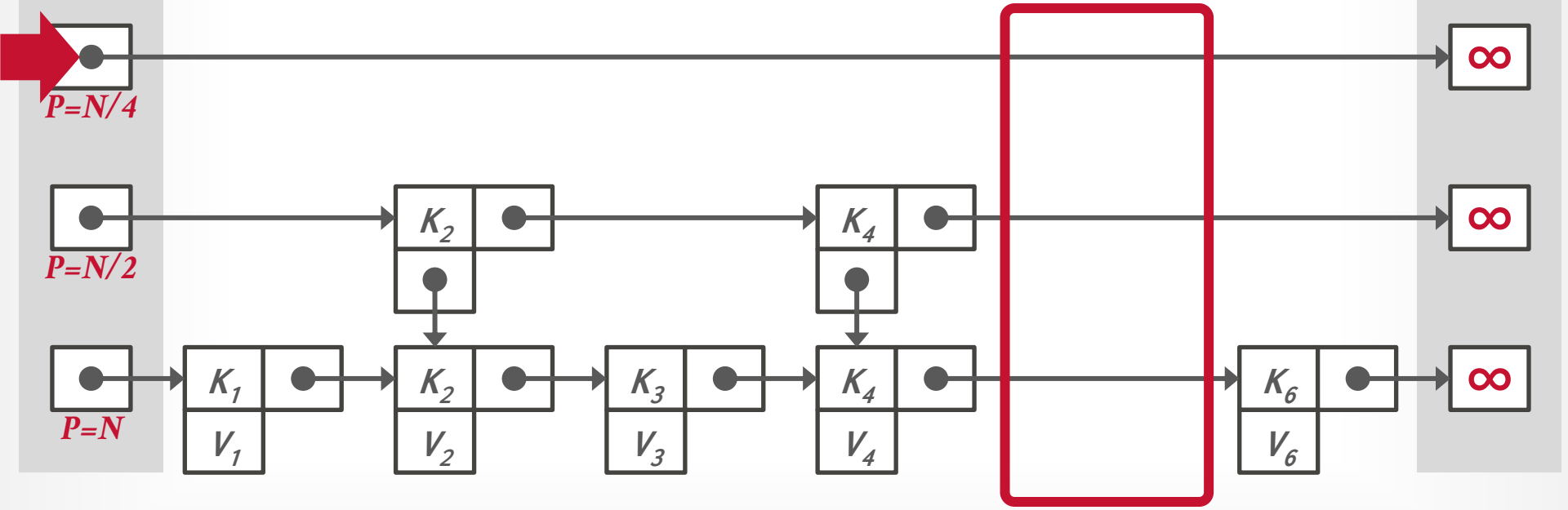
Flip a coin to decide how many levels to add the new key into.



SKIP LISTS: INSERT

Insert K_5

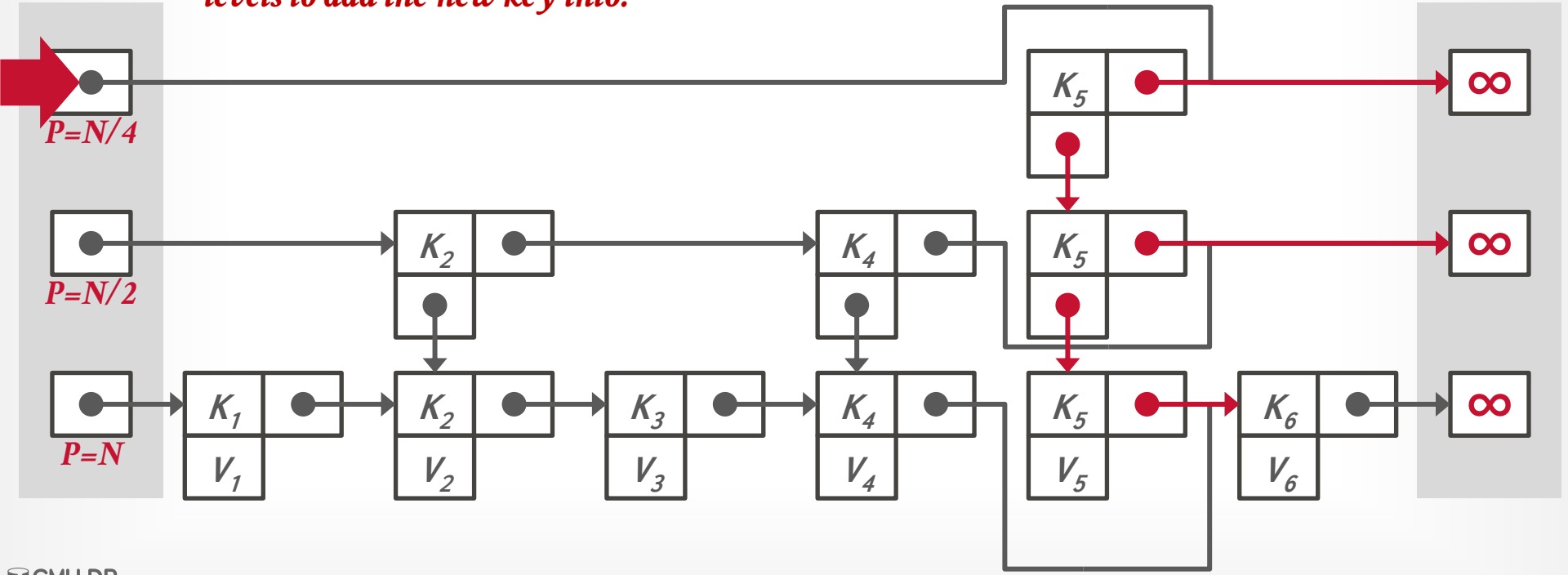
Flip a coin to decide how many levels to add the new key into.



SKIP LISTS: INSERT

Insert K_5

Flip a coin to decide how many levels to add the new key into.

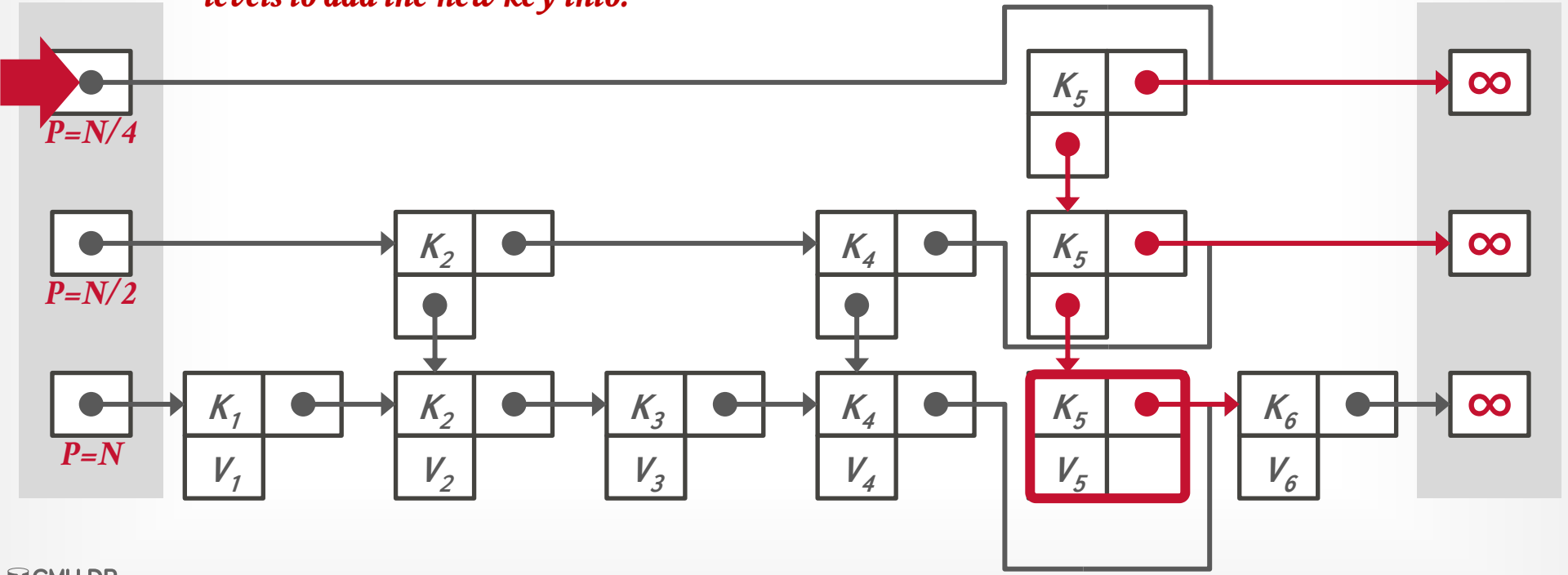


SKIP LISTS: INSERT

Insert K_5

Flip a coin to decide how many levels to add the new key into.

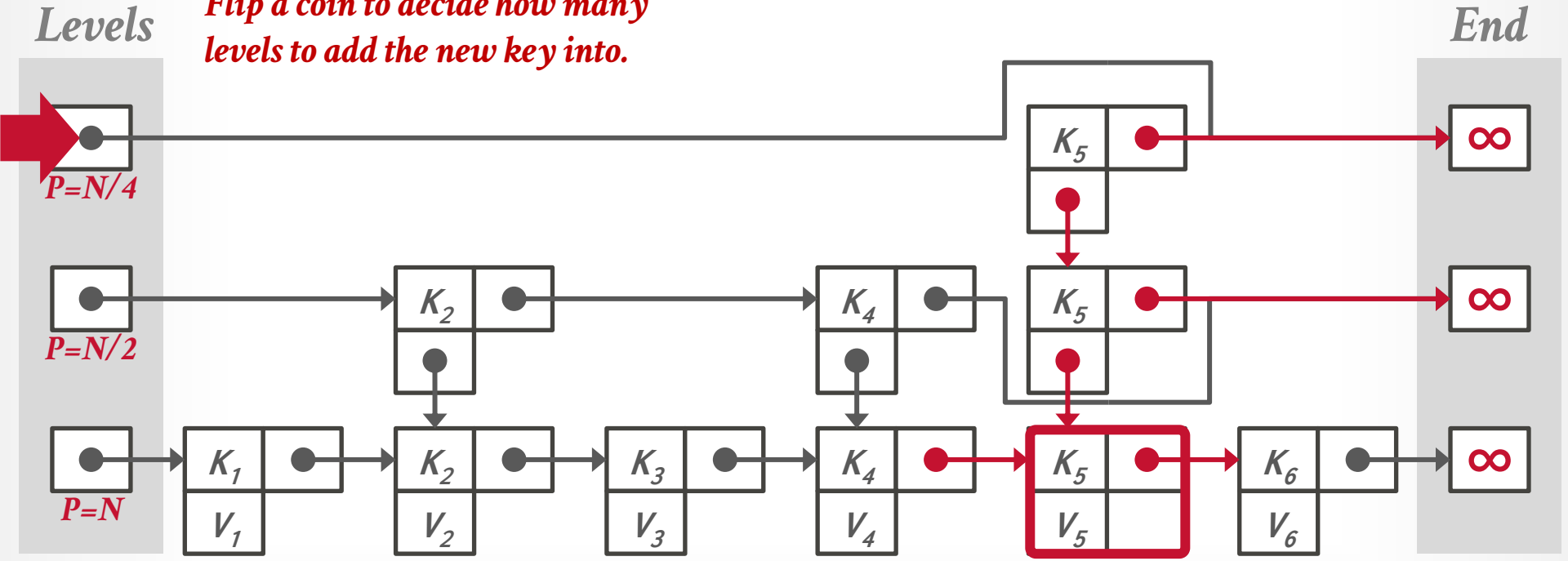
从下到上依次修改前后指针的指向.



SKIP LISTS: INSERT

Insert K_5

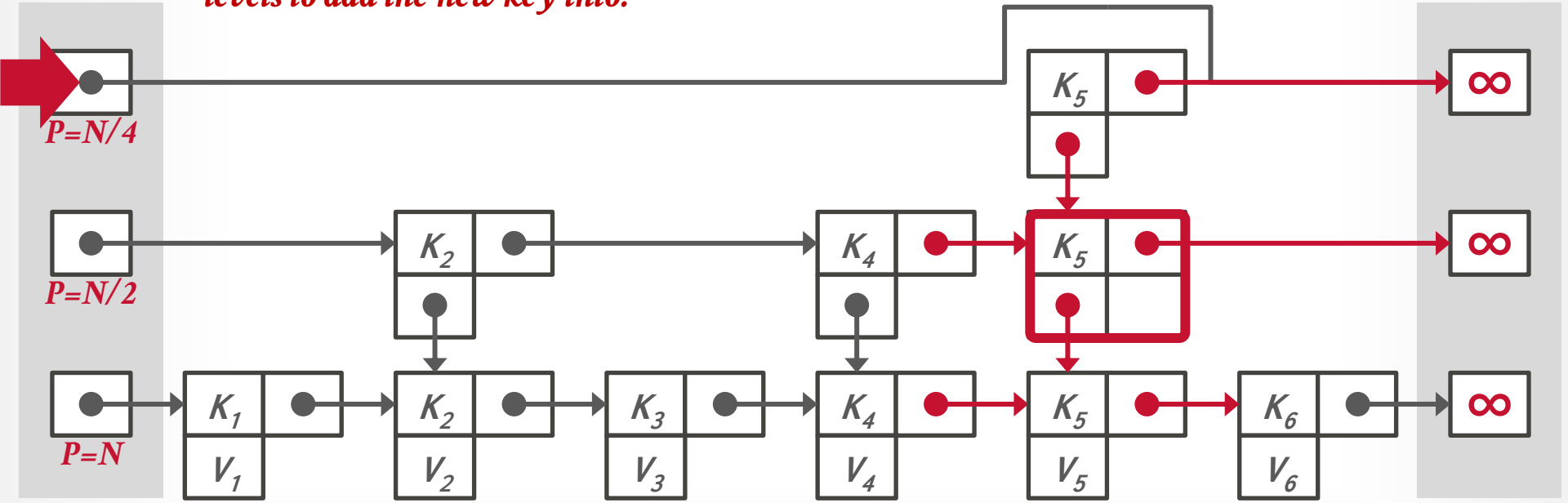
Flip a coin to decide how many levels to add the new key into.



SKIP LISTS: INSERT

Insert K_5

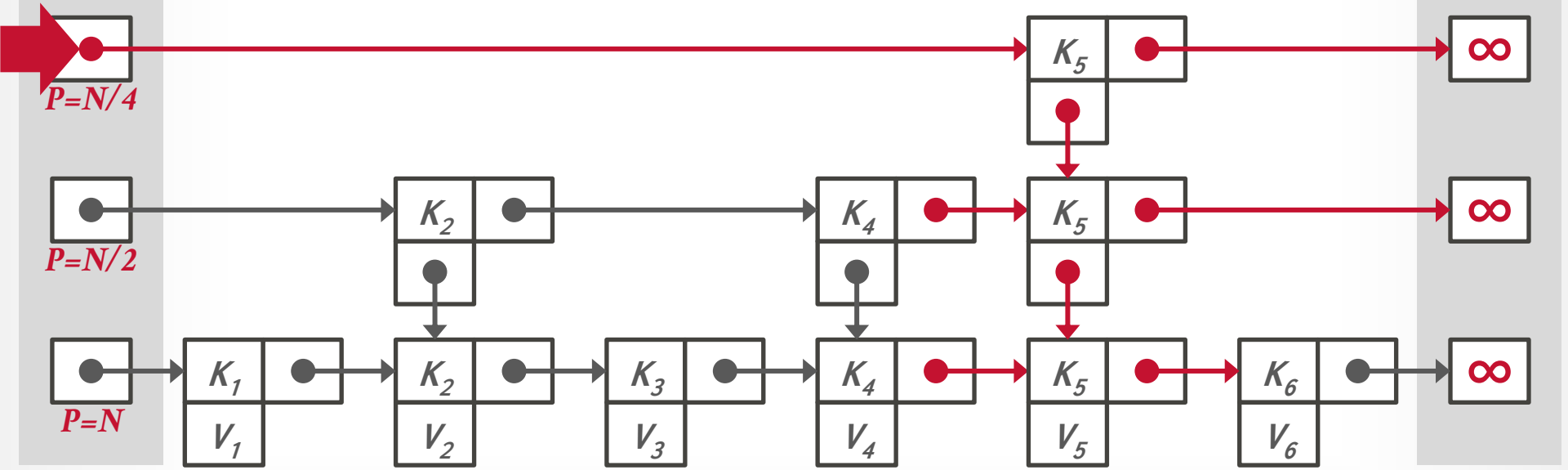
Flip a coin to decide how many levels to add the new key into.



SKIP LISTS: INSERT

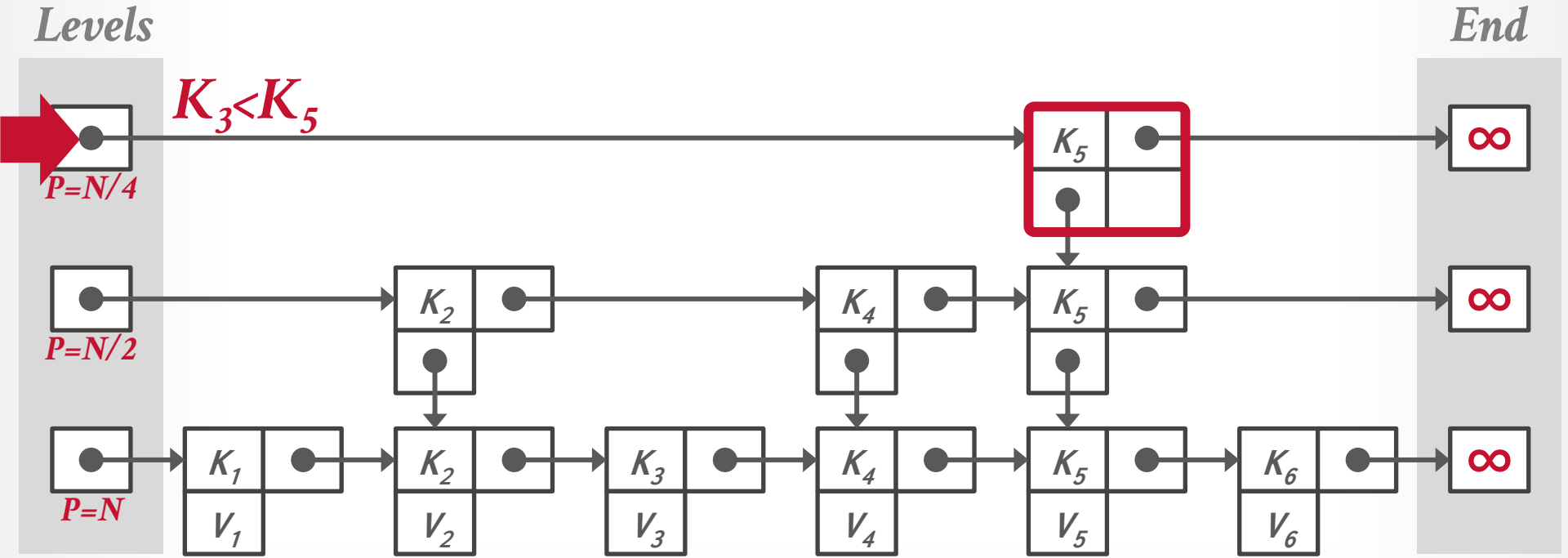
Insert K_5

Flip a coin to decide how many levels to add the new key into.



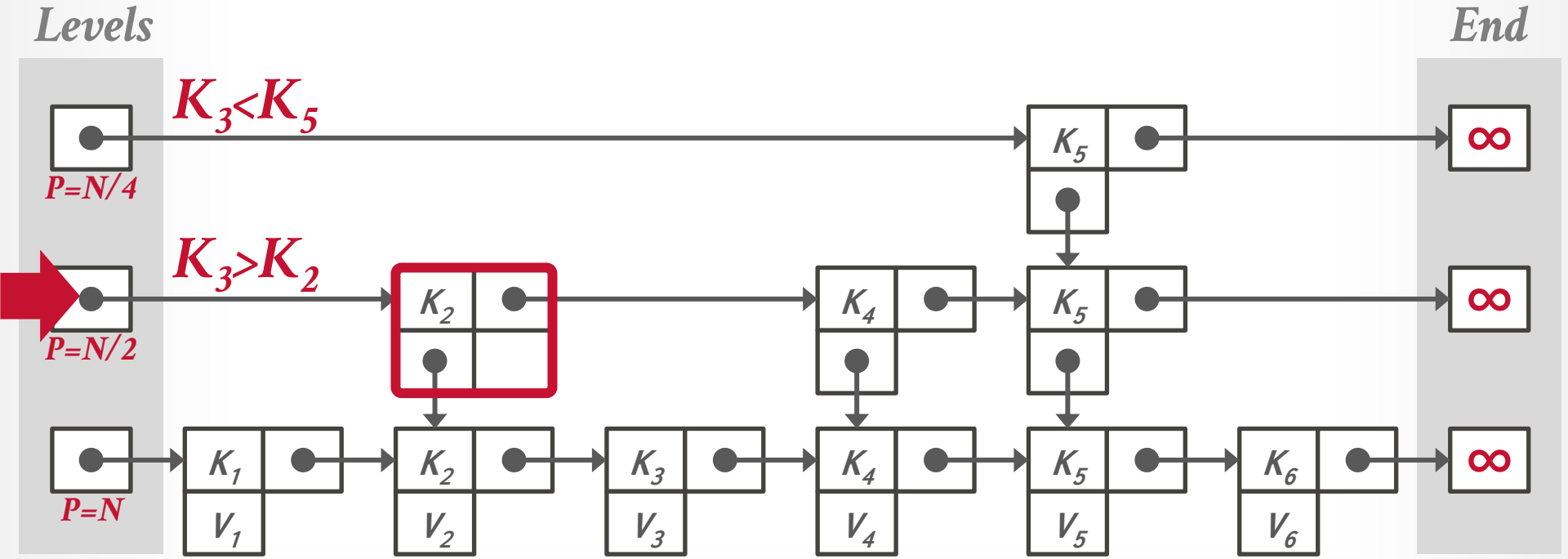
SKIP LISTS: SEARCH

Find K_3



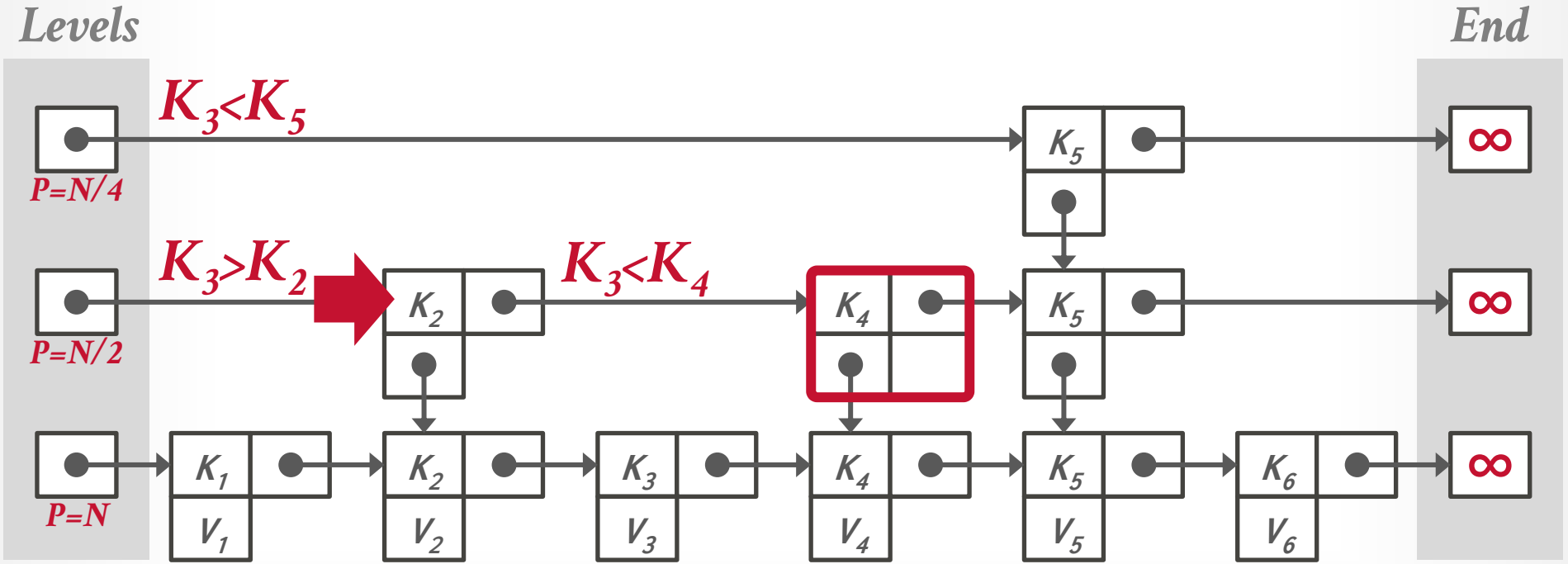
SKIP LISTS: SEARCH

Find K_3



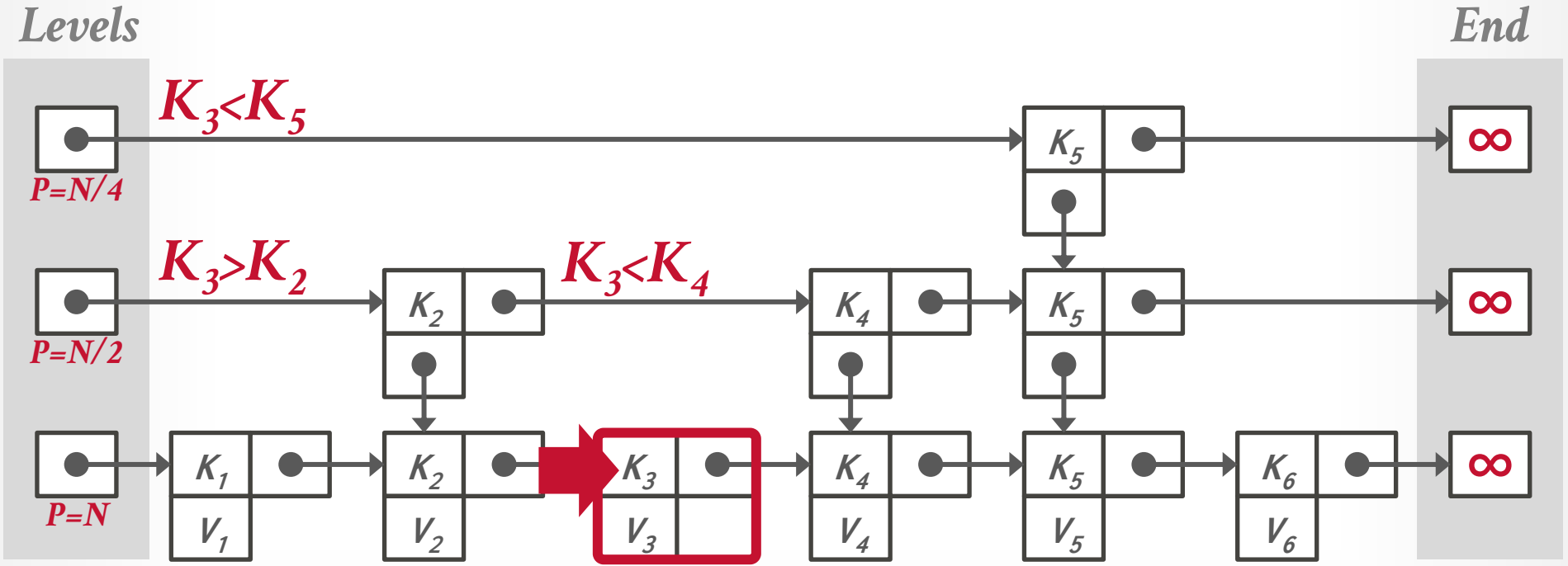
SKIP LISTS: SEARCH

Find K_3



SKIP LISTS: SEARCH

Find K_3



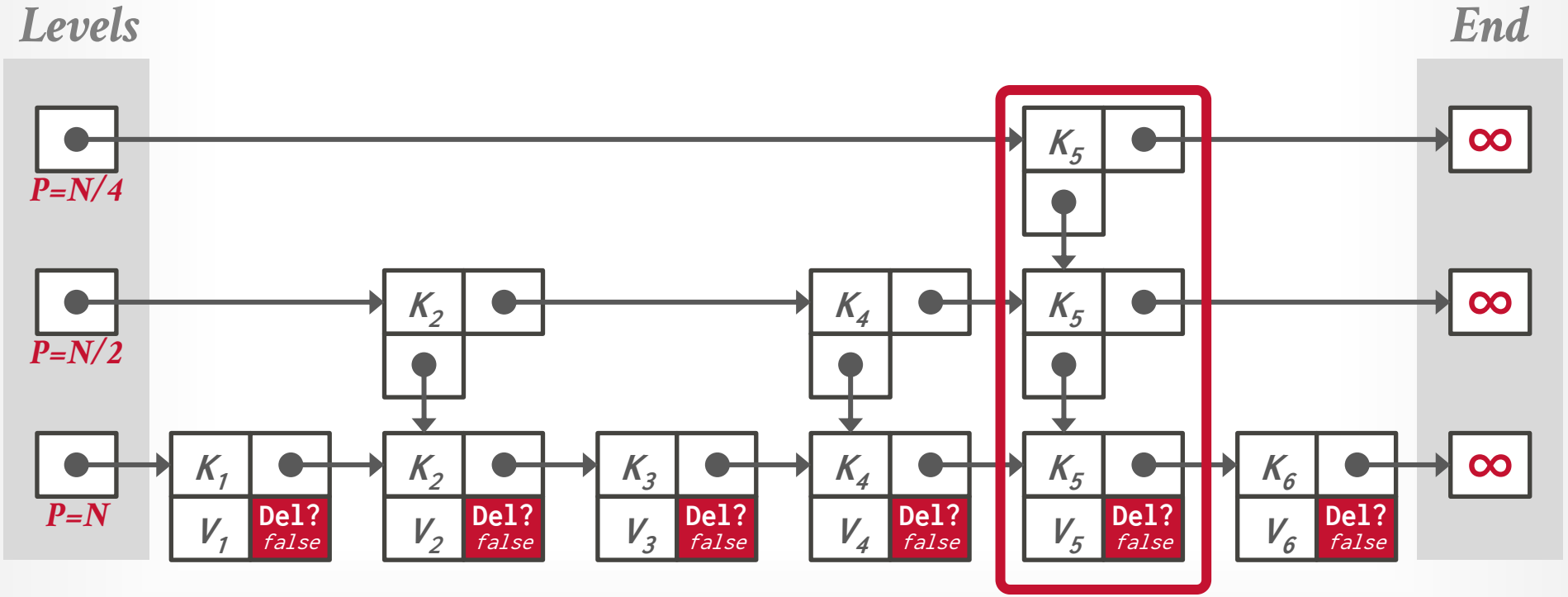
SKIP LISTS: DELETE

First **logically** remove a key from the index by setting a flag to tell threads to ignore.

Then **physically** remove the key once we know that no other thread is holding the reference.

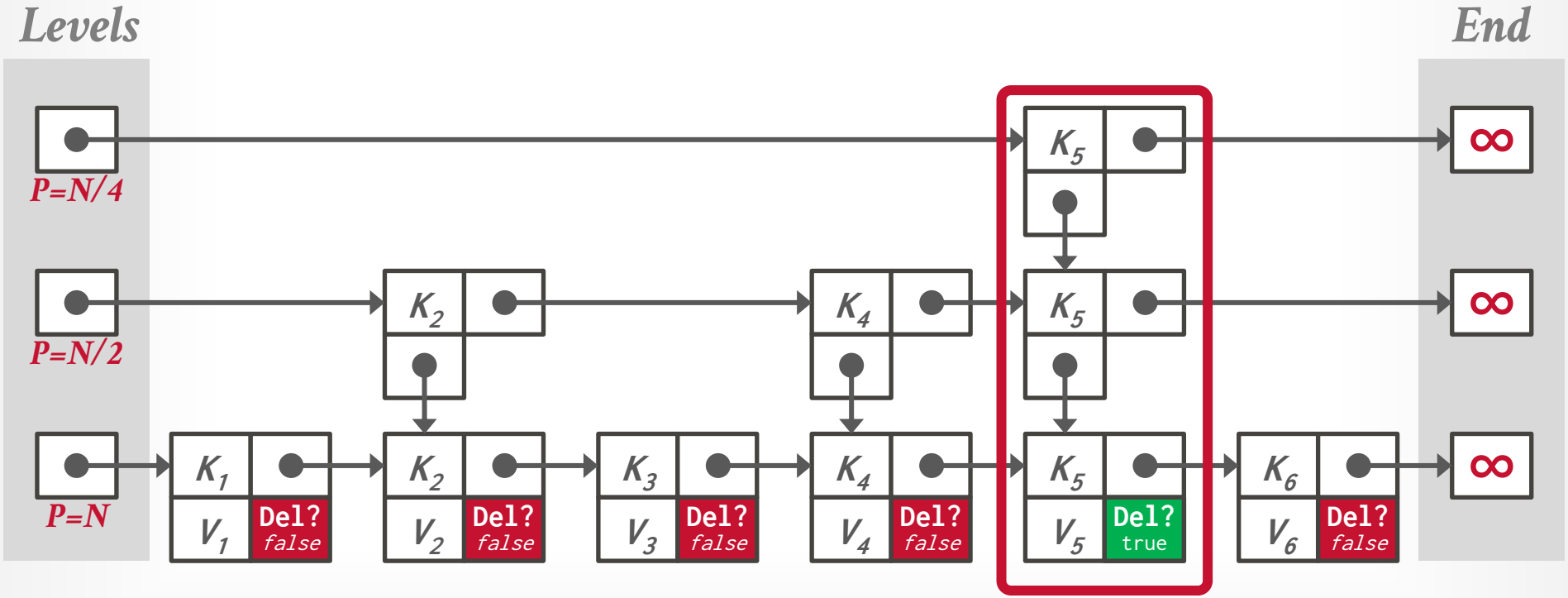
SKIP LISTS: DELETE

Delete K_5



SKIP LISTS: DELETE

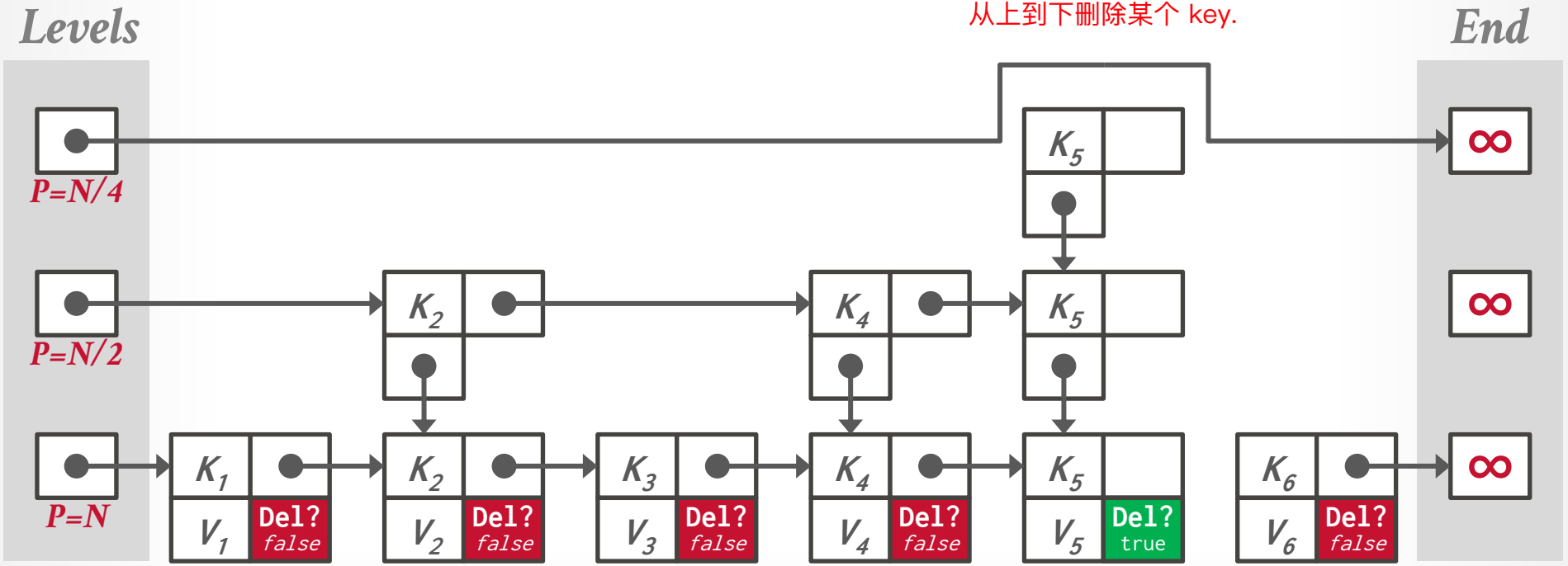
Delete K_5



SKIP LISTS: DELETE

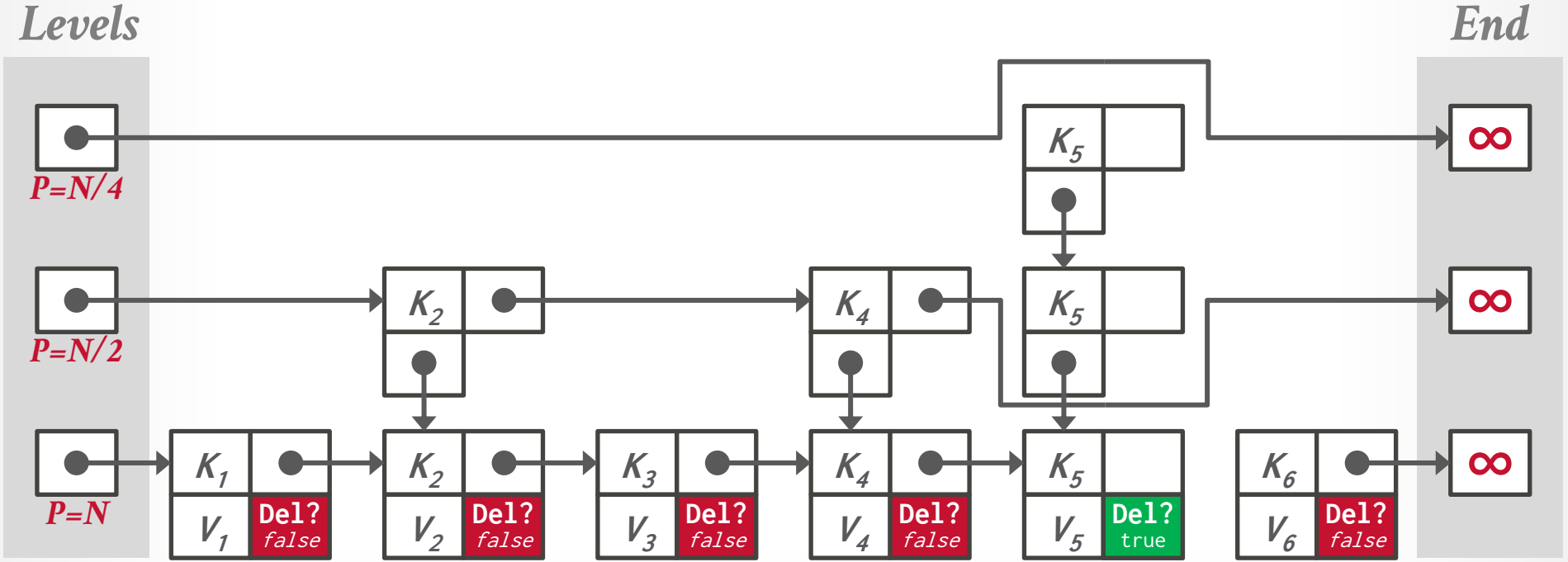
Delete K_5

从上到下删除某个 key.



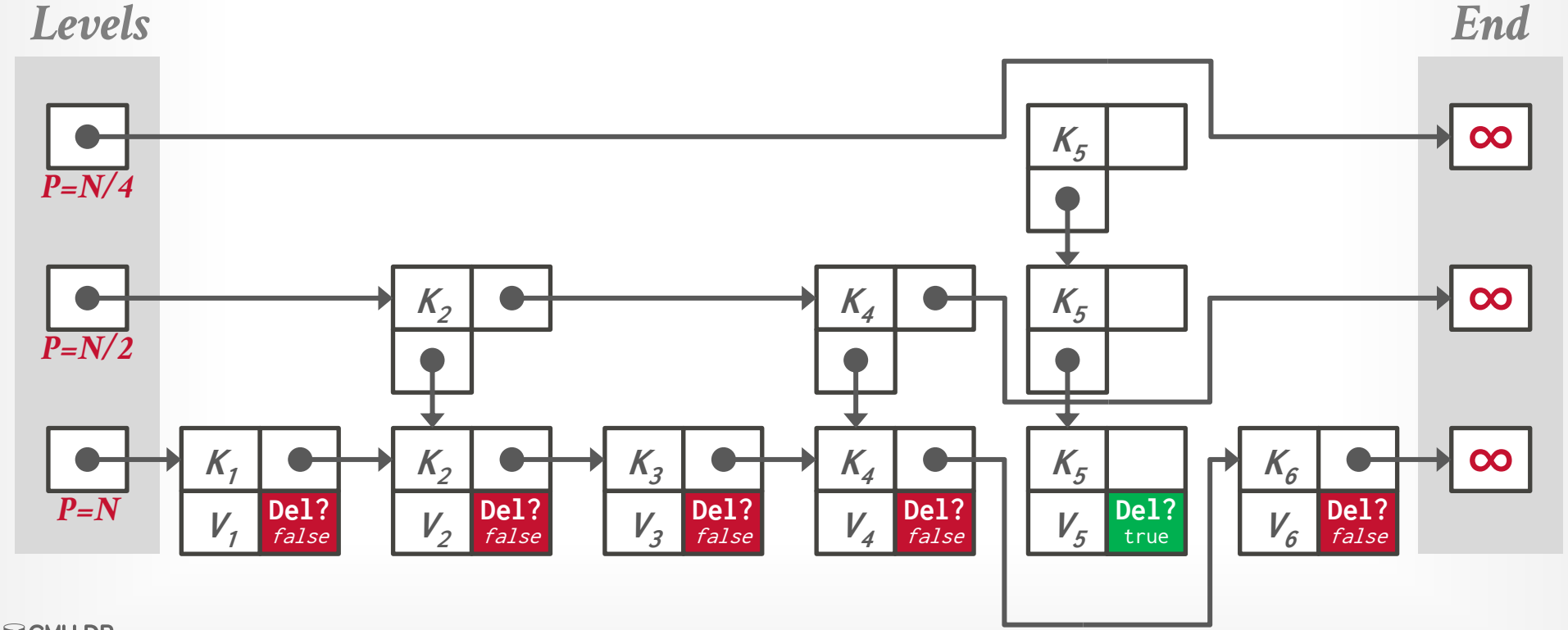
SKIP LISTS: DELETE

Delete K_5



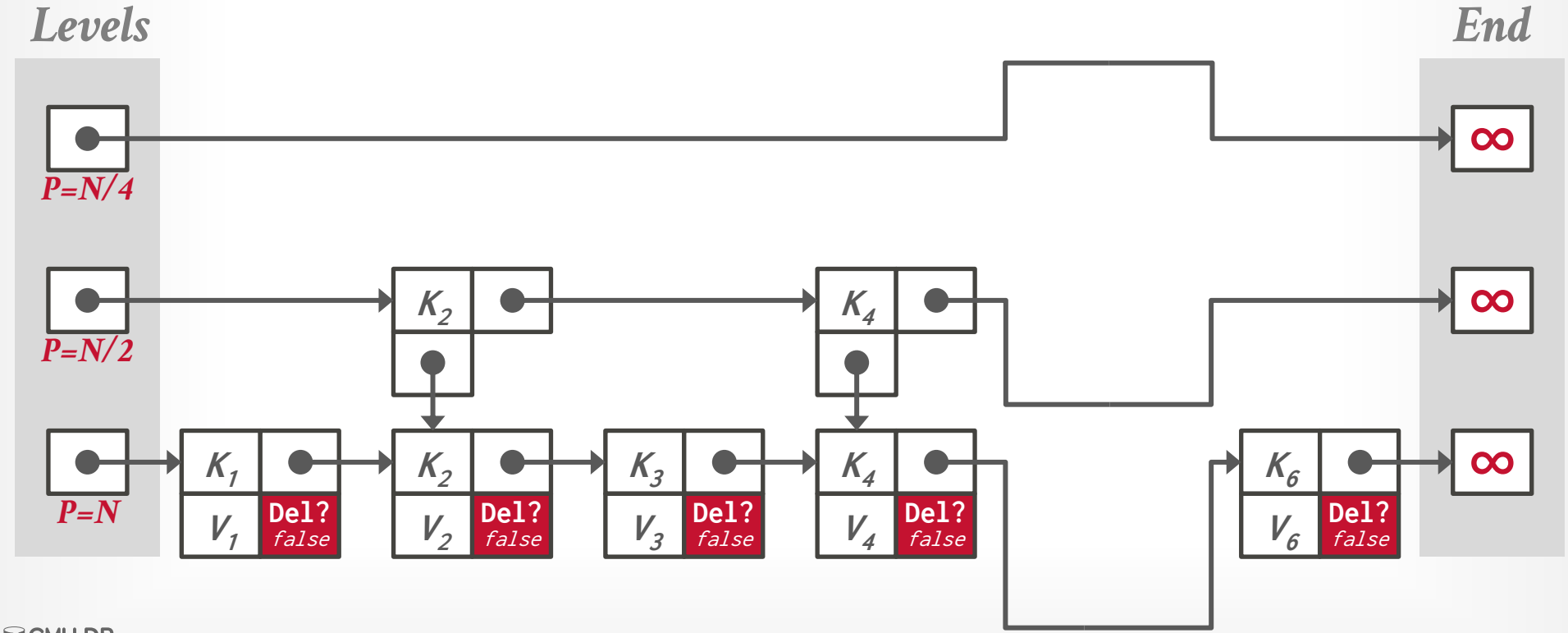
SKIP LISTS: DELETE

Delete K_5



SKIP LISTS: DELETE

Delete K_5



SKIP LISTS

Advantages:

→ Uses less memory than a typical B+Tree if you do not include reverse pointers.
→ Insertions and deletions do not require rebalancing.

Disadvantages:

→ Not disk/cache friendly because they do not optimize locality of references.
→ Reverse search is non-trivial.

OBSERVATION

The inner node keys in a B+Tree cannot tell you whether a key exists in the index. You must always traverse to the leaf node.

This means that you could have (at least) one buffer pool page miss per level in the tree just to find out a key does not exist.

TRIE INDEX

该数据结构不会在任何一个节点存储整个 key.

Use a digital representation of keys to examine prefixes one-by-one.

→ aka ***Digital Search Tree***, ***Prefix Tree***.

Shape depends on keys and lengths.

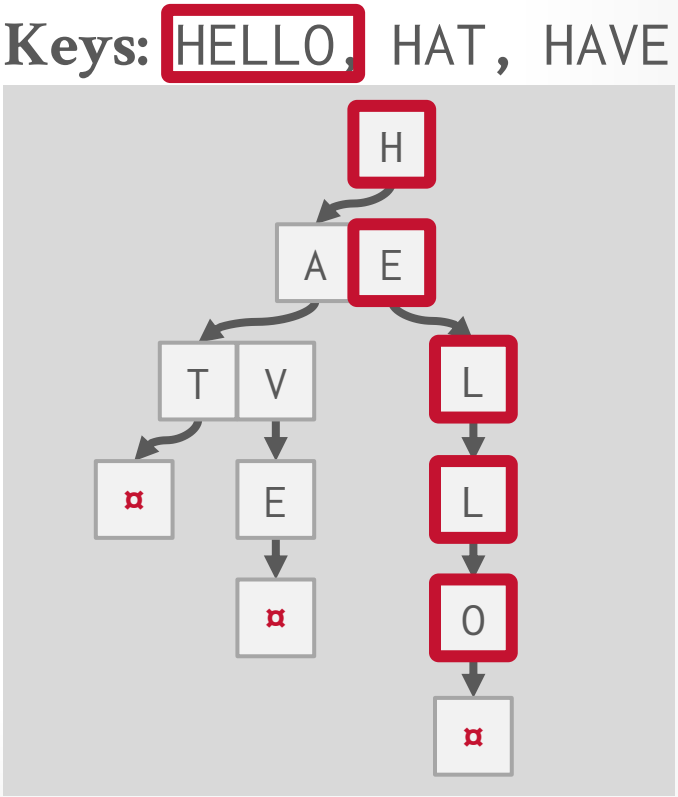
→ Does not depend on existing keys or insertion order.

→ Does not require rebalancing operations.

All operations have **O(k)** complexity where **k** is the length of the key.

→ Path to a leaf node represents a key.

→ Keys are stored implicitly and can be reconstructed from paths.



TRIE KEY SPAN

The **span** of a trie level is the number of bits that each partial key / digit represents.

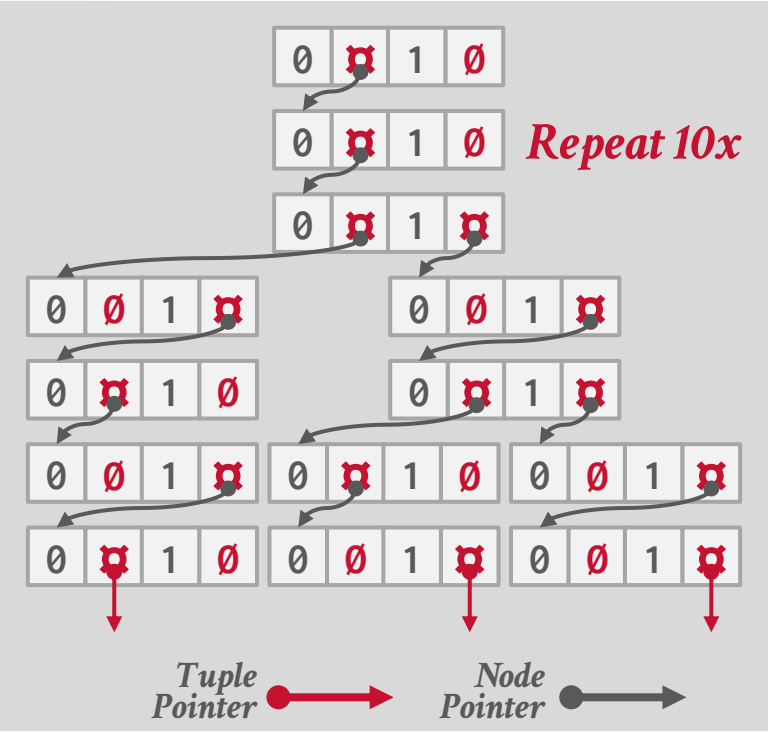
→ If the digit exists in the corpus, then store a pointer to the next level in the trie branch. Otherwise, store null.

This determines the **fan-out** of each node and the physical **height** of the tree.

→ ***n***-way Trie = Fan-Out of ***n***

TRIE KEY SPAN

1-bit Span Trie

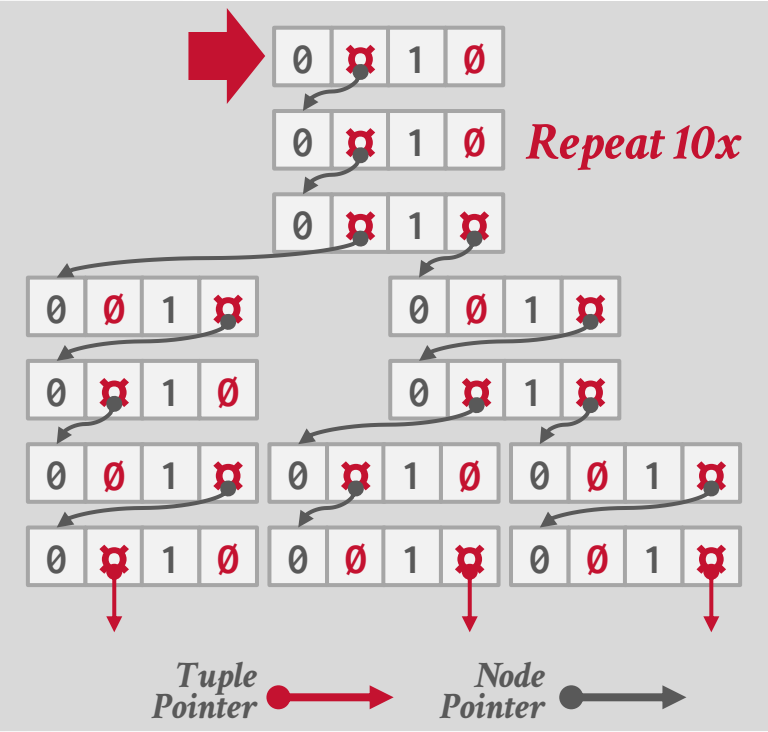


Keys: K10,K25,K31

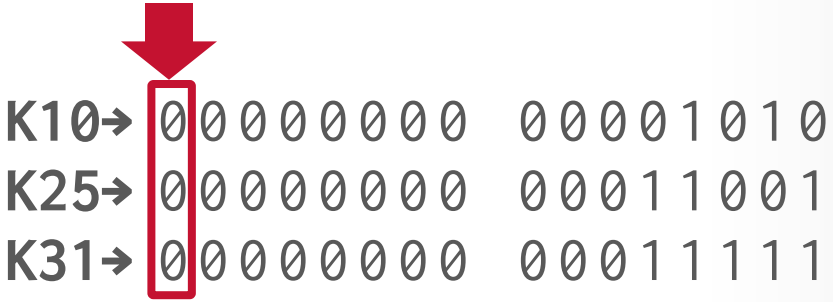
K10→ 00000000 00001010
K25→ 00000000 00011001
K31→ 00000000 00011111

TRIE KEY SPAN

1-bit Span Trie

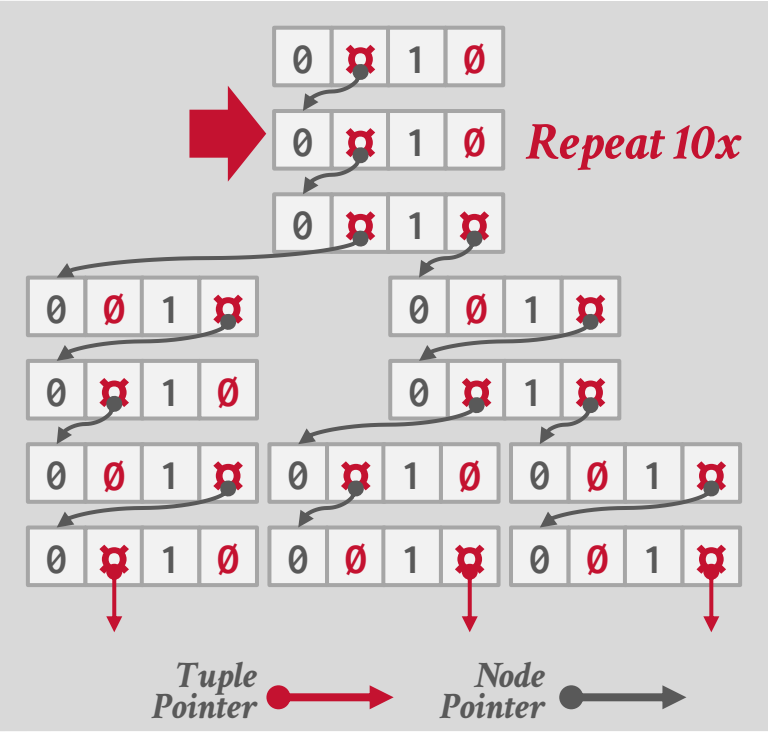


Keys: K10,K25,K31



TRIE KEY SPAN

1-bit Span Trie

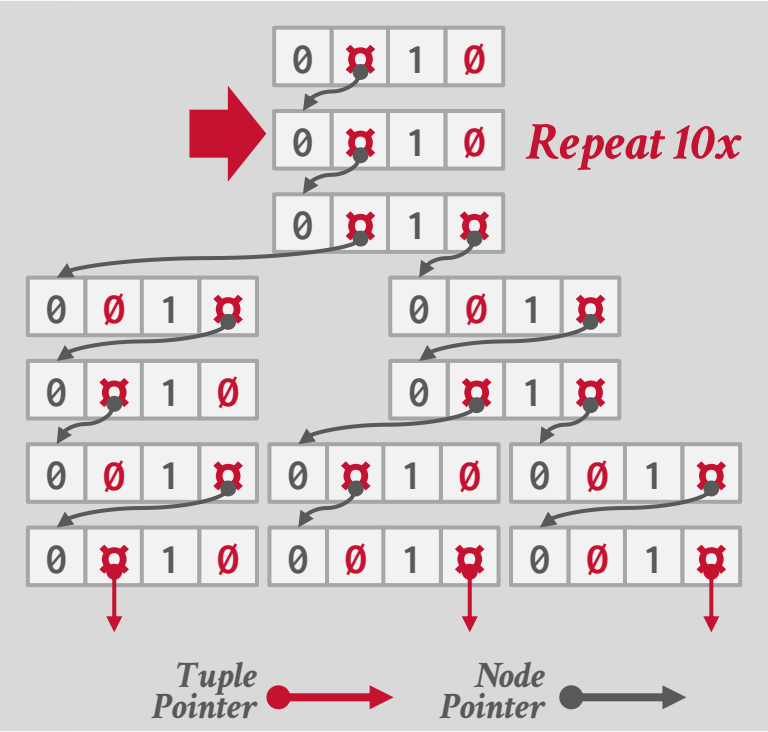


Keys: K10,K25,K31

K10→ 00000000 00001010
K25→ 00000000 00011001
K31→ 00000000 00011111

TRIE KEY SPAN

1-bit Span Trie

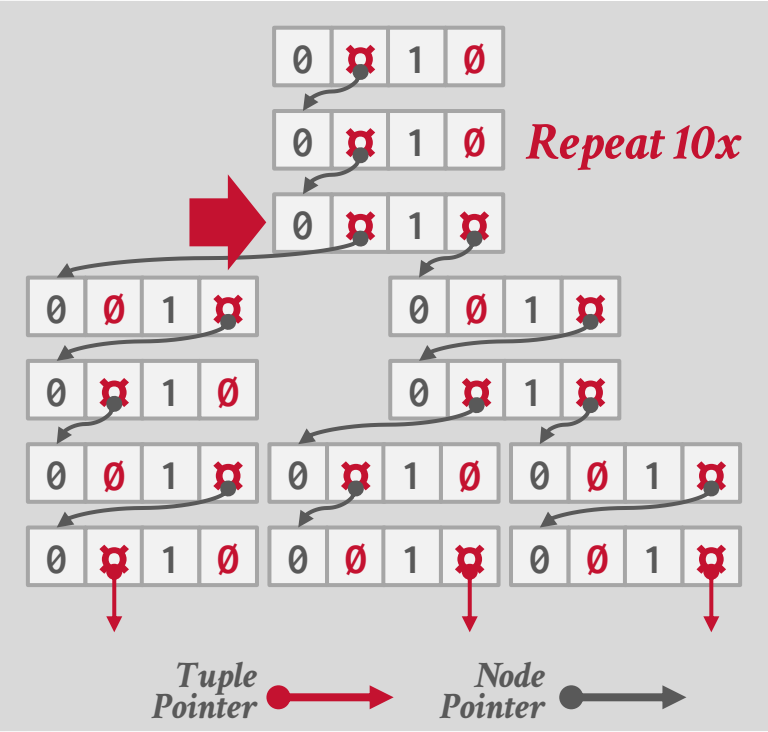


Keys: K10,K25,K31

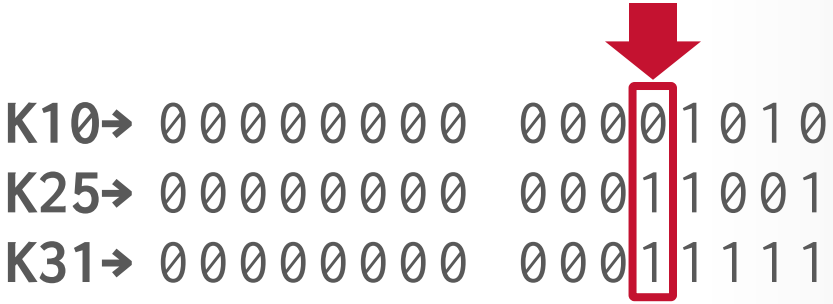
K10→ 00000000 00001010
K25→ 00000000 00011001
K31→ 00000000 00011111

TRIE KEY SPAN

1-bit Span Trie

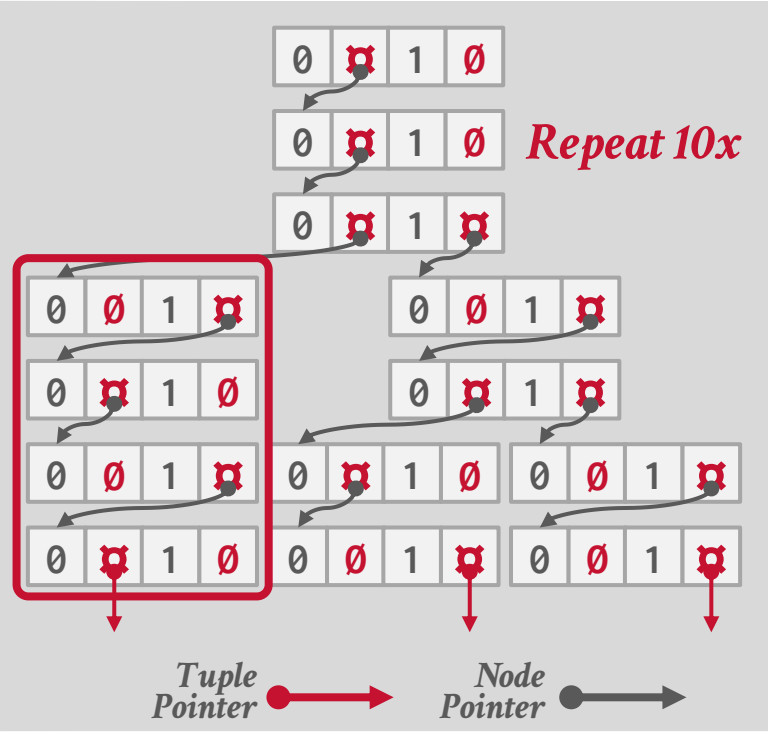


Keys: K10,K25,K31



TRIE KEY SPAN

1-bit Span Trie

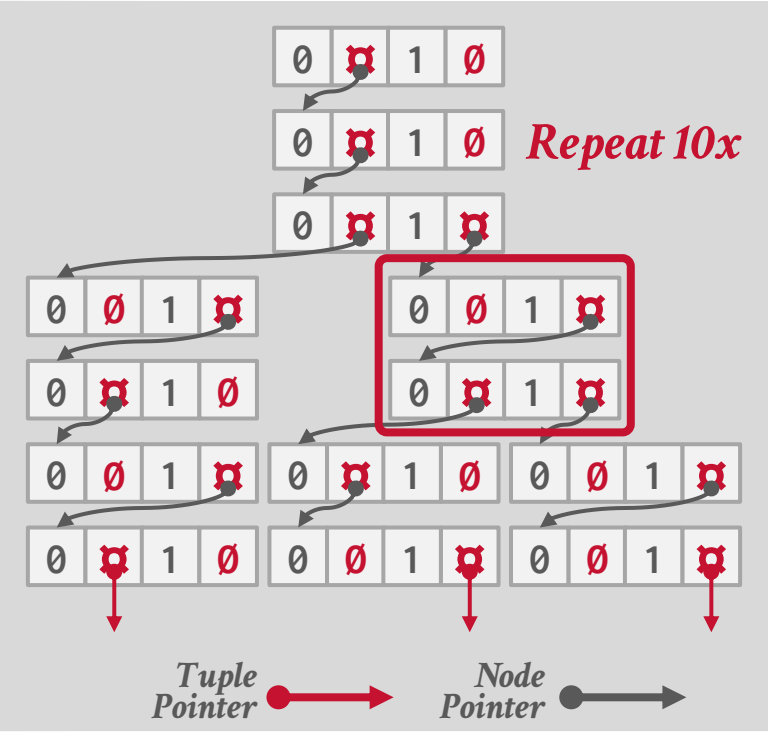


Keys: K10,K25,K31

K10→ 00000000 00001010
K25→ 00000000 00011001
K31→ 00000000 00011111

TRIE KEY SPAN

1-bit Span Trie



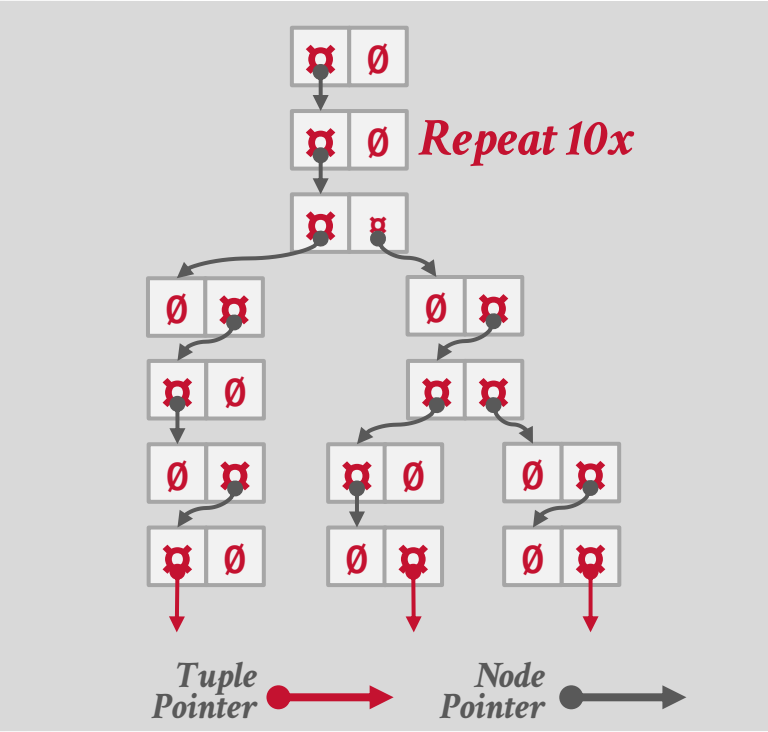
Keys: K10,K25,K31

垂直压缩: 如果存在重复路径, 则可以压缩这些层级来减少存储空间.

K10→ 00000000 00001010
K25→ 00000000 00011001
K31→ 00000000 00011111

TRIE KEY SPAN

1-bit Span Trie



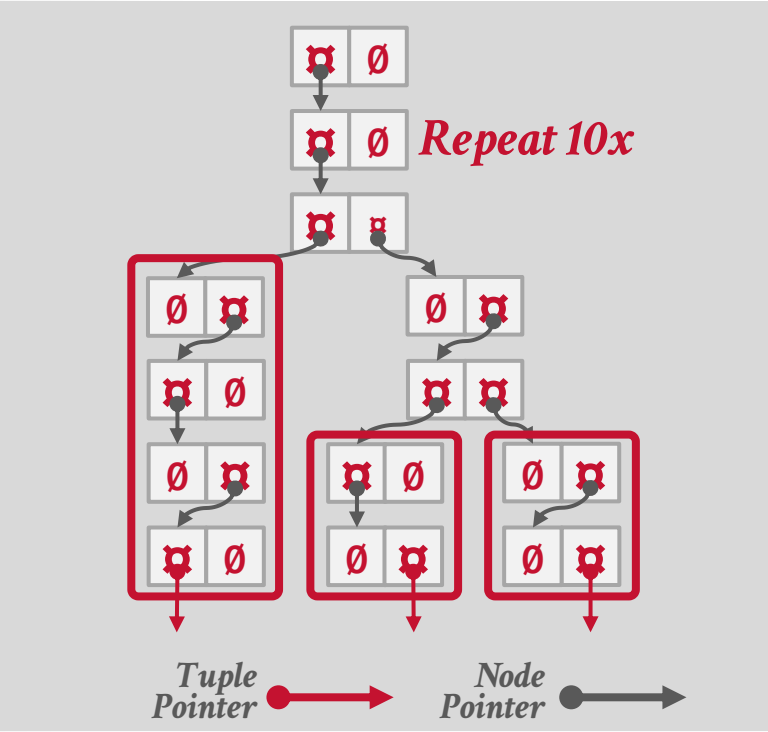
Keys: K10,K25,K31

水平压缩.

K10→ 00000000 00001010
K25→ 00000000 00011001
K31→ 00000000 00011111

TRIE KEY SPAN

1-bit Span Trie

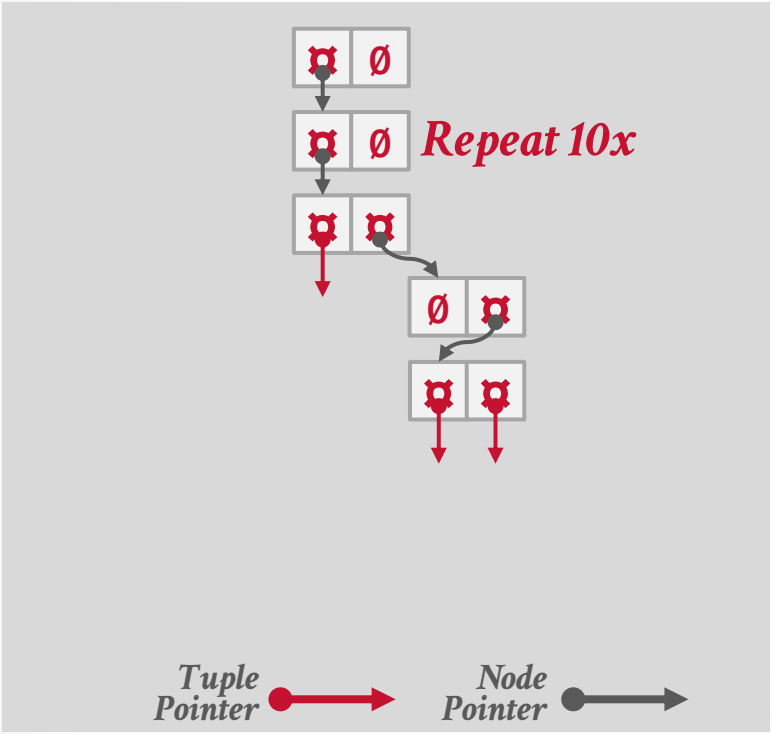


Keys: K10,K25,K31

K10→ 00000000 00001010
K25→ 00000000 00011001
K31→ 00000000 00011111

RADIX TREE 基数树

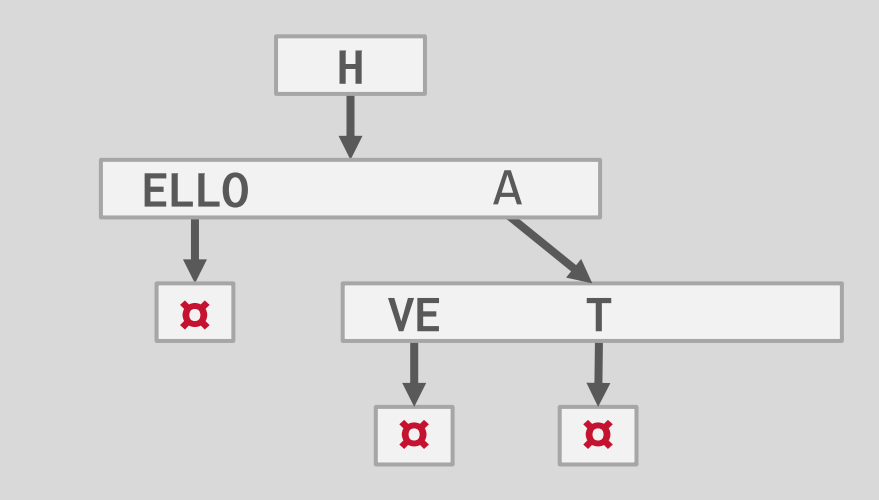
1-bit Span Radix Tree



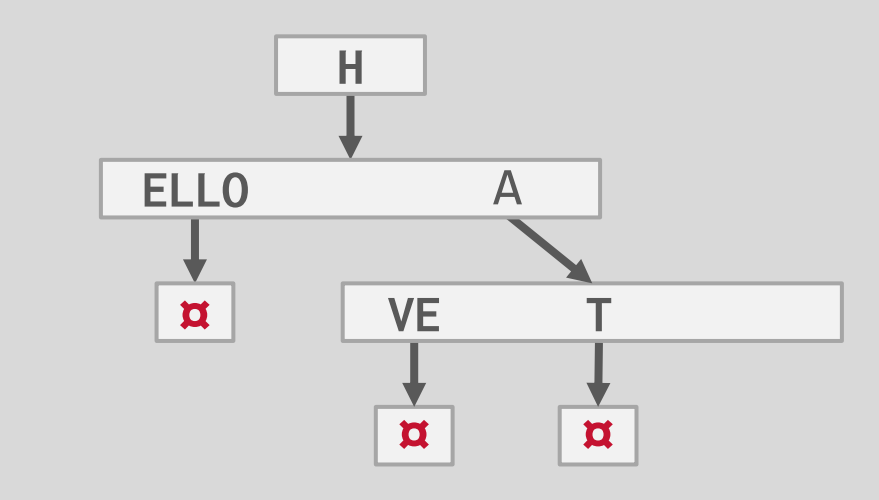
Vertically compressed trie that compacts nodes with a single child.
→ Also known as ***Patricia Tree***.

Can produce false positives, so the DBMS always checks the original tuple to see whether a key matches.

RADIX TREE: MODIFICATIONS

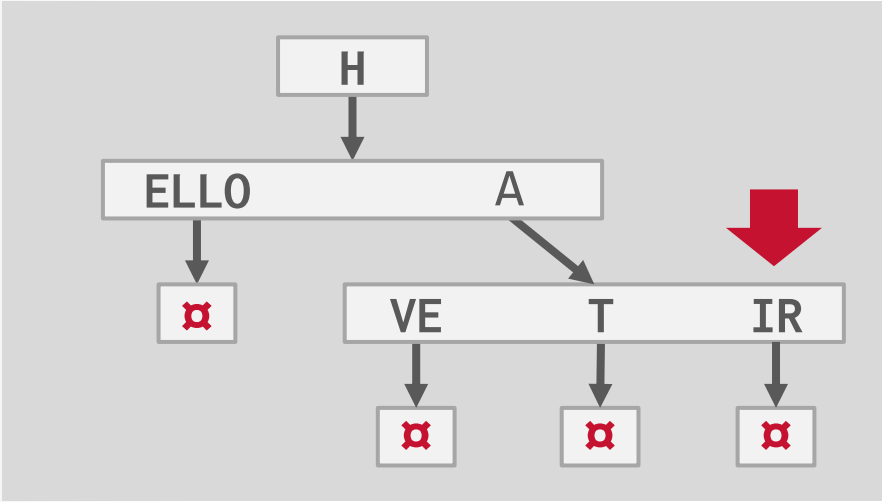


RADIX TREE: MODIFICATIONS



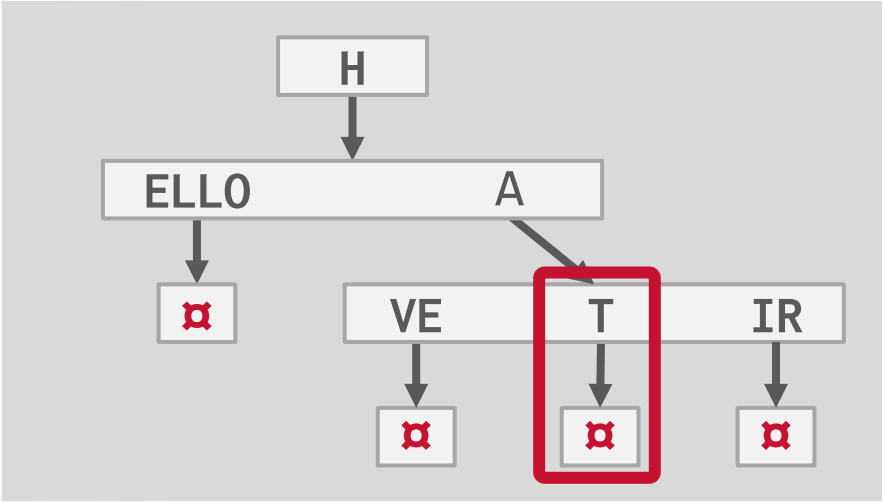
Insert HAIR

RADIX TREE: MODIFICATIONS



Insert HAIR

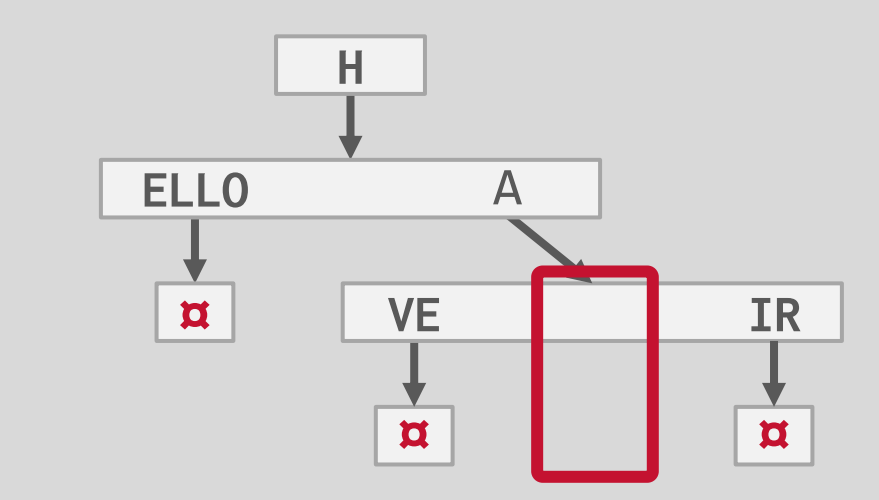
RADIX TREE: MODIFICATIONS



Insert HAIR

Delete HAT

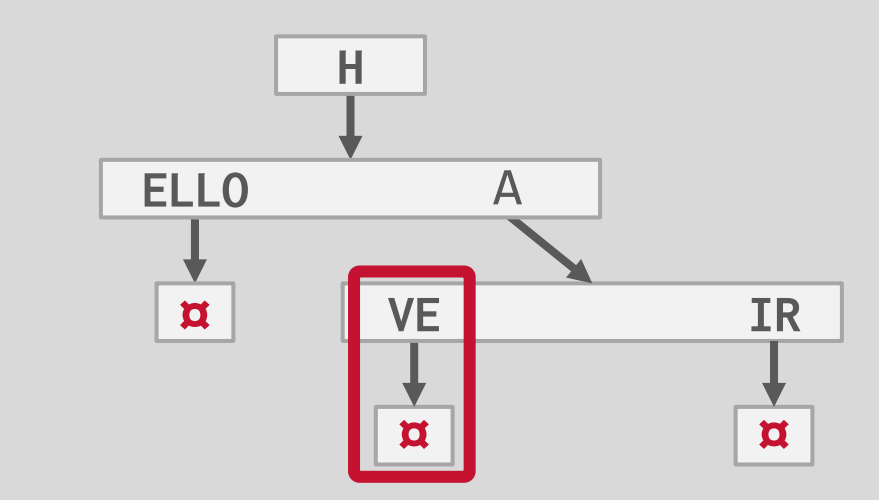
RADIX TREE: MODIFICATIONS



Insert HAIR

Delete HAT

RADIX TREE: MODIFICATIONS

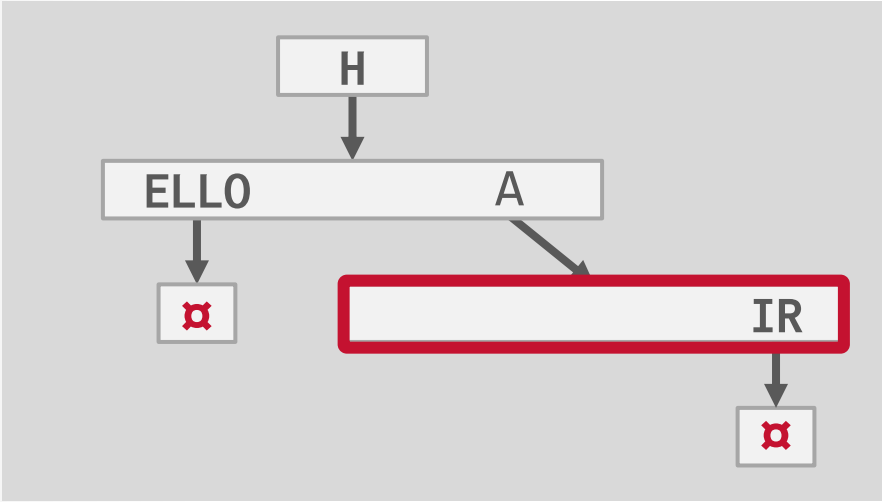


Insert HAIR

Delete HAT

Delete HAVE

RADIX TREE: MODIFICATIONS

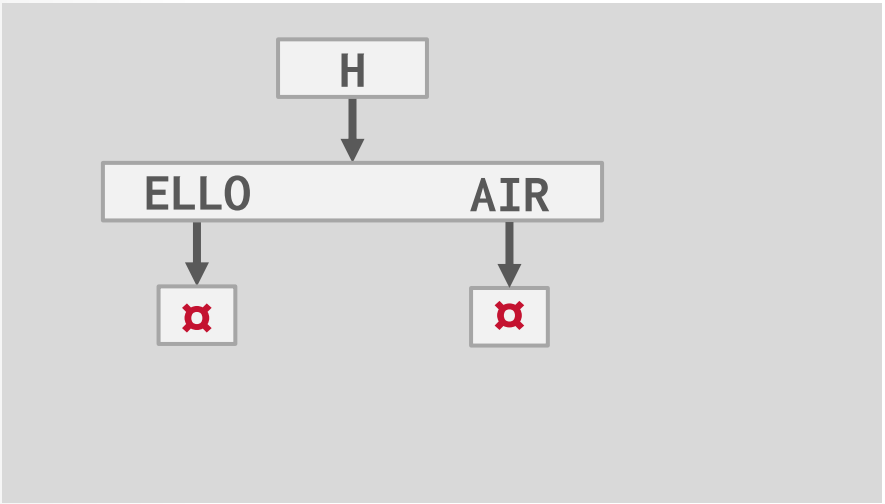


Insert HAIR

Delete HAT

Delete HAVE

RADIX TREE: MODIFICATIONS



Insert HAIR

Delete HAT

Delete HAVE

基数树比 B+Tree 更加紧凑, 不需要重复存储 key.

OBSERVATION

The indexes that we've discussed are useful for "point" and "range" queries:

→ Find all customers in the 15217 zipcode.

→ Find all orders between June 2024 and September 2024.

They are **not** good at keyword searches:

→ Example: Find all Wikipedia articles that contain the word "Pavlo"

OBSERVATION

The indexes that we've discussed are useful for "point" and "range" queries:

→ Find all customers in the 15217 zipcode.

→ Find all orders between June 2024 and September 2024.

They are **not** good at keyword searches:

→ Example: Find all Wikipedia articles that contain the word "Pavlo"

revisions(id,content,…)

id	content
11	Wu-Tang Clan is an American hip hop musical collective formed in Staten Island, New York City, in 1992...
22	Carnegie Mellon University (CMU) is a private research university in Pittsburgh, Pennsylvania. The institution was established in 1900 by Andrew Carnegie...
33	In computing, a database is an organized collection of data or a type of data store based on the use of a database management system (DBMS), the software...
44	Andrew Pavlo, best known as Andy Pavlo, is an associate professor of Computer Science at Carnegie Mellon University. He conducts research on database...

```
CREATE INDEX idx_rev_cntnt
    ON revisions (content);
```

```
SELECT pageID FROM revisions
 WHERE content LIKE '%Pavlo%';
```

INVERTED INDEX

倒排索引是为了实现关键字搜索的索引, 主要用在全文搜索的场景.

An **inverted index** stores a mapping of terms to records that contain those terms in the target attribute.

→ Sometimes called a ***full-text search index***.

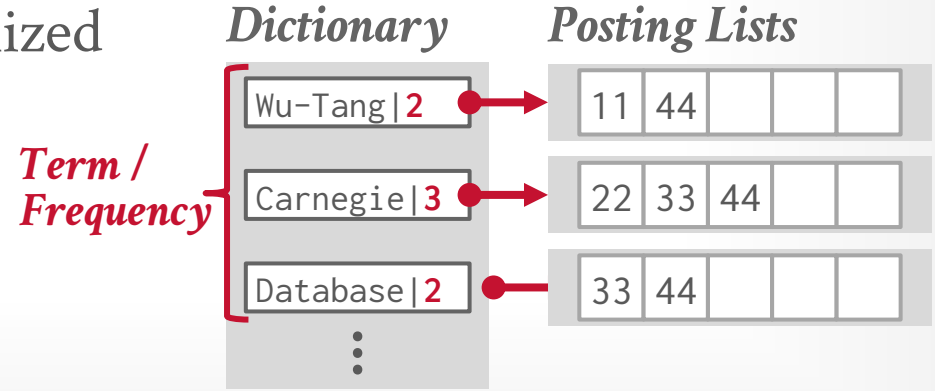
→ Originally called a ***concordance*** (1200s).

Many major DBMSs support these natively. But there are also specialized DBMSs and libraries.

Lucene elasticsearch Solr Xapian OpenSearch Sphinx tantivy vespa splunk>

revisions(id,content,…)

id	content
11	Wu-Tang Clan is an American hip hop musical collective formed in Staten Island, New York City, in 1992...
22	Carnegie Mellon University (CMU) is a private research university in Pittsburgh, Pennsylvania. The institution was established in 1900 by Andrew Carnegie...
33	In computing, a database is an organized collection of data or a type of data store based on the use of a database management system (DBMS), the software...
44	Andrew Pavlo, best known as Andy Pavlo, is an associate professor of Computer Science at Carnegie Mellon University. He conducts research on database...



INVERTED INDEX: LUCENE

有限状态转换器.

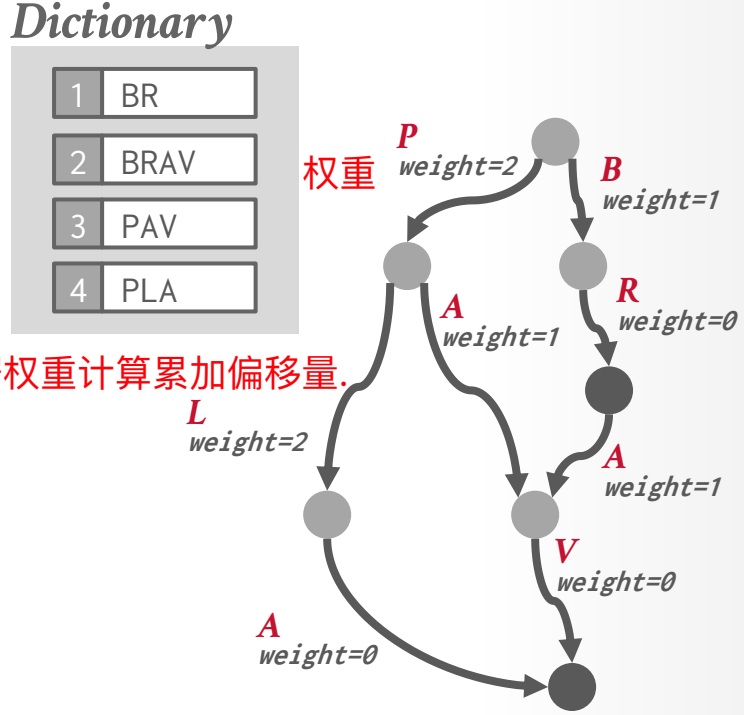
Uses a **Finite State Transducer** for determining offset of terms in dictionary.

为每次插入的记录创建字典, 后台进行合并成更大的字典以减少冗余的值.

Incrementally create dictionary segments and then merge them in the background.

→ Uses compression methods we previously discussed (e.g., delta, bit packing).

→ Also supports precomputed aggregations for terms and occurrences.



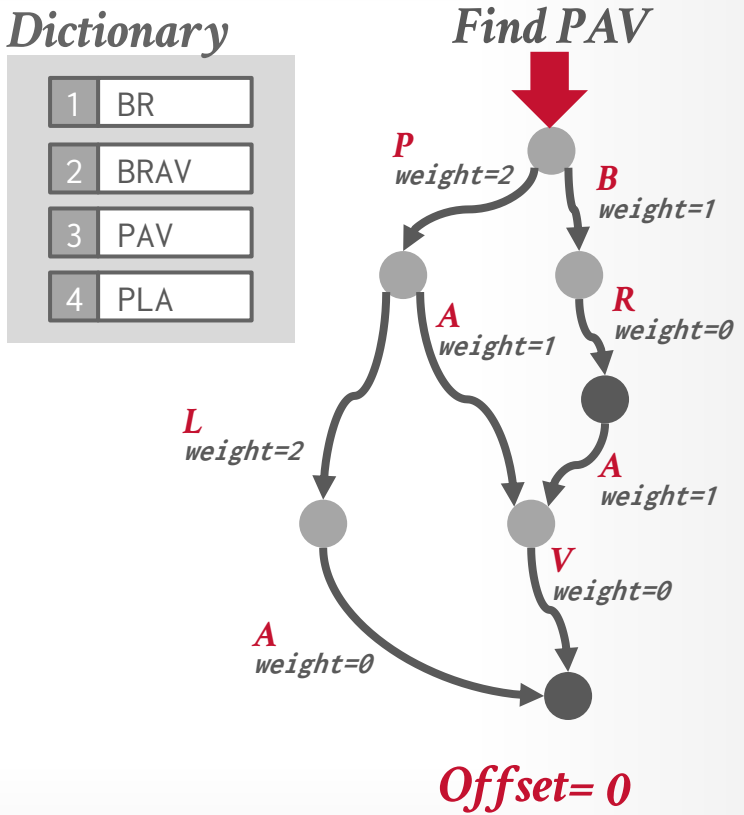
INVERTED INDEX: LUCENE

Uses a **Finite State Transducer** for determining offset of terms in dictionary.

Incrementally create dictionary segments and then merge them in the background.

→ Uses compression methods we previously discussed (e.g., delta, bit packing).

→ Also supports precomputed aggregations for terms and occurrences.



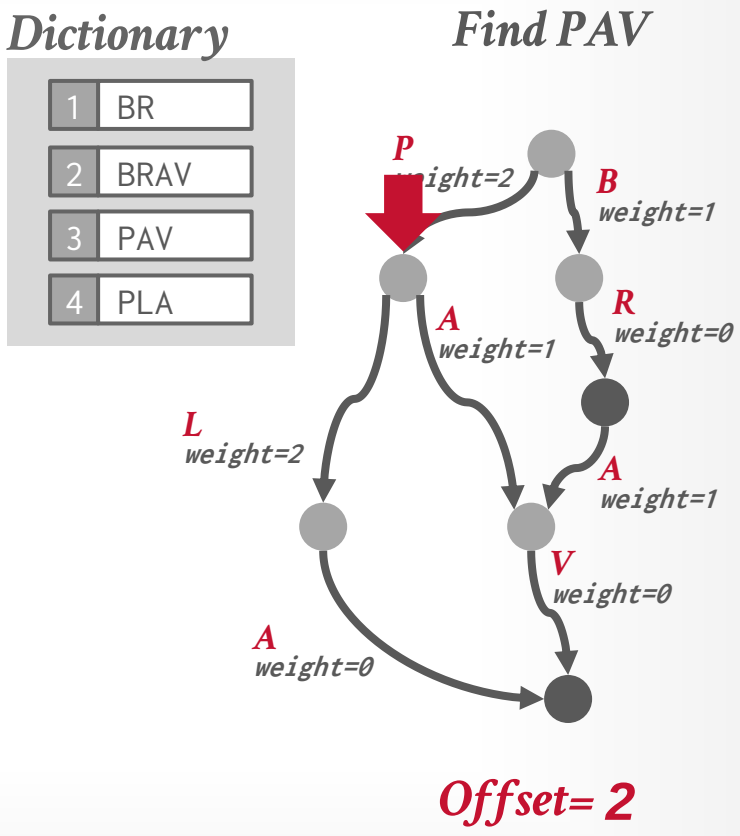
INVERTED INDEX: LUCENE

Uses a **Finite State Transducer** for determining offset of terms in dictionary.

Incrementally create dictionary segments and then merge them in the background.

→ Uses compression methods we previously discussed (e.g., delta, bit packing).

→ Also supports precomputed aggregations for terms and occurrences.



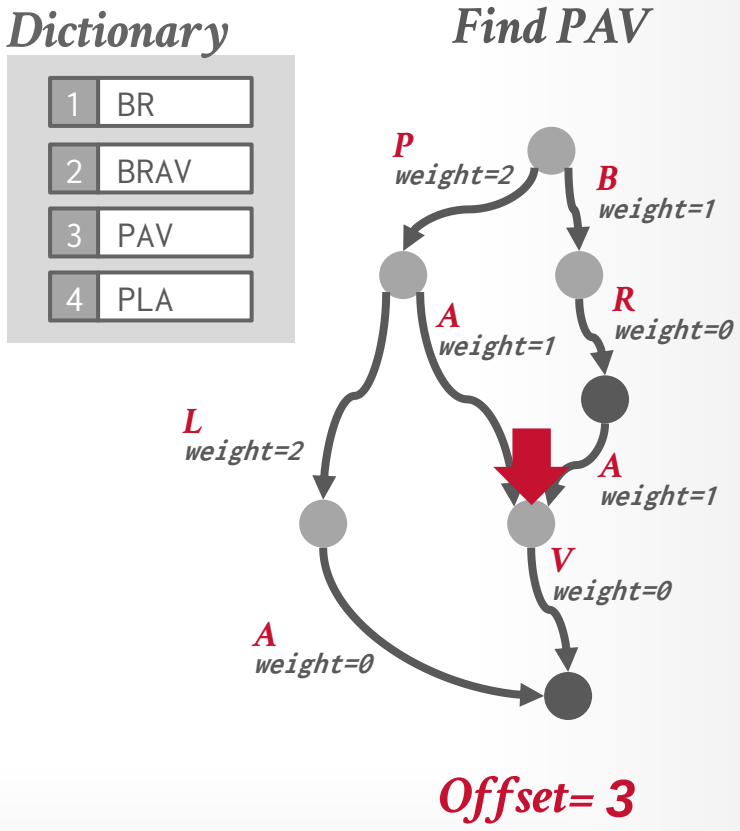
INVERTED INDEX: LUCENE

Uses a **Finite State Transducer** for determining offset of terms in dictionary.

Incrementally create dictionary segments and then merge them in the background.

→ Uses compression methods we previously discussed (e.g., delta, bit packing).

→ Also supports precomputed aggregations for terms and occurrences.



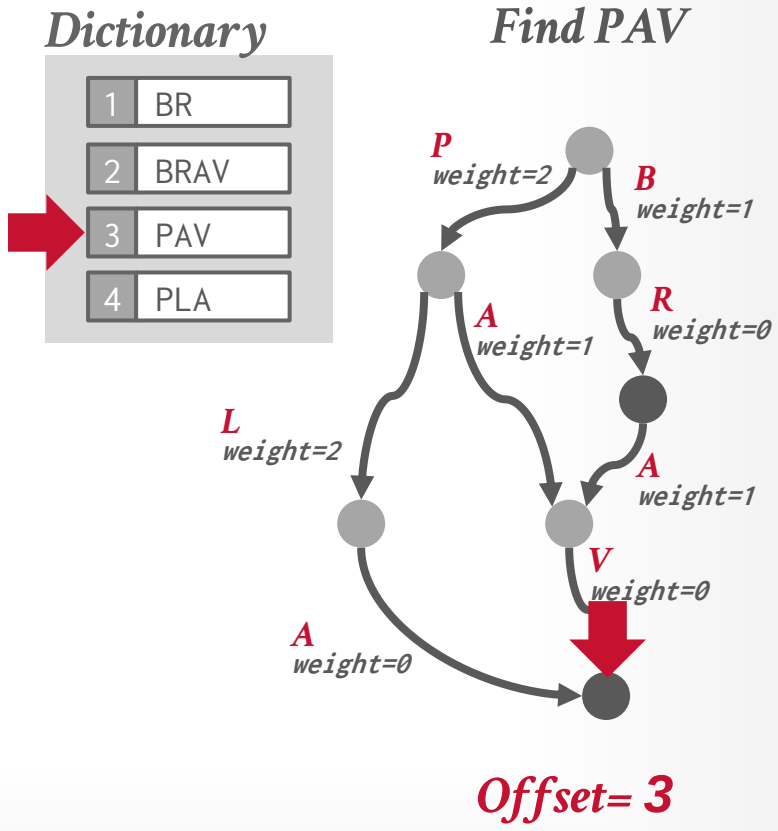
INVERTED INDEX: LUCENE

Uses a **Finite State Transducer** for determining offset of terms in dictionary.

Incrementally create dictionary segments and then merge them in the background.

→ Uses compression methods we previously discussed (e.g., delta, bit packing).

→ Also supports precomputed aggregations for terms and occurrences.



INVERTED INDEX: POSTGRESQL

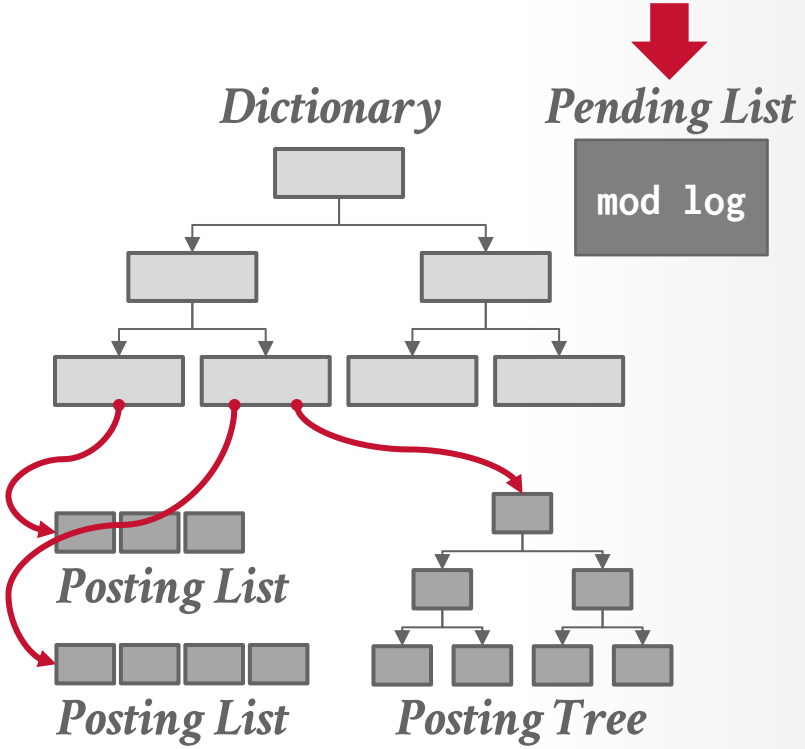
PostgreSQL's **Generalized Inverted Index** (GIN) uses a B+Tree for the term dictionary that map to a posting list data structure.

Posting list contents varies depending on number of records per term:

→ **Few**: Sorted list of record ids.

→ **Many**: Another B+Tree of record ids.

Uses a separate "pending list" log to avoid incremental updates.



INVERTED INDEX: POSTGRESQL

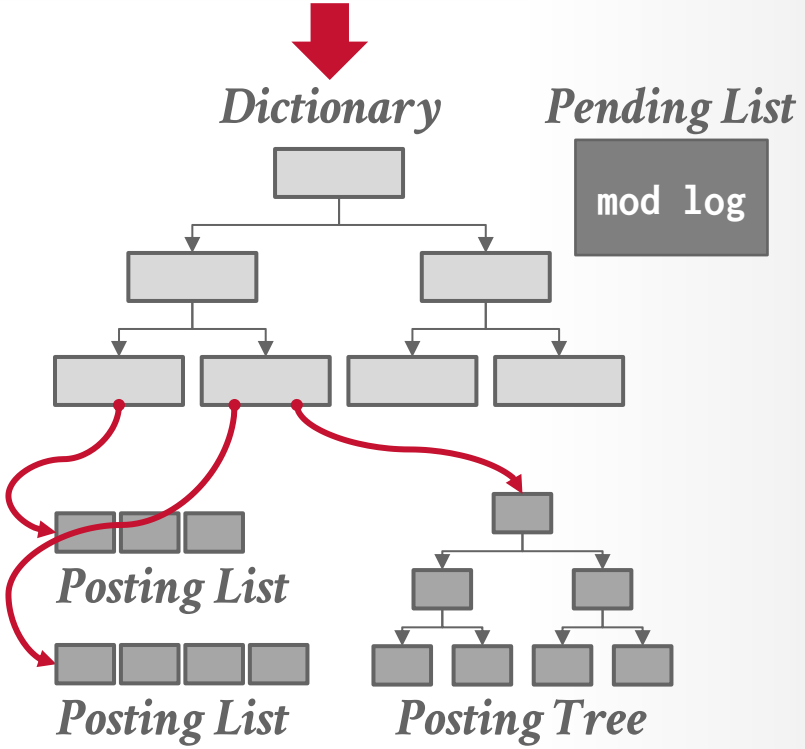
PostgreSQL's **Generalized Inverted Index** (GIN) uses a B+Tree for the term dictionary that map to a posting list data structure.

Posting list contents varies depending on number of records per term:

→ **Few**: Sorted list of record ids.

→ **Many**: Another B+Tree of record ids.

Uses a separate "pending list" log to avoid incremental updates.



OBSERVATION

Inverted indexes search data based on its contents.

→ There is a little magic to tweak terms based on linguistic models.

→ Example: Normalization ("**Wu-Tang**" matches "**Wu Tang**").

语言语义等价.

Instead of searching for records containing exact keywords (e.g., "Wu-Tang"), an application may want search for records that are related to topics (e.g., "hip-hop groups with songs about slinging").

VECTOR INDEXES

Specialized data structures to perform nearest-neighbor searches on **embeddings**.

→ An embedding is an array of floating point numbers.
→ May also need to filter data before / after vector searches.

The correctness of a query depends on whether the result "feels right".

turbopuffer <(°O°)>

VECTOR INDEXES

Specialized data structures to perform nearest-neighbor searches on **embeddings**.

→ An embedding is an array of floating point numbers.
→ May also need to filter data before / after vector searches.

The correctness of a query depends on whether the result "feels right".

turbopuffer <(°O°)>

VECTOR INDEXES: INVERTED FILE

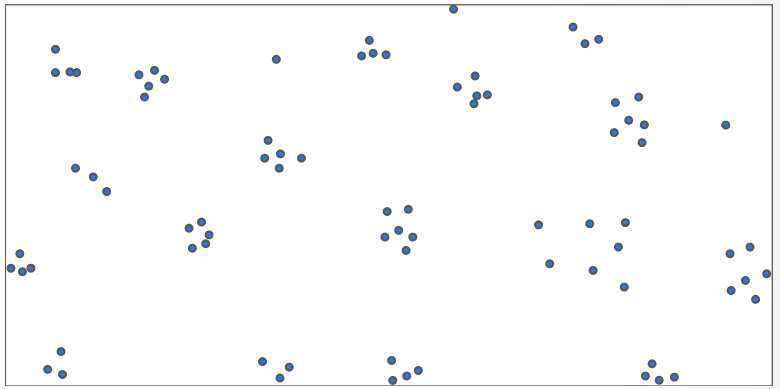
Partition vectors into smaller groups using a clustering algorithm.

To find a match, use same clustering algorithm to map into a group, then scan that group's vectors.

→ Also check nearby groups to improve accuracy.

Preprocess / quantize vectors to reduce dimensionality.

Example: IVFFlat



Source: Chi Zhang

VECTOR INDEXES: INVERTED FILE

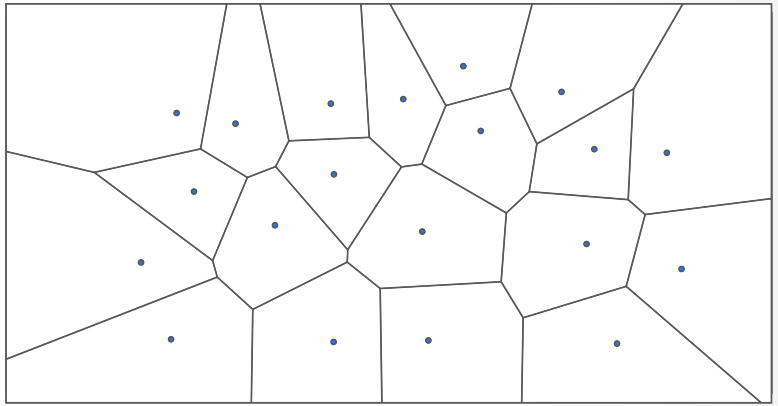
Partition vectors into smaller groups using a clustering algorithm.

To find a match, use same clustering algorithm to map into a group, then scan that group's vectors.

→ Also check nearby groups to improve accuracy.

Preprocess / quantize vectors to reduce dimensionality.

Example: IVFFlat



Source: Chi Zhang

VECTOR INDEXES: INVERTED FILE

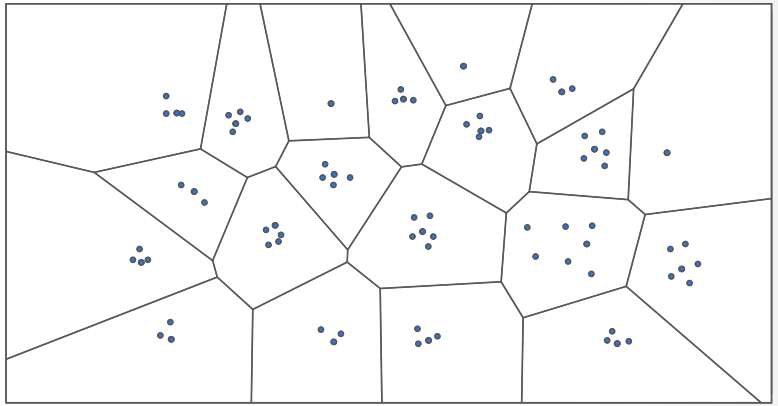
Partition vectors into smaller groups using a clustering algorithm.

To find a match, use same clustering algorithm to map into a group, then scan that group's vectors.

→ Also check nearby groups to improve accuracy.

Preprocess / quantize vectors to reduce dimensionality.

Example: IVFFlat



VECTOR INDEXES: INVERTED FILE

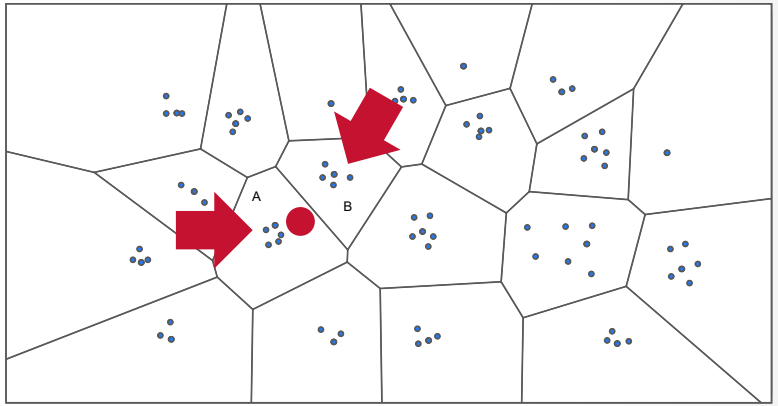
Partition vectors into smaller groups using a clustering algorithm.

To find a match, use same clustering algorithm to map into a group, then scan that group's vectors.

→ Also check nearby groups to improve accuracy.

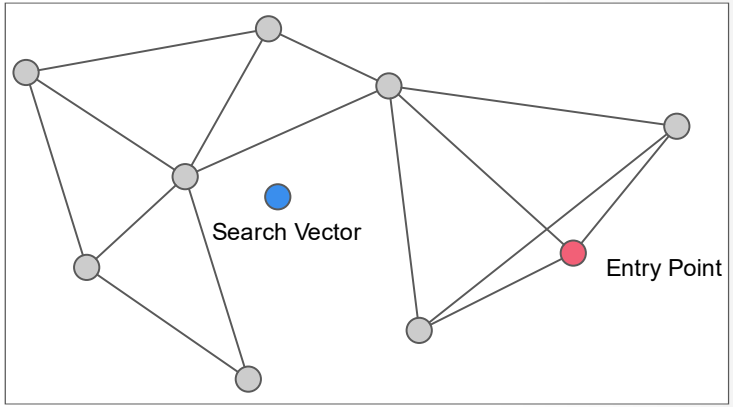
Preprocess / quantize vectors to reduce dimensionality.

Example: IVFFlat



VECTOR INDEXES: NAVIGABLE SMALL WORLDS

Build a graph where each node represents a vector and it has edges to its ***n*** nearest neighbors.
→ Can use multiple levels of graphs (HNSW)



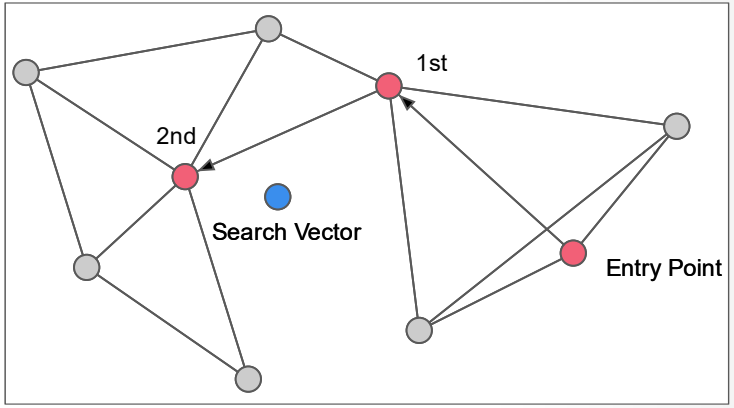
To find a match for a given vector, enter the graph and then greedily choose the next edge that moves closer to that vector.

Example: Faiss, hnswlib

Source: Chi Zhang

VECTOR INDEXES: NAVIGABLE SMALL WORLDS

Build a graph where each node represents a vector and it has edges to its ***n*** nearest neighbors.
→ Can use multiple levels of graphs (HNSW)



To find a match for a given vector, enter the graph and then greedily choose the next edge that moves closer to that vector.

Example: Faiss, hnswlib

Source: Chi Zhang

CONCLUSION

We will see filters again this semester.

B+Trees are still the way to go for tree indexes.

Inverted indexes are covered in CMU 11-442.

We did not discuss geo-spatial tree indexes:
→ Examples: R-Tree, Quad-Tree, KD-Tree
→ This is covered in CMU 15-826.

NEXT CLASS

How to make indexes thread-safe!