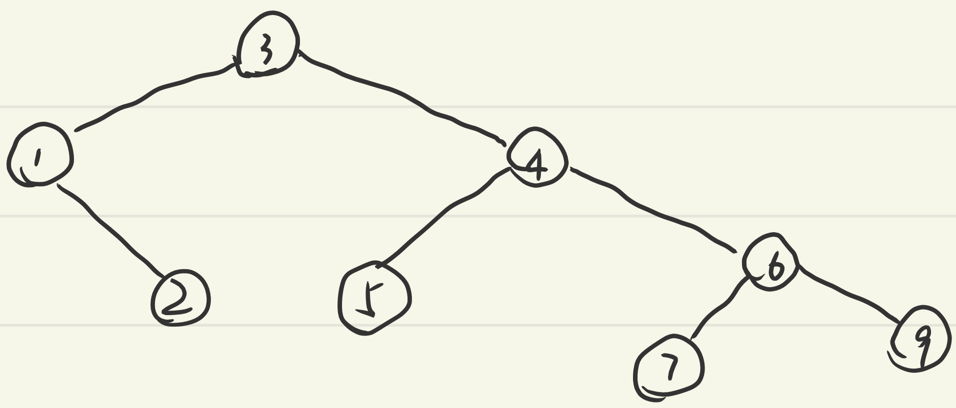
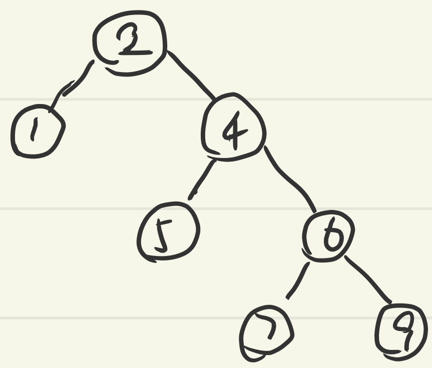
4.1

1. a.



b. 用左子树最大节点代替



2 Node 类中 属性 leftSize 存储左子树节点数+1

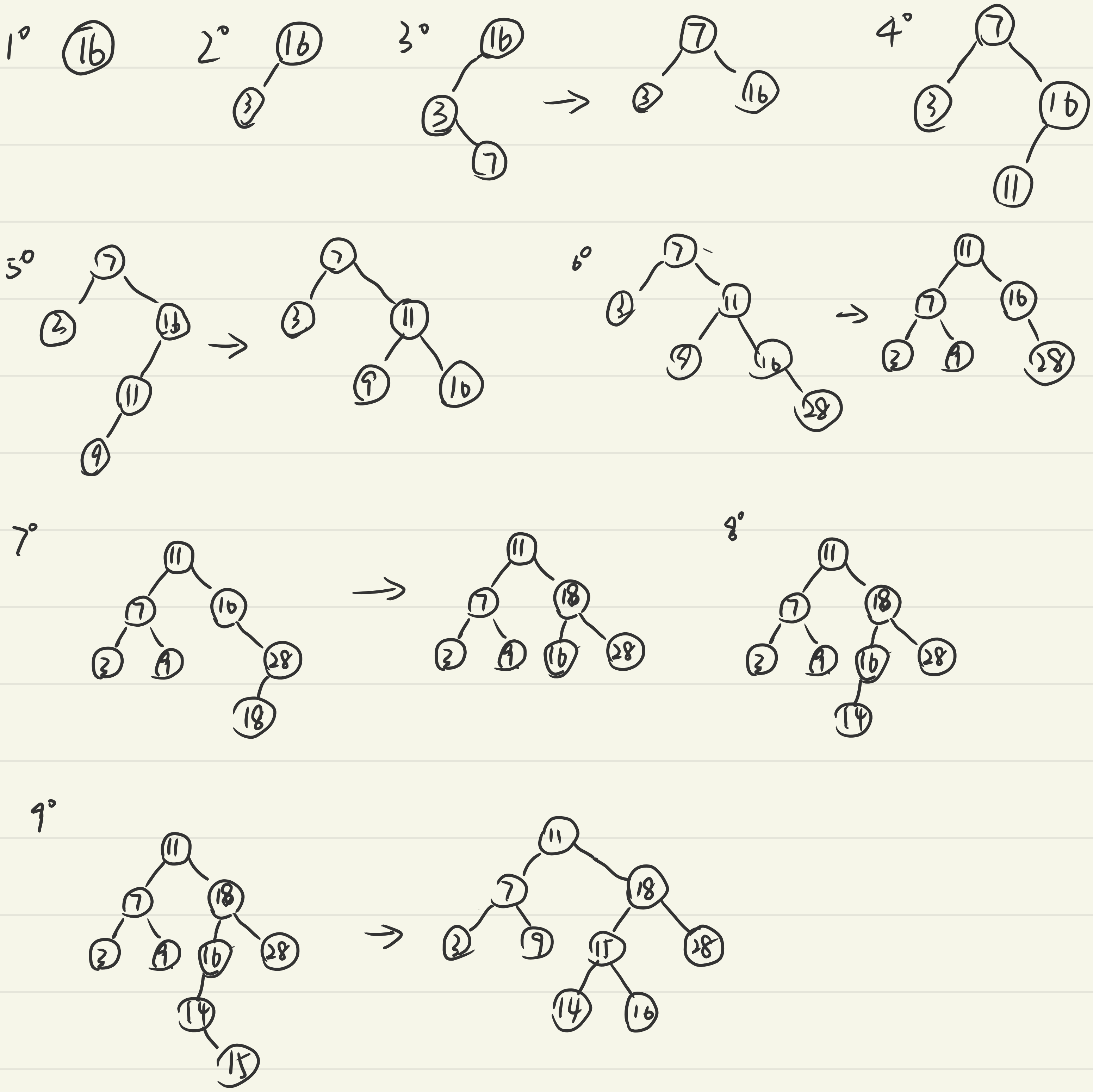
```
public static Node findkthMin(int k, Node root){
    if ( k == 0 ) return root;
    (if (root.left == null) return null;)

    return findkthMin(k-1, root.left);
}
```

```
if (k < root.leftSize) // 左子树
return findkthMin(k, root.left);
```

```
        return findKthMin(k, root.left);
    else if (k > root.leftsize)   //右子树
        return findKthMin(k - root.leftsize, root.right);
    else
        return root;
```

3.




```
4. public boolean isBST(BinaryNode root){
       if (root == null)   return true;
       boolean judgeLeft = true, judgeRight = true;
       if (root.left != null)
           judgeLeft = (root.left.element < root.element) && (isBST(root.left));
       if (root.right != null){
           judgeRight = (root.right.element > root.element) && (isBST(root.right));
```

judgeRight = (root.right.element > root.element) && (isBST(root.right));

reture judgeLeft && judgeRight;

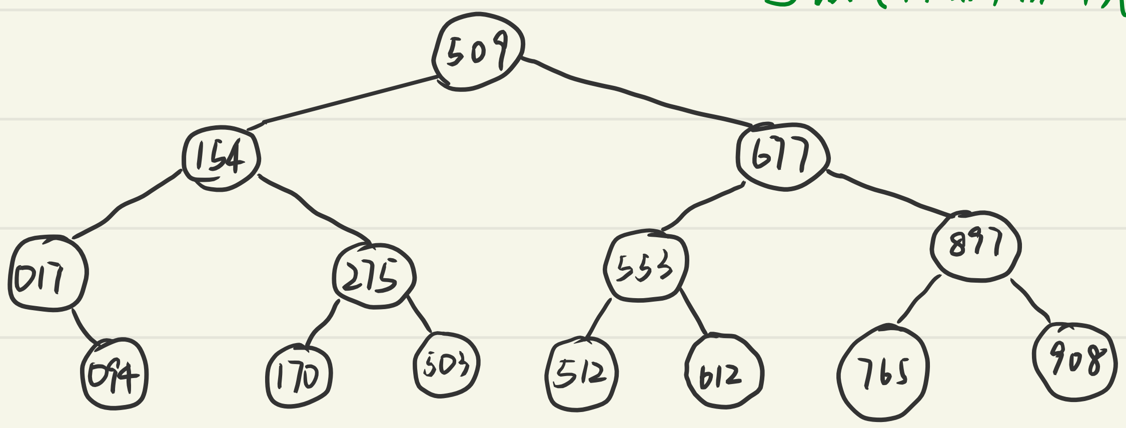
}

①得到中序遍历.检查是否为升序

② findMax (root.left) < root.element < findMin(root.right)　　递归

5 　二分查找判定树：　A[1, n] 是一个有序表，① 取 $m = (1+n)/2$，以为m为根，画出结点。

② 以 (1, m', m-1) 画左子树，以 (m+1, m*, n) 画右子树



$$\frac{\cancel{0}^{1} + \cancel{1}^{2}\times 2 + \cancel{2}^{3}\times 4 + \cancel{3}^{4}\times 7}{14} = \cancel{\frac{31}{14}}\ \frac{45}{14}$$

（根的搜索长度为1）

6. 不成立

对于 $\forall a \in A$ 在树中找到 a 在 S_2 上的最近祖先 f　则 a 在 f 的左子树上

b 在经过 f 到根结点的路径上　　因此有可能 f 在 b 的右子树上，因而 a 也在 b 的右子树上.

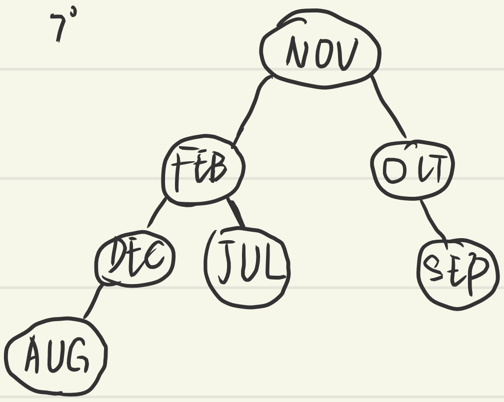
此时 $a > b$　因此 $a < b$ 不成立

同理 $b < c$ 不成立

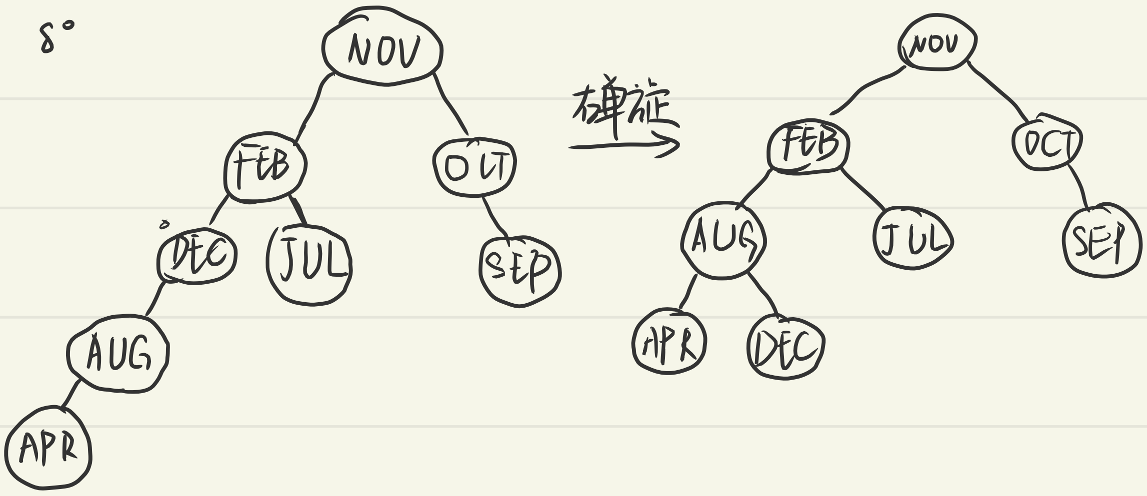
（而对于 $\forall a \in A$, $\forall c \in C$, 均有 $a < c$）

7.

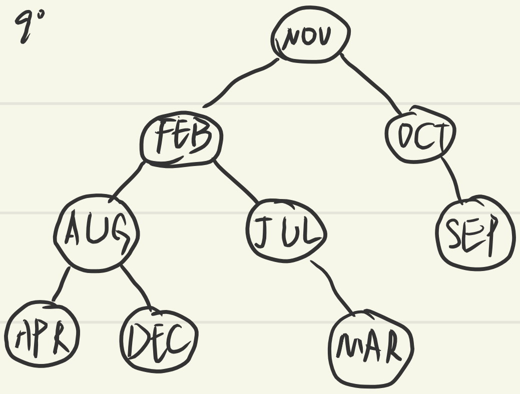
7°



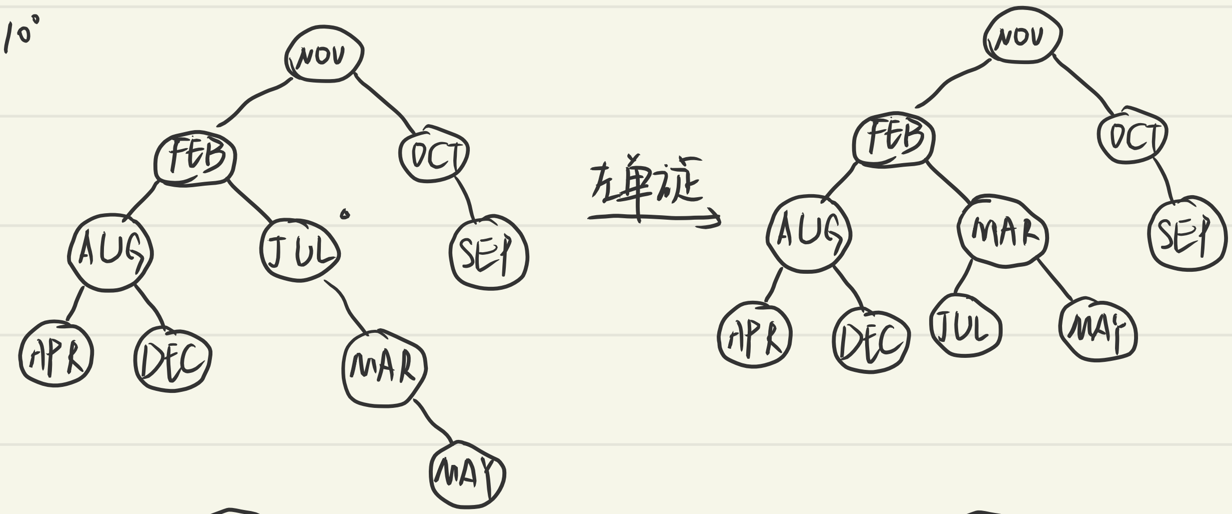
8°



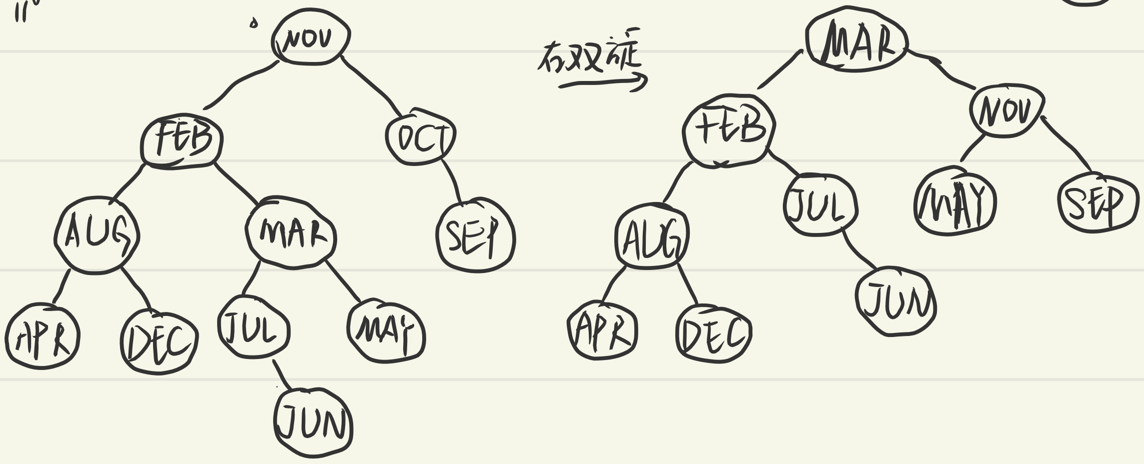
9°



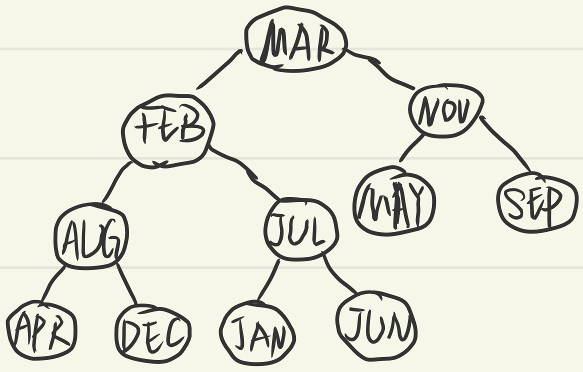
10°



11°



12°



8 设N_h是高度为h的AVL树的最小结点数。最坏情况下，树的一棵子树高度为h-1，另一棵子树高度为h-2，这两棵子树也是高度平衡的。因此有

空树 $N_0=0$，仅有根节点 $N_1=1$　　$N_h=N_{h-1}+N_{h-2}+1$ $(h>1)$

对于斐波那契数列 $F_0=0$ $F_1=1$ $F_n=F_{n-1}+F_{n-2}$　　$h\geq1$时有 $N_h=F_{h+2}-1$

F_n	0	1	1	2	3	5	8
N_n	0	1	2	4	7		

$$F_n=\frac{\phi_1^n-\phi_2^n}{\sqrt5}\qquad \phi_1=\frac{1+\sqrt5}{2}\qquad \phi_2=\frac{1-\sqrt5}{2}$$

$$\Rightarrow N_h=\frac{\phi_1^{h+2}-\phi_2^{h+2}}{\sqrt5}-1>\frac{\phi_1^{h+2}}{\sqrt5}-2\qquad \left|\frac{\phi_2^{h+2}}{\sqrt5}\right|<1$$ ⇐高度为h的AVL树的最少结点数

$$\Rightarrow \phi^{h+2}<\sqrt5(N_h+2)$$

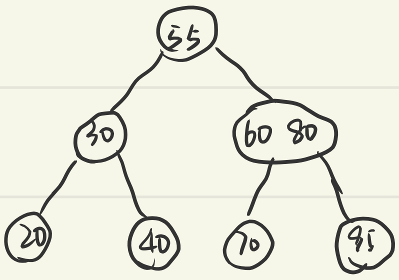
$$h+2<\log_\phi\sqrt5+\log_\phi(N_h+2)\qquad \log_2\phi=0.694$$

可得 $h<1.44\times\log_2(N_h+2)-0.328<1.44\times\log_2(n+2)$ (换底公式)

因此 有n个结点的AVL树，高度不超过 $1.44\times\log_2(n+2)$

设AVL树每一层结点个数都达到最多 (2^{i-1}, i为层号)　AVL树的高度最小, $h = \lceil \log_2(n+1) \rceil$

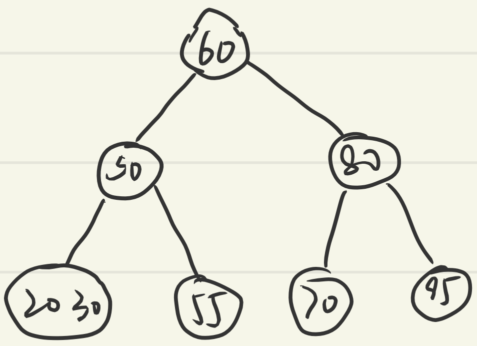
9. 1° delete 50



3阶. 子结点 [2, 3]

关键字 [1, 2]

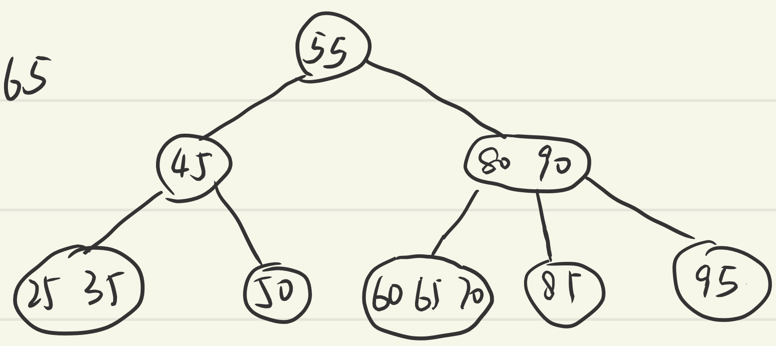
2° delete 40



10. 1° insert 65

3阶

key 1~2个



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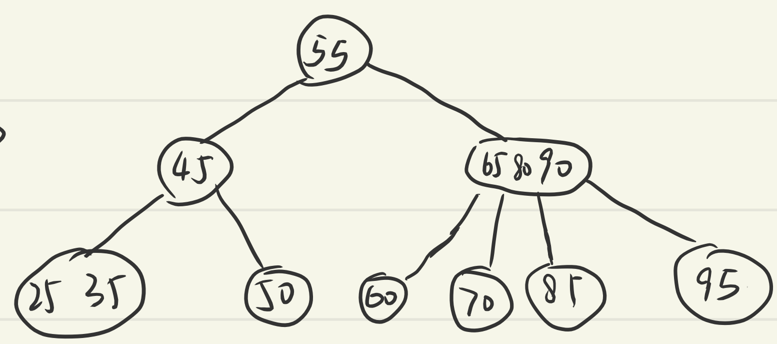

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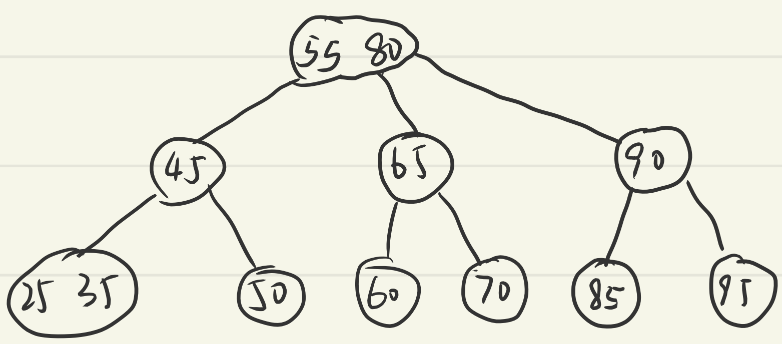
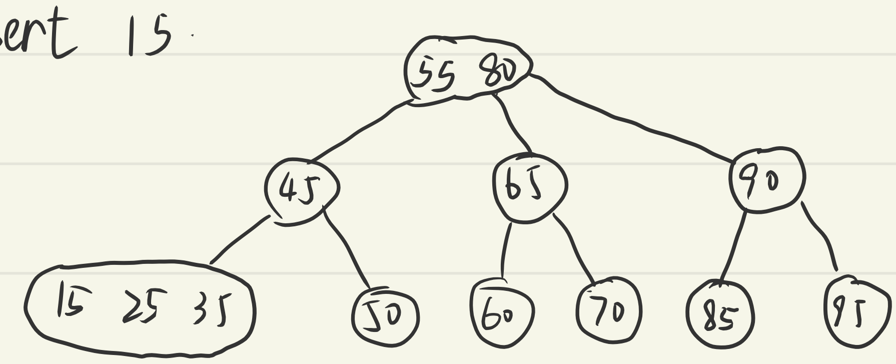


2° insert 15



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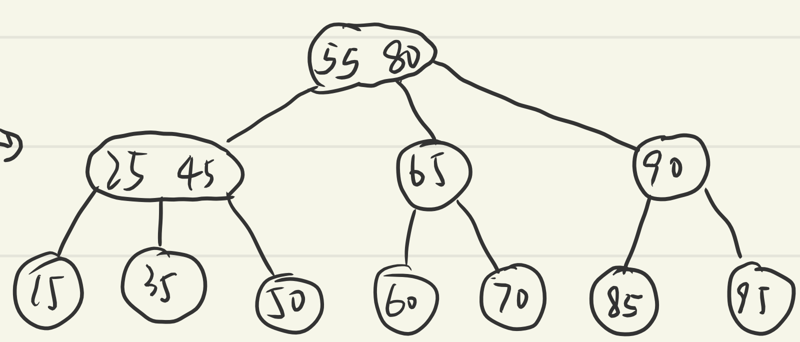
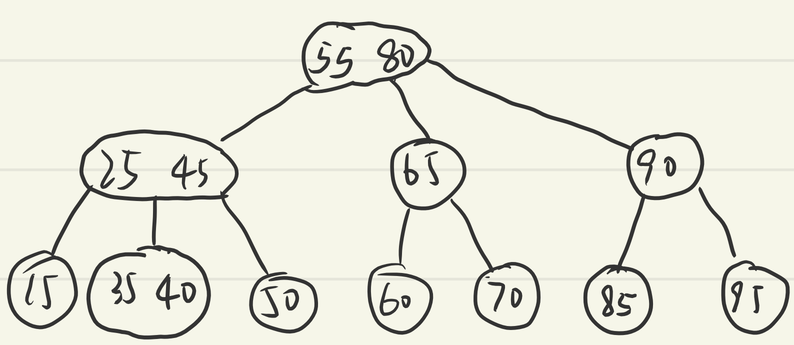
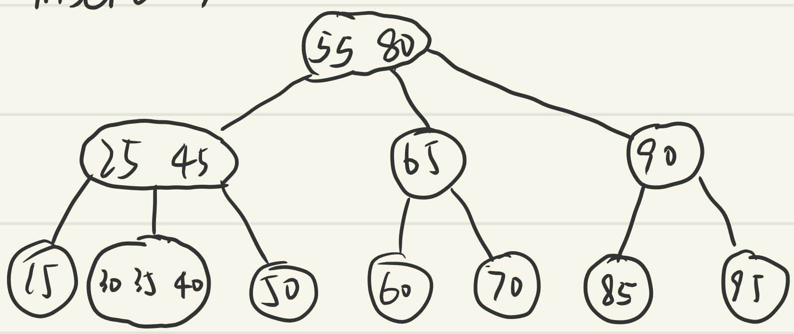


3° insert 40



4° insert 30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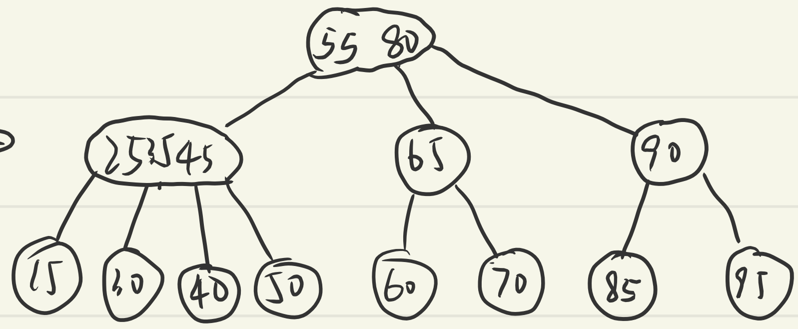
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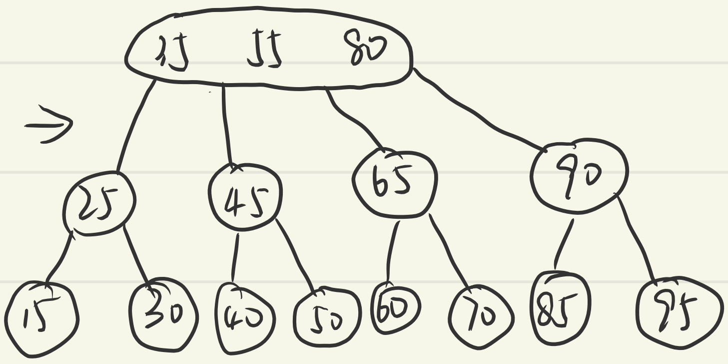
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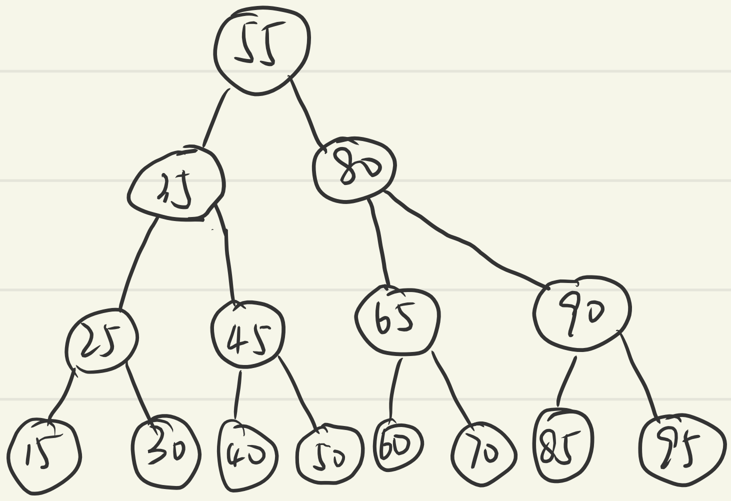
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