Chapter 6

Priority Queues

带优先集

6.1 Introduction

- A priority queue is a collection of zero or more elements. Each element has a priority or value.

6.1 Introduction

- In a min priority queue the find operation finds the element with minimum priority, while the delete operation delete this element.
- In a max priority queue, the find operation finds the element with maximum priority, while the delete operation delete this element.

6.1 Introduction

ADT of a max priority queue

```
AbstractDataType  MaxPriorityQueue
{
  instances
      finite collection of elements,each has a priority
  operations
      Create(): create an empty priority queue
      Size():  return number of element in the queue
      Max(): return element with maximum priority
      Insert(x): insert x into queue
      DeleteMax(x):delete the element with largest priority
                  from  the queue; return it in x;
}
```

6.2 Linear List Representation

Use an unordered linear list

Insertions are performed at the right end of the list, $\theta(1)$

A deletion requires a search for the element with largest priority, $\theta(n)$

6.3 Heaps

堆

1.definition: A max heap(min Heap)

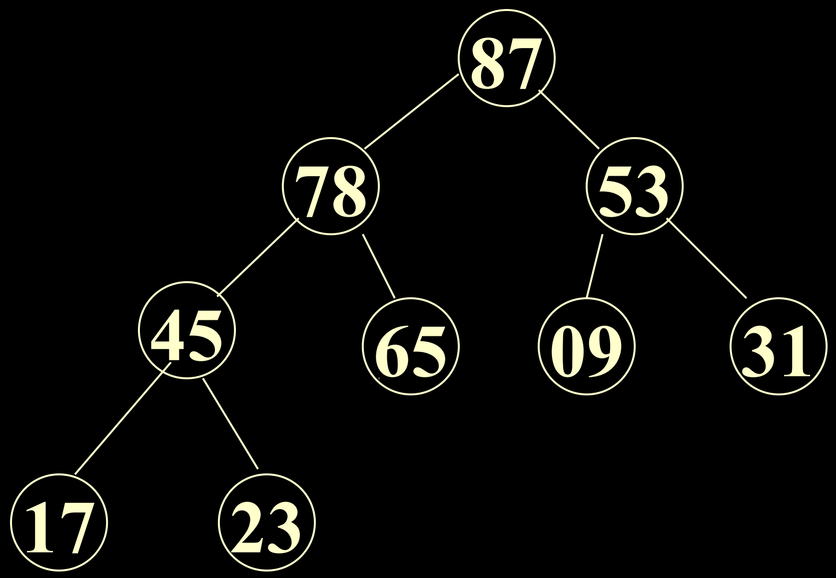
- is A complete binary tree 是完全二叉树
- The value in each node is greater(less) than or equal to those in its children(if any).

优先集：根≥子

6.3 Heaps

Example of a max heap

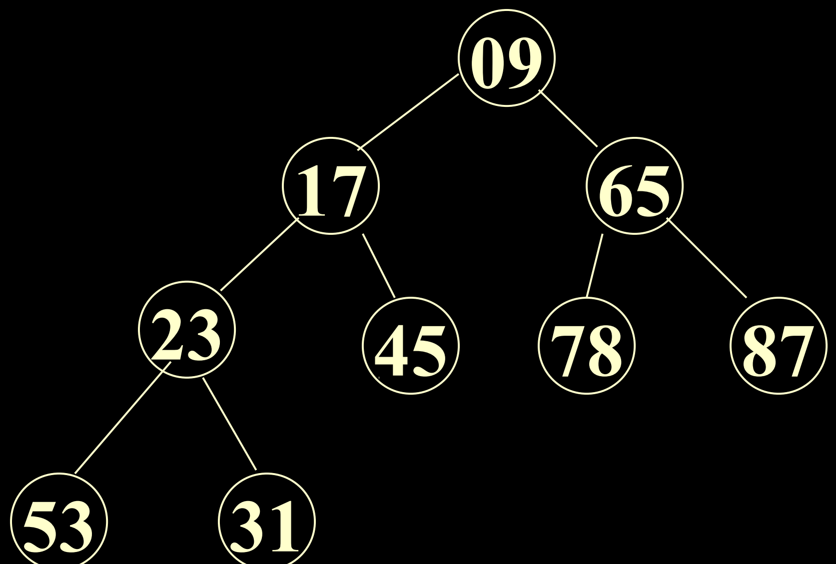
k={87,78,53,45,65,09,31,17,23}



6.3 Heaps

Example of a min heap

k={09,17,65,23,45,78,87,53,31}

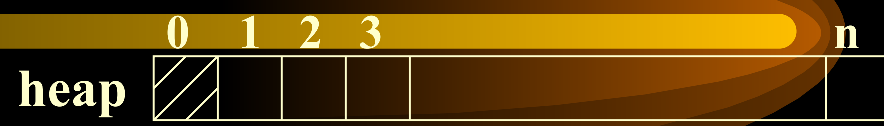


6.3 Heaps

2. class MaxHeap

（完全二叉树可用数组实现，之前讲过）

Data member of heap: <u>T * heap, int MaxSize, CurrentSize</u>



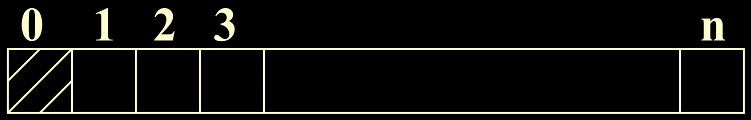

```
template<class T>class MaxHeap
{ public:
      MaxHeap(int MaxHeapSize=10);
      ~MaxHeap(){delete[]heap;}
      int size()const{return CurrentSize;}
      T Max(){ if (CurrentSize= =0)throw OutOfBounds();
                 return heap[1];}
      MaxHeap<T>&insert(const T&x);
      MaxHeap<T>& DeleteMax(T& x);
      void initialize(T a[], int size,  int ArraySize);
  private:
      int CurrentSize, MaxSize;
      T * heap;
}
```

6.3 Heaps

3.member function of MaxHeap

1)Constructor for MaxHeap

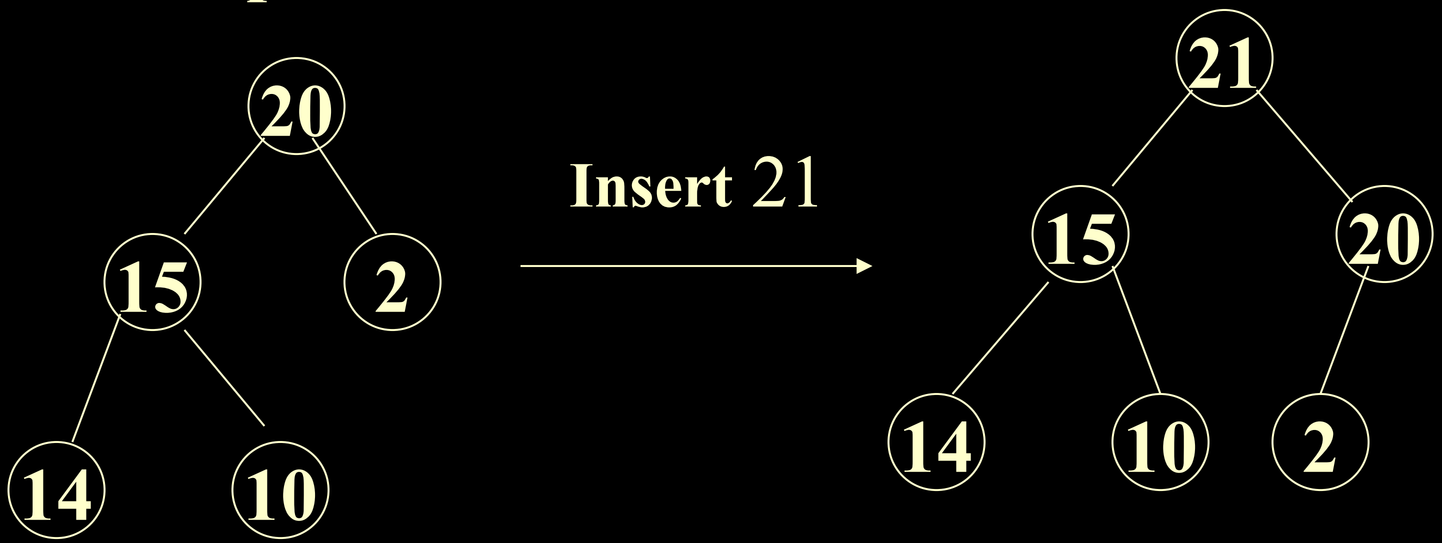
```
 template<class T>
 MaxHeap<T>::MaxHeap(int MaxHeapSize)
{  MaxSize=MaxHeapSize;
   Heap=new T[MaxSize+1];
   CurrentSize=0;
}
```



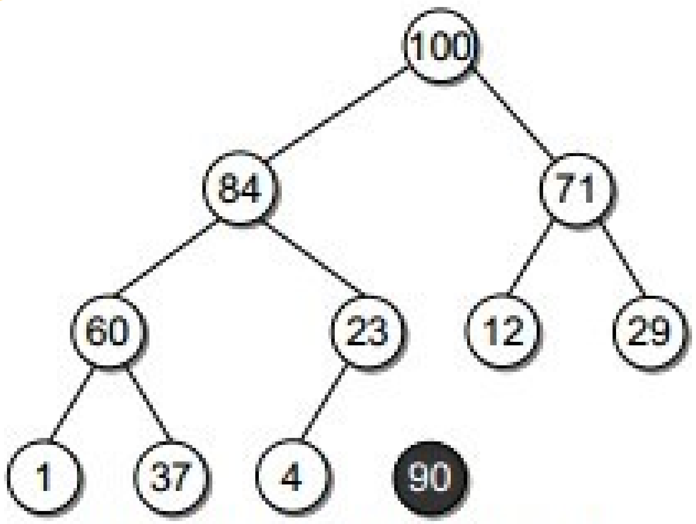
6.3 Heaps

2)Insertion

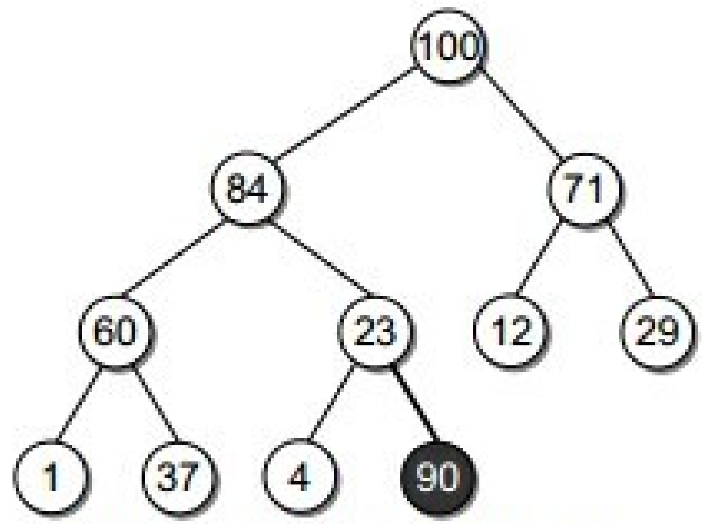
Example:



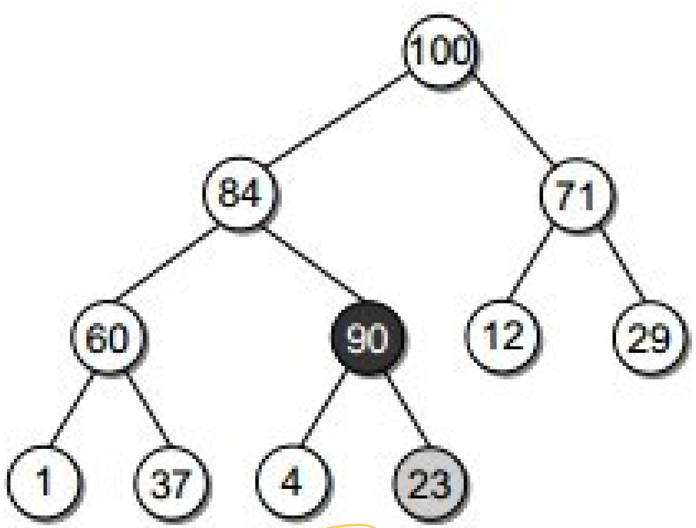
上滤 ($\log_2 n$)



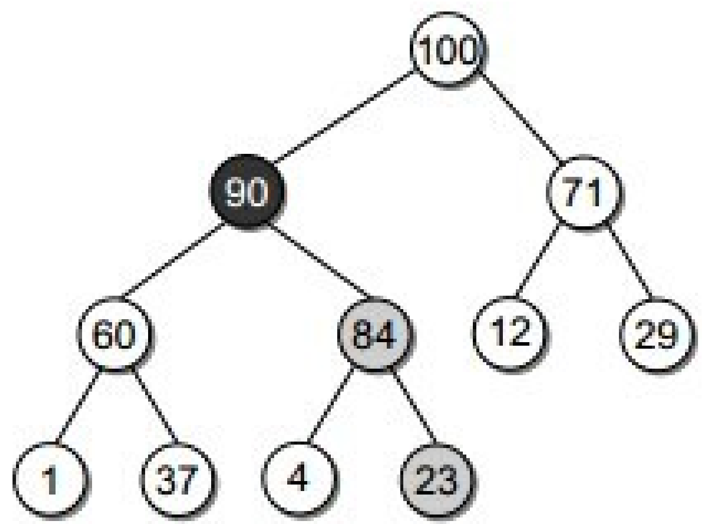
(a) create a new node for 90.



(b) link the node as the last child.



(c) sift-up: swap 23 and 90.



(d) sift-up: swap 84 and 90.

6.3 Heaps

Insertion

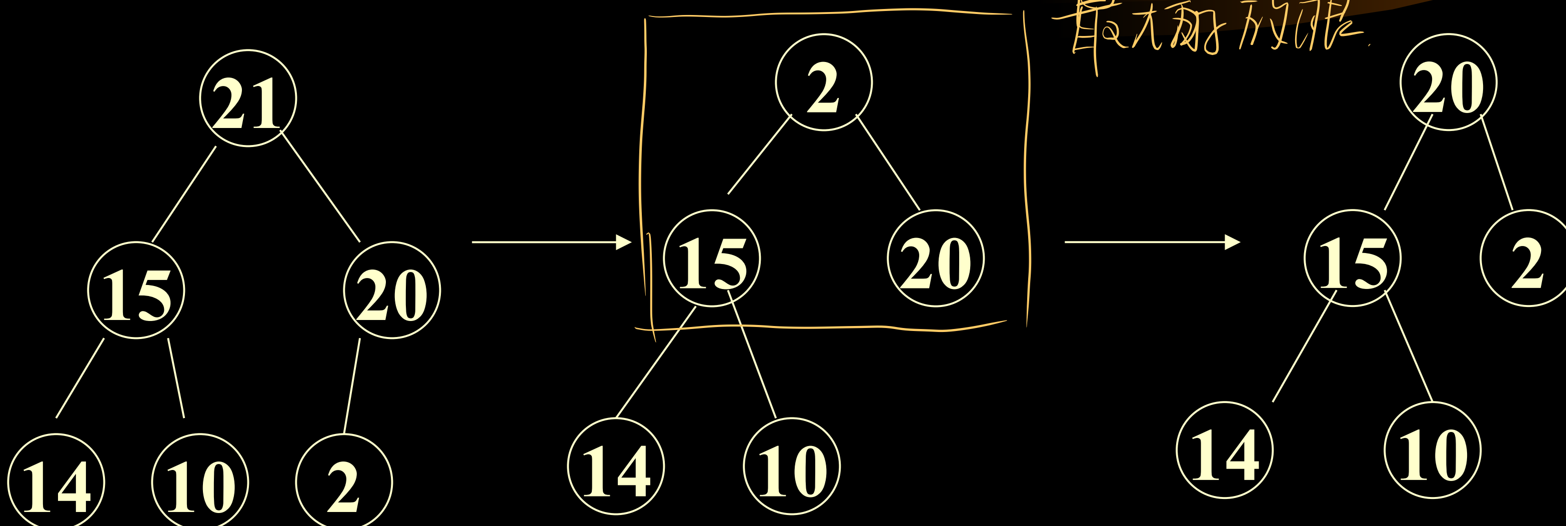
```
template<class T>MaxHeap<T>& MaxHeap<T>::
  Insert(const T& x)
{  if(CurrentSize= =MaxSize)throw NoMem();
    int i=++CurrentSize;
    while(i!=1&&x>heap[i/2])      父结点
      { heap[i]=heap[i/2];  i/=2; }
     heap[i]=x;
     return *this;
}
```

time complexity is $O(\log_2 n)$

6.3 Heaps

3)deletion 去根. 下滤 $O(\log_2 n)$

最大的放根.



6.3 Heaps

deletion from a max heap

```
template<class T>MaxHeap<T>& MaxHeap<T>:: DeleteMax(T& x)
  { if (CurrentSize= =0)throw OutOfBounds();
     x=heap[1];
     T y=heap[CurrentSize--];
     int i=1; ci=2; i的左子结点.
     while(ci<=CurrentSize)                 左右结点先比         指向左右结点的较大者
                 有右子结点、
       { if(ci<CurrentSize&&heap[ci]<heap[ci+1]) ci++;
                父
          if(y>=heap[ci]) break;
          heap[i]=heap[ci];
          父
          i=ci; ci*=2; 新的左子结点
       }  最后一个元素.
     heap[i]=y;    return *this;
  }      Time complexity is O(log2n)
```

6.3 Heaps

java program(MinHeap)

```
public class BinaryHeap
 {  public BinaryHeap( )
    public BinaryHeap( int capacity )
    public void insert( Comparable x ) throws Overflow
    public Comparable findMin( )
    public Comparable deleteMin( )

    public boolean isEmpty( )
    public boolean isFull( )
    public void makeEmpty( )

    private static final int DEFAULT_CAPACITY = 100;
    private int currentSize;
    private Comparable [ ] array;
    private void percolateDown( int hole )
    private void buildHeap( )
 }
```

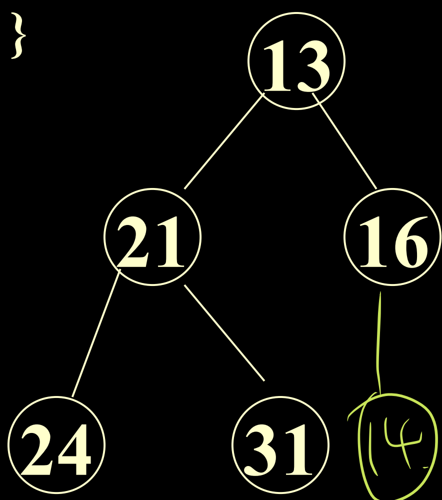
6.3 Heaps

```
public BinaryHeap( )
{   this( DEFAULT_CAPACITY );
}

public BinaryHeap( int capacity )
{   currentSize = 0;
    array = new Comparable[ capacity + 1 ];
}

 public void makeEmpty( )
{   currentSize = 0;
}
```

6.3 Heaps

```
public void insert( Comparable x ) throws Overflow
{  if( isFull( ) )
        throw new Overflow( );
    int hole = ++currentSize;
    for( ; hole > 1 && x.comparebleTo( array[ hole / 2 ] ) < 0;
            hole /= 2 )
        array[ hole ] = array[ hole / 2 ];
    array[ hole ] = x;
}
```

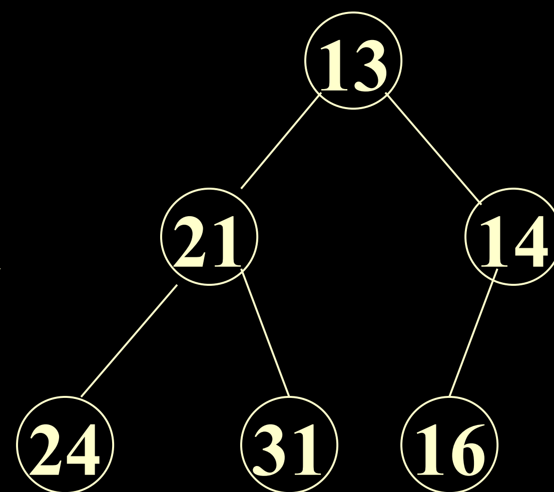
X比较小



Insert 14

hole / 2

hole.

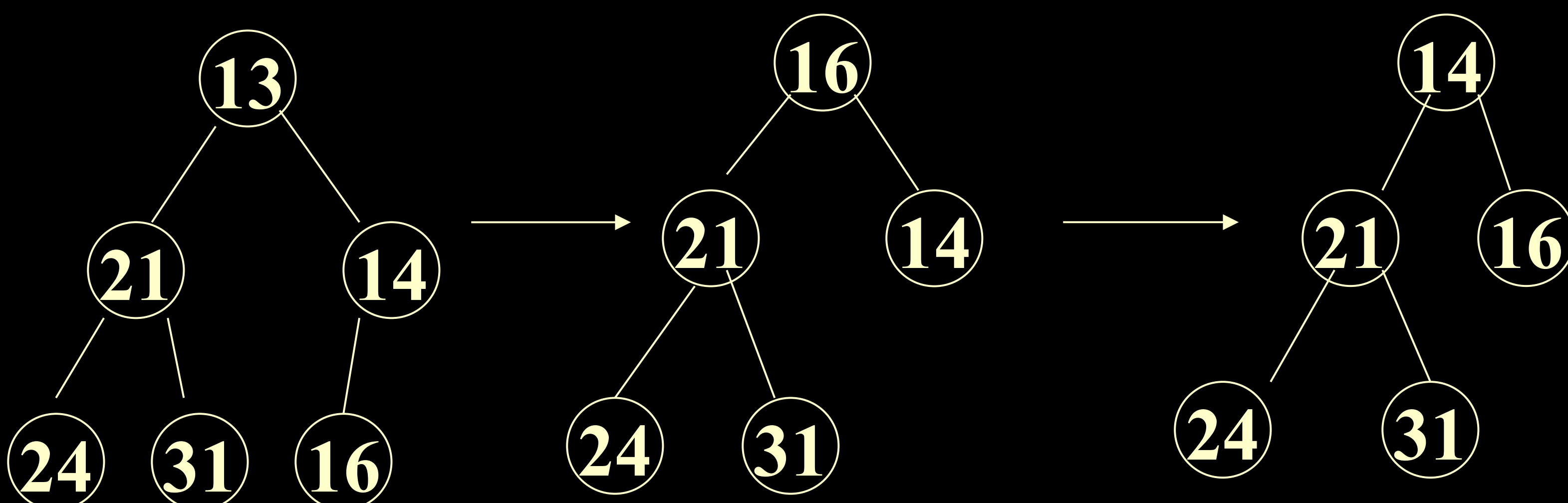


6.3 Heaps

```
public Comparable deleteMin( )
{  if( isEmpty( ) )
      return null;

   Comparable minItem = findMin( );
   array[ 1 ] = array[ currentSize-- ];
   percolateDown( 1 );

   return minItem;
}
```

6.3 Heaps

```
private void percolateDown( int hole )
{   int child;
    Comparable tmp = array[ hole ];
    for( ; hole *2 <= currentSize; hole = child )
    {   child = hole * 2;
        if ( child != currentSize &&  array[ child + 1 ].compareTo( array[ child ] ) < 0 )
                child++;
        if( array[child ].compareTo( tmp ) < 0 )
            array[ hole ] = array[ child ];
        else
            break;
    }
    array[ hole ] = tmp;
}
```



(叶).
一半数据上滤走 $\log_2 n$

一半数据无需下滤
$\frac{1}{4}$数据下滤1步
$\frac{1}{8}$ 2
……

因此 $O(n\log_2 n)$ → $O(n)$

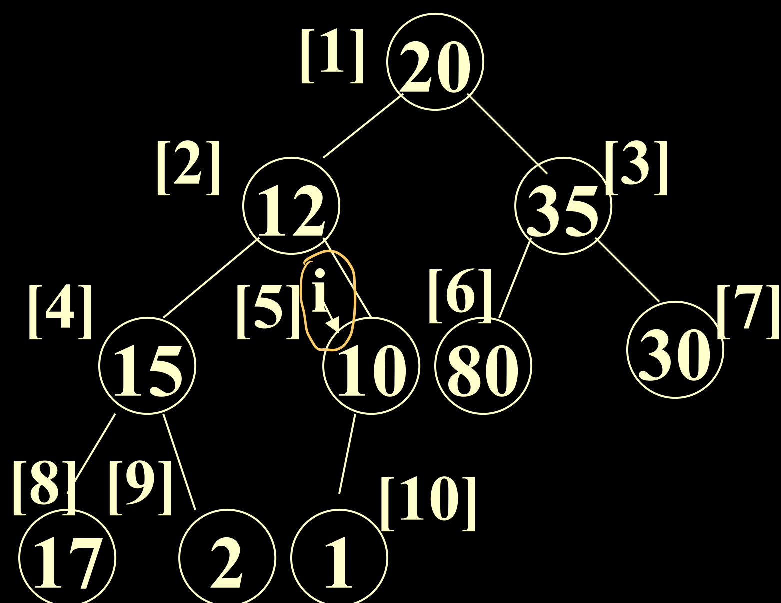
换个加入，做的实际是上滤算法
$O(n\log n) \rightarrow O(n)$

6.3 Heaps

4)Initialize a nonempty max heap

Example: {20,12,35,15,10,80,30,17,2,1}

书中称为由底向上： 做下滤



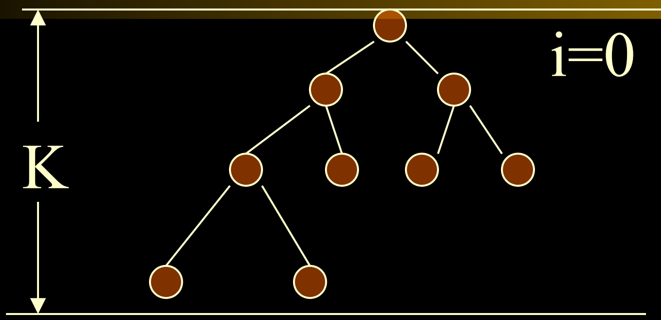
从最后一个元素的父结点开始

$i=[n/2], [n/2]-1, \ldots, 1$

Turn into max heap from these subtree roots

算法分析

初始建堆： n 个结点， $K=\lfloor \log_2 n \rfloor$，从 0 层开始



第 i 层交换的最大次数为 k-i

第 i 层有 2^i 个结点

总交换次数：$\sum_{i=0}^{k-1} 2^i\cdot(k-i)=\sum_{j=1}^{k} j\cdot 2^{k-j}=\sum_{j=1}^{k} j(2^k\cdot 2^{-j})$

令 k-i=j

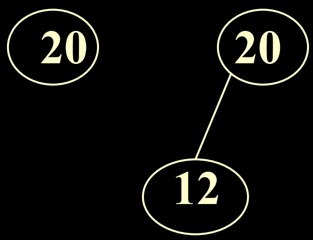
$$=2^k\cdot\sum_{j=1}^{k} j\cdot 2^{-j}\le 2^k\cdot 2\le 2^{\log n}\cdot 2=2n=O(n)$$

6.3 Heaps

4)Initialize a nonempty max heap

Example: {20,12,35,15,10,80,30,17,2,1}

还可以这样做：依次插入一个元素到堆中 . 书中称为由顶向下 (也可见书中例子).



Complexity of Initialize:

$$\sum_{k=1}^{n} \lfloor \lg k \rfloor <= \sum_{k=1}^{n} \lg k = \lg 1 + \ldots\ldots + \lg n = \lg(1\cdot 2 \ldots. n) =$$

$$\lg(n!) = O(n\lg n)$$

(提示： 因 $n! = n*(n-1) * (n-2) * \ldots.. *2 * 1$

显然 该 n 项都小于 n, 所以 $\lg(n!) <= \lg(n*n*n*\ldots.*n)$)

6.3 Heaps

initialize (C++ program)

```
Template<class T> void MaxHeap<T>::
                    Initialize (T a[],int size,int ArraySize)
 {  delete[] heap;
    heap=a;   CurrentSize=Size;    MaxSize=ArraySize;
    for( int i=CurrentSize/2; i>=1; i--)
       {  T y=heap[i];    int c=2*i;
          while(c<=CurrentSize)
            { if(c<CurrentSize && heap[c]<heap[c+1])  c++;
              if(y>=heap[c]) break;
              heap[c/2] = heap[c];
              c*=2;
            }
          heap[c/2]=y;
       }
 }
```

6.3 Heaps

java initialize

```
private void buildHeap( )
{  for( int i = currentSize / 2; i > 0 ; i-- )
        percolateDown( i );
}
```

考．目考细节

6.4.Applications of Priority Queues

1.heap sort 堆排序 $O(n\log_2 n)$

Method:

1)initialize a max heap with the n elements to be sorted $O(n)$ 建最大堆

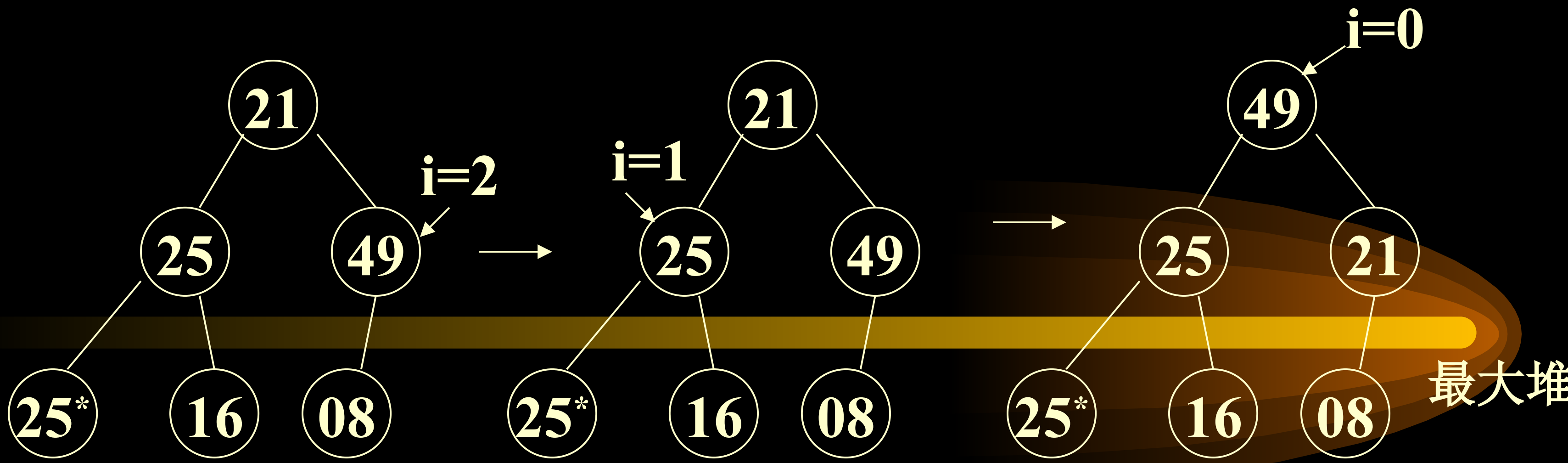
2)each time we delete one element, then adjust the heap $O(\log_2 n)$ 依次得到剩下的最大元素

Time complexity is $O(n)+O(n*\log_2 n)=O(n*\log_2 n)$

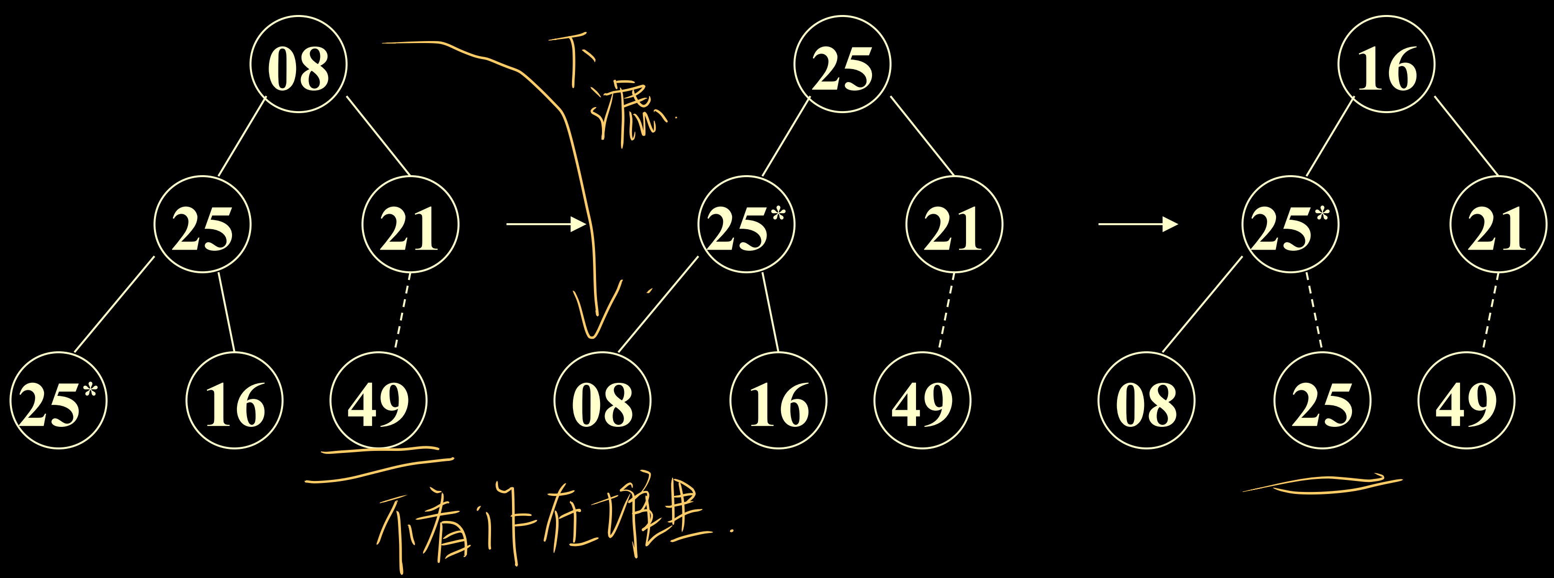
6.4.Applications of Priority Queues

heap sort

Example :{21,25,49,25*,16,08}


21
25
49
i=2
25*
16
08
21
i=1
25
49
25*
16
08
i=0
49
25
21
25*
16
08
最大堆

调整


08
25
21
25*
16
49
下滤
25
25*
21
08
16
49
16
25*
21
08
25
49
不看作在堆里

25和25*顺序交换了

从以上例子可以看出堆排序是不稳定的

heap sort

算法：c++

```
Template<class  Type>void HeapSort(datalist<Type>&list)
{for(int i=(list.currentsize)/2;i>=1;i--)
         FilterDown(i,list.currentsize);
 for(i=list.currentsize;i>1;i--)
         {Swap(list.Vector[1],list.vector[i]);
          FilterDown(1,i-1);
         }
}
```

建堆：依次下滤

heap sort

java program

```
public static void heapsort( Comparable [ ] a )
{  for( int i = a.length / 2; i >= 1; i-- )
        percDown( a, i, a.length );
   for( int i = a.length ; i > 1; i-- )
   {  swapReferences( a, 1, i );
        percDown( a, 1, i-1);
    }
}
```

☆☆☆ 常考代码!

heap sort

```
private static void percDown( Comparable [ ] a, int i, int n )
```

下滤 ↑ size

下滤对象

```
{   int child;
    Comparable tmp;
```

存储 a[i], 让 a[i] 作一个 hole

2i

```
    for( tmp = a[ i ]; leftChild( i ) < n; i = child )
    {   child = leftChild( i );
        if( child != n – 1 && a[child ].compareTo( a[ child + 1 ]) < 0 )
            child++;
```

存右子节点 且左子节点比右子节点小

← 选择右子节点.

```
        if( tmp.compareTo( a[ child ] ) < 0 )
            a[ i ] = a[ child ];
        else  break;
    }
    a[ i ] = tmp;
}
private static int leftChild( int i )
{   return 2 * i + 1;
}
```

(根从0开始)

6.4. Applications of Priority Queues

2. The Selection Problem

堆 用于：不必要排序，但每次要知道最值 如 kruskal

6.4.Applications of Priority Queues

2. The Selection Problem 选择问题

在 N 个元素中找出第 K 个最大元素。

1A 算法：读入 N 个元素放入数组， 并将其选择排序，返回适当的元素。 下标为 n-k.

运行时间： $O(N^2)$

1B 算法：

1) 将 K 个元素读入数组， 并对其排序（按递减次序）。
   最小者在第 K 个位置上。
2) 一个一个地处理其余元素：
   每读入一个元素与数组中第 K 个元素 (在 K 个元素中为最小) 比较，
   如果 >，则删除第 K 个元素，再将该元素放在合适的位置上。
   如果 <，则舍弃。

最后在数组 K 位置上的就是第 K 个最大元素。

例如： 3, 5, 8, 9, 1, 10 找第 3 个最大元素。

6.4.Applications of Priority Queues

运行时间 (1B 算法) :

最坏情况

$O(K^2 + (N - K)*K)$

$= O(N*K)$

当 $K = \lceil N / 2 \rceil$, $O(N^2)$

K越小这个算法越好

试验： 在 N = 100 万个元素中， 找第 500,000 个最大元素。

以上两个算法在合理时间内均不能结束，都要处理若干天才算完 .

<u>用堆来实现：</u>

6A 算法：假设求第 K 个最小元素

1 ）将 N 个元素建堆（最小） $O(N)$

2) 执行 K 次 delete $O(K*\log N)$

$\Big\}$ $O(N + K * \log N)$

如果 $K = \lceil N / 2 \rceil$, $\theta (N * \log N)$

如果 $K = N$, $O(N * \log N)$ —— 堆排序

6.4.Applications of Priority Queues

1B和6A结合.

6B 算法：假设求第 K 个最大元素

1) 读入前 K 个元素， 建立最小堆   O(K)

2) 其余元素一一读入：

每读入一个元素与堆中第 K 个最大元素比（实际上是堆中最小元素）

O(1)

大于，则将小元素去掉（堆顶），该元素进入，进行一次调整。

O(log K)

小于，则舍弃。

$O(K + (N-K) * \log K) = O(N*\log K)$

当 $K = \lceil N / 2 \rceil$， $\theta(N * \log N)$

对 6A, 6B, 用同样的数据进行测试， 只需几秒钟左右给出问题解。

Chapter 6

exercises:

1. a. Show the result of inserting 10, 12, 1, 14, 6, 5, 8, 15, 3, 9, 7, 4, 11, 13, and 2,one at a time, into an initially empty binary heap.

   b. Show the result of using the linear-time algorithm to build a binary heap using the same input.
2. Show the result of performing three deleteMin operations in the heap of the previous exercise.

Chapter 6

3. 判别以下序列是否是堆？如果不是，将它调整为堆。

   1) { 100, 86, 48, 73, 35, 39, 42, 57, 66, 21 }

   2) { 12, 70, 33, 65, 24, 56, 48, 92, 86, 33 }

   3) { 103, 97, 56, 38, 66, 23, 42, 12, 30, 52, 06, 20 }

   4) { 05, 56, 20, 23, 40, 38, 29, 61, 35, 76, 28, 100 }

4. 设待排序的关键码序列为 { 12, 2, 16, 30, 28, 10, 16^*, 20, 6, 18 }，使用堆排序方法进行排序。写出建立的初始堆，以及调整的每一步。