# Chapter 5

## Hashing

# 散列

- 散列函数(Hash function)
  - Address=hash(key)

目标：每个数据有独一无二的散列值

# 散列

- 散列函数 (Hash function)
  - Address=hash(key)
- 散列表 (Hash Table )的设计目标

增删改查都是常数复杂度.

代价：空间复杂度

# 5.1 General Idea

- Sequention search : O(n)
- Binary search: $O(\log_2 n)$
- hashing method: O(C)

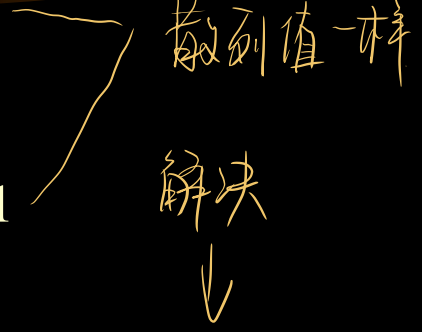
  Address=hash(key)

  also called : name-address function

# 5.1 General Idea

- **Example:**

H( x ) = x % 7
x1 = 8849 H( x1 ) = 8849 % 7 = 1
x2 = 8850 H( x2 ) = 8850 % 7 = 2
x3 = 8851 H( x3 ) = 8851 % 7 = 3
beta = 66698465 H( beta ) = 66698465 % 7 = 1
y2 = 8950 H( y2 ) = 8950 % 7 = 4



| | name | type | address | link |
|---|---|---|---|---|
| 0 | | | | |
| 1 | x1 | float | 1000 | |
| 2 | x2 | float | 1004 | |
| 3 | x3 | float | 1008 | |
| 4 | y2 | int | 2000 | |
| 5 | | | | |
| 6 | | | | |
| | | | | |


长了就不符合常数复杂度设计思想

# 5.1 General Idea

- **problems**

  **Find a proper hash function**

  **How to solve a collision**

  **Select a suitable load factor α.**

装填因子

$$\alpha = n/b$$

**n is number of elements in the hash table**

**b is the number of buckets in the hash table**

$\alpha > 1$ 碰撞频率大

$\alpha < 1$ 碰撞频率小

# 5.2 Hash Function

1. 取余法

H( Key ) = Key % M

其中： M <= 基本区长度的最大质数

| 基本区长 | M |
|---|---|
| 8 | 7 |
| 16 | 13 |
| 2048 | 2039 |

为什么取最大质数？

1) 若取偶数，如 10，100，…., 2, 4, …， 冲突率是比较大的；

2）若取含有质因子的M，如 M=21 （3*7） 含质因子3和7，对下面的例子：

key: 28 35 63 77 105

则 7 14 0 14 0 关键码中含质因子7的哈希值均为7的倍数。

2.平方取中法

H( Key ) = $Key^2$ 的中间部分，其长度取决于表的大小。

设表长 = $2^9$ = $(512)_{10}$ 地址 000~777(八进制）

$(1073)_{10}$ = $(2061)_8$ $(4310541)_8$

$(2062)_8$ $4314704$

$(2161)_8$ $4734741$

$(2162)_8$ $4741304$

$(1100)_8$ $1210000$

# 5.2 Hash Function

3. 乘法杂凑函数

$$H(Key) = \lfloor M * ((\varphi * Key) \% 1) \rfloor$$

例：设表长 $= 2^9 = (512)_{10}$　　地址 000~777(八进制），则

$$H(1) = \lfloor 2^9 * (0.618)_{10} \rfloor = \lfloor 2^9 * (0.4743\ldots)_8 \rfloor = 474$$

# 5.2 Hash Function

有些书中的

1. Hash1:

to add up the ASCII( or Unicode ) value of the characters in the string.

```
public static int hash( String Key, int tableSize )
{   int hashVal = 0;
    for( int i = 0; i < Key.length( ); i++ )
        hashVal += Key.charAt( i );
    return hashVal % tableSize;
}
```

Example:

Suppose TableSize = 10007,

Suppose all the keys are eight or fewer characters long, 8*127=1016

hash function typically can only assume value between 0~1016 引起浪费

目标：散列尽量均匀

# 5.2 Hash Function

2. Hash2:

$$h_{key} = k_0 + 37k_1 + 37^2k_2 + \ldots..$$

```
public static int hash( String key, int tableSize ) // good  hash fanction
{   int hashVal = 0;
    for( int i = key.length( )-1; i > =0; i-- )
        hashVal = 37 * hashVal + key.charAt( i );

    hashVal %= tableSize;
    if( hashVal < 0 )  //  函数允许溢出，这可能会引进负数
        hashVal += tableSize;
    return hashVal;
}
```

# 5.3 how to solve a collision

## —— linear Probing

碰撞的两个(或多个)关键码称为同义词，即H(k1) = H(k2)，k1不等于k2

### 1. Open Addressing

#### 1) linear Probing

If hash(key)=d and the bucket is already occupied then we will examine successive buckets d+1, d+2,……m-1, 0, 1, 2, ……d-1, in the array

# 5.3 how to solve a collision

## —— linear Probing

**Example 1:** a hash table with 11 buckets,

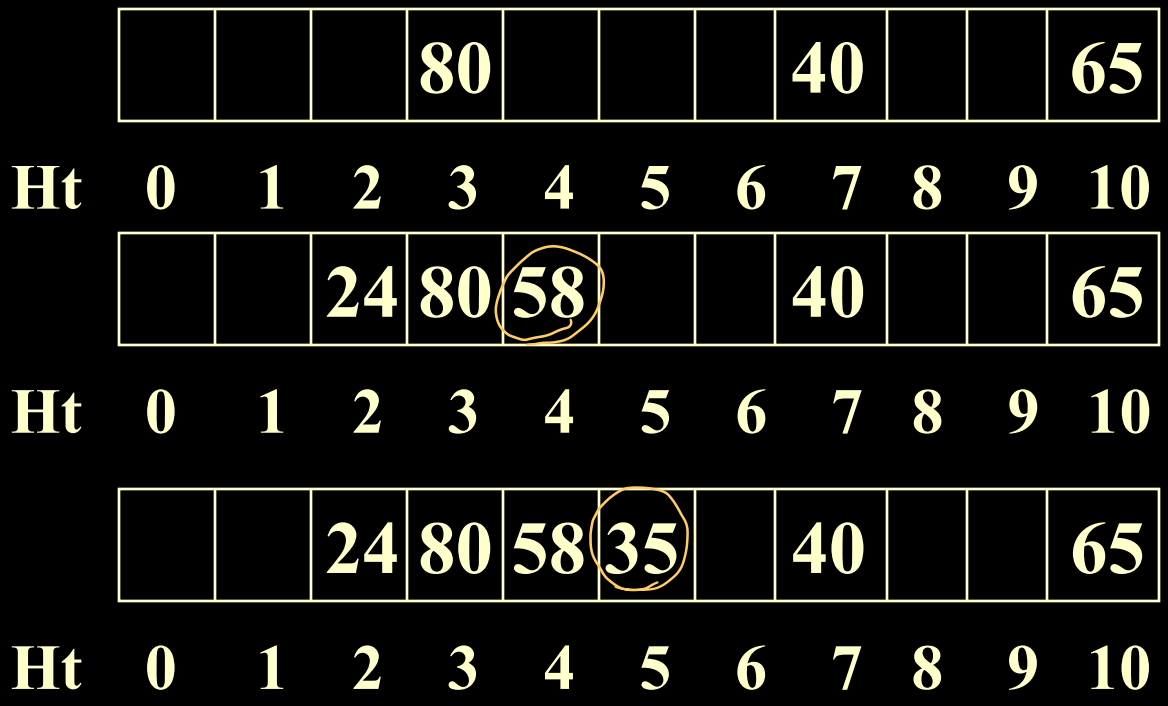
H(k) = k % 11,

Then 80, 40, 65, 24, 58, 35

H(80) = 3, H(40) = 7, H(65) = 10,

H(24) = 2, H(58) = 3, H(35) = 2

# 5.3 how to solve a collision
## ——linear Probing



# 5.3 how to solve a collision
## ——linear Probing

**Performance analysis**

**the adding sequence is 80,40,65,24,58,35**

$ASL_{succ}=(1+1+1+1+2+4)/6=10/6$

查找的复杂度.

# 5.3 how to solve a collision
## ——linear Probing

**example 2 :**

keys : Burke, Ekers, Broad, Blum, Attlee, Hecht, Alton, Ederly

hash( key ) = ord( x ) – ord('A')

x为取key第一个字母在字母表中的位置。例如：

hash(Attlee) = 0

H( Burke ) = 1 , H( Ekers ) = 4 , H( Broad ) = 1, H( Blum ) = 1,
H( Attlee ) = 0 , H( Hecht ) = 7 , H(Alton ) = 0 , H( Ederly ) = 4,

设散列表长 m = 26 ( 0~25 )

这种 很依赖于加入的顺序

Problem 1

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | ... | 25 |
|---|---|---|---|---|---|---|---|---|---|---|
| Attlee | Burke | Broad | Blum | Ekers | Alton | Ederly | Hecht | ..... | ..... | |
| 1 | 1 | 2 | 3 | 1 | 6 | 3 | 1 | | | |

分析比较次数：

搜索成功的平均搜索长度

( 1 + 1 + 2 + 3 + 1 + 1 + 6 + 3 )*1/8 = 18/8

# 5.3 how to solve a collision
# linear Probing

**example 3 :**

**hash( key ) = key % 10 ;**

**{ 89, 18, 49, 58, 69 }**

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 49 | 58 | 69 | | | | | | 18 | 89 |
| 2 | 4 | 4 | | | | | | 1 | 1 |

Problem 2

数据碰撞问题

**“clustering problem”** 堆积----指不同的同义词表合为一张了。从而增加了插入，查找的时间。

# 5.3 how to solve a collision

## linear Probing

**H(k) = k % 11,**

**Then 80, 40, 65, 24, 58, 35**

**H(80) = 3, H(40) = 7, H(65) = 10,**

**H(24) = 2, H(58) = 3, H(35) = 2**

| | | 24 | 80 | 58 | 35 | | 40 | | | 65 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

Ht

要删除58，如果真的删了，则后面要查找35就找不到了。

problem 3: 删除时

标记位置是否为空

# 5.3 how to solve a collision
## ——linear Probing

C++ Implementation

- Assume that each element to be stored in the hash table is of type E and has a field key of type k.
- the hash table is implemented using two arrays: ht and empty .
- empty[i] is true iff ht[i] does not have an element in it. It is defined for the deletion operation

# 5.3 how to solve a collision
## ——linear Probing

```
template<class E,class K>class HashTable
{  public:
       HashTable(int divisor =11);
       ~HashTable() {delete[]ht;  delete [] empty;}
       bool Search(const K&k ,E& e)const;
       HashTable<E,K>& Insert(const E&e);
    private:
       int hSearch(const K& k)const;
       int D;   //hash function divisor
       E *ht ; //hash table array
       bool *empty ; //1D array
};
```

# 5.3 how to solve a collision
## ——linear Probing

- **Constructor for hashtable**

```
template<class E,class K>
HashTable<E,K>::HashTable(int divisor)
{  D=divisor;
   ht=new E[D];
   empty= new bool[D];
   for(int i=0;i<D;i++)
        empty[i]=true;
 }
```

# 5.3 how to solve a collision
## ——linear Probing

```
template<class E,class K>
int HashTable<E,K>::hSearch(const K&k)const
{  int i=k%D;  //home bucket，做hash
   int j=i;  //start at home bucket
   do
     {  if(empty[j] || ht[j].key = =k) return j; //fit
        j=(j+1)%D; //next bucket
     } while(j!=i);  //returned to home?
   return j; //table full;
}
```

# 5.3 how to solve a collision
## ——linear Probing

- **Search function**

查找某元素

```
Template<class E,class K>
 bool HashTable<E,K>::Search(const
    K&k,E&e)const
 {//put element that matches k in e.
   //return false if no match.
    int b=hSearch(k) ;
    if(empty[b]|| ht[b].key!=k)return false;
    e=ht[b];
    return true;
  }
```

# 5.3 how to solve a collision
## ——linear Probing

- **Insertion into a hash table**

```
template<class E,class K>
HashTable<E,K>& HashTable<E,K>::Insert(const E& e)
{  K k=e.key;  //extract key
   int b=hSearch(k);
   if(empty[b]){empty[b]=false;  ht[b]=e;          //插入
                return *this;}
   if(ht[b]= =k)throw BadInput();//duplicate      //已有此
   throw NoMem();  //table full                  //表已满
}
```

# 5.3 how to solve a collision

## ——Quadratic probing

### 2) Quadratic probing 二次探测

If hash(k)=d and the bucket is already occupied then we will examine successive buckets d+1, d+$2^2$, d+$3^2$……, in the array

example :

( 89,18, 49, 58, 69 ) [个位数为key]

hash( k ) = k % 10;

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 49 | | 58 | 69 | | | | | 18 | 89 |
| 2 | | 3 | 3 | | | | | 1 | 1 |

ASLsucc=(1+1+2+3+3)/5

# 5.3 how to solve a collision

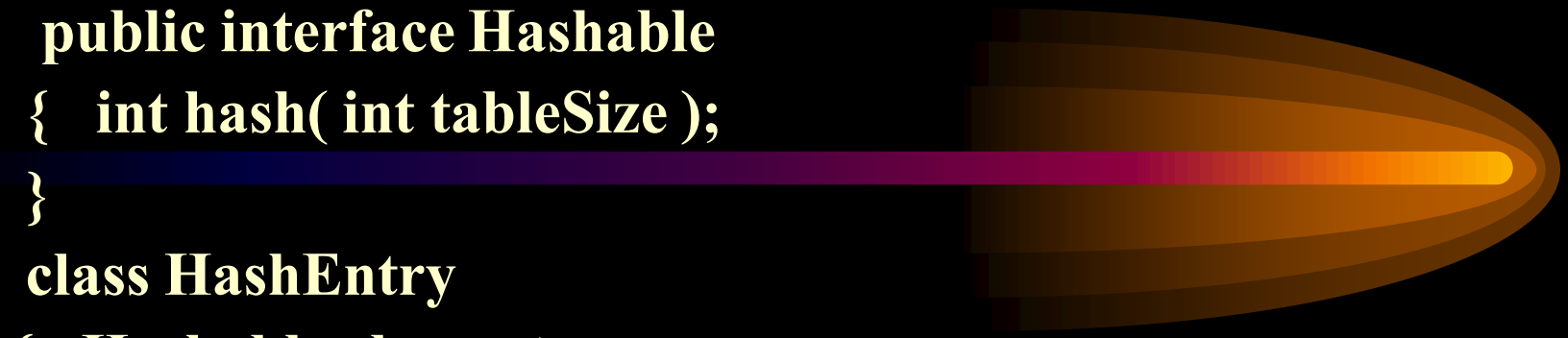
## ——Quadratic probing

- Java Implementation

element isActive

HashEntry 

第一种情况： null
第二种情况： 非null且该项是活动的，isActive为true
第三种情况：非null 且该项标记为被删除， isActive为false



```
public interface Hashable
{   int hash( int tableSize );
}
class HashEntry
{   Hashable element;
    boolean isActive;

    public HashEntry( Hashable e ) { this( e, true ) ; }
    public HashEntry( Hashable e, boolean i )
   {   element = e;
       isActive = i;
    }
}
```

# 5.3 how to solve a collision
## ——Quadratic probing

```
public class QuadraticProbingHashTable
{    public QuadraticProbingHashable( )
     public QuadraticProbingHashable( int size )
     public void makeEmpty( )

     public Hashable find( Hashable x )
     public void insert( Hashable x )
     public void remove( Hashable x )

     public static int hash( String key, int tableSize )
     private static final int DEFAULT_TABLE_SIZE = 11;

     protected HashEntry [ ] array;
     private int currentSize;
```

# 5.3 how to solve a collision

## ——Quadratic probing

```
    private void allocateArray(int arraySize )
    private boolean isActive( int currentPos )
    private int findPos( Hashable x )
    private void rehash( )
    private static int nextPrime( int n )
    private static boolean isPrime( int n )
}
```

# 5.3 how to solve a collision
## ——Quadratic probing

- Constractor

```
public QuadraticProbingHashTable( )
{   this( DEFAULT_TABLE_SIZE );
}
public QuadraticProbingHashTable( int size )
{   allocateArray( size );
    makeEmpty( );
}
```

# 5.3 how to solve a collision
## ——Quadratic probing

- **Some other function**

```
private void allocateArray( int arraySize )
{   array = new HashEntry[ arraySize ];
}

public void makeEmpty( )
{   currentSize = 0;
     for( int i = 0; i < array.length; i++ )
          array[ i ] = null;
}
```

# 5.3 how to solve a collision
## ——Quadratic probing

- **Find function**

```
public Hashable find( Hashable x )
{   int currentPos = findPos( x );
    return isActive( currentPos ) ? array[ currentPos ].element : null ;
}
private int findPos( hashable x )
{   int collisionNum = 0;
    int currentPos = x.hash( array.length );
    while( array[ currentPos ] != null &&
                        !array[ currentPos ].element.equals( x ) )
        currentPos += 2 * ++collisionNum – 1;
        if( currentPos >= array . length )
            currentPos -= array . length;
    }
    return currentPos;
}
```

# 5.3 how to solve a collision

## ——Quadratic probing

```
private boolean isActive( int currentPos )
{   return array[ currentPos ] != null &&
    array[ currentPos ].isActive;
}
```

- **Insert function**

```
public void insert( Hashable x )
{   int currentPos = findPos( x );
    if( isActive( currentPos ) ) return;
    array[ currentPos ] = new HashEntry( x, true );
    if( ++currentSize > array . length / 2 )
        rehash( );
}
```

# 5.3 how to solve a collision
## ——Quadratic probing

- **Remove function**

```
public final void remove( Hashable x )
{   int currentPos = findPos( x );
    if( isActive( currentPos) )
        array[ currentPos ] . isActive = false;
}
```

只设标志位

数据只覆盖，不设 null.

# 5.3 how to solve a collision
## ———Double Hashing

3) Double Hashing 双散列

If $hash_1(k)=d$ and the bucket is already occupied then we will counting $hash_2(k)=c$, examine successive buckets d+c, d+2c, d+3c……, in the array

**example:**

hash1(k) = k % 10; ⇒地址

hash2(k) = R – (k % R ); ⇒步长

其中 R < TableSize的质数

( 89, 18, 49, 58, 69 )

hash2(49)=7-49%7=7 (R=7)

hash2(58)=7-58%7=5

hash2(69)=7-69%7=1

首位+步长查到有空置

(到还没有的话就要rehash)

作业T1 a.

89后7

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 69 | | | 58 | | | 49 | | 18 | 89 |

# rehashing

**example:**

{ 13, 15, 24, 6 }

h(x) = x % 7;

13 % 7 = 6

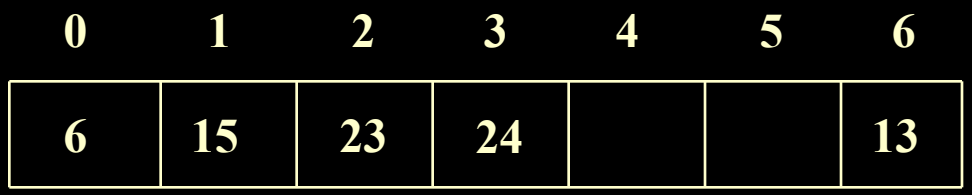
15 % 7 = 1

24 % 7 = 3

6 % 7 = 6

23

23 % 7 = 2

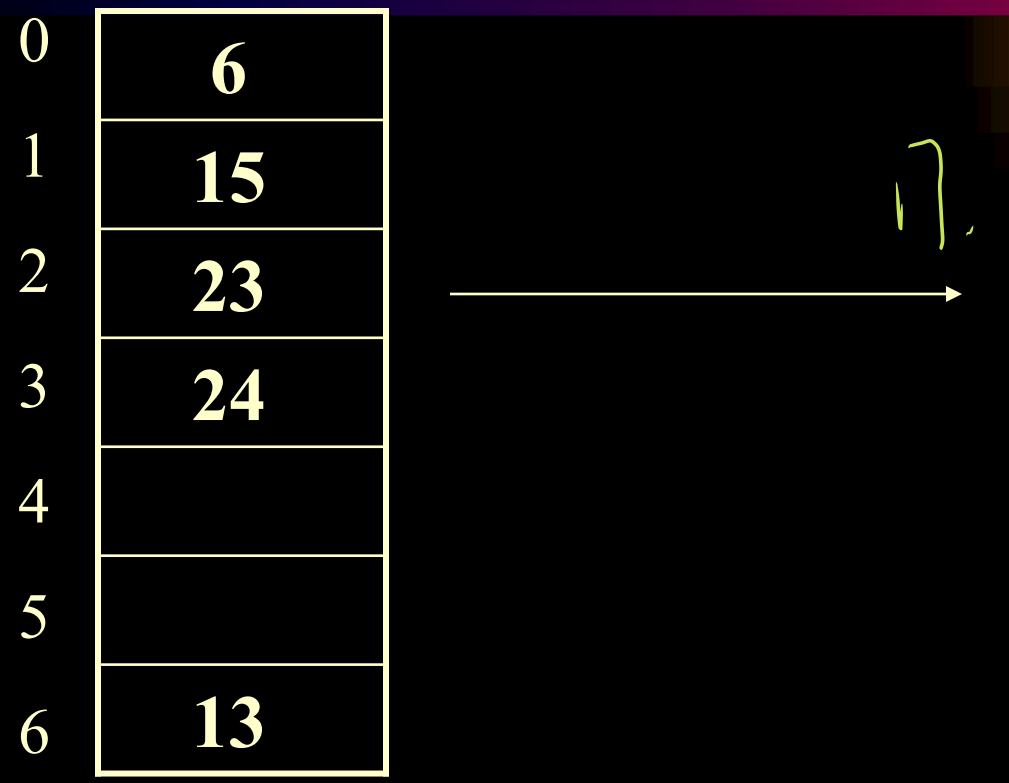



当表项数 > 表的70% 时,可再散列.

即, 取比(2*原表长=14)大的质数17再散列.

6%17=6, 15%17=15, 23%17=6, 24%17=7, 13%17=13

# rehashing



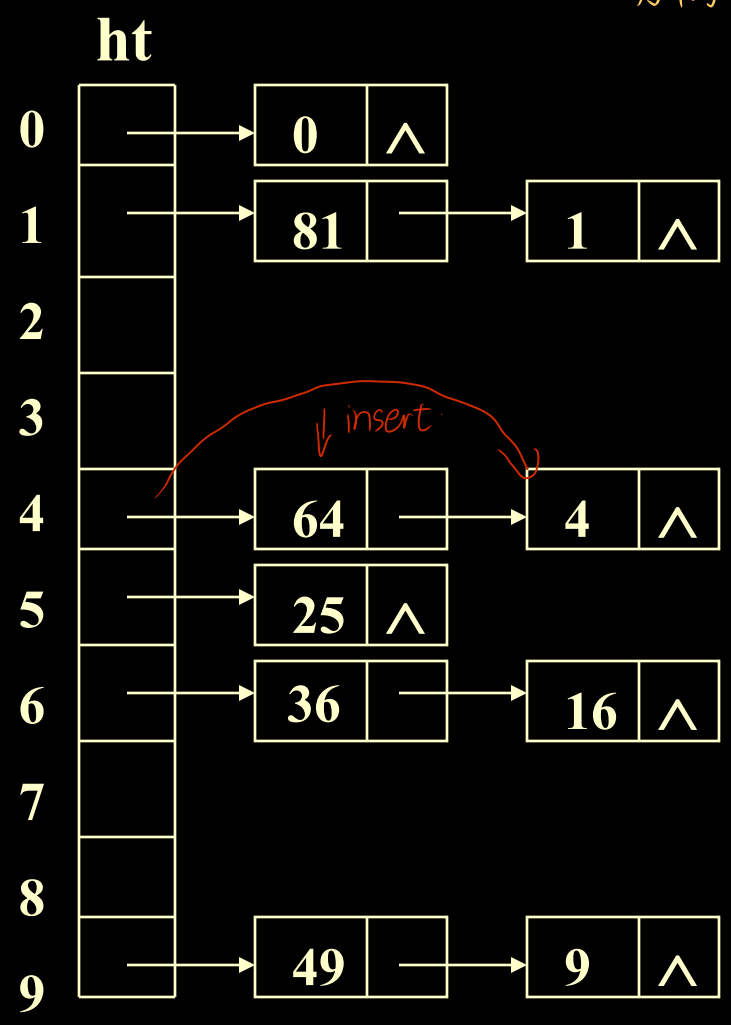
# rehashing

```
private void rehash( )
{   HashEntry [ ] oldArray = array ;

    allocateArray(  nextPrime( 2* oldArray.length ) );
    currentSize = 0;

    for( int i = 0; i < oldArray.length;  i++ )
        if( oldArray[ i ] != null && oldArray[ i ] . isActive )
            insert( oldArray[ i ] . Element );
}
```

# 5.4 how to solve a collision

## —— Separate Chaining

### 2. Separate Chaining 分离链接法/链地址法




0, 1, 4, 9, 16, 25, 36, 49, 64, 81

Hash(x) = x % 10

先进的在后面

# 5.4 how to solve a collision

## ——Separate Chaining

```
public class SeparateChainingHashTable
{   public  SeparateChainingHashTable( )
    public SeparateChainingHashTable( int size )
    public void insert( Hashable x )
    public void remove( Hashable x )
    public Hashable find( Hashable x )
    public void makeEmpty( )
    public static int hash( String key, int tableSize )

    private static final int DEFAULT_TABLE_SIZE = 101;

    private LinkedList [ ] theLists;

    private static int nextPrime( int n )
    private static boolean isPrime( int n )
 }
```

# 5.4 how to solve a collision

## ——Separate Chaining

```
public interface Hashable
{   int hash( int tableSize );
}

public class Employee implements Hashable
{   public int hash( int tableSize )
       {  return SeparateChainingHashTable.hash( name, tableSize ); }
    public boolean equals( object rhs )
       {  return name.equals( ( Employee) rhs ).name ); }

    private String name;
    private double salary;
    private int seniority;
}
```

# 5.4 how to solve a collision

## ——Separate Chaining

```
public SeparateChainingHashTable( )
{   this( DEFAULT_TABLE_SIZE );
}

public SeparateChainingHashTable( int size )
{  theLists = new LinkedList[ nextPrime( size ) ];
    for( int i = 0; i < theLists.length; i++ )
        theLists[ i ] = new LinkedList( );
}
public void makeEmpty( )
{   for( int i = 0; i < theLists.length; i++ )
        theLists[ i ].makeEmpty( );
}
```

# 5.4 how to solve a collision

## ——Separate Chaining

```
public void remove( Hashable x )
{  theLists[ x.hash( theLists.length ) ].remove( x );
}
public Hashable find( Hashable x )
{  return ( Hashable ) theLists[ x.hash( theLists.length ) ]. Find( x ).
     Retrieve( );
}

public void insert( Hashable x )
{   LinkedList whichList = theLists[ x.hash( theLists.length ) ];
     LinkedListItr itr = whichList.find( x );

     if( itr.isPastEnd( ) )
        whichList.insert( x, whichList.zeroth( ) );
}
```

在链表中查找？

# Chapter 5

**exercises:**

1 3 3 9 4 7 9

**1. Given input { 4371, 1323, 6173, 4199, 4344, 9679, 1989 }and a hash function h( x ) = x (mod 10), show the resulting:**

**a. Separate chaining hash table.**

**b. Hash table using linear probing.**

**c. Hash table using quadratic probing.**

**d. Hash table with second hash function $h_2(x) = 7 - (x \bmod 7)$.**

**2. 设散列表为HT[13]，散列函数为** 就是指 开放地址的线性探测

**H(key) = key % 13 。用线性开地址法解决冲突，对下列关键码序列 12,23,45,57,20,03,78,31,15,36 :**

**1) 画出其散列表。**

**2）计算等概率下搜索成功的平均搜索长度。**

**3）如果采用链表散列解决冲突，画出该链表。**