Q1 Let L be the sum of squares. $L=\sum_{i=1}^{n}(X_{i,n}-\hat{a}Q_s(P_{i,n})-\hat{b})^2$

L will be minimized when $\frac{\partial L}{\partial \hat{a}}=0$ and $\frac{\partial L}{\partial \hat{b}}=0$.

① $\frac{\partial L}{\partial \hat{b}}=\sum_{i=1}^{n}-2[X_{i,n}-\hat{a}Q_s(P_{i,n})-\hat{b}]=0$ $\quad n\hat{b}-\sum_{i=1}^{n}X_{i,n}+\hat{a}\sum_{i=1}^{n}Q_s(P_{i,n})=0$

$\hat{b}=\frac{\sum_{i=1}^{n}X_{i,n}}{n}-\frac{\hat{a}\sum_{i=1}^{n}Q_s(P_{i,n})}{n}$ $\quad \hat{b}=\bar{x}-\hat{a}\bar{q}$

② $\frac{\partial L}{\partial \hat{a}}=\sum_{i=1}^{n}-2Q_s(P_{i,n})(X_{i,n}-\hat{a}Q_s(P_{i,n})-\hat{b})=0$

$\sum_{i=1}^{n}[Q_s(P_{i,n})X_{i,n}-\hat{a}[Q_s(P_{i,n})]^2-(\bar{x}-\hat{a}\bar{q})Q_s(P_{i,n})]=0$

$\sum_{i=1}^{n}[Q_s(P_{i,n})X_{i,n}-\hat{a}[Q_s(P_{i,n})]^2-\bar{x}Q_s(P_{i,n})+\hat{a}\bar{q}Q_s(P_{i,n})]=0$

$\sum_{i=1}^{n}[Q_s(P_{i,n})X_{i,n}-\bar{x}Q_s(P_{i,n})]-\hat{a}\sum_{i=1}^{n}[[Q_s(P_{i,n})]^2-\bar{q}Q_s(P_{i,n})]=0$

$\hat{a}=\frac{\sum_{i=1}^{n}[Q_s(P_{i,n})X_{i,n}-\bar{x}Q_s(P_{i,n})]+\sum_{i=1}^{n}[\bar{x}\bar{q}-X_{i,n}\bar{q}]}{\sum_{i=1}^{n}[[Q_s(P_{i,n})]^2-\bar{q}Q_s(P_{i,n})]+\sum_{i=1}^{n}[\bar{q}^2-Q_s(P_{i,n})\bar{q}]}$ $\left(\sum_{i=1}^{n}[\bar{x}\bar{q}-X_{i,n}\bar{q}]\right.$ and $\sum_{i=1}^{n}[\bar{q}^2-Q_s(P_{i,n})\bar{q}]$ are equal to $\left.0\right)$

$\hat{a}=\frac{\sum_{i=1}^{n}(X_{i,n}-\bar{x})[Q_s(P_{i,n})-\bar{q}]}{\sum_{i=1}^{n}[Q_s(P_{i,n})-\bar{q}]^2}$

Q2 $r_Q=\frac{Cov(Q_s,X)}{\sigma_{Q_s}\sigma_X}=\frac{\frac{\sum_{i=1}^{n}[Q_s(P_{i,n})-\bar{q}](X_{i,n}-\bar{x})}{n-1}}{\sqrt{\frac{\sum_{i=1}^{n}[Q_s(P_{i,n})-\bar{q}]^2}{n-1}}\sqrt{\frac{\sum_{i=1}^{n}(X_{i,n}-\bar{x})^2}{n-1}}}$

$=\frac{\sum_{i=1}^{n}[Q_s(P_{i,n})-\bar{q}](X_{i,n}-\bar{x})}{\sqrt{\sum_{i=1}^{n}[Q_s(P_{i,n})-\bar{q}]^2}\sqrt{\sum_{i=1}^{n}(X_{i,n}-\bar{x})^2}}$