Atomic Components

Formal syntax

Let V be a set of variables, X a set of clocks and $\mathcal{E}(V)$ the set of expressions where variables are in V. Variables can be integer or Boolean and expressions can use comparison operators $(<, \leq, >, \geq, =, \neq)$, arithmetics operators $(+, -, *, /$ (euclidean division), $\%)$, logical Boolean operators $(!, \&\&, ||)$, Boolean bitwise operators $(\tilde{}, \&, |, \hat{})$ and function calls with returned value.
A **clock constraint** over X is a conjunction of atomic constraints of the form $x \sim n$ with $x \in X, \sim \in \{\leq, <, \geq, >\}$ and $n \in \mathbb{N}$.
A **guard** over $V \cup X$ is a (possibly empty) conjunction of atomic guard of the form c or b or $c \vee b$ with c a clock constraint and $b \in \mathcal{E}(V)$ evaluates to a Boolean value without side effects. We write $\mathcal{G}(V, X)$ to denote the set of guards over $V \cup X$ and $\mathcal{G}(V) = \mathcal{G}(V, \emptyset)$.
An **invariant** over $V \cup X$ is defined as a guard where $\sim$ is restricted to $\{\leq, <\}$. We write $\mathcal{I}(V, X)$ to denote the set of invariants over $V \cup X$.
An **update** on $V \cup X$ is a sequence of assignments of the form $x = n$ or $v = e$ or $f(p)$ with $x \in X, n \in \mathcal{E}(V)$ evaluates to a natural number, $v \in V, e \in \mathcal{E}(V)$, p a list of variables in V and f a function that can assign values to variables in p. We write $\mathcal{F}(V, X)$ to denote the set of updates on $V \cup X$ and $\mathcal{F}(V) = \mathcal{F}(V, \emptyset)$.
A **port** is a pair $\langle p, x_p \rangle$, denoted p, where:

- p is an identifier,
- x_p is a set of variables.

An **atomic component** is a tuple $A = \langle V_A, X_A, P_A, L_A, l_A^0, E_A, I_A \rangle$ where:

- V_A is a finite set of variables with v_0 the initial value of v for each $v \in V_A$,
- X_A is a finite set of clocks,
- P_A is a finite set of ports, each $p \in P_A$ is associated with a set of variables $x_p \subseteq V_A$,
- L_A is a finite set of locations,
- $l_A^0 \in L_A$ is the initial location,
- E_A is a finite set of transitions, each transition is a tuple $\langle l, p, g, f, l', u \rangle$ with $l, l' \in L_A, p \in P_A, f \in \mathcal{F}(V_A, X_A)$, $u \in \{e, l\}$ an urgency (e for eager and l for lazy) and $g \in \mathcal{G}(V_A, X_A)$ if $u = l$ and $g \in \mathcal{G}(V_A)$ otherwise,
- $I_A : L_A \rightarrow \mathcal{I}(V_A, X_A)$ is a function that assigns an invariant to each location.

We denote by $\mathcal{T}(A, p) = \{\langle l, p', g, f, l', u \rangle \in E_A \mid p' = p\}$ the set of transitions of p in A.

Semantics

A **valuation** $v : V \cup X \to \mathcal{D}$ is a function mapping variables in V and clocks in X to their codomain, we denote by $\mathcal{D}$ the union of these codomains. For $\delta \in \mathbb{R}^+$, $v+\delta$ is defined as $(v+\delta)(x) = \begin{cases} v(x) & \text{if } x \in V \\ v(x)+\delta & \text{if } x \in X \end{cases}$. For $f \in \mathcal{F}(V,X)$, $f(v)$ is the valuation v updated by f.

For two functions $v : X \to Y$ and $v' : X' \to Y'$, the **substitution function** noted $v/v' : X \cup X' \to Y \cup Y'$ is defined as $(v/v')(x) = \begin{cases} v'(x) & \text{if } x \in X' \\ v(x) & \text{if } x \in X \setminus X' \end{cases}$.

For a function $f : X \to Y$ and a set $X' \subseteq X$, the **restriction** of f to X' noted $f|_{X'} : X' \to Y$ is defined as $f|_{X'}(x) = f(x)$ for $x \in X'$.

For two sets X and Y, the set of all functions $f : X \to Y$ is denoted Y^X.

The semantics of an atomic component A is a LTS $\langle Q_A, q_A^0, \Sigma_A, \to_A \rangle$ where:

- Q_A is a set of states of the form $\langle l, v \rangle$ with $l \in L_A$ and $v \in \mathcal{D}^{V_A \cup X_A}$ a valuation of variables and clocks of A,
- q_A^0 is the initial state $\langle l^0, v_0 \rangle$ with $v_0(x) = x_0$ for each $x \in V_A$ and $v_0(x) = 0$ for each $x \in X_A$,
- Σ_A is the set of labels of the form d or $\langle p, v_p, u \rangle$ with $d \in \mathbb{R}_{>0}, p \in P_A, v_p \in \mathcal{D}^{x_p}$ a valuation of variables of the port p and $u \in \{e, l\}$ an urgency
- $\to_A$ is the transition relation defined by the rules:

  PORT
  $$\frac{\langle l, p, g, f, l', u \rangle \in E_A \qquad v \vDash g \qquad v' = f(v/v_p)\ (1) \qquad v' \vDash I_A(l')}{\langle l, v \rangle \xrightarrow{\langle p, v_p, u \rangle}_A \langle l', v' \rangle}$$

  (1) v' is the valuation v updated by v_p then by f

  DELAY
  $$\frac{\begin{array}{c} v' = v + \delta \\ v' \vDash I_A(l) \qquad \neg\big(\exists q \in Q_A, \exists p \in P_A, \exists v_p \in \mathcal{D}^{x_p} : \langle l, v \rangle \xrightarrow{\langle p, v_p, e \rangle}_A q\big)\ (2) \end{array}}{\langle l, v \rangle \xrightarrow{\delta}_A \langle l, v' \rangle}$$

  (2) expresses that there is no port transition from $\langle l, v \rangle$ with an eager urgency

Composite Components

Formal syntax

Let $(A_1, \ldots, A_n)$ be a vector of atomic components and $I \subseteq [1, n]$.

A **connector** is a tuple $\gamma = \langle T_\gamma, S_\gamma, A_\gamma \rangle$ where:

- T_γ and S_γ are two disjoint sets of ports with $T_\gamma \cup S_\gamma = \{p_i \mid i \in I\}$ and $p_i \in P_{A_i}$ for each $i \in I$, I defines the indices of the atomics components from where the ports of the connector come and there is at most one port per atomic component in the connector,

- $\forall i \in I : \big(\exists \langle l, p, g, f, l', u\rangle \in \mathcal{T}(A_i, p_i) : u = e\big) \Rightarrow \forall j \in I \backslash \{i\}, \forall \langle l, p, g, f, l', u\rangle \in \mathcal{T}(A_j, p_j) : g \in \mathcal{G}(V_{A_j})$, i.e. if a port has an eager transition then the transitions of the other ports cannot have clocks in their guard,
- A_γ is a set of actions, each action is a tuple $\langle P, g, f\rangle$ with
  - $P = \{p_j \mid j \in J\} \subseteq T_\gamma \cup S_\gamma$ for a $J \subseteq I$ where either $P \cap T_\gamma \neq \emptyset$ or $P = T_\gamma \cup S_\gamma$, T_γ is a set of trigger ports that can initiate an interaction without synchronizing and S_γ is a set of synchron ports that needs synchronization with other ports to initiate an interaction,
  - $g \in \mathcal{G}(\bigcup_{j \in J} x_{p_j})$,
  - $f \in \mathcal{F}(\bigcup_{j \in J} x_{p_j})$.

We denote by $\mathcal{A}(\gamma) = \{P \mid \langle P, g, f\rangle \in A_\gamma\}$ the set of **interactions** of γ.
A **priority** model π is a partial order on a set of connector.
A **composite component** is tuple $C = \langle (A_1, \ldots, A_n), \Gamma_C, \pi_C\rangle, n \in \mathbb{N}$ where:

- $(A_1, \ldots, A_n)$ is a vector of atomic components with the sets in $\{V_{A_i} \mid 1 \leq i \leq n\}$ and $\{X_{A_i} \mid 1 \leq i \leq n\}$ pairwise disjoint,
- Γ_C is a set of connectors on $(A_1, \ldots, A_n)$,
- π_C a priority model on Γ_C.

We denote by $\mathcal{B}(C) = \bigcup_{1 \leq i \leq n} P_{A_i} \setminus \bigcup_{\gamma \in \Gamma_C} T_\gamma \cup S_\gamma$ the set of **internal ports** of C.

Semantics

The semantics of a composite component C is a LTS $\langle Q_C, q_C^0, \Sigma_C, \rightarrow_C\rangle$ where:

- $Q_C = \prod_{1 \leq i \leq n} Q_{A_i}$,
- $q_C^0 = \langle q_{A_1}^0, \ldots, q_{A_n}^0\rangle$,
- $\Sigma_C = \mathbb{R}_{>0} \cup \big(\bigcup_{\gamma \in \Gamma_C} \mathcal{A}(\gamma) \cup \mathcal{B}(C)\big) \times \{e, l\}$,
- $\rightarrow_C$ is defined by the rules:

INTERNAL PORT

$$\frac{\begin{array}{c} i \in [1, n] \\ p \in \mathcal{B}(C) \cap P_{A_i} \qquad q_i \xrightarrow{\langle p, v_\emptyset, u\rangle}_{A_i} q_i' \qquad \forall j \in [1, n] \setminus \{i\} : q_j = q_j' \end{array}}{\langle q_1, \ldots, q_n\rangle \xrightarrow{\langle p, u\rangle}_C \langle q_1', \ldots, q_n'\rangle}$$

where $v_\emptyset$ is the empty function to $\mathcal{D}$

INTERACTION

$$\frac{\begin{array}{c} I \subseteq [1, n] \qquad a = \{p_i \mid i \in I\} \wedge p_i \in P_{A_i} \qquad \exists \gamma \in \Gamma_C : \langle a, g, f\rangle \in A_\gamma \\ v \vDash g \qquad \forall i \in I, \exists u_i \in \{e, l\} : q_i \xrightarrow{\langle p_i, v_{p_i}, u_i\rangle}_{A_i} q_i' \wedge v_{p_i} = f|_{x_{p_i}}(v) \ (3) \\ u = \begin{cases} e & \text{if } \exists i \in I : u_i = e \\ l & \text{otherwise} \end{cases} \qquad \forall i \notin I : q_i = q_i' \\ \neg\big(\exists \gamma' \in \Gamma_C, \exists a' \in \mathcal{A}(\gamma'), \exists u \in \{e, l\}, \exists Q \in Q_C : \gamma \prec_{\pi_C} \gamma' \wedge \langle q_1, \ldots, q_n\rangle \xrightarrow{\langle a', u\rangle}_C Q\big) \ (4) \\ \neg\big(\exists a' \in \mathcal{A}(\gamma), \exists u \in \{e, l\}, \exists Q \in Q_C : a \subset a' \wedge \langle q_1, \ldots, q_n\rangle \xrightarrow{\langle a', u\rangle}_C Q\big) \ (5) \end{array}}{\langle q_1, \ldots q_n\rangle \xrightarrow{\langle a, u\rangle}_C \langle q_1', \ldots, q_n'\rangle}$$

where $v = v_I|_{\bigcup_{i \in I} x_{p_i}}$ with v_I defined as $v_I(x) = v_i(x)$ for each $x \in V_{A_i}$ if $i \in I$, it is the valuation of variables of the ports in a
(3) v_{p_i} is the valuation of variables in p_i updated by f
(4) and (5) express that there is no interaction with a higher priority or including a from $\langle q_1, \ldots, q_n \rangle$

DELAY

$$\frac{\forall i \in [1,n] : q_i \xrightarrow{\delta}_{A_i} q'_i \qquad \neg\big(\exists p \in \bigcup_{\gamma \in \Gamma_C} \mathcal{A}(\gamma) \cup \mathcal{B}(C), \exists Q \in Q_C : \langle q_1, \ldots, q_n \rangle \xrightarrow{\langle p,e \rangle}_C Q\big) \ (6)}{\langle q_1, \ldots q_n \rangle \xrightarrow{\delta}_C \langle q'_1, \ldots, q'_n \rangle}$$

(6) expresses that there is no internal port or interaction transition from $\langle q_1, \ldots q_n \rangle$ with an eager urgency

Differences

- No hierarchy:
  - connectors have no exported port
  - components have no compound component, exported data and exported port
- No guard on priorities and only priorities on connectors:
  - atoms have no priority on ports, export data and merged ports
  - priority rules in components are in the form `I < J` where `I` and `J` are in the form `C:*` or `*:*` where `C` is a connector
- There is no sub-expression in the expressions of the interactions in connectors
- No delayable urgency, invariant from and resume
- Guard and invariant conditions are restricted to the grammar:
  $G ::= c \mid b \mid c \vee b \mid G \wedge G$
  $c ::= x \sim n \mid c \wedge c$
  where b is an expression that evaluates to a Boolean value, x is a clock, $n \in \mathbb{N}$, $\sim \in \{<, \leq, >, \geq\}$ for guards and $\sim \in \{<, \leq\}$ for invariants
- No internal transition
- No if-then-else statement in actions
- Only Boolean and integer variables
- An eager transition has no clock in its guard and transitions in an eager interaction have no clock in their guard