DATA130004: Homework 9

Due in class on December 19, 2018

1. Exercises 7.1
2. Exercises 9.1, 9.2, 9.3 and 9.4
3. In Bayesian inference, if the posterior distributions $p(\theta|x)$ are in the same family as the prior probability distribution $\pi(\theta)$, the prior and posterior are then called conjugate distributions, and the prior is called a conjugate prior for the likelihood function. Let $x = (x_1, \ldots, x_n)^\top$ consist of n i.i.d. random samples. Prove the following well-known conjugate results by specifying the posterior distribution type and parameters.
   (a) When the variance parameter is known, the Gaussian family is conjugate to itself (or self-conjugate) with respect to a Gaussian likelihood function. That is, if the likelihood function is Gaussian, choosing a Gaussian prior over the mean will ensure that the posterior distribution is also Gaussian.
   (b) Gamma distribution is the conjugate prior for the rate parameter of Poisson distribution. That is, given data are generated according to Poisson(λ), specify a Gamma(α, β) prior for λ, then the posterior distribution $p(\lambda|x)$ is still Gamma.