Confidence Intervals and Hypothesis Testing

STAT-UB.0001 – Statistics for Business Control

1. Of the Stern MBA students who filled out an online class survey, 42 reported their GMAT scores. The sample mean of the reported scores was 720, and the sample standard deviation was 35.

   (a) What is a reasonable population to associate with this sample?

   (b) What is the meaning of the "population mean"?

   (c) Find a 95% confidence interval for the population parameter.

   (d) Under what conditions is the confidence interval valid?

2. Use the following sample means and sample standard deviations of other questions from that class survey to form 95% confidence intervals for the population mean of each variable.

   (a) Dinners per month: $\bar{x} = 9.0$, $s = 4.6$, $n = 47$.

   (b) Age (years): $\bar{x} = 26.7$, $s = 6.1$, $n = 47$.

   (c) Time planned for studying per week (hours): $\bar{x} = 15.75$, $s = 10.5$, $n = 46$.

3. In Problem 2, what assumptions do we need for the confidence intervals to be valid? How could we check these assumptions?

4. In each of the following situations, find α and $t_{\alpha/2,n-1}$.
   (a) An 80% confidence interval with $n = 10$.

   (b) A 99% confidence interval with $n = 25$.

   (c) A 90% confidence interval with $n = 30$.

5. A random sample of 36 measurements was selected from a population with unknown mean μ. The sample mean is $\bar{x} = 12$ and the sample standard deviation is $s = 18$. Calculate an approximate 95% confidence interval for μ. Use the approximation $t_{\alpha/2,n-1} = t_{0.025,35} \approx 2$.

6. Complete Problem 5, with a 99% confidence interval instead of a 95% confidence interval.

7. Complete Problem 5, with an 80% confidence interval instead of a 95% confidence interval.

8. How reliable is the SoHo Halal Guy's Yelp rating? The SoHo Halal Guy at Broadway and Houston (`http://www.yelp.com/biz/soho-halal-guy-new-york`) currently has 53 Yelp reviews (4 1-star; 1 2-star; 6 3-star; 17 4-star; and 25 5-star). The average star rating is 4.1 and the sample standard deviation of the star ratings is 1.2. How much should we trust the number "4.1"? We will use a confidence interval to quantify the uncertainty associated with this number.

   (a) What is a reasonable population to associate with this sample?

   (b) What is the meaning of the population mean, μ?

   (c) Find a 95% confidence interval for the population mean, μ.

   (d) Under what conditions is the confidence interval valid?

9. La Colombe at Lafayette and 4th St (`http://www.yelp.com/biz/la-colombe-new-york-2/`) currently has 612 Yelp reviews (16 1-star; 24 2-star; 50 3-star; 185 4-star; and 337 5-star). The average star rating is 4.31 and the sample standard deviation of the star ratings is 0.96. Find a 95% confidence interval for the expected rating of a random La Colombe Yelp reviewer.

Confidence Interval for Proportion

10. A CNN/ORC post-debate poll surveyed 547 voters who watched the third presidential debate on October 19, 2016. The results are at `http://www.cnn.com/2016/10/19/politics/hillary-clinton-wins-third-presidential-debate-according-to-cnn-orc-poll`. Of the respondents, 52% thought that Clinton did the best job, while 39% thought that Trump did.

    (a) What is a reasonable population to associate with this sample?

    (b) There are a few population parameters of interest. Choose one.

    (c) Find a 95% confidence interval for the population parameter.

    (d) Under what conditions is the confidence interval valid?

11. Use the following data from a Stern MBA class survey to estimate the relevant population proportions. Give 95% confidence intervals for these proportions.

    (a) Gender: 17 Female, 30 Male.

    (b) Drinks at least one cup of coffee on a typical day: 37 Yes, 10 No.

    (c) Political affiliation: 36 Democrat, 6 Republican, 5 Other. (For this problem there are three different choices for the population parameter; choose one of them.)

12. In Problem 11, what are the relevant populations?

13. In Problem 11, what assumptions do we need for the confidence intervals to be valid?

Hypothesis Test: Introduction

14. An analyst claims to have a reliable model for Yahoo's quarterly revenues. His model predicted that the most recent quarterly revenues could be described as a normal random variable with mean \$1.5B and standard deviation \$0.1B. In actuality, the revenues were \$1.0B. Is there evidence of a problem with the analyst's model? Why or why not?

15. David has a coin, which he claims to be fair (50% chance of "heads," and 50% chance of "tails"). He flips the coin 10 times, and gets "heads" all 10 times. Do you believe him that the coin is fair? Why or why not?

Test on a Population Mean

16. (Adapted from Stine and Foster, 4M 16.2). Does stock in IBM return a different amount on average than T-Bills? We will attempt to answer this question by using a dataset of the 264 monthly returns from IBM between 1990 and 2011. Over this period, the mean of the monthly IBM returns was 1.26% and the standard deviation was 8.27%. We will take as given that the expected monthly returns from investing in T-Bills is 0.3%.

    (a) What is the sample? What are the sample mean and standard deviation?

    (b) What is the relevant population? What are the interpretations of population mean and standard deviation?

    (c) What are the null and alternative hypotheses for testing whether or not IBM gives a different expected return from T-Bills (0.3%)?

(d) Use an appropriate test statistic to summarize the evidence against the null hypothesis.

(e) If the null hypothesis were true (there were no difference in expected monthly returns between IBM and T-Bills) what would be the chance of observing data at least as extreme as observed?

(f) Is there compelling evidence (at significance level 5%) of a difference in expected monthly returns between IBM and T-Bills?

(g) What assumptions do you need for the test to be valid? Are these assumptions plausible?

Test Statistic and Observed Significance Level (p-value)

17. In each of the following examples, for the hypothesis test with

$$H_0 : \mu = \mu_0$$
$$H_a : \mu \neq \mu_0$$

find the test statistic (t) and the p-value.

(a) $\mu_0 = 5$; $\bar{x} = 7$; $s = 10$; $n = 36$.

(b) $\mu_0 = 90$; $\bar{x} = 50$; $s = 200$; $n = 64$.

(c) $\mu_0 = 50$; $\bar{x} = 49.4$; $s = 2$; $n = 100$.

18. For each example from problem 17:

(a) Indicate whether a level 5% test would reject H_0.

(b) Indicate whether a level 1% test would reject H_0.