

Permutations and Combinations

1. **New York state lotto.** You pick six of the numbers 1 through 54, and then in a televised drawing six of the numbers are selected. If all six of your numbers are selected then you win a share of the first place prize. If five or four of your numbers are selected you win a share of the second or third prize.

(a) How many ways are there to select 6 numbers for the lotto ticket?

(b) How many ways are there to select a first prize number?

(c) What is the probability of selecting a first prize number?

(d) What is the probability of selecting a second prize number?

2. **Quality assurance.** Suppose we have a batch of 100 light bulbs, which contains 5 defective bulbs. If we pick 10 for testing, what is the probability that no bulbs in the sample are defective? We can answer this question in three steps.
- (a) How many ways are there of picking 10 bulbs for testing out of 100?
 - (b) How many ways are there of picking 10 non-defective bulbs?
 - (c) What is the probability that there are no defective bulbs in your sample of 10?
3. **The Birthday Problem.** A class has 20 students. What is the probability that at least two students have the same birthday? Assume that each person in the class was assigned a random birthday between January 1 and December 31.

Probability Distribution Function and Expectation

4. Consider the following game:

1. You pay \$6 to flip a coin.
 2. If the coin lands heads, you get \$10; otherwise, you get nothing.
- (a) Would you play this game? Why or why not?

(b) What is the random experiment involved in the game? What are the sample space? What are the probabilities of the sample points?

(c) Let W be the random variable equal to the amount of money you win from playing the game. If you lose money, W will be negative. Find the value of W for each of the sample points.

(d) Describe W in terms of its probability distribution function (PDF).

(e) What are your expected winnings? That is, what is μ , the expectation of W ?

5. Suppose you flip two coins. Let X be the random variable which counts the number of heads on the two tosses.

(a) List all of the sample points of the experiment, along with the corresponding values of X .

(b) Compute the probability distribution function of X .

(c) Compute the expectation of X .

(d) What is the interpretation of the expectation of X ?

6. Let X be a random variable describing the number of cups of coffee a randomly-chosen member of the class drinks on a typical day. There is a 22% chance that the student has one cup, a 16% chance that the student has two cups, a 16% chance that the student has three cups, an 11% chance that the student has four cups, and a 3% chance that the student has five cups. Also, there is a 32% chance that the student doesn't drink any coffee.

(a) Let $p(x)$ be the probability distribution function of X . Fill in the following table:

x	0	1	2	3	4	5
$p(x)$						

(b) Find $E(X)$, the expectation of X .

(c) What is the interpretation of the expectation of X ?

Variance and Standard Deviation

7. This is a continuation of problem 6.

- (a) Find $\text{var}(X)$ and $\text{sd}(X)$, the variance and standard deviation of X , the number of cups of coffee that a random student from the class drinks on a typical day.

- (b) What is the interpretation of the standard deviation of X ?

8. Consider the following game:

- 1. You pay \$6 to pick a card from a standard 52-card deck.
- 2. If the card is a diamond (\diamond), you get \$22; if the card is a heart (\heartsuit), you get \$6; otherwise, you get nothing.

Perform the following calculations to decide whether or not you would play this game.

- (a) Let W be the random variable equal to the amount of money you win from playing the game. If you lose money, W will be negative. Find the PDF of W .

- (b) What are your expected winnings? That is, what is μ , the expectation of W ?

- (c) What is the standard deviation of W ?

- (d) What are the interpretations of the expectation and standard deviation of W ?

Properties of Expectation

9. **Affine Transformations.** Let X be a random variable with expectation $\mu_X = 2$. What is the expectation of $5X + 2$?
10. **Sums of Random Variables.** Let X and Y be random variables with $\mu_X = 1$, $\mu_Y = -5$. What is $E(X + Y)$?
11. Let X and Y be random variables with $\mu_X = -2$, $\mu_Y = 3$.
- (a) Find the expectation of $-3X + 2$.
 - (b) Find the expectation of $X + Y$.
12. You invite four people to go out to dinner on Friday night. The attendance probabilities for the four potential guests are 50%, 20%, 30%, and 90%.
- (a) Find the expected number of guests.
 - (b) The dinner will be a *prix fixe* meal, costing \$50 per person. What is the expected total cost for yourself and your guests?
 - (c) What is the interpretation of your answer to part (b)?