Multiplicative Rule

1. Suppose you run a lottery in the class of 20 students. You put 1 red ball and 19 green balls into a blackbox. One by one, each student randomly picks a ball from the blackbox, and whoever gets the red ball wins the lottery.

    (a) What is the probability for the first student to win the lottery?

    (b) What is the probability for the second student to win the lottery?

    (c) What is the probability for the last student to win the lottery?

Independence

2. Suppose that you flip two fair coins. Let $A =$ "the first coin shows Heads," $B =$ "The second coin shows Heads." Find the probability of getting Heads on both coins, i.e. find $P(A \cap B)$.

3. Suppose that you roll two dice. What is the probability of getting exactly one 6?

4. Suppose that you sell fire insurance policies to two different buildings in Manhattan, located in different neighborhoods. You estimate that the buildings have the following chances of being damaged by fire in the next 10 years: 5%, and 1%. Assume that fire damages to the two buildings are independent events. Compute the probability that exactly one building gets damaged by fire in the next 10 years.

5. Consider the following experiment. A hat contains two coins:

- one coin, the "fair" coin, has 50% chance of heads and 50% chance of tails on every flip;
- the other coin, the "heads" coin, has heads on both sides, so it always lands heads on every flip.

You reach into the hat and pull out a random coin, equally likely to get the fair coin or the heads coin. Then, you flip this coin twice.

Define events A and B as

$$A = \text{the first flip lands heads}$$
$$B = \text{the second flip lands heads.}$$

(a) Based on your intuition, do you think that A and B independent events?

(b) Compute $P(A)$.

(c) Compute $P(A \cap B)$.

(d) Use your answers to parts (b) and (c) to either prove or disprove that A and B are independent.

Bayes' Rule

6. With probability 0.15, a person will pass the job interview for a Data Analyst position. Among those who passed the interview, 60% had taken college courses in Statistics. It happens also that 30% of all those who interviewed had college courses in Statistics. Find the probability that a person with college courses in Statistics will pass the job interview.

7. Amazon.com maintains a list of all registered customers, along with their email addresses. During July, they sent coupons to 20% of their customers. They recorded that 5% of their customers made purchases in July, and 40% of all purchases were made with coupons. In this problem we will compute the proportion of customers sent a coupon in July who made a purchase in that month. For simplicity, we will assume that customers either make 0 or 1 purchases in July.

   (a) Consider a random customer, and define two events:

   $$\text{Coupon} = \text{the customer received a coupon in July,}$$
   $$\text{Purchase} = \text{the customer made a purchase in July.}$$

   Express all percentages given in the problem statement as probabilities or conditional probabilities of these two events. Example: $P(\text{Coupon}) = 0.20$.

   (b) Use Bayes' rule to compute the proportion of custoers sent a coupon in July who made a purchase that month.

8. Suppose that 1% of population have a special disease. A blood test detects the disease with probability 0.95 when it is present, but also falsely detects it when it's not present with probability 0.02. Test shows that a person has the disease; what is the probability that he indeed has it?

9. A desk lamp produced by a company was found to be defective (D). There are three factories (A, B, C) where such desk lamps are manufactured. A Quality Control Manager is responsible for investigating the source of found defects. This is what the manager knows about the company's desk lamp production and the possible source of defects:

Factory	% of total production	Probability of defective lamps
A	0.35	0.015
B	0.35	0.010
C	0.30	0.020

If a randomly selected lamp is defective, what is the probability that the lamp was manufactured in factory C?

The Multiplication Rule

10. A man has 4 pair of pants, 6 shirts, 8 pairs of socks, and 3 pairs of shoes. How many ways can he get dressed?

11. A restaurant offers soup or salad to start, and has 11 entrées to choose from, each of which is served with rice, baked potato, or zucchini. How many meals can you have if you can choose to eat one of their 4 desserts or have no desert?

12. How many answer sheets are possible for a true/false test with 15 questions?

Permutations

13. How many ways can 5 people stand in line?

14. How many different batting orders are possible for 9 baseball players?

15. How many ways can 8 books be put on a shelf?

More Permutations

16. Twelve people belong to a club. How many ways can they pick a president, vice-president, secretary, and treasurer?

17. In a horse race the first three finishers are said to win, place, and show. How many finishes are possible for a race with 11 horses?

18. Five different awards are to be given to a class of 30 students. How many ways can this be done if (a) each student can receive any number of awards, (b) each sutent can receive at most one award?

Combinations

19. A club has 12 members.

    (a) A club has 12 people. How many ways can they pick 2 people to be on a committee to plan a party?

    (b) How many ways can they pick 4 people to be on a committee to plan a party?

20. A restaurant offers 15 possible toppings for its pizza. How many different pizzas with 3 toppings can be ordered?

21. We are going to pick 5 cards out of a deck of 52. In how many ways can this be done?

Advanced Problems

22. **New York state lotto.** You pick six of the numbers 1 through 54, and then in a televised drawing six of the numbers are selected. If all six of your numbers are selected then you win a share of the first place prize. If five or four of your numbers are selected you win a share of the second or third prize.

    (a) How many ways are there to select 6 numbers for the lotto ticket?

    (b) How many ways are there to select a first prize number?

    (c) What is the probability of selecting a first prize number?

    (d) What is the probability of selecting a second prize number?

23. **Quality assurance.** Suppose we have a batch of 100 light bulbs, which contains 5 defective bulbs. If we pick 10 for testing, what is the probability that no bulbs in the sample are defective? We can answer this question in three steps.

(a) How many ways are there of picking 10 bulbs for testing out of 100?

(b) How many ways are there of picking 10 non-defective bulbs?

(c) What is the probability that there are no defective bulbs in your sample of 10?

24. **The Birthday Problem.** A class has 20 students. What is the probability that at least two students have the same birthday? Assume that each person in the class was assigned a random birthday between January 1 and December 31.