Basic Probability

STAT-UB.0001 Statistics for Business Control

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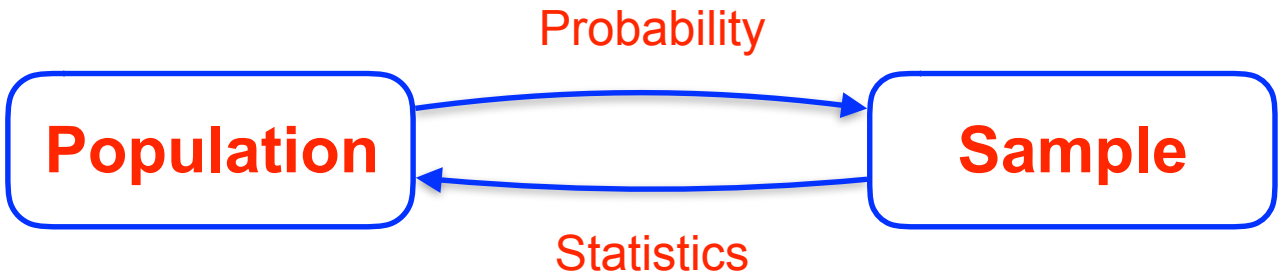
Review

- Goal of this class: use a sample to make a statement about a population.
  - Representative sample and unbiased sample.
  - Sources of bias.
- Descriptive statistics: summarize data.
  - Graphical summary.
  - Numerical summary: mean, median, std, empirical rules.

Probability

Probability is the basic language of statistics.



Probability vs Statistics

- Probability: a fair coin is tossed 10 time; what is the probability of getting 8 heads?
- Statistics: a coin is tossed 10 times, showing 8 heads; is the coin fair?

Experiments, Sample Points, and Sample Spaces

- Random experiment: the process of observation leading to an outcome that cannot be predicted with certainty.
- Sample point: a possible outcome of an experiment.
- Sample space of experiments: the set of all sample points, denoted by Ω, or S.

Example: flip a coin; roll a 6-sided dice.

Probability

Given a sample space, $\Omega = \{e_1, e_2, ..e_n\}$. A probability $\mathbb{P}$ is a function with two properties:

- The probability of any sample point is nonnegative,

$$\mathbb{P}(e_i) \geq 0.$$

- The sum of probabilities is 1,

$$\mathbb{P}(e_1) + .. + \mathbb{P}(e_n) = 1.$$

Events

Event is a set of sample points. Example:

- A fair die rolls odd.
- A fair die rolls 4 or higher.

Events are usually denoted by capital letters, e.g. A. If $A = \{f_1, \ldots, f_m\}$, then

$$\mathbb{P}(A) = \mathbb{P}(f_1) + \cdots + \mathbb{P}(f_m).$$

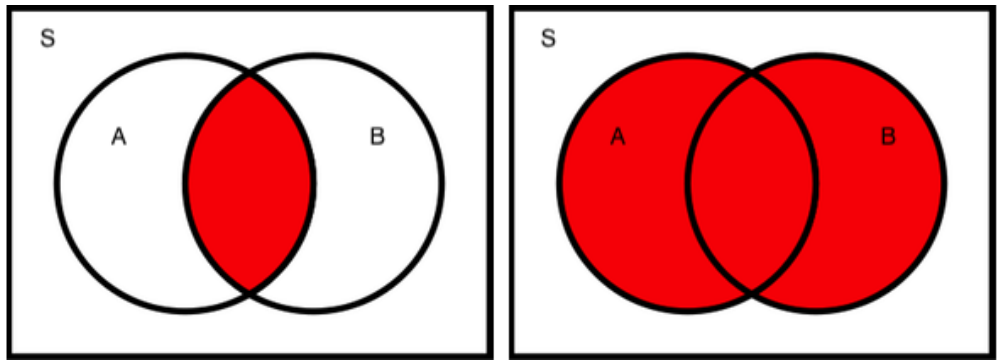
Example: Find the probabilities of the two events above.

Compound Events: Union and Intersections

A and B are two events.

- Union ($A \cup B$, "A or B"): event A or event B occurs, or both occur.
- Intersection ($A \cap B$, "A and B"): event A and event B both occur.

Figure: Left: $A \cap B$. Right: $A \cup B$



Example: A = "a die rolls odd" and B = "a die rolls 4 or higher". What is $A \cup B$? What is $A \cap B$?

Additive Rule

Additive rule helps computing the probability of compound events:

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B).$$

Example: A = "a die rolls odd" and B = "a die rolls 4 or higher".

- $\mathbb{P}(A)$
- $\mathbb{P}(B)$
- $\mathbb{P}(A \cap B)$
- $\mathbb{P}(A \cup B)$

Mutually Exclusive Events

- Events A and B are mutually exclusive (ME): A and B cannot occur together, $A \cap B = \emptyset$ (empty).
- Additive rule for *mutually exclusive* events:

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B).$$

Example:

- "a die rolls 3" and "a die rolls 4"
- "a die rolls odd" and "a die rolls 4 or higher"

Complementary Event

- Complement of event A (A^c, $\bar{A}$, "not A"): event A does not occur.
- A and A^c are mutually exclusive events.
- Complement rule:

$$\mathbb{P}(A^c) = 1 - \mathbb{P}(A).$$

Example: Suppose you flip a fair coin five times. What is the probability of getting at least one head?

Interpretations of Probability

- Long-run relative frequency: when an experiment is repeated n times (n is large),

$$\mathbb{P}(A) \approx (\text{no. of times } A \text{ occured})/n.$$

- In daily conversation, the probability is a statement about how much you believe something to be true.
  - Example: there is 10% probability of rain tomorrow.
  - It's different from the way we use probability in this class.

Conditional Probability

$\mathbb{P}(A \mid B)$: the probability of event A, given that event B occurred. It is formally defined as

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}.$$

Other interpretations:

- If I know the sample point is in B, what is the chance it is also in A.
- Equivalently, among all the sample points in B, what is the proportion of sample points that are also in A.

Multiplicative Rule

Multiplicative rule helps to compute the prob. of interactions:

$$\mathbb{P}(A \cap B) = \mathbb{P}(B)\mathbb{P}(A \mid B) = \mathbb{P}(A)\mathbb{P}(B \mid A).$$

Example: Box A contains 8 red and 2 green balls; box B contains 15 red and 30 green balls. Flip a fair coin, and select a ball from box A if heads, or from box B if tails. Find

- $\mathbb{P}(\text{red} \mid \text{heads})$
- $\mathbb{P}(\text{red} \mid \text{tails})$
- $\mathbb{P}(\text{red and heads})$
- $\mathbb{P}(\text{red and tails})$
- $\mathbb{P}(\text{red})$

Summary

- Random experiment, sample point, sample space
- Probability
- Events, union and intersection, mutually exclusive events, complementary event
- Additive rule
- Conditional probability, multiplicative rule