Random Variables

STAT-UB.0001 Statistics for Business Control

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Jul 12, 2018

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Review

- Independence, Bayes' Rule
- Counting: Permutations, Combinations

Random Variable

Random Variable

A variable whose value depends uniquely on the outcome of a random experiment. Usually denoted by capital letters, e.g. X, Y, Z.

More interpretations:

- A random variable maps sample points of a random experiment to values.
- Once the experiment is performed, the value of the random variable is referred to as an *observation*.
- A data set contains observations of a random variable resulting from repeated trials of the random experiment.

Random Variable: More Examples

- Example 1
  - Random experiment: Roll two dices.
  - $X =$ sum of the two dices.
  - Observe 2 and 4 from one trial of the experiment, $X = 6$.
- Example 2
  - Random experiment: flip a coin three times.
  - $X =$ number of heads from the three flips.
  - Observe HHT from one trial of the experiment, $X = 2$.

Two Types of Random Variables

Discrete random variable

Can take a countable (finite or infinite) number of values, often obtained from "counting".

Example: Number of heads when flipping three coins.

Continuous random variable

Can take any value in an interval of real numbers. Uncountable. Often obtained from "measuring".

Example: Average height of 3 students randomly picked from this class.

Case Study

Consider the following game:

1. You pay \$6 to flip a coin.
2. If the coin lands heads, you get \$10; otherwise, you get nothing.

Let W be the random variable equal to the amount of money you win (can be negative).

Sample point	W	Probability
H		
T		

Probability Distribution Function (PDF) for Discrete RV

Probability Distribution Function (PDF)

For a discrete random variable X, we denote the probability that X takes the value x by

$$p(x) = \mathbb{P}(X = x).$$

$p(x)$ is referred to as the *probability distribution function* (PDF) of random variable X.

- A PDF can be described with a table listing every possible values of random variable, and the probability of each value.
- In the previous case study, what is the PDF of W?

Notations for RV and its Values

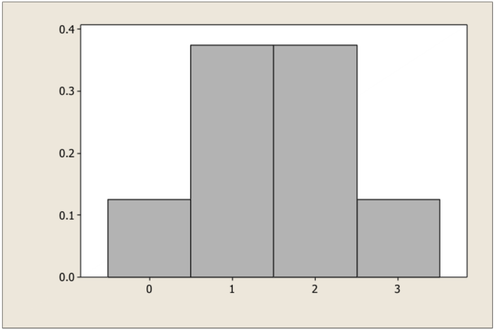
- Use capital letters (e.g. X, W) for the random variable.
- Use lower case letters (e.g. x, w) for any particular value that you are interested in.

The PDF $p(x)$ is the probability of X takes the value of x.

PDF

Sometimes it's useful to look at a graphical representation of the PDF.

Figure: PDF for X = number of heads in three coin flips.



* Total shaded area is 1.

Expected Value

Expected value

The expected value (or mean, expectation) of a discrete random variable X is

$$\mathbb{E}(X) = \sum_x x \cdot p(x)$$

Sometimes write μ, or μ_X.

- $\mathbb{E}(X)$ can be interpreted as the long run average of X.
- $\mathbb{E}(X)$ is not informative to predict a single run of the experiment.
- The sample mean $\bar{x}$ is an estimate for $\mathbb{E}(X)$.

Variance and Standard Deviation

Variance

The variance of a discrete random variable X is

$$\mathrm{var}(X) = \sum_x (x - \mu)^2 p(x)$$

Also write σ^2, or σ_X^2.

To compute the variance of a discrete random variable X:

1. Compute $\mu = \mathbb{E}(X)$.
2. For each possible x, compute $(x - \mu)^2 p(x)$.
3. Add up these values.

Variance and Standard Deviation

Standard deviation

The standard deviation (SD) of X is

$$\mathrm{sd}(X) = \sqrt{\mathrm{var}(X)} = \sqrt{\sum_x (x - \mu)^2 p(x)}$$

Also write σ, or σ_X.

Variance and Standard Deviation

Interpretation of $\text{var}(X)$:

- The expected value of $(X - \mu)^2$:

$$\text{var}(X) = \mathbb{E}\Big[(X - \mu)^2\Big]$$

- Long run average squared deviation from the mean.
- The sample variance s^2 is an estimate for σ^2.

Empirical Rule

If the probability distribution is bell-shaped, then,

$$\begin{aligned} \mathbb{P}(\mu - \sigma < X < \mu + \sigma) &\approx 0.68 \\ \mathbb{P}(\mu - 2\sigma < X < \mu + 2\sigma) &\approx 0.95 \\ \mathbb{P}(\mu - 3\sigma < X < \mu + 3\sigma) &\approx 0.997 \end{aligned}$$

Properties of Expected Value

1. (Aaffine transformation) Let a, b be two constants, and let X be a random variable. Then,

$$\mathbb{E}(aX + b) = a\mathbb{E}(X) + b$$

   - Example: Measure the room temprature in Fahrenheit and convert to Celcius ($C = (F - 32)/1.8$).

2. (Sum) Let X and Y be two random variables. Then,

$$\mathbb{E}(X + Y) = \mathbb{E}(X) + \mathbb{E}(Y)$$

   - Example: Count number of male and female students in the class, and compute total number of students in the class.

Summary

Random Variable

- Probability distribution function (PDF)
- Expected value of a random variable
- Variance and standard deviation of a random variable
- Properties of expected value