Sample Points and Sample Spaces

1. In the following two experiments, what are the sample points and the sample space?

   (a) You flip a coin.

   (b) You roll a 6-sided die.

2. Suppose that a customer visits a restaurant and leaves a review on Yelp with 1–5 stars. What are the sample points and the sample space for the customer's star rating?

3. Suppose that two customers visit a restaurant, and that they both leave Yelp reviews with 1–5 stars each. What are the sample points and the sample space for the pair of star ratings?

4. Suppose you randomly pick a respondent from the class survey, then record their undergraduate major and gender. What are the sample points and the sample space? Assume that major is either "Business," "Humanities/Social Science," or "Science/Engineering."

Events

5. Suppose that a customer leaves a Yelp rating (1–5 stars) for a restaurant. Describe the event "the rating is odd (not even)."

6. Suppose you randomly pick a respondent from the class survey, then record their undergraduate major and gender. Assume that undergraduate major is listed as "Business", "Hum./Soc. Sci.", or "Sci./Eng.", and that gender is listed as "Male" or "Female".

   (a) List the sample points in the event "the major is Business."

   (b) List the sample points in the event "the gender is Male."

Probability

7. Suppose you randomly pick a respondent from the class survey and record their undergraduate major and gender.

   (a) Use the following table of recorded survey response frequencies to compute the probabilities of the sample points.

   | | Gender | | |
Undergrad Major	Female	Male	Total
Business	9	6	15
Hum./Soc. Sci.	10	12	22
Sci./Eng.	2	8	10
Total	21	26	47

   (b) Find the probability that the undergraduate major is Business.

   (c) Find the probability that the gender is Male.

   (d) Find the probability the undergraduate major is Humanities/Social Science.

8. Suppose that a customer's Yelp rating is random, and that the probabilities for the possible star ratings are $p_1 = 10\%$, $p_2 = 30\%$, $p_3 = 25\%$, $p_4 = 20\%$, $p_5 = 15\%$. Find the probability that the rating is odd.

Compound Events and the Additive Rule

9. Suppose you pick a random survey respondent and record their undergraduate major and gender.

   (a) List the sample points in the event "the major is Business or the gender is Male."

   (b) Compute the probability of the event in part (a) by adding the probabilities of all of the sample points in the event.

   (c) Express the event "the major is Business or the gender is Male" as a union of two other events.

   (d) Compute the probability of the event using the additive rule.

10. Suppose that two customers give ratings (1–5 stars) to the same restaurant on Yelp.

   (a) Express the event "at least one customer gives a 1 star rating" as a union of two other events.

   (b) Suppose that both customers randomly assign their ratings, giving equal probabilities to all possible star ratings. In this case, all 25 sample points have equal probability. Compute the probability of the event in part (a).

11. Suppose that two customers give ratings to the same restaurant on Yelp.

   (a) Express the event "the average of their ratings is 3.5 or 4" as a union of two other events. *Hint: this is the same event as "the sum of their ratings is 7 or 8."*

   (b) As in problem 10(b), suppose that both customers randomly assign their ratings with equal probability for all possible star ratings, so that all 25 sample points have equal probability. Compute the probability of the event in part (a).

Complementary Events and the Complement Rule

12. Suppose that 60% of NYU undergraduates own iPhones. If you pick a random NYU undegraduate, what is the probability that he or she will *not* own an iPhone?

13. Suppose you flip five coins. What is the probability of getting at least one head?
    Hint: what is the complement of this event?

Conditional Probability

14. Here is a table of the tabulated frequencies for undergrad major and gender for the respondents to a class survey.

Undergrad Major	Gender		Total
	Female	Male	
Business	9	6	15
Hum./Soc. Sci.	10	12	22
Sci./Eng.	2	8	10
Total	21	26	47

(a) Express the following statements as conditional probabilities:

- $\frac{9}{21} \approx 43\%$ of the females have undergrad major in Business.
- $\frac{9}{15} = 60\%$ of those having undergrad major in Business are female.

(b) Compute $P(\text{Male} \mid \text{Sci./Eng.})$ and $P(\text{Sci./Eng.} \mid \text{Male})$. Explain the difference between these two quantities.

15. The following table lists the pick-up and drop-off locations of approximately 170 million yellow cab taxi trips made in New York City in 2013. Numbers are reported in thousands.

Pick-up	Drop-off					
	Bronx	Brooklyn	Manhattan	Queens	Staten Is.	Total
Bronx	53	1	37	4	0	95
Brooklyn	8	2,707	1,598	273	2	4,588
Manhattan	638	5,458	143,656	5,906	22	155,680
Queens	122	1,022	5,058	2,281	8	8,491
Staten Is.	0	0	0	0	3	3
Total	821	9,188	150,349	8,464	35	168,857

(a) Find P(drop-off Brooklyn | pick-up Manhattan) and P(pick-up Manhattan | drop-off Brooklyn). Explain the difference between these two quantities.

(b) Express the following statement as a conditional probability: "29% of the trips with drop-off locations in Brooklyn originated in the same borough."

The Multiplicative Rule

16. Out of the 58 students enrolled in the class, 24 are female (41%) and 34 are male (59%). Suppose that we randomly select two different students.

    (a) What is the probability that both students are male?

    (b) What is the probability that both students are female?

    (c) What is the probability that one of the students is male and one of the students is female?

17. Of the 48 students who filled out the survey, 33 indicated that they drink at least one cup of coffee per day, while 15 indicated that they do not drink coffee on a typical day. Suppose that we randomly select two different survey respondents.

    (a) What is the probability that both students regularly drink coffee?

    (b) What is the probability that neither student regularly drinks coffee?

    (c) What is the probability that exactly one student regularly drinks coffee?

18. A class has 20 students. What is the probability that at least two students have the same birthday? Assume that each person in the class was assigned a random birthday between January 1 and December 31.