Homework #4 – Due Thursday, Aug. 2
STAT-UB.0001 – Statistics for Business Control

Problem 1

Find the probability that a standard normal random variable is:

(a) Greater than zero

Solution:
$$P(Z > 0) = 0.5$$

(b) Greater than -1.4

Solution:
$$P(Z > -1.4) = 0.91924$$

(c) Less than -0.7

Solution:
$$P(Z < -0.7) = 0.2420$$

(d) Between -1 and 2

Solution:
$$P(-1 \leq Z \leq 2) = 0.97725 - 0.1587 = 0.81855$$

(e) Less than -0.5 or greater than 1.7

Solution:
$$P(Z \leq -0.5) + P(Z > 1.7) = 0.3085 + 0.04457 = 0.35307$$

(f) Equal to 1.

Solution:

$$P(Z = 1) = 0$$

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Problem 2

Find a value of a standard normal random variable Z (call it z_0) such that

(a) $P(Z < z_0) = .30$

Solution:

$$z_0 = -0.5244$$

(b) $P(Z > z_0) = .16$

Solution:

$$z_0 = 0.9945$$

(c) $P(-z_0 < Z < z_0) = .90$

Solution:

$$z_0 = 1.6449$$

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Problem 3

Suppose that X is normally distributed with mean 11 and standard deviation 2. Find

(a) $P(10 < X < 12)$

Solution:

$$\begin{aligned} P(10 < X < 12) &= P\left(\frac{10-11}{2} < \frac{X-11}{2} < \frac{12-11}{2}\right) \\ &= P(-0.5 < Z < 0.5) \\ &= 0.3829 \end{aligned}$$

(b) $P(X > 7.6)$.

Solution:

$$\begin{aligned} P(X > 7.6) &= P\left(\frac{X-11}{2} > \frac{7.6-11}{2}\right) \\ &= P(Z > -1.7) \\ &= 1 - 0.04457 \\ &= 0.95543. \end{aligned}$$

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Problem 4

A Pepsi machine in a Burger King store can be regulated so that it dispenses an average of μ ounces per cup. If the amount dispensed is normally distributed with standard deviation 0.3 ounces, what should be the setting for μ so that 8-ounce cups will overflow only 1% of the time?

Solution: Let X be the amount dispensed from the machine. We want to find μ such that $P(X > 8) = 0.01$. Thus,

$$\begin{aligned} P\left(\frac{X-\mu}{0.3} > \frac{8-\mu}{0.3}\right) &= 0.01 \\ P\left(Z > \frac{8-\mu}{0.3}\right) &= 0.01, \end{aligned}$$

so that

$$\frac{8-\mu}{0.3} = 2.3263,$$

and hence

$$\begin{aligned} \mu &= 8 - (0.3)(2.3263) \\ &= 7.30211. \end{aligned}$$

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Problem 5

Suppose that annual stock returns for a particular company are normally distributed with a mean of 18% and a standard deviation of 6%. You are going to invest in this stock for one year. (Note: In reality, annual returns tend to be more nearly normally distributed than daily returns.) Find that the probability that your one-year return will exceed 26%.

Solution: Let X be the annual return, in percent. This is a normal random variable with mean $\mu = 18$ and standard deviation $\sigma = 6$. The probability of interest is

$$\begin{aligned} P(X > 26) &= P\left(\frac{X-\mu}{\sigma} > \frac{26-\mu}{\sigma}\right) \\ &= P\left(Z > \frac{26-18}{6}\right) \\ &= P(Z > 1.33) \\ &\approx 0.0968 \end{aligned}$$

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Problem 6

If the population standard deviation is 2.5 and we take a random sample of size 81, what is $\mathrm{sd}(\bar{X})$? Note: this quantity is known as the "standard error of the mean."

Solution:

$$\mathrm{sd}(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{2.5}{\sqrt{81}} = 0.2778.$$

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Problem 7

Suppose that daily returns on a portfolio are independent, with a mean of 0.03% and a standard deviation of 1%. Approximately what is the probability that the average daily return over the next 100 days will be between 0.2% and 0.3%?

Solution:

Let $\bar{X}$ denote the average return over the next 100 days, in percent. Then, by the central limit theorem, $\bar{X}$ is approximately normal with mean and standard deviation

$$\begin{aligned} \mu_{\bar{X}} &= \mu = 0.03, \\ \sigma_{\bar{X}} &= \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{100}} = 0.1. \end{aligned}$$

The probability of interest is

$$\begin{aligned} P(0.2 < X < 0.3) &= P\left(\frac{0.2 - 0.03}{0.1} < \frac{X - \mu_{\bar{X}}}{\sigma_{\bar{X}}} < \frac{0.3 - 0.03}{0.1}\right) \\ &= P(1.7 < Z < 2.7) \\ &= 0.996533 - 0.95543 \\ &= 0.041103. \end{aligned}$$

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Problem 8

If we throw n dice where n is large, why can we think of the distribution of the sum as being approximately normal?

Solution: The sum is equal to $n\bar{X}$, where $\bar{X}$ is the average value of the n rules. By the central limit theorem, $\bar{X}$ is approximately normal if n is large. Further, if we scale a normal random variable by a constant (n), then we get a normal random variable. Thus, $n\bar{X}$, the sum, is approximately normal.

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Problem 9

Suppose that an auto factory has 8 assembly lines, operating independently. For each assembly line, the number of autos produced per day has a normal distribution with mean of 20 and a standard deviation of 4. Approximately what is the probability that 120 or fewer autos will be produced tomorrow?

Solution: Let $\bar{X}$ be the average number produced by the 8 assembly lines. Then, by the central limit theorem, $\bar{X}$ is approximately normal with mean and standard deviation

$$\begin{aligned} \mu_{\bar{X}} &= 20, \\ \sigma_{\bar{X}} &= \frac{4}{\sqrt{8}} = 1.414. \end{aligned}$$

To produce a total of 120 or fewer autos, the 8 factories must produce an average of $120/8 = 15$

or fewer. The probability of interest is

$$\begin{aligned} P(\bar{X} < 15) &= P\left(\frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} < \frac{15 - 20}{1.414}\right) \\ &\approx P(Z < -3.536) \\ &\approx 0.0002326. \end{aligned}$$

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