Homework #5 – Due Tuesday, Aug. 7
STAT-UB.0001 – Statistics for Business Control

Problem 1

For each of the following values of α, and n, find $t_{\alpha/2,n-1}$. Round the answer to two digits after the decimal point.

(a) $\alpha = 0.10$, $n = 25$.

> **Solution:**
> $$t_{.050,24} = 1.711 \approx 1.71.$$

(b) $\alpha = 0.02$, $n = 10$.

> **Solution:**
> $$t_{.010,9} = 2.821 \approx 2.82.$$

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Problem 2

Consider the time it takes for a call center to answer its calls. A random sample of 7 calls revealed a sample mean time of 191 seconds and a sample standard deviation of 11.4 seconds.

(a) What is the sample?

> **Solution:** The times to answer the $n = 7$ sampled calls.

(b) What is the population?

> **Solution:** The times to answer all calls at the call center.

(c) Explain what the population mean represents in this problem.

> **Solution:** The expected time it takes the call center to answer a call. (Equivalently, the average of all of the times taken to answer all calls at the call center.)

(d) Construct a 95% confidence interval for the population mean.

Solution: For a 95% confidence interval and 7 observations, we have $\alpha = .05$ and

$$t_{\alpha/2,n-1} = t_{.025,6} = 2.447.$$

The 95% confidence interval for the population mean is

$$\begin{aligned}\bar{x} \pm t_{\alpha/2,n-1}\frac{s}{\sqrt{n}} &= 191 \pm 2.447\frac{11.4}{\sqrt{7}} \\ &= 191 \pm 10.5 \\ &= (180.5, 201.5).\end{aligned}$$

(e) Construct a 99% confidence interval for the population mean.

Solution: For a 99% confidence interval and 7 observations, we have $\alpha = .01$ and

$$t_{\alpha/2,n-1} = t_{.005,6} = 3.707.$$

The 99% confidence interval for the population mean is

$$\begin{aligned}\bar{x} \pm t_{\alpha/2,n-1}\frac{s}{\sqrt{n}} &= 191 \pm 3.707\frac{11.4}{\sqrt{7}} \\ &= 191 \pm 16 \\ &= (175, 207).\end{aligned}$$

(f) State any assumptions you needed to do this problem. Do you think that the assumptions are reasonable? Why or why not?

Solution: Since the sample size is small ($n < 30$), we need to assume that the population is normal. That is, we need to assume that the histogram of the times to answer all calls looks like a bell curve. Personally, I do not think that this is reasonable; I expect that the time to answer a call has a high skew to the right. (You will still get full credit if you think that the assumption is reasonable, as your reasoning makes sense.)

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Problem 3

Consider (again) the time it takes for a call center to answer its calls. Assume the time to answer a call is normally distributed. The call center claims that the mean time to answer a call is 3 minutes. In a random sample of 7 calls, the average time for the call center to answer was 191 seconds, with a sample standard deviation of 11.4 seconds.

(a) Provide the null and alternative hypotheses for testing the call center's claim.

> **Solution:** Let μ be the population mean.
>
> $$\begin{aligned} H_0 &: \mu = 180 \\ H_a &: \mu \neq 180. \end{aligned}$$

(b) Compute the test statistic.

> **Solution:** The sample size is $n = 7$. The sample mean and standard deviation are $\bar{x} = 191$ and $s = 11.4$. The hypothesized null mean is $\mu_0 = 180$.
>
> $$\begin{aligned} t &= \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \\ &= \frac{191 - 180}{11.4/\sqrt{7}} \\ &= 2.55 \end{aligned}$$

(c) Compute the p-value.

> **Solution:** We will approximate the p-value using a z table:
>
> $$\begin{aligned} p &\approx P(|Z| > 2.55) \\ &= 0.009322 \end{aligned}$$
>
> Note that the value $z = 2.55$ is not in the Z-table, so we use the value for $z = 2.6$ instead. It would be acceptable to use $z = 2.5$ instead, in which case you would get $p \approx 0.01242$.
>
> (A more accurate solution would use a t distribution with $n - 1 = 6$ degrees of freedom; this gives a p-value of $p = 0.04349$.)

(d) Test the call center's claim, at the 1% level of significance.

> **Solution:** Since $p \geq 0.01$, we do not reject H_0. The data is consistent with the claim.

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Problem 4

You would like to estimate the proportion of Stern students who participates in course faculty evaluations (CFEs). If you want to construct a 95% confidence interval for this proportion, how many students should you survey to guarantee that the width of your confidence interval will be less than 0.05? *Hint: use the fact that* $\widehat{p}(1-\widehat{p}) \leq \frac{1}{4}$, *for any* $\widehat{p}$ *taking values between* $[0,1]$.

> **Solution:** Suppose you sampled n students, and the sample proportion of participating in CFE is $\widehat{p}$. The 95% CI for the population proportion is
>
> $$\left(\widehat{p} - 2\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}, \widehat{p} + 2\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}\right).$$
>
> The width of this CI is therefore $4\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}$, which is upper bounded by
>
> $$4\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}} \leq 4\sqrt{\frac{1}{4n}} = \frac{2}{\sqrt{n}}.$$
>
> To ensure that the width is less than 0.05, it is sufficient to have
>
> $$\frac{2}{\sqrt{n}} \leq 0.05 \Rightarrow n \geq 1600.$$
>
> Therefore you need to survey at least 1600 students to ensure that the width of your 95% confidence interval will be less than 0.05.

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Problem 5

Consider the `HeightWeight.csv` file containing data on 200 records of human heights and weights of 18 years old children. Here, we focus on the the Weight (in lb) column. Use Minitab to complete the following questions.

(a) Use *Stat* $\Rightarrow$ *Basic Statistics* $\Rightarrow$ *1-Sample t*, and choose `Weight` to create a 95% confidence interval for the population mean.

Solution:

One-Sample T: Weight

Menu

Descriptive Statistics

N	Mean	StDev	SE Mean	95% CI for μ
200	127.222	11.961	0.846	(125.554, 128.890)

μ: mean of Weight

(b) Use the Minitab output (`N`, `Mean`, `StDev`) to check the calculation of the confidence interval in (a). Also verify the calculation of `SE Mean`, the (estimated) standard deviation for the sample mean.

Solution: We have

$$n = 200, \bar{x} = 127.222, s = 11.961$$
$$z_{0.025} = 1.96$$
$$1.96\frac{11.961}{\sqrt{200}} = 1.658$$
$$\bar{x} \pm 1.658 = (125.56, 128.88)$$

The CI is close to Minitab output (up to rounding errors).

(c) Get a 99% confidence interval for the population mean, proceeding as in (a) but adding *Options* ⇒ *Confidence Level:* 99.0.

Solution:

One-Sample T: Weight

Menu

Descriptive Statistics

N	Mean	StDev	SE Mean	99% CI for μ
200	127.222	11.961	0.846	(125.022, 129.422)

μ: mean of Weight

(d) According to Wikipedia, the average weight for adults is 136.7 lb. Use Minitab to get the p-value corresponding to the null hypothesis that the average weight for the 18 years old children is the same as the average weight of the adults. Interpret the p-value. Proceed as in (a) but select *Perform hypothesis test*.

Solution:

Test

Null hypothesis	H_0: μ = 136.7
Alternative hypothesis	H_1: $\mu \neq$ 136.7

T-Value	P-Value
-11.21	0.000

The p-value is less than 0.05, so we reject the null hypothesis that 18 years old children have the same average weight as adults.

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